# Exercise 6.1

**1:**

Solve $24x < 100$, when (i) x is a natural number (ii) x is an integer.

**Solution:**

The given inequality is $24x < 100$.

$24x < 100$

$\Rightarrow \dfrac{24x}{24} < \dfrac{100}{24}$         [Dividing both sides by same positive number]

$\Rightarrow x < \dfrac{25}{6}$

(i) It is evident that 1, 2, 3 and 4 are the only natural numbers less than $\dfrac{25}{6}$

Thus, when x is a natural number, the solutions of the given inequality are 1, 2, 3 and 4

Hence, in this case, the solution set is $\{1, 2, 3, 4\}$.

(ii) The integers less than $\dfrac{25}{6}$ are …. $-3, -2, -1, 0, 1, 2, 3, 4$.

Thus, when x is an integer , the solutions of the given inequality are $… -3, -2, -1, 0, 1, 2, 3, 4$

Hence, in this case, the solution set is $\{-3, -2, -1, 0, 1, 2, 3, 4\}$

**2:**

Solve $-12x > 30$, when

(i) x is a natural number

(ii) x is an integer

**Solution:**

The given inequality is $-12x > 30$.

$-12x > 30$.

$\Rightarrow \dfrac{-12x}{-12} < \dfrac{30}{-12}$         [Dividing both sides by same negative number]

$\Rightarrow x < -\dfrac{5}{2}$

(i) There is no natural number less than $\left(-\dfrac{5}{2}\right)$.

Thus, when x is a natural number, there is no solution of the given inequality.

(ii) The integers less than $\left(-\dfrac{5}{2}\right)$ are … $-5, -4, -3$.

Thus, when x is an integer, the solutions of the given inequality are …., $-5, -4, -3$

Hence, in this case, the solution set is $\{…., -5, -4, -3\}$.

**3:**

Solve $5x - 3 < 7$, when

(i) x is an integer

(ii) x is a real number

**Solution:**

The given inequality is $5x - 3 < 7$.

$5x - 3 < 7$

$\quad 5x - 3 + 3 < 7 + 3$

$\Rightarrow 5x < 10$

$\Rightarrow \dfrac{5x}{5} < \dfrac{10}{5}$

$\Rightarrow x < 2$

(i) The integers less than 2 are $\ldots, -4, -3, -2, -1, 0, 1$.

Thus, when x is an integer, the solutions of the given inequality are $\ldots, -4, -3, -2, -1, 0, 1$.

Hence, in this case, the solution set is $\{\ldots, -4, -3, -2, -1, 0, 1\}$.

(ii) When x is a real number, the solutions of the given inequality are given by $x < 2$, that is, all real numbers x which are less than 2.

Thus, the solution set of the given inequality is $x \in (-\infty, 2)$.

**4:**

Solve $3x + 8 > 2$, when

(i) x is an integer

(ii) x is a real number

**Solution:**

The given inequality is $3x + 8 > 2$

$3x + 8 > 2$

$\quad 3x + 8 - 8 > 2 - 8$

$\Rightarrow 3x > -6$

$\Rightarrow \dfrac{3x}{3} > \dfrac{-6}{3}$

$\Rightarrow x > -2$

(i) The integers greater than $-2$ are $-1, 0, 1, 2, \ldots$

Thus, when x is an integer, the solutions of the given inequality are $-1, 0, 1, 2, \ldots$

Hence, in this case, the solution set is $\{-1, 0, 1, 2, \ldots\}$.

(ii) When x is a real number, the solutions of the given inequality are all the real numbers, which are greater than $-2$.

Thus, in this case, the solution set is $(-2, \infty)$.

Solve the inequalities in Exercises 5 to 16 for real $x$.

**5:**

$$4x+3<5x+7$$

**Solution:**
$4x+3<5x+7$
$\Rightarrow 4x+3-7<5x+7-7$
$\Rightarrow 4x-4<5x$
$\Rightarrow 4x-4-4x<5x-4x$
$\Rightarrow -4<x$
Thus, all real numbers x, which are greater than $-4$, are the solutions of the given inequality.
Hence, the solution set of the given inequality is $(-4,\infty)$.

**6:**

$$3x-7>5x-1$$

**Solution:**
$3x-7>5x-1$
$\Rightarrow 3x-7+7>5x-1+7$
$3x>5x+6$
$\Rightarrow 3x-5x>5x+6-5x$
$\Rightarrow -2x>6$
$\Rightarrow \dfrac{-2x}{-2}<\dfrac{6}{-2}$
$\Rightarrow x<-3$
Thus, all real numbers x, which are less than $-3$, are the solutions of the given inequality.
Hence, the solution set of the given inequality is $(-\infty,-3)$.

