Exercise 5.3

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Solve each of the following equations:

1. $x^2 + 3 = 0$ **2.** $2x^2 + x + 1 = 0$ **3.** $x^2 + 3x + 9 = 0$

4. $-x^2 + x - 2 = 0$ **5.** $x^2 + 3x + 5 = 0$ **6.** $x^2 - x + 2 = 0$

7. $\sqrt{2}x^2 + x + \sqrt{2} = 0$ **8.** $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

9. $x^2 + x + \frac{1}{\sqrt{2}} = 0$ **10.** $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

Solution:

1. $x^2 + 3 = 0$

The given quadratic equation is $x^2 + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 1$, $b = 0$, and $c = 3$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12$

Therefore, the required solutions are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2 \times 1} = \frac{\pm\sqrt{12}i}{2} \qquad \left[\sqrt{1} = i\right]$$

$$= \frac{\pm 2\sqrt{3}i}{2} = \pm\sqrt{3}i$$

2. $2x^2 + x + 1 = 0$

The given quadratic equation is $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 2$, $b = 1$ and $c = 1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times 2} = \frac{-1 \pm \sqrt{7}i}{4} \qquad \left[\sqrt{-1} = i\right]$$

3. $x^2 + 3x + 9 = 0$

The given quadratic equation is $x^2 + 3x + 9 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 1$, $b = 3$, and $c = 9$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}i}{2} \qquad \left[\sqrt{-1} = i\right]$$

4. $-x^2+x-2=0$

The given quadratic equation is $-x^2+x-2=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=-1$, $b=1$, and $c=-2$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=1^2-4\times(-1)\times(-2)=1-8=-7$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2(-1)}=\frac{-1\pm\sqrt{7}i}{-2} \qquad \left[\sqrt{-1}=i\right]$$

5. $x^2+3x+5=0$

The given quadratic equation is $x^2+3x+5=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=1$, $b=3$, and $c=5$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=3^2-4\times1\times5=9-20=-11$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm\sqrt{-11}}{2\times1}=\frac{-3\pm\sqrt{11}i}{2} \qquad \left[\sqrt{-1}=i\right]$$

6. $x^2-x+2=0$

The given quadratic equation is $x^2-x+2=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=1$, $b=-1$, and $c=2$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=(-1)^2-4\times1\times2=1-8=-7$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-1)\pm\sqrt{-7}}{2\times1}=\frac{1\pm\sqrt{7}i}{2} \qquad \left[\sqrt{-1}=i\right]$$

7. $\sqrt{2}x^2+x+\sqrt{2}=0$

The given quadratic equation is $\sqrt{2}x^2+x+\sqrt{2}=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=\sqrt{2}$, $b=1$, and $c=\sqrt{2}$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\times\sqrt{2}}=\frac{-1\pm\sqrt{7}i}{2\sqrt{2}} \qquad \left[\sqrt{-1}=i\right]$$

8. $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

The given quadratic equation is $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

On comparing the given equation with $ax^2+bx+c=0$,

We obtain $a=\sqrt{3}$, $b=-\sqrt{2}$, and $c=3\sqrt{3}$

Therefore, the discriminant of the given equation is

$$D=b^2-4ac=\left(-\sqrt{2}\right)^2-4\left(\sqrt{3}\right)\left(3\sqrt{3}\right)=2-36=-34$$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\left(-\sqrt{2}\right)\pm\sqrt{-34}}{2\times\sqrt{3}}=$$

9. $x^2+x+\frac{1}{\sqrt{2}}=0$

9. $x^2+x+\frac{1}{\sqrt{2}}=0$

The given quadratic equation is $x^2+x+\frac{1}{\sqrt{2}}=0$

This equation can also be written as $\sqrt{2}x^2+\sqrt{2}x+1=0$

On comparing the given equation with $ax^2+bx+c=0$, we obtain $a=\sqrt{2}$, $b=\sqrt{2}$, and $c=1$

$\therefore$ Discriminant $(D)=b^2-4ac=\left(\sqrt{2}\right)^2-4\times\left(\sqrt{2}\right)\times1=2-4\sqrt{2}$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}=\frac{-\sqrt{2}\pm\sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$=\left(\frac{-\sqrt{2}\pm\sqrt{2}\left(\sqrt{2\sqrt{2}-1}\right)i}{2\sqrt{2}}\right)\qquad\left[\sqrt{-1}=i\right]$$

$$=\frac{-1\pm\left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

10. $x^2+\frac{x}{\sqrt{2}}+1=0$

The given quadratic equation is $x^2+\frac{x}{\sqrt{2}}+1=0$

This equation can also be written as $\sqrt{2}x^2+x+\sqrt{2}=0$

On comparing the given equation with $ax^2+bx+c=0$,

We obtain $a=\sqrt{2}$, $b=1$, and $c=\sqrt{2}$

$\therefore$ Discriminant $(D)=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\frac{-1\pm\sqrt{7}i}{2\sqrt{2}}\qquad\left[\sqrt{-1}=i\right]$$