Exercise 6.2

Solve the following inequalities graphically in two-dimensional plane:

1: $x+y<5$

Solution:

The graphical representation of $x+y=5$ is given as dotted line in the figure below.

This line divides the $xy-$plane in two half planes, I and II.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
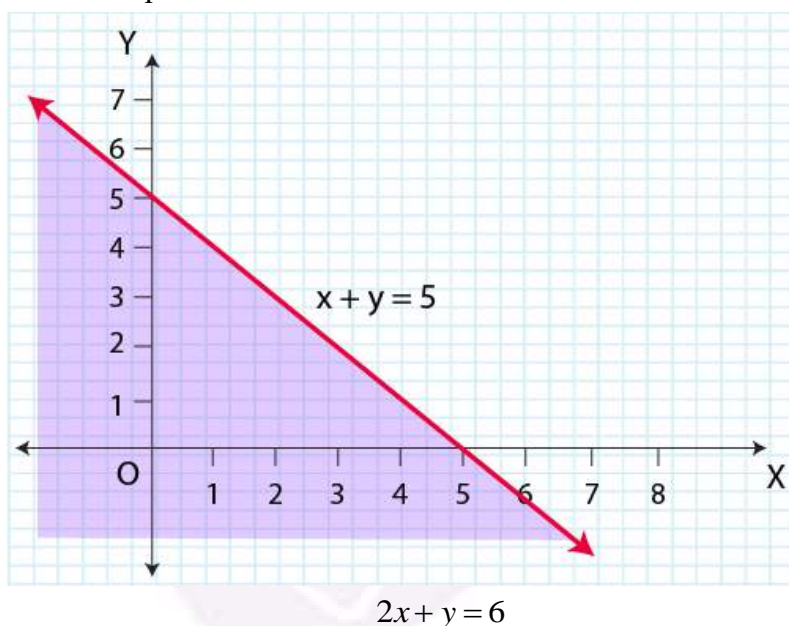
We select the point as (0, 0).

It is observed that,

$0+0<5$ or $0<5$

Therefore, half plane II is not the solution region of the given inequality. Also, it is evident that any point on the line does not satisfy the given strict inequality.

Thus, the solution region of the given inequality is the shaded half plane I excluding the points on the line.

This can be represented as follows.

$2x+y=6$

2: $2x+y\geq 6$

Solution:

The graphical representation of   is given in the figure below.

This line divides the xy-plane in two half planes, I and II.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
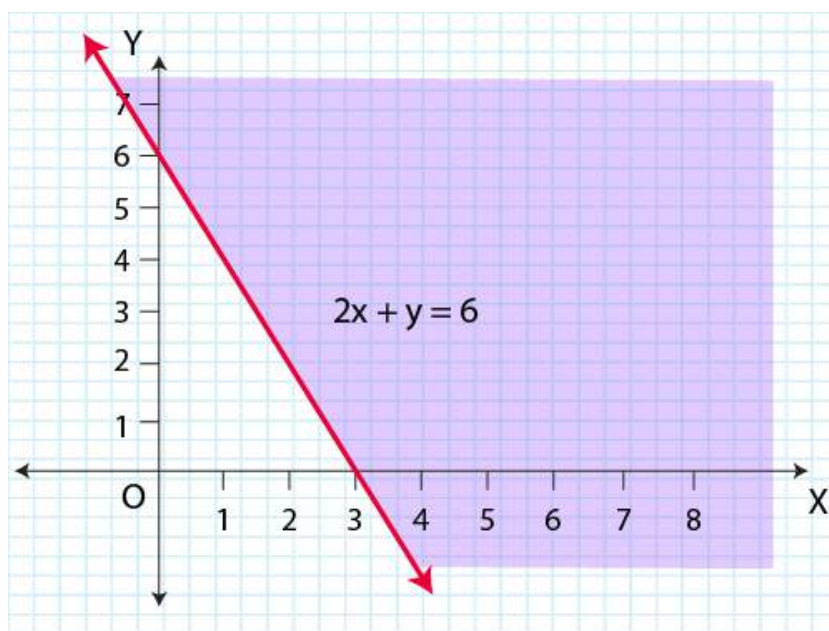
We select the point as (0, 0).

It is observed that,

$2(0)+0\geq 6$ or $0\geq 6$, which is false

Therefore, half plane I is not the solution region of the given inequality. Also, it is evident that any point on the line satisfies the given inequality.

Thus, the solution region of the given inequality is the shaded half plane II including the points on the line.

This can be represented as follows.

