Exercise 5.1

Express each of the complex number given in the Exercises 1 to 10 in the form $a + ib$.

1. $(5i)\left(-\frac{3}{5}i\right)$ **2.** $i^9 + i^{19}$ **3.** i^{-39}

4. $3(7 + i7) + i(7 + i7)$ **5.** $(1 - i) - (-1 + i6)$

6. $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$ **7.** $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

8. $(1 - i)^4$ **9.** $\left(\frac{1}{3}+3i\right)^3$ **10.** $\left(-2-\frac{1}{3}i\right)^3$

Solution:

1.

$(5i)\left(\frac{-3}{5}i\right)=-5\times\frac{3}{5}\times i\times i$

$=-3i^2$

$=-3(-1)$ $\left[i^2=-1\right]$

$= 3$

2.

$i^9+i^{19}=i^{4\times2+1}+i^{4\times4+3}$

$=\left(i^4\right)^2\cdot i+\left(i^4\right)^4\cdot i^3$

$=1\times i+1\times(-i)$ $\left[i^4=1,\ i^3=-i\right]$

$=i+(-i)$

$=0$

3.

$i^{-39}=i^{-4\times9-3}=\left(i^4\right)^{-9}\cdot i^{-3}$

$=(1)^{-9}\cdot i^{-3}$ $\left[i^4=1\right]$

$=\frac{1}{i^3}=\frac{1}{-i}$ $\left[i^3=-i\right]$

$=\frac{-1}{i}\times\frac{i}{i}$

$=\frac{-i}{i^2}=\frac{-i}{-1}=i$ $\left[i^2=-1\right]$

$$3(7+i7)+i(7+i7)=21+21i+7i+7i^2$$
$$=21+28i+7\times(-1) \qquad [\because\ i^2=-1]$$
$$=14+28i$$

5.

$$(1-i)-(-1+i6)=1-i+1-6i$$
$$=2-7i$$

6.

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

7.

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

8.

$$(1-i)^4=\left[(1-i)^2\right]^2$$
$$=\left[1^2+i^2-2i\right]^2$$
$$=[1-1-2i]^2$$
$$=(2i)^2$$
$$=(-2i)\times(-2i)$$
$$=4i^2=-4 \qquad [i^2=-1]$$

9.

$$\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$

$$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$$

$$= \frac{1}{27} + 27(-i) + i + 9i^2 \qquad [i^3 = -i]$$

$$= \frac{1}{27} - 27i + i - 9 \qquad [i^2 = -1]$$

$$= \left(\frac{1}{27} - 9\right) + i(-27+1)$$

$$= \frac{-242}{27} - 26i$$

10.

$$\left(-2-\frac{1}{3}i\right)^3 = (-1)^3\left(2+\frac{1}{3}i\right)^3$$

$$= -\left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8 + \frac{i^3}{27} + 2i\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8 - \frac{i}{27} + 4i + \frac{2i^2}{3}\right] \qquad [i^3 = -i]$$

$$= -\left[8 - \frac{i}{27} + 4i - \frac{2}{3}\right] \qquad [i^2 = -1]$$

$$= -\left[\frac{22}{3} + \frac{107i}{27}\right]$$

$$= -\frac{22}{3} - \frac{107}{27}i$$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.

11. $4 - 3i$ **12.** $\sqrt{5} + 3i$ **13.** $-i$

Solution:

11.

Let $z = 4 - 3i$

Then,

$z = 4 + 3i$ and $|\bar{z}| = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{z}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

$z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = \left(\sqrt{5}\right)^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

13.

Let $z = -i$

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

14:

Express the following expression in the form of $a + ib$.

$$\frac{\left(3 + i\sqrt{5}\right)\left(3 - i\sqrt{5}\right)}{\left(\sqrt{3} + \sqrt{2}i\right) - \left(\sqrt{3} - i\sqrt{2}\right)}$$

Solution:

$$\frac{\left(3 + i\sqrt{5}\right)\left(3 - i\sqrt{5}\right)}{\left(\sqrt{3} + \sqrt{2}i\right) - \left(\sqrt{3} - i\sqrt{2}\right)}$$

$$= \frac{(3)^2 - \left(i\sqrt{5}\right)^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i} \qquad \left[(a+b)(a-b) = a^2 - b^2\right]$$

$$= \frac{9 - 5i^2}{2\sqrt{2}i}$$

$$= \frac{9 - 5(-1)}{2\sqrt{2}i} \qquad \left[i^2 = -1\right]$$

$$= \frac{9 + 5}{2\sqrt{2}i} \times \frac{i}{i}$$

$$= \frac{14i}{2\sqrt{2}i^2}$$

$$= \frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$