Exercise 3.1

1:

Find the radian measures corresponding to the following degree measures:

(i) $25°$ (ii) $-47°30'$ (iii) $240°$ (iv) $520°$

Solution:

(i)

We know that $180° = \pi$ radian

$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$

(ii)

$-47°30'$

$-47°30' \; -47\frac{1}{2}$

$= \frac{-95}{2}$ degree

Since $180° = \pi$ radian

$\frac{-95}{2} \text{ degree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{radian} = \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$

$\therefore -47°30' = \frac{-19}{72}\pi$ radian

(iii)

We know that $180° = \pi$ radian

$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$

(iv)

We know that $180° = \pi$ radian

$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$

2:

Find the degree measures corresponding to the following radian measures (Use $\pi = \frac{22}{7}$).

(i) $\frac{11}{16}$ (ii) -4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Solution:

(i) $\frac{11}{16}$

We know that π radian $= 180°$

$\therefore \frac{11}{16} \text{ radian} = \frac{180}{\pi} \times \frac{11}{16} \text{ degree} = \frac{45 \times 11}{\pi \times 4} \text{ degree}$

$= \frac{45 \times 11 \times 7}{22 \times 4}$ degree $= \frac{315}{8}$ degree

$= 36\frac{3}{8}$ degree

$= 39° + \frac{3 \times 60}{8}$ minutes $\quad [1° = 60']$

$= 39° + 22' + \frac{1}{2}$ minutes

$= 39°22'30'' \quad [1' = 60'']$

(ii) -4

We know that π radian $= 180°$

-4 radian $= \frac{180}{\pi} \times (-4)$ degree $= \frac{180 \times 7(-4)}{22}$ degree

$= \frac{-2520}{11}$ degree $= -229\frac{1}{11}$ degree

$= -229° + \frac{1 \times 60}{11}$ minutes $\quad [1° = 60']$

$= -229° + 5' + \frac{5}{11}$ minutes

$= -229°5'27'' \quad [1' = 60'']$

(iii)

We know that π radian $= 180°$

$\therefore \frac{5\pi}{3}$ radian $= \frac{180}{\pi} \times \frac{5\pi}{3}$ degree $= 300°$

(iv) $\frac{7\pi}{6}$

We know that π radian $= 180°$

$\therefore \frac{7\pi}{6}$ radian $= \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$

3:

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:

Number of revolutions made by the wheel in 1 minute = 360

$\therefore$ Number of revolutions made by the wheel in 1 second $= \frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian,

i.e., 12π radian

Thus, in one second, the wheel turns an angle of 12π radian.

4:
Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm.

Solution:
We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then

$$\theta = \frac{1}{r}$$

Therefore, for $r = 100\,cm,\ l = 22\,cm,$ we have

$$\theta = \frac{22}{100}\text{ radian} = \frac{180}{\pi}\times\frac{22}{100}\text{ degree} = \frac{180\times7\times22}{22\times100}\text{ degree}$$

$$= \frac{126}{10}\text{ degree} = 12\frac{3}{5}\text{ degree} = 12°36' \qquad [1° = 60']$$

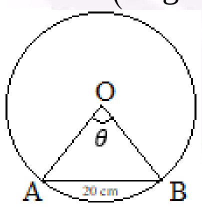
Thus, the required angle is $12°36'$.

5:
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution:
Diameter of the circle = 40 cm

$\therefore$ Radius (r) of the circle $= \frac{40}{2}cm = 20\,cm$

Let AB be a chord $(\text{length} = 20\,cm)$ of the circle.



In $\Delta OAB, OA = OB = \text{Radius of circle} = 20\,cm$

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

$$\therefore \theta = 60° = \frac{\pi}{3}\text{ radian}$$

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre then

$$\theta = \frac{l}{r}$$

$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3}$ cm

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3} cm$.

6:

If in two circles, arcs of the same length subtend angles $60°$ and $75°$ at the centre, find the ratio of their radii.

Solution:

Let the radii of the two circles be r_1 and r_2. Let an arc of length l subtend an angle of $60°$ at the centre of the circle of radius r_1, while let an arc of length/subtend an angle of $75°$ at the centre of the circle of radius r_2.