3: $3x + 4y \leq 12$

Solution:

The graphical representation of $3x + 4y = 12$ is given in the figure below.
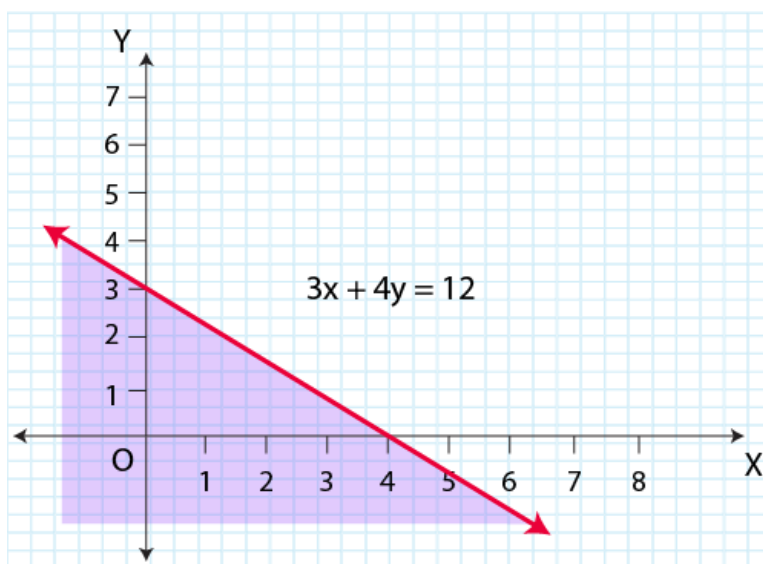This line divides the xy-plane in two half planes, I and II.
Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
We select the point as (0, 0).
Thus, the solution region of the given inequality is the shaded half plane I including the points on the line.
This can be represented as follows.

4: $y + 8 \geq 2x$

Solution:
The graphical representation of $y + 8 = 2x$ is given in the figure below.

This line divides the xy-plane in two half planes.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
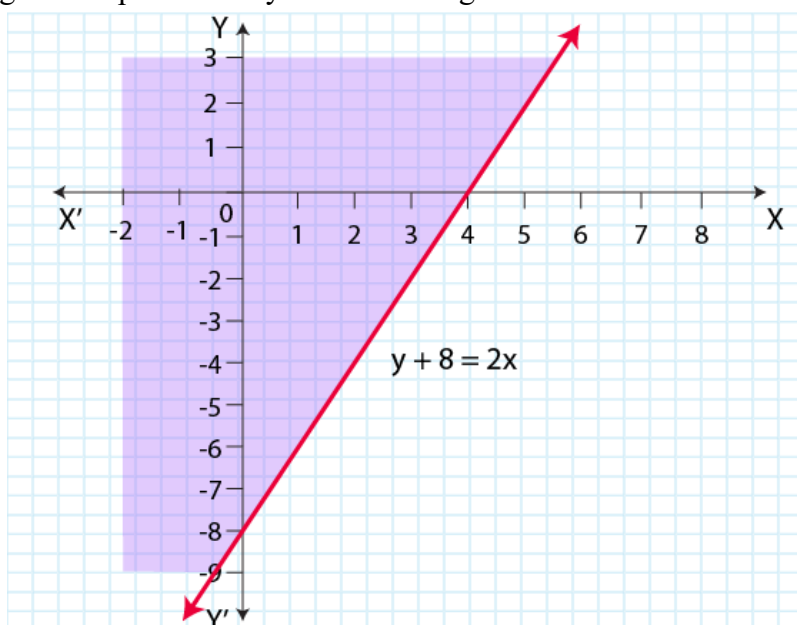
We select the point as (0, 0).

It is observed that,

$0+8 \geq 2(0)$ or $8 \geq 0$, which is true

Therefore, lower half plane is not the solution region of the given inequality. Also, it is evident that any point on the line satisfies the given inequality.

Thus, the solution region of the given inequality is the half plane containing the point (0, 0) including the line.

The solution region is represented by the shaded region as follows.

5: $x - y \leq 2$

Solution:

The graphical representation of $x - y = 2$ is given in the figure below.

