Exercise 5.1

Express each of the complex number given in the Exercises 1 to 10 in the form $a + ib$.

1. $(5i)\left(-\frac{3}{5}i\right)$ **2.** $i^9 + i^{19}$ **3.** i^{-39}

4. $3(7 + i7) + i(7 + i7)$ **5.** $(1 - i) - (-1 + i6)$

6. $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$ **7.** $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

8. $(1 - i)^4$ **9.** $\left(\frac{1}{3}+3i\right)^3$ **10.** $\left(-2-\frac{1}{3}i\right)^3$

Solution:

1.

$$(5i)\left(\frac{-3}{5}i\right) = -5\times\frac{3}{5}\times i\times i$$
$$= -3i^2$$
$$= -3(-1) \qquad [i^2 = -1]$$
$$= 3$$

2.

$$i^9 + i^{19} = i^{4\times2+1} + i^{4\times4+3}$$
$$= (i^4)^2\cdot i + (i^4)^4\cdot i^3$$
$$= 1\times i + 1\times(-i) \qquad [i^4 = 1,\ i^3 = -i]$$
$$= i + (-i)$$
$$= 0$$

3.

$$i^{-39} = i^{-4\times9-3} = (i^4)^{-9}\cdot i^{-3}$$
$$= (1)^{-9}\cdot i^{-3} \qquad [i^4 = 1]$$
$$= \frac{1}{i^3} = \frac{1}{-i} \qquad [i^3 = -i]$$
$$= \frac{-1}{i}\times\frac{i}{i}$$
$$= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad [i^2 = -1]$$

$3(7+i7)+i(7+i7)=21+21i+7i+7i^2$

$=21+28i+7\times(-1) \qquad [\because\ i^2=-1]$

$=14+28i$

5.

$(1-i)-(-1+i6)=1-i+1-6i$

$=2-7i$

6.

$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$

$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$

$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$

$=\frac{-19}{5}-\frac{21}{10}i$

7.

$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$

$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$

$=\frac{17}{3}+i\frac{5}{3}$

8.

$(1-i)^4=\left[(1-i)^2\right]^2$

$=\left[1^2+i^2-2i\right]^2$

$=[1-1-2i]^2$

$=(2i)^2$

$=(-2i)\times(-2i)$

$=4i^2=-4 \qquad [i^2=-1]$

9.

$$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$

$$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$$

$$=\frac{1}{27}+27(-i)+i+9i^2 \qquad [i^3=-i]$$

$$=\frac{1}{27}-27i+i-9 \qquad [i^2=-1]$$

$$=\left(\frac{1}{27}-9\right)+i(-27+1)$$

$$=\frac{-242}{27}-26i$$

10.

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$

$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad [i^3=-i]$$

$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad [i^2=-1]$$

$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$

$$=-\frac{22}{3}-\frac{107}{27}i$$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.

11. $4-3i$ **12.** $\sqrt{5}+3i$ **13.** $-i$

Solution:

11.

Let $z=4-3i$

Then,

$\bar{z}=4+3i$ and $|z|^2=4^2+(-3)^2=16+9=25$

Therefore, the multiplicative inverse of $4-3i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{4+3i}{25}=\frac{4}{25}+\frac{3}{25}i$$

$z=\sqrt{5}+3i$

Then, $\bar{z}=\sqrt{5}-3i$ and $|z|^2=\left(\sqrt{5}\right)^2+3^2=5+9=14$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5}-3i}{14}=\frac{\sqrt{5}}{14}-\frac{3i}{14}$$

13.

Let $z=-i$

Then, $\bar{z}=i$ and $|z|^2=1^2=1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{i}{1}=i$$

14:

Express the following expression in the form of $a+ib$.

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Solution:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Exercise 5.2 Page: 108

Find the modulus and the arguments of each of the complex numbers in Exercises 1 to 2.