Now, $60° = \frac{\pi}{3}$ radian and $75° = \frac{5\pi}{12}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ. radian at the centre then

$\theta = \frac{l}{r}$ or $l = r\theta$

$\therefore l = \frac{r_1\pi}{3}$ and $l = \frac{r_2 5\pi}{12}$

$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$

$\Rightarrow r_1 = \frac{r_2 5}{4}$

$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$

Thus, the ratio of the radii is 5:4.

7:

Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length.

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

We know that in a circle of radius r unit, if an arc of length l unit subtends

An angle θ radian at the centre, then $\theta = \frac{l}{r}$

It is given that $r = 75cm$

(i) Here, $l = 10cm$

$\theta = \frac{10}{75}$ radian $= \frac{2}{15}$ radian

(ii) Here, $l = 15cm$

$$\theta = \frac{15}{75}\text{radian} = \frac{1}{5}\text{radian}$$

(iii) Here, $l = 21\text{cm}$

$$\theta = \frac{21}{75}\text{radian} = \frac{7}{25}\text{radian}$$

Exercise 3.2 Page: 63

Find the values of other five trigonometric functions in Exercises 1 to 5.
1:

$\cos x = -\frac{1}{2}, x$ lies in third quadrant.

Solution:

$\cos x = -\frac{1}{2}$

$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

Since x lies in the 3rd quadrant, the value of $\sin x$ will be negative.

$\therefore \sin x = -\frac{\sqrt{3}}{2}$

$\text{cosec}\, x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$.

2:

$\sin x = \frac{3}{5}$, x lies in second quadrant.

Solution:

$\sin x = \frac{3}{5}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$

$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$

$\Rightarrow \cos^2 x = \frac{16}{25}$

$\Rightarrow \cos x = \pm\frac{4}{5}$

Since x lies in the 2nd quadrant, the value of $\cos x$ will be negative

$\therefore \cos x = -\frac{4}{5}$

$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$

$\cot x = \frac{1}{\tan x} = -\frac{4}{3}.$

3:

$\cot x = \frac{3}{4}$, x lies in third quadrant.

Solution:

$\cot x = \frac{3}{4}$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

4:

$\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Solution:

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 = 1$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since x lies in the 4th quadrant, the value of sin x will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}.$$

5:

$\tan x = -\frac{5}{12}$, x lies in second quadrant.

Solution:

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since x lies in the 2nd quadrant, the value of sec x will be negative.

$\therefore \sec x = -\dfrac{13}{12}$

$\cos x = \dfrac{1}{\sec x} = \dfrac{1}{\left(-\dfrac{13}{12}\right)} = -\dfrac{12}{13}$

$\tan x = \dfrac{\sin x}{\cos x}$

$\Rightarrow -\dfrac{5}{12} = \dfrac{\sin x}{\left(-\dfrac{12}{13}\right)}$

$\Rightarrow \sin x = \left(-\dfrac{5}{12}\right) \times \left(-\dfrac{12}{13}\right) = \dfrac{5}{13}$

$\text{cosec}\, x = \dfrac{1}{\sin x} = \dfrac{1}{\left(\dfrac{5}{13}\right)} = \dfrac{13}{5}$.

Find the values of the trigonometric functions in Exercises 6 to 10.

6:

$\sin 765^\circ$.

Solution:

It is known that the value of sinx repeat after an interval of 2n or 360

$\therefore \sin 765^\circ = \sin\left(2 \times 360^\circ + 45^\circ\right) = \sin 45^\circ = \dfrac{1}{\sqrt{2}}$.

7:

$\text{cosec}\left(-1410^\circ\right)$

Solution:

It is known that the values of $\text{cosec}\, x$ repeat after an interval of 360° or .

$\therefore \text{cosec}\left(-1410^\circ\right) = \text{cosec}\left(-1410^\circ + 4 \times 360^\circ\right)$

$= \text{cosec}\left(-1410^\circ + 1440^\circ\right)$

$= \text{cosec}\, 30^\circ = 2$.

8:

$\tan \dfrac{19\pi}{3}$.

Solution:

It is known that the values of $\tan x$ repeat after an interval of n or 180°.

$\therefore \tan \dfrac{19\pi}{3} = \tan 6\dfrac{1}{3}\pi = \tan\left(6\pi + \dfrac{\pi}{3}\right) = \tan \dfrac{\pi}{3} = \tan 60^\circ = \sqrt{3}$.

