Miscellaneous Exercise

1:

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Solution:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$
$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$
$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$
$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$
$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$
$$=\left[-1+\frac{i}{i^2}\right]^3$$
$$=\left[-1-i\right]^3$$
$$=(-1)^3\left[1+i\right]^3$$
$$=-\left[1^3+i^3+3\cdot1\cdot i\left(1+i\right)\right]$$
$$=-\left[1+i^3+3i+3i^2\right]$$
$$=-\left[1-i+3i-3\right]$$
$$=-\left[-2+2i\right]$$
$$=2-2i$$

2:

For any two complex numbers z_1 and z_2, prove that

Re $\left(z_1z_2\right)=\operatorname{Re}z_1\operatorname{Re}z_2-\operatorname{Im}z_1\operatorname{Im}z_2$

Solution:

Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$

$$\therefore\ z_1z_2=\left(x_1+iy_1\right)\left(x_2+iy_2\right)$$
$$=x_1\left(x_2+iy_2\right)+iy_1\left(x_2+iy_2\right)$$
$$=x_1x_2+ix_1y_2+iy_1x_2+i^2y_1y_2$$

$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$

$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$

$\Rightarrow \operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$

$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$

Hence, proved.

3:

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form

Solution:

$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$

$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$

$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$

$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)}$ [On multiplying numerator and denominator by $(14+5i)$]

$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$

$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$

This is the required standard form.

4:

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Solution:

$x - iy = \sqrt{\frac{a-ib}{c-id}}$

$= \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}}$ [On multiplying numerator and denominator by $(c+id)$]

$= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$

$$\therefore (x-iy)^2 = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$\Rightarrow x^2 - y^2 - 2ixy = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2 - y^2 = \frac{ac+bd}{c^2+d^2}, -2xy = \frac{ad-bc}{c^2+d^2} \text{ ......(1)}$$

$$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{ad-bc}{c^2+d^2}\right) \qquad \text{[Using (1)]}$$

$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2+d^2)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2+d^2)^2}$$

$$= \frac{a^2(c^2+d^2) + b^2(c^2+d^2)}{(c^2+d^2)^2}$$

$$= \frac{(c^2+d^2)(a^2+b^2)}{(c^2+d^2)^2}$$

$$= \frac{a^2+b^2}{c^2+d^2}$$

Hence, proved.

5:

Convert the following in the polar form:

(i) $\frac{1+7i}{(2-i)^2}$, (ii) $\frac{1+3i}{1-2i}$

Solution:

(i) Here, $z = \frac{1+7i}{(2-i)^2}$

$$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$$

$$= -1+i$$

Let $r\cos\theta=-1$ and

On squaring and adding, we obtain $r^2\left(\cos^2\theta+\sin^2\theta\right)=1$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=2$

$\Rightarrow r^2=2$ $\left[\cos^2\theta+\sin^2\theta=1\right]$

$\Rightarrow r=\sqrt{2}$ [Conventionally, $r>0$]

$\therefore\ \sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$

$\Rightarrow \cos\theta=\dfrac{-1}{\sqrt{2}}$ and $\sin\theta=\dfrac{1}{\sqrt{2}}$

$\therefore\ \theta=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$ [As lies in II quadrant]

$\therefore\ z=r\cos\theta+i\,r\sin\theta$

$=\sqrt{2}\cos\dfrac{3\pi}{4}+i\sqrt{2}\sin\dfrac{3\pi}{4}=\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$

This is the required polar form.

(ii) Here, $z=\dfrac{1+3i}{1-2i}$

$=\dfrac{1+3i}{1-2i}\times\dfrac{1+2i}{1+2i}$

$=\dfrac{1+2i+3i-6}{1+4}$

$=\dfrac{-5+5i}{5}=-1+i$

Let $r\cos\theta=-1$ and $r\sin\theta$

$=1$ on squaring and adding, we obtain $r^2\left(\cos^2\theta+\sin^2\theta\right)$

$=1+1$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=2$

$\Rightarrow r^2=2$ $\left[\cos^2\theta+\sin^2\theta=1\right]$

$\Rightarrow r=\sqrt{2}$

$\therefore\ \sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$

$\Rightarrow \cos\theta=\dfrac{-1}{\sqrt{2}}$ and $\sin\theta=\dfrac{1}{\sqrt{2}}$

$\therefore\ \theta=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$ [As lies in II quadrant]

$\therefore\ z=r\cos\theta+i\,r\sin\theta$

$=\sqrt{2}\cos\dfrac{3\pi}{4}+i\sqrt{2}\sin\dfrac{3\pi}{4}=\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$

This is the required polar form.