1. $z = -1 - i\sqrt{3}$ **2.** $z = -\sqrt{3} + i$

Solution:

1. $z = -1 - i\sqrt{3}$

Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + \left(-\sqrt{3}\right)^2$$

$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 3$$

$$\Rightarrow r^2 = 4 \qquad \left[\cos^2\theta + \sin^2\theta = 1\right]$$

$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$

$\therefore$ Modulus = 2

$\therefore 2\cos\theta = -1$ and $2\sin\theta = -\sqrt{3}$

$\Rightarrow \cos\theta = \dfrac{-1}{2}$ and $\sin\theta = \dfrac{-\sqrt{3}}{2}$

Since both the values of $\sin\theta$ and $\cos\theta$ negative and $\sin\theta$ and $\cos\theta$ are negative in III quadrant,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}i$ are 2 and $-\dfrac{2\pi}{3}$ respectively.

2. $z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$$

$$\Rightarrow r^2 = 3 + 1 = 4 \qquad \left[\cos^2\theta + \sin^2\theta = 1\right]$$

$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$

$\therefore$ Modulus = 2

$\therefore 2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \dfrac{-\sqrt{3}}{2}$ and $\sin\theta = \dfrac{1}{2}$

$\therefore \theta = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$ [As θ lies in the II quadrant]

Thus, the modulus an argument of the complex number $-\sqrt{3}+i$ are 2 and $\dfrac{5\pi}{6}$ respectively.

.ch of the complex numbers given in Exercises 3 to 8 in the polar form:

3. $1-i$ **4.** $-1+i$ **5.** $-1-i$

6. -3 **7.** $\sqrt{3}+i$ **8.** i

Solution:

3:

$1-i$

Let $r\cos\theta=1$ and $r\sin\theta=-1$

On squaring and adding, we obtain

$$r^2\cos^2\theta+r^2\sin^2\theta=1^2+(-1)^2$$

$$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=1+1$$

$$\Rightarrow r^2=2$$

$$\Rightarrow r=\sqrt{2} \qquad [\text{Conventionally, } r>0]$$

$$\therefore\ \sqrt{2}\cos\theta=1 \text{ and } \sqrt{2}\sin\theta=-1$$

$$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}} \text{ and } \sin\theta=-\frac{1}{\sqrt{2}}$$

$$\therefore\ \theta=-\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$

$$\therefore\ 1-i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\left(-\frac{\pi}{4}\right)+i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)=\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]$$

This is the required polar form.

4:

$-1+i$

Let $r\cos\theta=-1$ and $r\sin\theta=1$

On squaring and adding, we obtain

$$r^2\cos^2\theta+r^2\sin^2\theta=(-1)^2+1^2$$

$$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=1+1$$

$$\Rightarrow r^2=2$$

$$\Rightarrow r=\sqrt{2} \qquad [\text{Conventionally, r} > 0]$$

$$\therefore\ \sqrt{2}\cos\theta=-1 \text{ and } \sqrt{2}\sin\theta=1$$

$$\Rightarrow \cos\theta=-\frac{1}{\sqrt{2}} \text{ and } \sqrt{2}\sin\theta=1$$

$$\therefore\ \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

It can be written,

$$\therefore\ -1+i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\frac{3\pi}{4}+i\sqrt{2}\sin\frac{3\pi}{4}=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

5:

$-1-i$

Let $r\cos\theta=-1$ and $r\sin\theta=-1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=(-1)^2+(-1)^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+1$

$\Rightarrow r^2=2$

$\Rightarrow r=\sqrt{2}$ [Conventionally, $r>0$]

$\therefore \sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=-1$

$\Rightarrow \cos\theta=-\frac{1}{\sqrt{2}}$ and $\sin\theta=-\frac{1}{\sqrt{2}}$

$\therefore \theta=-\left(\pi-\frac{\pi}{4}\right)=-\frac{3\pi}{4}$ [As lies in the III quadrant]

$\therefore -1-i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\frac{-3\pi}{4}+i\sqrt{2}\sin\frac{-3\pi}{4}$

$=\sqrt{2}\left(\cos\frac{-3\pi}{4}+i\sin\frac{-3\pi}{4}\right)$

This is the required polar form.

6:

-3

Let $r\cos\theta=-3$ and $r\sin\theta=0$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=(-3)^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=9$

$\Rightarrow r^2=9$

$\Rightarrow r=\sqrt{9}=3$ [Conventionally, r > 0]

$\therefore 3\cos\theta=-3$ and $3\sin\theta=0$

$\Rightarrow \cos\theta=-1$ and $\sin=0$

$\therefore \theta=\pi$

$\therefore -3=r\cos\theta+ir\sin\theta=3\cos\pi+i3\sin\pi=3(\cos\pi+i\sin\pi)$

This is the required polar form.