9:

$$\sin\left(-\frac{11\pi}{3}\right)$$

Solution:

It is known that the values of sinx repeat after an interval of $2n$ or $360°$.

$$\therefore \sin\left(-\frac{11\pi}{3}\right)=\sin\left(-\frac{11\pi}{3}+2\times 2\pi\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}.$$

10:

$$\cot\left(-\frac{15\pi}{4}\right)$$

Solution:

It is known that the values of cot x repeat after an interval of n or $180°$.

$$\therefore \cot\left(-\frac{15\pi}{4}\right)=\cot\left(-\frac{15\pi}{4}+4\pi\right)=\cot\frac{\pi}{4}=1.$$

Exercise 3.3

Page: 73

1:

$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

Solution:

L.H.S $=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$

$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$

= R.H.S.

2:

Prove that $2\sin^2\frac{\pi}{6}+\text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Solution:

L.H.S. $=2\sin^2\frac{\pi}{6}+\text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$=2\left(\frac{1}{2}\right)^2+\text{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$

$=2\times\frac{1}{4}+\left(-\text{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$

$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$

$=\frac{1}{2}+\frac{4}{4}=\frac{1}{2}+1=\frac{3}{2}$

= R.H.S.

3:

Prove that $\cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Solution:

L.H.S. = $\cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$=\left(\sqrt{3}\right)^2+\text{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$

$=3+\text{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$

$=3+2+1=6$

$=$ R.H.S

4:

Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Solution:

L.H.S. $=2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$=2\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}^2+2\left(\frac{1}{\sqrt{2}}\right)^2+2(2)^2$

$=2\left\{\sin\frac{\pi}{4}\right\}^2+2\times\frac{1}{2}+8$

$=1+1+8$

= 10

= R.H.S

5:

Find the value of :

(i) $\sin 75°$
(ii) $\tan 15°$

Solution:

(i) $\sin 75° = \sin(45° + 30°)$

$$= \sin 45° \cos 30° + \cos 45° \sin 30°$$

$$[\sin(x+y) = \sin x \cos y + \cos x \sin y]$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\tan 15° = \tan(45° - 30°)$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

6:

Prove that $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) = \sin(x+y)$

Solution:

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} + \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} - \cos\left\{\frac{\pi}{4}-x\right\} - \left(\frac{\pi}{4}-y\right)\right]$$

$$\begin{bmatrix} \because \ 2\cos A\cos B = \cos(A+B)+\cos(A-B) \\ -2\sin A\sin B = \cos(A+B)-\cos(A-B) \end{bmatrix}$$

$$= 2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{4}-(x+y)\right]$$

$$= \sin(x+y)$$

= R.H.S.

7:

Prove that $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

Solution:

It is known that $\tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $(A-B) = \dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}} = \frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2 = \text{R.H.S.}$$

8:

Prove that $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$

Solution:

$$\text{L.H.S.} = \frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

$= \cot^2 x$
= R.H.S.

9:
$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Solution:

L.H.S. = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$= \sin x\cos x[\tan x+\cot x]$

$= \sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$

$= (\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$

= 1 = R.H.S.

10:
Prove that $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

Solution:

L.H.S. $= \sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$

$= \frac{1}{2}[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x]$

$= \frac{1}{2}\begin{bmatrix}\cos\{(n+1)x-(n+2)x\}-cis\{(n+1)x+(n+2)x\} \\ +\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{bmatrix}$

$$\begin{bmatrix}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B) \\ 2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{bmatrix}$$

$= \frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$

$= \cos(-x)=\cos x=$ R.H.S.

11:

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Solution:

It is known that $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $= \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$

$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\left(\frac{3\pi}{4}-x\right)\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$

$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$

$= -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x$

$= -2\sin\frac{\pi}{4}\sin x$

$= -2\times\frac{1}{\sqrt{2}}\times\sin x$

$= -\sqrt{2}\sin x$

= R.H.S.

12:

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

It is known that

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $= \sin^2 6x - \sin^2 4x$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$

$= (2\sin 5x\cos x)(2\cos 5x\sin x) = (2\sin 5x\cos 5x)(2\sin x\cos x)$

$= \sin 10x \sin 2x$

= R.H.S.

13:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

It is known that

$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $=\cos^2 2x-\cos^2 6x$

$=(\cos 2x+\cos 6x)(\cos 2x-6x)$

$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right]$

$=\left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$

$=\left[2\cos 4x\cos 2x\right]\left[-2\sin 4x(-\sin 2x)\right]$

$=(2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

$=\sin 8x\sin 4x=$ R.H.S

14:

Prove that $\sin 2x+2\sin 4x+\sin 6x=4\cos^2 x\sin 4x$

Solution:

L.H.S. $=\sin 2x+2\sin 4x+\sin 6x$

$=[\sin 2x+\sin 6x]+2\sin 4x$

$=\left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]+2\sin 4x$

$\left[\because\ \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$=2\sin 4x\cos(-2x)+2\sin 4x$

$=2\sin 4x\cos 2x+2\sin 4x$

$=2\sin 4x(\cos 2x+1)$

$=2\sin 4x\left(2\cos^2 x-1+1\right)$

$=2\sin 4x\left(2\cos^2 x\right)$

$=4\cos^2 x\sin\ 4x$

$=$ R.H.S.

15:

Prove that $\cot 4x(\sin 5x+\sin 3x)=\cot x(\sin 5x-\sin 3x)$

Solution:

L.H.S $=\cot 4x(\sin 5x+\sin 3x)$

$=\frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

$$=2\cos 4x\cos x$$

R.H.S. $=\cot x(\sin 5x-\sin 3x)$

$$=\frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{\cos x}{\sin x}\left[2\cos 4x\sin x\right]$$

$$=2\cos 4x.\cos x$$

L.H.S. = R.H.S.

16:

Prove that $\dfrac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\dfrac{\sin 2x}{\cos 10x}$

Solution:

It is known that

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$=\frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$=-\frac{\sin 2x}{\cos 10x}$$

$$=\text{R.H.S.}$$

17:

Prove that: $\dfrac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

Solution:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$=\tan 4x=\text{R.H.S.}$$

18:

Prove that $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\frac{x-y}{2}$

Solution:

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\sin x-\sin y}{\cos x+\cos y}$$

$$=\frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$=\tan\left(\frac{x-y}{2}\right)=\text{R.H.S.}$$

19:

Prove that $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Solution:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$=\frac{\sin 2x}{\cos 2x}$$

$$=\tan 2x$$

= R.H.S.

20:

Prove that $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Solution:

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \cos^2 A-\sin^2 A=\cos 2A$$

$$\therefore \text{L.H.S.}=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$$

$$=\frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$=\frac{2\cos 2x\sin(-x)}{-\cos 2x}$$

$$=-2\times(-\sin x)$$

$$=2\sin x=\text{R.H.S.}$$

21:

Prove that $\dfrac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

Solution:

L.H.S. $=\dfrac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$

$$=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$$

$$=\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin 3x}$$

$$\left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{2\cos 3x\cos+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$

$\cot 3x=\text{R.H.S.}$

22:

Prove that $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

Solution:

L.H.S. $=\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x$

$=\cot x\cot 2x-\cot 3x(\cot 2x+\cot x)$

$=\cot x\cot 2x-\cot(2x+x)(\cot 2x+\cot x)$

$$=\cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot x+\cot 2x}\right](\cot 2x+\cot x)$$

$$\left[\because \cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}\right]$$

$=\cot x\cot 2x-(\cot 2x\cot x-1)=1=\text{R.H.S.}$

23:

Prove that $\tan 4x=\dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Solution:

It is known that $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$

$\therefore$ L.H.S. $=\tan 4x=\tan 2(2x)$

$$=\frac{2\tan 2x}{1-\tan^2(2x)}$$

$$=\frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$=\frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$=\frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{\left(1-\tan^2 x\right)^2-4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$=\frac{4\tan x\left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2-4\tan^2 x}$$

$$=\frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$=\frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}=\text{R.H.S.}$$

24:

Prove that: $\cos 4x=1-8\sin^2 x\cos^2 x$

Solution:

L.H.S. $=\cos 4x$

$=\cos 2(2x)$

$=1-2\sin^2 2x\left[\cos 2A=1-2\sin^2 A\right]$

$=1-2(2\sin x\cos x)^2\left[\sin 2A=2\sin A\cos A\right]$

$=1-8\sin^2 x\cos^2 x$

$=$ R.H.S.