This line divides the xy-plane in two half planes.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
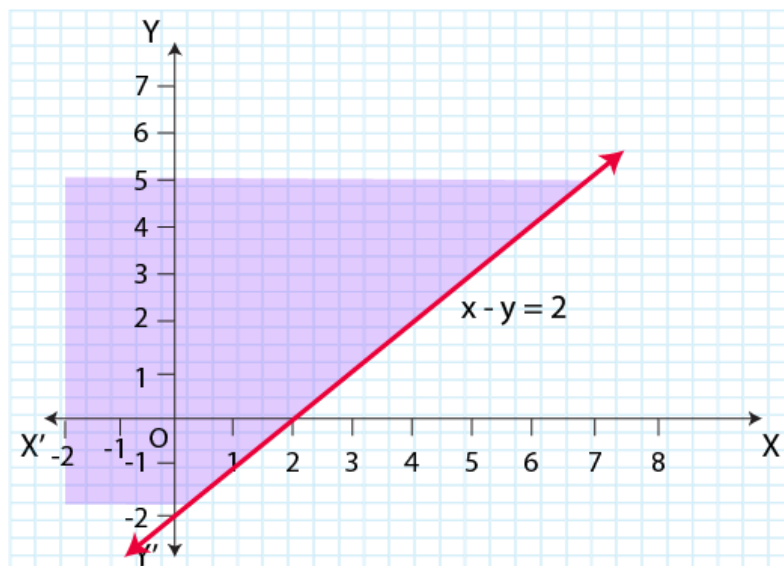
We select the point as (0, 0).

It is observed that,

$0-0 \leq 2$ or $0 \leq 2$, which is true

Therefore, the lower half plane is not the solution region of the given inequality. Also, it is clear that any point on the line satisfies the given inequality.

Thus, the solution region of the given inequality is the half plane containing the point (0, 0) including the line.

The solution region is represented by the shaded region as follows.

6: $2x - 3y > 6$

Solution:

The graphical representation of $2x - 3y = 6$ is given as dotted line in the figure below.

This line divides the xy-plane in two half planes.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
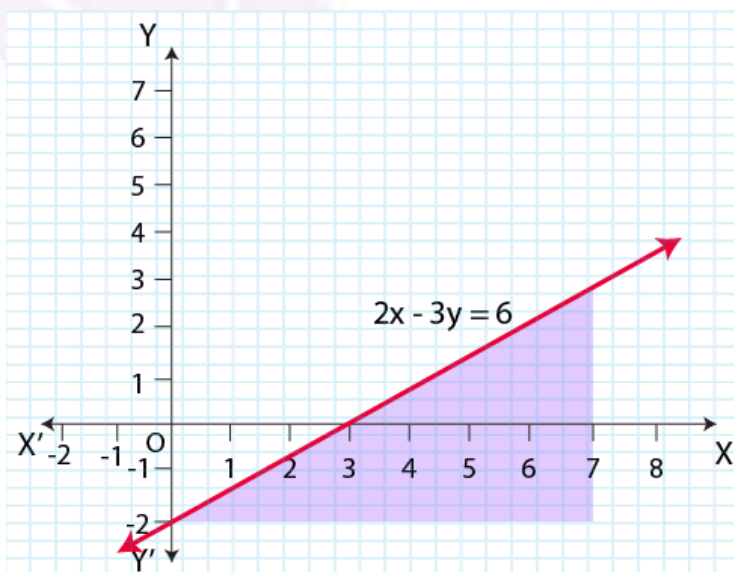
We select the point as (0, 0).

It is observed that,

$2(0) - 3(0) > 6$ or $0 > 6$, which is false

Therefore, the upper half plane is not the solution region of the given inequality. Also, it is clear that any point on the line does not satisfy the given inequality.

Thus, the solution region of the given inequality is the half plane that does not contain the point (0, 0) including the line.

The solution region is represented by the shaded region as follows.

7: $-3x + 2y \geq -6$

Solution:

The graphical representation of $-3x + 2y = -6$ is given in the figure below.

This line divides the xy-plane in two half planes.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
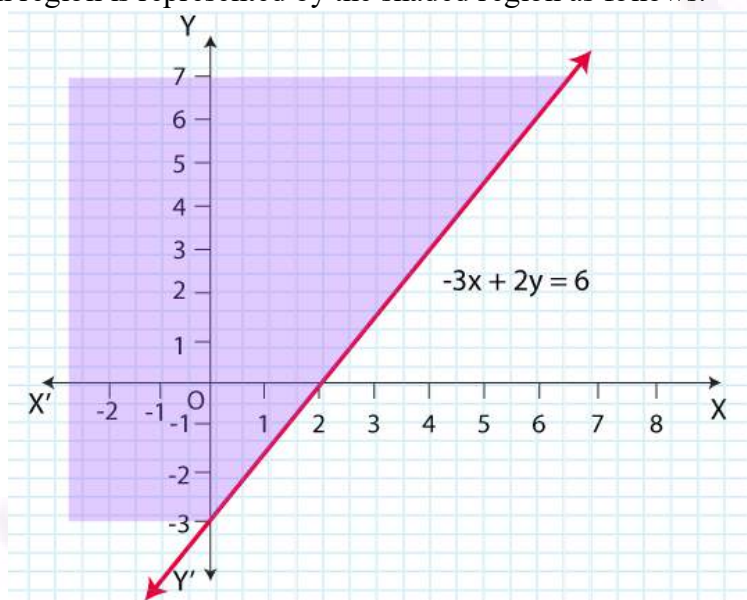
We select the point as (0, 0).