7

$\sqrt{3}+i$

Let $r\cos\theta=\sqrt{3}$ and

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=\left(\sqrt{3}\right)^2+1^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=3+1$

$\Rightarrow r^2=4$

$\Rightarrow r=\sqrt{4}=2$ [Conventionally, $r>0$]

$\therefore 2\cos\theta=\sqrt{3}$ and

$\Rightarrow \cos\theta=\dfrac{\sqrt{3}}{2}$ and $\sin\theta=\dfrac{1}{2}$

$\therefore \theta=\dfrac{\pi}{6}$ [As lies in the I quadrant]

$\therefore \sqrt{3}+i=r\cos\theta+i\,r\sin\theta=2\cos\dfrac{\pi}{6}+i\,2\sin\dfrac{\pi}{6}=2\left(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right)$

This is the required polar form.

8:

i

Let $r\cos\theta=0$ and $r\sin\theta=1$ On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=0^2+1^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1$

$\Rightarrow r^2=1$

$\Rightarrow r=\sqrt{1}=1$ [Conventionally, $r>0$]

$\therefore \cos\theta=0$ and $\sin\theta=1$

$\therefore \theta=\dfrac{\pi}{2}$

$\therefore i=r\cos\theta+i\,r\sin\theta=\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}$

This is the required polar form.

Exercise 5.3 Page: 109

Solve each of the following equations:

1. $x^2 + 3 = 0$ **2.** $2x^2 + x + 1 = 0$ **3.** $x^2 + 3x + 9 = 0$

4. $-x^2 + x - 2 = 0$ **5.** $x^2 + 3x + 5 = 0$ **6.** $x^2 - x + 2 = 0$

7. $\sqrt{2}x^2 + x + \sqrt{2} = 0$ **8.** $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

9. $x^2 + x + \frac{1}{\sqrt{2}} = 0$ **10.** $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

Solution:

1. $x^2 + 3 = 0$

The given quadratic equation is $x^2 + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 1$, $b = 0$, and $c = 3$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12$

Therefore, the required solutions are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2 \times 1} = \frac{\pm\sqrt{12}i}{2} \qquad \left[\sqrt{1} = i\right]$$

$$= \frac{\pm 2\sqrt{3}i}{2} = \pm\sqrt{3}i$$

2. $2x^2 + x + 1 = 0$

The given quadratic equation is $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 2$, $b = 1$ and $c = 1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times 2} = \frac{-1 \pm \sqrt{7}i}{4} \qquad \left[\sqrt{-1} = i\right]$$

3. $x^2 + 3x + 9 = 0$

The given quadratic equation is $x^2 + 3x + 9 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 1$, $b = 3$, and $c = 9$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}i}{2} \qquad \left[\sqrt{-1} = i\right]$$

4. $-x^2+x-2=0$

The given quadratic equation is $-x^2+x-2=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=-1$, $b=1$, and $c=-2$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=1^2-4\times(-1)\times(-2)=1-8=-7$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2(-1)}=\frac{-1\pm\sqrt{7}i}{-2} \qquad \left[\sqrt{-1}=i\right]$$

5. $x^2+3x+5=0$

The given quadratic equation is $x^2+3x+5=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=1$, $b=3$, and $c=5$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=3^2-4\times1\times5=9-20=-11$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm\sqrt{-11}}{2\times1}=\frac{-3\pm\sqrt{11}\,i}{2} \qquad \left[\sqrt{-1}=i\right]$$

6. $x^2-x+2=0$

The given quadratic equation is $x^2-x+2=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=1$, $b=-1$, and $c=2$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=(-1)^2-4\times1\times2=1-8=-7$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-1)\pm\sqrt{-7}}{2\times1}=\frac{1\pm\sqrt{7}i}{2} \qquad \left[\sqrt{-1}=i\right]$$