25:

Prove that: $\cos 6x=32x\cos^6 x-48\cos^4 x+18\cos^2 x-1$

Solution:

L.H.S. $=\cos 6x$

$=\cos 3(2x)$

$$= 4\cos^3 2x - 3\cos 2x \left[\cos 3A = 4\cos^3 A - 3\cos A\right]$$
$$= 4\left[\left(2\cos^2 x - 1\right)^3 - 3\left(2\cos^2 x - 1\right)\right]\left[\cos 2x = 2\cos^2 x - 1\right]$$
$$= 4\left[\left(2\cos^2 x\right)^3 - (1)^3 - 3\left(2\cos^2 x\right)^2 + 3\left(2\cos^2 x\right)\right] - 6\cos^2 x + 3$$
$$= 4\left[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x\right] - 6\cos^2 x + 3$$
$$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$$
$$= 32\cos^6 x - 48\cos^4 x + 18\ \cos^2 x - 1$$
$$= \text{R.H.S.}$$

Exercise 3.4 Page: 78

1:

Find the principal and general solutions of the question $\tan x = \sqrt{3}$.

Solution:

$\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$.

2:

Find the principal and general solutions of the equation $\sec x = 2$

Solution:

$\sec x = 2$

It is known that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Now, $\sec x = \sec\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}$ $\left[\sec x = \frac{1}{\cos x}\right]$

$\Rightarrow 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$.

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$.

3:

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Solution:

$\cot x = -\sqrt{3}$

It is known that $\cot\frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\cot x = \cot\frac{5\pi}{6}$

$\Rightarrow \tan x = \tan\frac{5\pi}{6}$ $\qquad \left[\cot x = \frac{1}{\tan x}\right]$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$

4:

Find the general solution of $\text{cosec}x = -2$

Solution:

$\text{cosec}\, x = -2$

It is known that

$\text{cosec}\frac{\pi}{6} = 2$

$\therefore \text{cosec}\left(\pi + \frac{\pi}{6}\right) = -\text{cosec}\frac{\pi}{6} = -2$ and $\text{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\text{cosec}\frac{\pi}{6} = -2$

i.e., $\text{cosec}\frac{7\pi}{6} = -2$ and $\text{cosec}\frac{11\pi}{6} = -2$

Therefore, the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\text{cosec}\, x = \text{cosec}\frac{7\pi}{6}$

$\Rightarrow \sin x = \sin \frac{7\pi}{6}$ $\left[\operatorname{cosec} x = \frac{1}{\sin x}\right]$

$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}$, where

Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}$, where .

5:
Find the general solution of the equation $\cos 4x = \cos 2x$

Solution:

$\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$

$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0$ or $\sin x = 0$

$\therefore 3x = n\pi$ or $\sin x = 0$

$\therefore 3x = n\pi$ or $x = n\pi$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{3}$ or $x = n\pi$, where $n \in Z$

6:
Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$.

Solution:

$\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+2}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$ $\left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0$ or $2\cos x - 1 = 0$

$\Rightarrow \cos 2x = 0$ or $\cos x = \frac{1}{2}$

$\therefore 2x = (2n+1)\frac{\pi}{2}$ *or* $\cos x = \cos\frac{\pi}{3}$, where $n \in Z$

$\Rightarrow x = (2n+1)\frac{\pi}{4}$ *or* $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

7:

Find the general solution of the equation $\sin 2x + \cos x = 0$.

Solution:

$\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x \cos x + \cos x = 0$

$\Rightarrow \cos x(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0 \quad \text{or} \quad 2\sin x + 1 = 0$

Now, $\cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}$, where $n \in Z$

$2\sin x + 1 = 0$

$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}, n \in Z$.