**7:**

$$3(x-1)\le 2(x-3)$$

**Solution:**
$3(x-1)\le 2(x-3)$
$\Rightarrow 3x-3\le 2x-6$
$\Rightarrow 3x-3+3\le 2x-6+3$
$\Rightarrow 3x\le 2x-3$
$\Rightarrow 3x-2x\le 2x-3-2x$
$\Rightarrow x\le -3$
Thus, all real numbers x, which are less than or equal to    , are the solutions of the
given inequality.
Hence, the solution set of the given inequality is $(-\infty,-3]$. $-3$

**8:**

$$3(2-x) \geq 2(1-x)$$

**Solution:**

$$3(2-x) \geq 2(1-x)$$
$$\Rightarrow 6-3x \geq 2-2x$$
$$\Rightarrow 6-3x+2x \geq 2-2x+2x$$
$$\Rightarrow 6-x \geq 2$$
$$6-x-6 \geq 2-6$$
$$\Rightarrow -x \geq -4$$
$$\Rightarrow x \leq 4$$

Thus, all real numbers x, which are greater than or equal to 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 4]$.

**9:**

$$x : x + \frac{x}{2} + \frac{x}{3} < 11$$

**Solution:**

$$x + \frac{x}{2} + \frac{x}{3} < 11$$
$$\Rightarrow x\left(1 + \frac{1}{2} + \frac{1}{3}\right) < 11$$
$$\Rightarrow x\left(\frac{6+3+2}{6}\right) < 11$$
$$\Rightarrow \frac{11x}{6} < 11$$
$$\Rightarrow \frac{11x}{6 \times 11} < \frac{11}{11}$$
$$\Rightarrow \frac{x}{6} < 1$$
$$\Rightarrow x < 6$$

Thus, all real numbers x, which are less than 6, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 6)$.

**10**

$$: \frac{x}{3} > \frac{x}{2} + 1$$

**Solution:**

$$\frac{x}{3} > \frac{x}{2} + 1$$

$$\Rightarrow \frac{x}{3} - \frac{x}{2} > 1$$

$$\Rightarrow \frac{2x - 3x}{6} > 1$$

$$\Rightarrow -\frac{x}{6} > 1$$

$$\Rightarrow -x > 6$$

$$\Rightarrow x < -6$$

Thus, all real numbers x, which are less than $-6$, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, -6)$.

**11:**

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

**Solution:**

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

$$\Rightarrow 9(x-2) \leq 25(2-x)$$

$$\Rightarrow 9x - 18 \leq 50 - 25x$$

$$\Rightarrow 9x - 18 + 25x \leq 50$$

$$\Rightarrow 34x - 18 \leq 50$$

$$\Rightarrow 34x \leq 50 + 18$$

$$\Rightarrow 34x \leq 68$$

$$\Rightarrow \frac{34x}{34} \leq \frac{68}{34}$$

$$\Rightarrow x \leq 2$$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given hence the solution set of the given inequality is $(-\infty, 2]$.

**12:**

$$\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$$

**Solution:**

$$\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x-6)$$

$$\Rightarrow 3\left(\frac{3x}{5}+4\right) \geq 2(x-6)$$

$$\Rightarrow \frac{9x}{5}+12 \geq 2x-12$$

$$\Rightarrow 12+12 \geq 2x-\frac{9x}{5}$$

$$\Rightarrow 24 \geq \frac{10x-9x}{5}$$

$$\Rightarrow 24 \geq \frac{x}{5}$$

$$\Rightarrow 120 \geq x$$

Thus, all real numbers x, which are less than or equal to 120, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 120]$.

**13:**

$$2(2x+3)-10 < 6(x-2)$$

**Solution:**

$$2(2x+3)-10 < 6(x-2)$$

$$\Rightarrow 4x+6-10 < 6x-12$$

$$\Rightarrow 4x-4 < 6x-12$$

$$\Rightarrow -4+12 < 6x-4x$$

$$\Rightarrow 8 < 2x$$

$$\Rightarrow 4 < x$$

Thus, all real numbers x, which are greater than 4, are the solutions of the given inequality. Hence, the solution set of the given inequality is $(4, \infty)$.

**14:**

$$37-(3x+5) \geq 9x-8(x-3)$$

**Solution:**

$$37-(3x+5) \geq 9x-8(x-3)$$

$$\Rightarrow 37-3x-5 \geq 9x-8x+24$$

$$\Rightarrow 32-3x \geq x+24$$

$$\Rightarrow 32-24 \geq x+3x$$

$$\Rightarrow 8 \geq 4x$$

$$\Rightarrow 2 \geq x$$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

**15:**

Solve the given inequality for real $x: \dfrac{x}{4} < \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

**Solution:**

$\dfrac{x}{4} < \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

$\Rightarrow \dfrac{x}{4} < \dfrac{5(5x-2) - 3(7x-3)}{15}$

$\Rightarrow \dfrac{x}{4} < \dfrac{25x - 10 - 21x + 9}{15}$

$\Rightarrow \dfrac{x}{4} < \dfrac{4x - 1}{15}$

$\Rightarrow 15x < 4(4x - 1)$

$\Rightarrow 15x < 16x - 4$

$\Rightarrow 4 < 16x - 15x$

$\Rightarrow 4 < x$

Thus, all real numbers x, which are greater than 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(4, \infty)$.