7. $\sqrt{2}x^2+x+\sqrt{2}=0$

The given quadratic equation is $\sqrt{2}x^2+x+\sqrt{2}=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=\sqrt{2}$, $b=1$, and $c=\sqrt{2}$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\times\sqrt{2}}=\frac{-1\pm\sqrt{7}i}{2\sqrt{2}} \qquad \left[\sqrt{-1}=i\right]$$

8. $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

The given quadratic equation is $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

On comparing the given equation with $ax^2+bx+c=0$,

We obtain $a=\sqrt{3}$, $b=-\sqrt{2}$, and $c=3\sqrt{3}$

Therefore, the discriminant of the given equation is

$$D=b^2-4ac=\left(-\sqrt{2}\right)^2-4\left(\sqrt{3}\right)\left(3\sqrt{3}\right)=2-36=-34$$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\left(-\sqrt{2}\right)\pm\sqrt{-34}}{2\times\sqrt{3}}=$$

9. $x^2+x+\frac{1}{\sqrt{2}}=0$

9. $x^2+x+\frac{1}{\sqrt{2}}=0$

The given quadratic equation is $x^2+x+\frac{1}{\sqrt{2}}=0$

This equation can also be written as $\sqrt{2}x^2+\sqrt{2}x+1=0$

On comparing the given equation with $ax^2+bx+c=0$, we obtain $a=\sqrt{2}$, $b=\sqrt{2}$, and $c=1$

$\therefore$ Discriminant $(D)=b^2-4ac=\left(\sqrt{2}\right)^2-4\times\left(\sqrt{2}\right)\times 1=2-4\sqrt{2}$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}=\frac{-\sqrt{2}\pm\sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$=\left(\frac{-\sqrt{2}\pm\sqrt{2}\left(\sqrt{2\sqrt{2}-1}\right)i}{2\sqrt{2}}\right)\qquad\left[\sqrt{-1}=i\right]$$

$$=\frac{-1\pm\left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

10. $x^2+\frac{x}{\sqrt{2}}+1=0$

The given quadratic equation is $x^2+\frac{x}{\sqrt{2}}+1=0$

This equation can also be written as $\sqrt{2}x^2+x+\sqrt{2}=0$

On comparing the given equation with $ax^2+bx+c=0$,

We obtain $a=\sqrt{2}$, $b=1$, and $c=\sqrt{2}$

$\therefore$ Discriminant $(D)=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\frac{-1\pm\sqrt{7}\,i}{2\sqrt{2}}\qquad\left[\sqrt{-1}=i\right]$$

Miscellaneous Exercise

1:

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Solution:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$
$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$
$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$
$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$
$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$
$$=\left[-1+\frac{i}{i^2}\right]^3$$
$$=[-1-i]^3$$
$$=(-1)^3[1+i]^3$$
$$=-\left[1^3+i^3+3\cdot1\cdot i(1+i)\right]$$
$$=-\left[1+i^3+3i+3i^2\right]$$
$$=-[1-i+3i-3]$$
$$=-[-2+2i]$$
$$=2-2i$$

2:

For any two complex numbers z_1 and z_2, prove that

$\text{Re}\,(z_1z_2)=\text{Re}\,z_1\,\text{Re}\,z_2-\text{Im}\,z_1\,\text{Im}\,z_2$

Solution:

Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$

$$\therefore\ z_1z_2=(x_1+iy_1)(x_2+iy_2)$$
$$=x_1(x_2+iy_2)+iy_1(x_2+iy_2)$$
$$=x_1x_2+ix_1y_2+iy_1x_2+i^2y_1y_2$$

$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$

$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$

$\Rightarrow \text{Re}(z_1z_2) = x_1x_2 - y_1y_2$

$\Rightarrow \text{Re}(z_1z_2) = \text{Re}\, z_1 \,\text{Re}\, z_2 - \text{Im}\, z_1 \,\text{Im}\, z_2$

Hence, proved.

3:

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form

Solution:

$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$

$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$

$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$

$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)}$ [On multiplying numerator and denominator by $(14+5i)$]

$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$

$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$

This is the required standard form.