8:

Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

Solution:

$\sec^2 2x = 1 - \tan 2x$

$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x(\tan 2x + 1) = 0$

$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$

Now, $\tan 2x = 0$

$\Rightarrow \tan 2x = \tan 0$

$\Rightarrow 2x = n\pi + 0$, where

$\Rightarrow x = \frac{n\pi}{2}$, where

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \frac{3\pi}{4}$, where $n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}, n \in Z$

9:

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

Solution:

$\sin x+\sin 3x+\sin 5x=0$

$(\sin x+\sin 5x)+\sin 3x=0$

$\Rightarrow\left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right]+\sin 3x=0 \qquad \left[\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x\cos(-2x)+\sin 3x=0$

$\Rightarrow 2\sin 3x\cos 2x+\sin 3x=0$

$\Rightarrow \sin 3x(2\cos 2x+1)=0$

$\Rightarrow \sin 3x=0 \quad$ or $\quad 2\cos 2x+1=0$

Now, $\sin 3x=0 \Rightarrow 3x=n\pi$, where $n\in Z$

i.e., $x=\frac{n\pi}{3}$, where $n\in Z$

$2\cos 2x+1=0$

$\Rightarrow \cos 2x=\frac{-1}{2}=-\cos\frac{\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)$

$\Rightarrow \cos 2x=\cos\frac{2\pi}{3}$

$\Rightarrow 2x=2n\pi\pm\frac{2\pi}{3}$, where $n\in Z$

$\Rightarrow x=n\pi\pm\frac{\pi}{3}$, where $n\in Z$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi\pm\frac{\pi}{3}, n\in Z$.

Miscellaneous Exercise

1: Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Solution:

L.H.S. $=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=2\cos\frac{\pi}{3}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)$

$\left[\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$

$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$

— $=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$

$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$

$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$

$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right]$

$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$

$=2\cos\frac{\pi}{13}\times 2\times 0\times\cos\frac{5\pi}{26}$

$=0=$ R.H.S.

2:

Prove that: $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x=0$

Solution:

$=\cos 2x-\cos 2x$

L.H.S. $=(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x$

$=\sin 3x\sin x+\sin^2 x+\cos 3x\cos x-\cos^2 x$

$=\cos 3x\cos x+\sin 3x\sin x-(\cos^2-\sin^2 x)$

$=\cos(3x-x)-\cos 2x$ $\quad\left[\cos(A-B)=\cos A\cos B+\sin A\sin B\right]$

= 0

= R.H.S.

3:

Prove that: $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Solution:

L.H.S. $=(\cos x+\cos y)^2+(\sin x-\sin y)^2$

$=\cos^2 x+\cos^2 y+2\cos x\cos y+\sin^2 x+\sin^2 y-2\sin x\sin y$

$=(\cos^2 x+\sin^2 x)+(\cos^2 y+\sin^2 y)+2(\cos x\cos y-\sin x\sin y)$

$=1+1+2\cos(x+y)$ $\quad[\cos(A+B)=(\cos A\cos B-\sin A\sin B)]$

$=2+2\cos(x+y)$

$=2[1+\cos(x+y)]$

$=2\left[1+2\cos^2\left(\frac{x+y}{2}\right)-1\right]$ $\quad[\cos 2A=2\cos^2 A-1]$

$=4\cos^2\left(\frac{x+y}{2}\right)=$ R.H.S

4:

Prove that: $(\cos x-\cos y)^2+(\sin x-\sin y)^2=4\sin^2\frac{x-y}{2}$

Solution:

L.H.S. $(\cos x-\cos y)^2+(\sin x-\sin y)^2$

$=\cos^2 x+\cos^2 y-2\cos x\cos y+\sin^2 x+\sin^2 y-2\sin x\sin y$

$=(\cos^2 x+\sin^2 x)+(\cos^2 y+\sin^2 y)-2[\cos x\cos y+\sin x\sin y]$

$=1+1-2[\cos(x-y)]$ $\quad[\cos(A-B)=\cos A\cos B+\sin A\sin B]$

$=2[1-\cos(x-y)]$

$=2\left[1-\left\{1-2\sin^2\left(\frac{x-y}{2}\right)\right\}\right]$ $\quad[\cos 2A=1-2\sin^2 A]$

$=4\sin^2\left(\frac{x-y}{2}\right)=$ R.H.S

5:

Prove that: $\sin x+\sin 3x+\sin 5x+\sin 7x=4\cos x\cos 2x\sin 4x$

Solution:

It is known that $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$

L.H.S. $=\sin x+\sin 3x+\sin 5x+\sin 7x$

$=(\sin x+\sin 5x)+(\sin 3x+\sin 7x)$

$=2\sin\left(\frac{x+5x}{2}\right).\cos\left(\frac{x-5x}{2}\right)+2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$