**16:**

Solve the given inequality for real $x: \dfrac{(2x-1)}{3} \geq \dfrac{(3x-2)}{4} - \dfrac{(2-x)}{5}$

**Solution:**

$\dfrac{(2x-1)}{3} \geq \dfrac{(3x-2)}{4} - \dfrac{(2-x)}{5}$

$\Rightarrow \dfrac{(2x-1)}{3} \geq \dfrac{5(3x-2) - 4(2-x)}{20}$

$\Rightarrow \dfrac{(2x-1)}{3} \geq \dfrac{15x - 10 - 8 + 4x}{20}$

$\Rightarrow \dfrac{(2x-1)}{3} \geq \dfrac{19x - 18}{20}$

$\Rightarrow 20(2x-1) \geq 3(19x - 18)$

$\Rightarrow 40x - 20 \geq 57x - 54$

$\Rightarrow -20 + 54 \geq 57x - 40x$

$\Rightarrow 34 \geq 17x$

$\Rightarrow 2 \geq x$

Thus, all real numbers x, which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

Solve the inequalities in Exercises 17 to 20 and show the graph of the solution in each case on number line
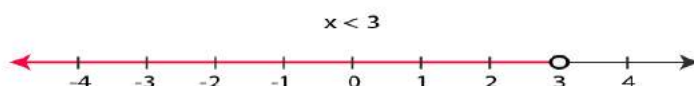
**17:**

$3x - 2 < 2x + 1$

**Solution:**

$3x - 2 < 2x + 1$

$\Rightarrow 3x - 2x < 1 + 2$

$\Rightarrow x < 3$

The graphical representation of the solutions of the given inequality is as follows:



**18:**

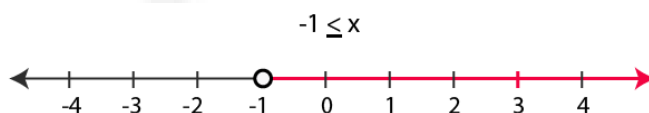$5x - 3 \geq 3x - 5$

**Solution:**

$5x - 3 \geq 3x - 5$

$5x - 3x \geq -5 + 3$

$\Rightarrow 2x \geq -2$

$\Rightarrow \dfrac{2x}{2} \geq \dfrac{-2}{2}$

$\Rightarrow x \geq -1$

The graphical representation of the solutions of the given inequality is as follows.



**19:**

$3(1 - x) < 2(x + 4)$

**Solution:**

$3(1 - x) < 2(x + 4)$

$\Rightarrow 3 - 3x < 2x + 8$
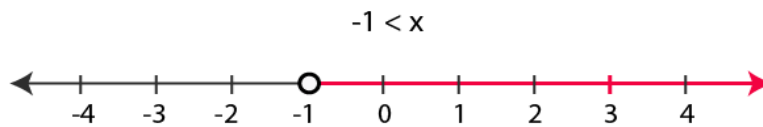
$\Rightarrow 3 - 8 < 2x + 3x$

$$\Rightarrow -5 < 5x$$
$$\Rightarrow \frac{-5}{5} < \frac{5x}{5}$$
$$\Rightarrow -1 < x$$

The graphical representation of the solutions of the given inequality is as follows:



$-1 < x$

**20:**

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

**Solution:**

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$
$$\Rightarrow \frac{x}{2} \geq \frac{5(5x-2)-3(7x-3)}{15}$$
$$\Rightarrow \frac{x}{2} \geq \frac{25x-10-21x+9}{15}$$
$$\Rightarrow \frac{x}{2} \geq \frac{4x-1}{15}$$
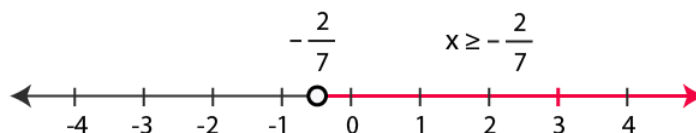$$\Rightarrow 15x \geq 2(4x-1)$$
$$\Rightarrow 15x \geq 8x-2$$
$$\Rightarrow 15x-8x \geq 8x-2-8x$$
$$\Rightarrow 7x \geq -2$$
$$\Rightarrow x \geq -\frac{2}{7}$$

The graphical representation of the solutions of the given inequality is as follows.