4:

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Solution:

$x - iy = \sqrt{\frac{a-ib}{c-id}}$

$= \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}}$ [On multiplying numerator and denominator by $(c+id)$]

$= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$

$$\therefore (x-iy)^2=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$\Rightarrow x^2-y^2-2ixy=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2-y^2=\frac{ac+bd}{c^2+d^2},-2xy=\frac{ad-bc}{c^2+d^2} \text{......(1)}$$

$$\left(x^2+y^2\right)^2=\left(x^2-y^2\right)^2+4x^2y^2$$

$$=\left(\frac{ac+bd}{c^2+d^2}\right)^2+\left(\frac{ad-bc}{c^2+d^2}\right) \quad \text{[Using (1)]}$$

$$=\frac{a^2c^2+b^2d^2+2acbd+a^2d^2+b^2c^2-2adbc}{\left(c^2+d^2\right)^2}$$

$$=\frac{a^2c^2+b^2d^2+a^2d^2+b^2c^2}{\left(c^2+d^2\right)^2}$$

$$=\frac{a^2\left(c^2+d^2\right)+b^2\left(c^2+d^2\right)}{\left(c^2+d^2\right)^2}$$

$$=\frac{\left(c^2+d^2\right)\left(a^2+b^2\right)}{\left(c^2+d^2\right)^2}$$

$$=\frac{a^2+b^2}{c^2+d^2}$$

Hence, proved.

5:

Convert the following in the polar form:

(i) $\frac{1+7i}{(2-i)^2}$, (ii) $\frac{1+3i}{1-2i}$

Solution:

(i) Here, $z=\frac{1+7i}{(2-i)^2}$

$$=\frac{1+7i}{(2-i)^2}=\frac{1+7i}{4+i^2-4i}=\frac{1+7i}{4-1-4i}$$

$$=\frac{1+7i}{3-4i}\times\frac{3+4i}{3+4i}=\frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$=\frac{3+4i+21i-28}{3^2+4^2}=\frac{-25+25i}{25}$$

$$=-1+i$$

Let $r\cos\theta = -1$ and

On squaring and adding, we obtain $r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 2$

$\Rightarrow r^2 = 2$ $\quad\left[\cos^2\theta + \sin^2\theta = 1\right]$

$\Rightarrow r = \sqrt{2}$ $\quad$[Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$[As lies in II quadrant]

$\therefore z = r\cos\theta + i\,r\sin\theta$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

(ii) Here, $z = \frac{1+3i}{1-2i}$

$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$

$= \frac{1+2i+3i-6}{1+4}$

$= \frac{-5+5i}{5} = -1+i$

Let $r\cos\theta = -1$ and $r\sin\theta$

$= 1$ on squaring and adding, we obtain $r^2\left(\cos^2\theta + \sin^2\theta\right)$

$= 1+1$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 2$

$\Rightarrow r^2 = 2$ $\quad\left[\cos^2\theta + \sin^2\theta = 1\right]$

$\Rightarrow r = \sqrt{2}$

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$[As lies in II quadrant]

$\therefore z = r\cos\theta + i\,r\sin\theta$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

Solve each of the equation in Exercises 6 to 9.

6: $3x^2-4x+\frac{20}{3}=0$

Solution:

The given quadratic equation is $3x^2-4x+\frac{20}{3}=0$

This equation can also be written as $9x^2-12x+20=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain $a=9, b=-12$ and $c=20$

Therefore, the discriminant of the given equation is

$D=b^2-4ac=(-12)^2-4\times9\times20=144-720=-576$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(12)\pm\sqrt{-576}}{2\times9}=\frac{12\pm\sqrt{576}\,i}{18} \qquad \left[\sqrt{-1}=i\right]$$

$$=\frac{12\pm24i}{18}=\frac{6(2\pm4i)}{18}=\frac{2\pm4i}{3}=\frac{2}{3}\pm\frac{4}{3}i$$

7: $x^2-2x+\frac{3}{2}=0$

Solution:

The given quadratic equation is $x^2-2x+\frac{3}{2}=0$

This equation can also be written as $2x^2-4x+3=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain $a=2, b=-4$ and $c=3$

Therefore, the discriminant of the given equation is

$D=b^2-4ac=(-4)^2-4\times2\times3=16-24=-8$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-4)\pm\sqrt{8}}{2\times2}=\frac{4\pm2\sqrt{2}\,i}{4} \qquad \left[\sqrt{-1}=i\right]$$

$$=\frac{2\pm\sqrt{2}i}{2}=1\pm\frac{\sqrt{2}}{2}i$$

8: $27x^2-10x+1=0$

Solution:

The given quadratic equation is $27x^2-10x+1=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain $a=27, b=-10$ and $c=1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-10)^2 - 4\times 27\times 1 = 100 - 108 = -8$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-10)\pm\sqrt{-8}}{2\times 27} = \frac{10\pm 2\sqrt{2}\,i}{54} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{5\pm\sqrt{2}i}{27} = \frac{5}{27}\pm\frac{\sqrt{2}}{27}i$$

9:

$$21x^2 - 28x + 10 = 0$$

Solution:

The given quadratic equation is $21x^2 - 28x + 10 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 21, b = -28$ and $c = 10$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-28)^2 - 4\times 21\times 10 = 784 - 840 = -56$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-28)\pm\sqrt{-56}}{2\times 21} = \frac{28\pm\sqrt{56}\,i}{42} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{28\pm 2\sqrt{14}\,i}{42} = \frac{28}{42}\pm\frac{2\sqrt{14}}{42}i = \frac{2}{3}\pm\frac{\sqrt{14}}{21}i$$

10:

If $z_1 = 2 - i,\ z_2 = 1 + i,$ find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$

Solution:

$z_1 = 2 - i,\ z_2 = 1 + i$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i}\times\frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{(1^2 - i^2)}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$=|1+i|=\sqrt{1^2+1^2}=\sqrt{2}$

Thus, the value of $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$ is $\sqrt{2}$.

11:

If $a+ib=\frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$

Solution:

$a+ib=\frac{(x+i)^2}{2x^2+1}$

$=\frac{x^2+i^2+2xi}{2x^2+1}$

$=\frac{x^2-1+i2x}{2x^2+1}$

$=\frac{x^2-1}{2x^2+1}+i\left(\frac{2x}{2x^2+1}\right)$

On comparing real and imaginary parts, we obtain

$a=\frac{x^2-1}{2x^2+1}$ **and** $b=\frac{2x}{2x^2+1}$

$\therefore a^2+b^2=\left(\frac{x^2-1}{2x^2+1}\right)^2+\left(\frac{2x}{2x^2+1}\right)^2$

$=\frac{x^4+1-2x^2+4x^2}{(2x+1)^2}$

$=\frac{x^4+1+2x^2}{(2x^2+1)^2}$

$=\frac{(x^2+1)^2}{(2x^2+1)^2}$

$\therefore a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$

Hence, proved.

12:

Let $z_1=2-i,\ z_2=-2+i$. Find (i) $\text{Re}\left(\frac{z_1z_2}{\bar{z}_1}\right)$, (ii) $\text{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$

Solution:

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$\overline{z_1} = 2 + i$

$\therefore \frac{z_1 z_2}{\overline{z_1}} = \frac{-3+4i}{2+i}$

On multiplying numerator and denominator by $(2-i)$, we obtain

$\frac{z_1 z_2}{\overline{z_1}} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$

$= \frac{-2+11i}{5} = \frac{-2}{5} + \frac{11}{5}i$

On comparing real parts, we obtain

$\text{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right) = \frac{-2}{5}$

(ii) $\frac{1}{z_1 \overline{z_1}} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2 + (1)^2} = \frac{1}{5}$

On comparing imaginary parts, we obtain

$\text{Im}\left(\frac{1}{z_1 \overline{z_1}}\right) = 0$

13:

Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

Solution:

Let $z = \frac{1+3i}{1-3i}$, then

$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$

$= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i$

Let $z = r\cos\theta + ir\sin\theta$

i.e., $r\cos\theta = \frac{-1}{2}$ and $r\sin\theta = \frac{1}{2}$

On squaring and adding, we obtain

$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\Rightarrow r = \frac{1}{\sqrt{2}}$ $\qquad$ [Conventionally, $r > 0$]

$\therefore \frac{1}{\sqrt{2}}\cos\theta = \frac{-1}{2}$ and $\frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As lies in the II quadrant]

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

14:

Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.