Solve each of the equation in Exercises 6 to 9.

6: $3x^2 - 4x + \frac{20}{3} = 0$

Solution:

The given quadratic equation is $3x^2 - 4x + \frac{20}{3} = 0$

This equation can also be written as $9x^2 - 12x + 20 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 9, b = -12$ and $c = 20$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}\, i}{18} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3} i$$

7:

$x^2 - 2x + \frac{3}{2} = 0$

Solution:

The given quadratic equation is $x^2 - 2x + \frac{3}{2} = 0$

This equation can also be written as $2x^2 - 4x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 2, b = -4$ and $c = 3$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}\, i}{4} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{2 \pm \sqrt{2} i}{2} = 1 \pm \frac{\sqrt{2}}{2} i$$

8: $27x^2 - 10x + 1 = 0$

Solution:

$$27x^2 - 10x + 1 = 0$$

The given quadratic equation is

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 27, b = -10$ and $c = 1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-10)^2 - 4\times 27\times 1 = 100 - 108 = -8$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-10)\pm\sqrt{-8}}{2\times 27} = \frac{10\pm 2\sqrt{2}\,i}{54} \qquad \left[\sqrt{-1}=i\right]$$

$$= \frac{5\pm\sqrt{2}i}{27} = \frac{5}{27}\pm\frac{\sqrt{2}}{27}i$$

9:

$$21x^2 - 28x + 10 = 0$$

Solution:

The given quadratic equation is $21x^2 - 28x + 10 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 21, b = -28$ and $c = 10$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-28)^2 - 4\times 21\times 10 = 784 - 840 = -56$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-28)\pm\sqrt{-56}}{2\times 21} = \frac{28\pm\sqrt{56}\,i}{42} \qquad \left[\sqrt{-1}=i\right]$$

$$= \frac{28\pm 2\sqrt{14}\,i}{42} = \frac{28}{42}\pm\frac{2\sqrt{14}}{42}i = \frac{2}{3}\pm\frac{\sqrt{14}}{21}i$$

10:

If $z_1 = 2 - i,\ z_2 = 1 + i,$ find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$

Solution:

$z_1 = 2 - i,\ z_2 = 1 + i$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i}\times\frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{(1^2 - i^2)}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$= |1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

Thus, the value of $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$ is $\sqrt{2}$.

11:

If $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Solution:

$a+ib = \frac{(x+i)^2}{2x^2+1}$

$= \frac{x^2+i^2+2xi}{2x^2+1}$

$= \frac{x^2-1+i2x}{2x^2+1}$

$= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$

On comparing real and imaginary parts, we obtain

$a = \frac{x^2-1}{2x^2+1}$ **and** $b = \frac{2x}{2x^2+1}$

$\therefore a^2+b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2$

$= \frac{x^4+1-2x^2+4x^2}{(2x+1)^2}$

$= \frac{x^4+1+2x^2}{(2x^2+1)^2}$

$= \frac{(x^2+1)^2}{(2x^2+1)^2}$

$\therefore a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Hence, proved.

12:

Let $z_1 = 2-i,\ z_2 = -2+i$. Find (i) $\mathrm{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) $\mathrm{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$

Solution:

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$\overline{z_1} = 2 + i$

$\therefore \dfrac{z_1 z_2}{\overline{z_1}} = \dfrac{-3+4i}{2+i}$

On multiplying numerator and denominator by $(2-i)$, we obtain

$$\frac{z_1 z_2}{\overline{z_1}} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$

$$= \frac{-2+11i}{5} = \frac{-2}{5} + \frac{11}{5}i$$

On comparing real parts, we obtain

$$\mathrm{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right) = \frac{-2}{5}$$

(ii) $\dfrac{1}{z_1 \overline{z_1}} = \dfrac{1}{(2-i)(2+i)} = \dfrac{1}{(2)^2 + (1)^2} = \dfrac{1}{5}$

On comparing imaginary parts, we obtain

$$\mathrm{Im}\left(\frac{1}{z_1 \overline{z_1}}\right) = 0$$

13:

Find the modulus and argument of the complex number $\dfrac{1+2i}{1-3i}$

Solution:

Let $z = \dfrac{1+3i}{1-3i}$, then

$$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$$

$$= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i$$

Let $z = r\cos\theta + ir\sin\theta$

i.e., $r\cos\theta = \dfrac{-1}{2}$ and $r\sin\theta = \dfrac{1}{2}$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow r = \frac{1}{\sqrt{2}} \qquad [\text{Conventionally, } r > 0]$$

$\therefore \ \frac{1}{\sqrt{2}}\cos\theta = \frac{-1}{2}$ and $\frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \ \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As lies in the II quadrant]

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

14:

Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.

Solution:

Let $z=(x-iy)(3+5i)$

$z = 3x+5xi-3yi-5yi^2 = 3x+5xi-3yi+5y = (3x+5y)+i(5x-3y)$

$\therefore \ \bar{z} = (3x+5y)-i(5x-3y)$

It is given that, $\bar{z} = -6-24i$

$\therefore (3x+5y)-i(5x-3y) = -6-24i$

Equating real and imaginary parts, we obtain

$3x+5y=-6$ ....(i)

$5x-3y=24$ ....(ii)

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$\begin{aligned} 9x+15y &= -18 \\ 25x-15y &= 120 \\ \hline 34x &= 102 \end{aligned}$$

$\therefore x = \frac{102}{34} = 3$

Putting the value of x in equation (i), we obtain

$3(3)+5y=-6$

$\Rightarrow 5y = -6-9 = -15$

$\Rightarrow y = -3$

Thus, the values of x and y are 3 and -3 respectively.

15:

Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

Solution:

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$
$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$
$$=\frac{4i}{2}=2i$$
$$\therefore \left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

16:

If $(x+iy)^3=u+iv,$ then show that: $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$

Solution:

$$(x+iy)^3=u+iv$$
$$\Rightarrow x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$$
$$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i^2=u+iv$$
$$\Rightarrow x^3-iy^3+3x^2yi-3xy^2=u+iv$$
$$\Rightarrow \left(x^3-3xy^2\right)+i\left(3x^2y-y^3\right)=u+iv$$

On equating real and imaginary parts, we obtain

$$u=x^3-3xy^2,\ v=3x^2y-y^3$$
$$\therefore \frac{u}{x}+\frac{v}{y}=\frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y}$$
$$=\frac{x\left(x^2-3y^2\right)}{x}+\frac{y\left(3x^2-y^2\right)}{y}$$
$$=x^2-3y^2+3x^2-y^2$$
$$=4x^2-4y^2$$
$$=4\left(x^2-y^2\right)$$
$$\therefore \frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$$

Hence, proved.

17:

If α and β are different complex numbers with $|\beta|=1,$ then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Solution:

Let $\alpha=a+ib$ and $\beta=x+iy$

It is given that, $|\beta|=1$

$$\therefore \sqrt{x^2+y^2}=1$$

$$\Rightarrow x^2+y^2=1 \;.......(i)$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}}\right|=\left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$=\left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$=\left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$=\left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right| \qquad \left[\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}\right]$$

$$=\frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$=\frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$=\frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$=\frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad [\text{Using}(1)]$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=1$$

18:

Find the number of non-zero integral solutions of the equation $|1-i|^x=2^x$

Solution:

$$|1-i|^x=2^x$$

$$\Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x=2^x$$

$$\Rightarrow \left(\sqrt{2}\right)^x=2^x$$

$$\Rightarrow 2^{x/2}=2^x$$

$$\Rightarrow \frac{x}{2}=x$$

$$\Rightarrow x=2x$$

$\Rightarrow 2x - x = 0$

$\Rightarrow x = 0$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of nonzero integral solutions of the given equation is 0.

19:

If $(a+ib)(c+id)(e+if)(g+ih) = A+iB,$, then show that:

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$.

Solution:

$(a+ib)(c+id)(e+if)(g+ih) = A+iB$

$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$|(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |A+iB| \qquad \because [|z_1 z_2| = |z_1||z_2|]$

$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

On squaring both sides, we obtain

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$. Hence proved.

20:

If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of m.

Solution:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$$\Rightarrow i^m = i^{4k}$$

$\therefore m = 4k,$ where k is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of m is $4(=4\times1)$.