$= 2\sin 3x\cos(-2x) + 2\sin 5x\cos(-2x)$

$= 2\sin 3x\cos 2x + 2\sin 5x\cos 2x$

$= 2\cos 2x[\sin 3x + \sin 5x]$

$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right).\cos\left(\frac{3x-5x}{2}\right)\right]$

$= 2\cos 2x[2\sin 4x.\cos(-x)]$

$= 4\cos 2x\sin 4x\cos x = \text{R.H.S.}$

6:

Prove that: $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Solution:

It is known that

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right), \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$

$\text{L.H.S.} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right).\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\sin\left(\frac{9x+3x}{2}\right).\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right).\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\cos\left(\frac{9x+3x}{2}\right).\cos\left(\frac{9x-3x}{2}\right)\right]}$

$= \text{R.H.S.}$

$= \frac{[2\sin 6x.\cos x] + [2\sin 6x.\cos 3x]}{[2\cos 6x.\cos x] + [2\cos 6x.\cos 6x]}$

$= \frac{2\sin 6x[\cos x + \cos 3x]}{2\cos 6x[\cos x + \cos 3x]}$

$= \tan 6x$

7:

Prove that: $\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Solution:

$\text{L.H.S.} = \sin 3x + \sin 2x - \sin x$

$= \sin 3x + \left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right] \qquad \left[\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$= \sin 3x + \left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$

$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$

$= 2\sin\frac{3x}{2}.\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2}$ $\quad [\sin 2A = 2\sin A.\cos B]$

$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$

$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right]$

$\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$= 2\cos\left(\frac{3x}{2}\right).2\sin x\cos\left(\frac{x}{2}\right)$

$= 4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right) = \text{R.H.S.}$

8:

Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if $\tan x = \frac{-4}{3}, x$ in quadrant II

Solution:

Here, x is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

There, $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$

are lies in first quadrant.

It is given that $\tan x = -\frac{4}{3}$

$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\therefore \cos^2 x = \frac{9}{25}$

$\Rightarrow \cos x = \pm\frac{3}{5}$

As x is in quadrant II, $\cos x$ is negative.

$\cos x = \frac{-3}{5}$

Now, $\cos x = 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow \frac{-3}{5} = 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow 2\cos^2 \frac{x}{2} = 1 - \frac{3}{5}$

$\Rightarrow 2\cos^2 \frac{x}{2} = \frac{2}{5}$

$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{5}$

$\Rightarrow \cos \frac{x}{2} = \frac{1}{\sqrt{5}}$ $\qquad \left[\because \cos \frac{x}{2} \text{ is positive}\right]$

$\therefore \cos \frac{x}{2} = \frac{\sqrt{5}}{5}$

$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$

$\Rightarrow \sin^2 \frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$

$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$

$\Rightarrow \sin^2 \frac{x}{2} = \frac{2}{\sqrt{5}}$ $\qquad \left[\because \sin \frac{x}{2} \text{ is positive}\right]$

$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are $\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}$, and 2.

9:

Find, $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ for $\cos x = -\frac{1}{3}, x$ in quadrant III

Solution:

Here, x is in quadrant III.

i.e., $\pi < x < \frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative, where $\sin\frac{x}{2}$ as is positive.

It is given that $\cos x = -\frac{1}{3}$.

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\left(-\frac{1}{3}\right)}{2} = \frac{\left(1+\frac{1}{3}\right)}{2} = \frac{4/3}{2} = \frac{2}{3}$$

$$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now $\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \qquad \left[\because \cos\frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{6}}{3}, \frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$.

10:

Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ *for* $\sin x = \frac{1}{4}, x$ in quadrant II

Solution:

Here, x is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$ [cos x is negative in quadrant II]

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} - \frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4+\sqrt{15}}{8}$$

$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}}$ $\left[\because \sin\frac{x}{2} \text{ is negative}\right]$

$$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$$

$\Rightarrow \cos\frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}}$ $\left[\because \cos\frac{x}{2} \text{ is positve}\right]$

$$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\frac{\sqrt{8-2\sqrt{15}}}{4}} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

$$=\sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}}=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

Thus, the respective values are $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$

are $\frac{\sqrt{8+2\sqrt{15}}}{4}, \frac{\sqrt{8-2\sqrt{15}}}{4}$ and $4+\sqrt{15}$