Solution:

Let $z=(x-iy)(3+5i)$

$z=3x+5xi-3yi-5yi^2=3x+5xi-3yi+5y=(3x+5y)+i(5x-3y)$

$\therefore \bar{z}=(3x+5y)-i(5x-3y)$

It is given that, $\bar{z}=-6-24i$

$\therefore (3x+5y)-i(5x-3y)=-6-24i$

Equating real and imaginary parts, we obtain

$3x+5y=-6 \quad ....(i)$

$5x-3y=24 \quad ....(ii)$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$\begin{array}{r} 9x+15y=-18 \\ 25x-15y=120 \\ \hline 34x=102 \end{array}$$

$\therefore x=\frac{102}{34}=3$

Putting the value of x in equation (i), we obtain

$3(3)+5y=-6$

$\Rightarrow 5y=-6-9=-15$

$\Rightarrow y=-3$

Thus, the values of x and y are 3 and -3 respectively.

15:

Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Solution:

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$

$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$

$$=\frac{4i}{2}=2i$$

$$\therefore \left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

16:

If $(x+iy)^3=u+iv$, then show that: $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$

Solution:

$$(x+iy)^3=u+iv$$

$$\Rightarrow x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$$

$$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i^2=u+iv$$

$$\Rightarrow x^3-iy^3+3x^2yi-3xy^2=u+iv$$

$$\Rightarrow \left(x^3-3xy^2\right)+i\left(3x^2y-y^3\right)=u+iv$$

On equating real and imaginary parts, we obtain

$$u=x^3-3xy^2,\ v=3x^2y-y^3$$

$$\therefore \frac{u}{x}+\frac{v}{y}=\frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y}$$

$$=\frac{x\left(x^2-3y^2\right)}{x}+\frac{y\left(3x^2-y^2\right)}{y}$$

$$=x^2-3y^2+3x^2-y^2$$

$$=4x^2-4y^2$$

$$=4\left(x^2-y^2\right)$$

$$\therefore \frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$$

Hence, proved.

17:

If α and β are different complex numbers with $|\beta|=1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Solution:

Let $\alpha=a+ib$ and $\beta=x+iy$

It is given that, $|\beta| = 1$

$\therefore \sqrt{x^2 + y^2} = 1$

$\Rightarrow x^2 + y^2 = 1 \text{ ....}(i)$

$$\left|\frac{\beta - \alpha}{1 - \bar{\alpha}}\right| = \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right| \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{(x-a)^2 + (y-b)^2}}{\sqrt{(1-ax-by)^2 + (bx-ay)^2}}$$

$$= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - 2by + b^2x^2 + a^2y^2 - 2abxy}}$$

$$= \frac{\sqrt{(x^2 + y^2) + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2(x^2 + y^2) + b^2(y^2 + x^2) - 2ax - 2by}}$$

$$= \frac{\sqrt{1 + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2 + b^2 - 2ax - 2by}} \qquad [\text{Using}(1)]$$

$$\therefore \left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = 1$$

18:

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$

Solution:

$|1-i|^x = 2^x$

$\Rightarrow \left(\sqrt{1^2 + (-1)^2}\right)^x = 2^x$

$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$

$\Rightarrow 2^{x/2} = 2^x$

$\Rightarrow \frac{x}{2} = x$

$\Rightarrow x = 2x$

$\Rightarrow 2x - x = 0$

$\Rightarrow x = 0$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of nonzero integral solutions of the given equation is 0.

19:

If $(a+ib)(c+id)(e+if)(g+ih) = A+iB,$, then show that:

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2.$

Solution:

$(a+ib)(c+id)(e+if)(g+ih) = A+iB$

$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$|(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |A+iB| \qquad \because [|z_1z_2| = |z_1||z_2|]$

$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

On squaring both sides, we obtain

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$. Hence proved.

20:

If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of m.

Solution:

$\left(\frac{1+i}{1-i}\right)^m = 1$

$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$

$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$

$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$

$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$

$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$

$\Rightarrow i^m = 1$

$\Rightarrow i^m = i^{4k}$

$\therefore m = 4k,$ where k is some integer.
Therefore, the least positive integer is 1.
Thus, the least positive integral value of m is $4(= 4 \times 1)$.