$-\frac{2}{7}$    $x \geq -\frac{2}{7}$

**21:**
Ravi obtained 70 and 75 marks in first two-unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

**Solution:**
Let x be the marks obtained by Ravi in the third unit test.
Since the student should have an average of at least 60 marks,
$$\frac{70+75+x}{3} \geq 60$$

$\Rightarrow 145 + x \geq 180$

$\Rightarrow x \geq 180 - 145$

$\Rightarrow x \geq 35$

Thus, the student must obtain a minimum of 35 marks to have an average of at least 60 marks.

**22:**

To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.

**Solution:**

Let x be the marks obtained by Sunita in the fifth examination.

In order to receive grade 'A' in the course, she must obtain an average of 90 marks or more in five examinations.

Therefore,

$$\frac{87 + 92 + 94 + 95 + x}{5} \geq 90$$

$$\Rightarrow \frac{368 + x}{5} \geq 90$$

$$\Rightarrow 368 + x \geq 450$$

$$\Rightarrow x \geq 450 - 368$$

$$\Rightarrow x \geq 82$$

Thus, Sunita must obtain greater than or equal to 82 marks in the fifth examination.

**23**

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

**Solution:**

Let x be the smaller of the two consecutive odd positive integers. Then, the other integer is $x + 2$.

Since both the integers are smaller than 10,

$x + 2 < 10$

$\Rightarrow x < 10 - 2$

$\Rightarrow x < 8 \ .....(i)$

Also, the sum of the two integers is more than 11.

$\therefore x + (x + 2) > 11$

$\Rightarrow 2x + 2 > 11$

$\Rightarrow 2x > 11 - 2$

$\Rightarrow 2x > 9$

$\Rightarrow x > \dfrac{9}{2}$

$\Rightarrow x > 4.5 \ .......(ii)$

From (i) and (ii), we obtain
Since x is an odd number, x can take the values, 5 and 7.
Thus, the required possible pairs are (5, 7) and (7, 9).

**24:**
Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

**Solution:**
Let x be the smaller of the two consecutive even positive integers. Then, the other integer is $x+2$
Since both the integers are larger than 5,
$x > 5........(1)$
Also, the sum of the two integers is less than 23
$x + (x+2) < 23$
$\Rightarrow 2x + 2 < 23$
$\Rightarrow 2x < 23 - 2$
$\Rightarrow 2x < 21$
$\Rightarrow x < \dfrac{21}{2}$
$\Rightarrow x < 10.5 ......(2)$
From (1) and (2), we obtain $5 < x < 10.5$.
Since x is an even number, x can take the values, 6, 8 and 10.
Thus, the required possible pairs are $(6,8), (8,10)$ and $(10,12)$.

**25:**
The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

**Solution:**
Let the length of the shortest side of the triangle be x cm.
Then, length of the longest side = 3x cm
Length of the third side $(3x - 2) cm$
Since the perimeter of the triangle is at least 61 cm,
$x\, cm + 3x\, cm + (3x - 2) cm \geq 61\, cm$
$\Rightarrow 7x - 2 \geq 61$
$\Rightarrow 7x \geq 61 + 2$
$\Rightarrow 7x \geq 63$

$$\Rightarrow \frac{7x}{7} \geq \frac{63}{7}$$

$$\Rightarrow x \geq 9$$

Thus, the minimum length of the shortest side is 9 cm.

**26:**

A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

[Hint: It x is the length of the shortest board, then $x$, $(x+3)$ and 2x are the lengths of the second and third piece, respectively. Thus, $x = (x+3) + 2x \leq 91$ and $2x \geq (x+3) + 5$]

**Solution:**

Let the length of the shortest piece be x cm. Then, length of the second piece and the third piece are (x + 3) cm and 2x cm respectively.

Since the three lengths are to be cut from a single piece of board of length 91 cm,

$x\,cm\,(+x+3)cm + 2x\ cm \leq 91cm$

$$\Rightarrow 4x + 3 \leq 91$$

$$\Rightarrow 4x \leq 91 - 3$$

$$\Rightarrow 4x \leq 88$$

$$\Rightarrow \frac{4x}{4} \leq \frac{88}{4}$$

$$\Rightarrow x \leq 22 \ldots (1)$$

Also, the third piece is at least 5 cm longer than the second piece.

$$\therefore 2x \geq (x+3) + 5$$

$$\Rightarrow 2x \geq x + 8$$

$$\Rightarrow x \geq 8 \ldots (2)$$

From (1) and (2), we obtain

$$8 \leq x \leq 22$$

Thus, the possible length of the shortest board is greater than or equal to 8 cm but less than or equal to 22 cm.