It is observed that,

$-3(0) + 2(0) \geq -6$ or $0 \geq -6$, which is true

Therefore, the lower half plane is not the solution region of the given inequality. Also, it is evident that any point on the line satisfies the given inequality.

Thus, the solution region of the given inequality is the half plane containing the point (0, 0) including the line.

The solution region is represented by the shaded region as follows.

8: $3y - 5x < 30$

Solution:

The graphical representation of $3y - 5x = 30$ is given as dotted line in the figure below.

This line divides the xy-plane in two half planes.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.

We select the point as (0, 0).

It is observed that,

$3(0) - 5(0) < 30$ or $0 > 30$, which is true

Therefore, the upper half plane is not the solution region of the given inequality. Also, it is evident that any point on the line does not satisfy the given inequality.

Thus, the solution region of the given inequality is the half plane containing the point (0, 0) excluding the line.

The solution region is represented by the shaded region as follows.
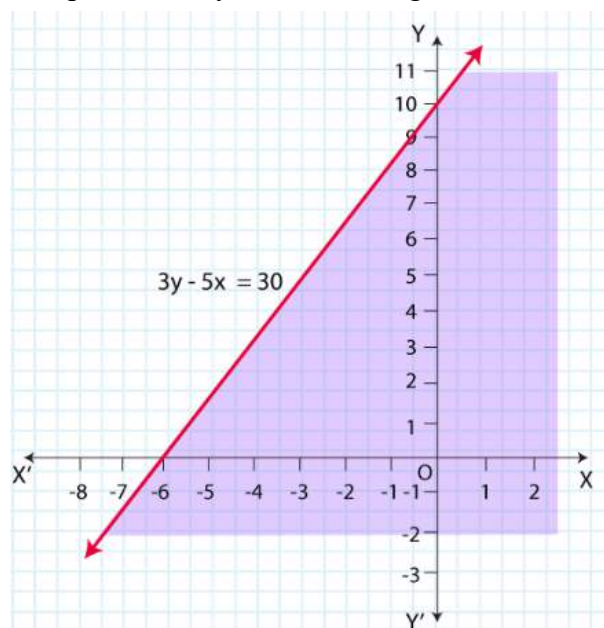
9: $y < -2$

Solution:

The graphical representation of $y = -2$ is given as dotted line in the figure below. This line divides the xy-plane in two half planes.

Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.

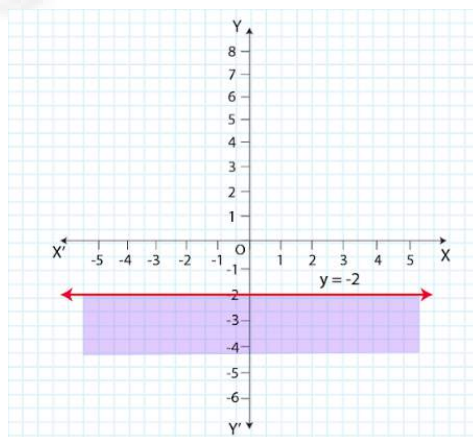We select the point as (0, 0).

It is observed that,

$0 < -2$, which is false

Also, it is evident that any point on the line does not satisfy the given inequality.

Hence, every point below the line, $y = -2$ (excluding all the points on the line), determines the solution of the given inequality.

The solution region is represented by the shaded region as follows.

10: $x > -3$

Solution:
The graphical representation of $x = -3$ is given as dotted line in the figure below. This line divides the xy-plane in two half planes.
Select a point (not on the line), which lies in one of the half planes, to determine whether the point satisfies the given inequality or not.
We select the point as (0, 0).
It is observed that,
$0 < -3$, which is true
Also, it is evident that any point on the line does not satisfy the given inequality.
Hence, every point on the right side of the line, $x = -3$ (excluding all the points on the line), determines the solution of the given inequality.
The solution region is represented by the shaded region as follows.