Exercise 7.4

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1:

If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$

Solution:

It is known that, ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $m = a + b$

Therefore,

$${}^nC_8 = {}^nC_2 \Rightarrow n = 8 + 2 = 10$$

$$\therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10\times9\times8!}{2\times1\times8!} = 45$$

2:

Determine n if

(i) ${}^{2n}C_3 : {}^nC_3 = 12:1$ (ii) ${}^{2n}C_3 : {}^nC_3 = 11:1$

Solution:

(i) $\frac{{}^{2n}C_3}{{}^nC_3} = \frac{12}{1}$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 12$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{(2n-1)}{(n-2)} = 3$$

$$\Rightarrow 2n-1 = 3(n-2)$$

$n=5$

(ii) $\frac{{}^{2n}C_3}{{}^nC_3} = \frac{11}{1}$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n} = 11$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!}$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 11$$

$\Rightarrow 3n = 18$
$\Rightarrow n = 6$

3:
How many chords can be drawn through 21 points on a circle?

Solution:
For drawing one chord a circle, only 2 points are required.
To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.
Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.
Thus, required number of chords $= {}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21\times 20}{2} = 210$

4:
In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Solution:
A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.
3 boys can be selected from 5 boys in ${}^5C_3$ ways.
3 girls can be selected from 4 girls in ${}^4C_3$ ways.
Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!}$

$= \frac{5\times 4\times 3!}{3!\times 2} \times \frac{4\times 3!}{3!}$
$= 10 \times 4 = 40$

5:
Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls each colour.

Solution:
There are a total of 6 red balls, 5 white balls, and 4 blue balls.
9 balls have to be selected in such a way that each selection consists of 3 balls of each colour.
Here,
3 balls can be selected from 6 red balls in ${}^6C_3$ ways.
3 balls can be selected from 5 white balls in ${}^5C_3$ ways.

3 balls can be selected from 5 blue balls in ${}^{5}C_{3}$ ways.

Thus, by multiplication principle, required number of ways of selecting 9 balls.

$$={}^{6}C_{3}\times{}^{5}C_{3}\times{}^{5}C_{3}=\frac{6!}{3!3!}\times\frac{5!}{3!2!}\times\frac{5!}{3!2!}$$

$$=\frac{6\times5\times4\times3!}{3!\times3\times2}\times\frac{5\times4\times3!}{3!\times2\times1}\times\frac{5\times4\times3!}{3!\times2\times1}$$

$$=20\times10\times10=2000$$

6:

Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Solution:

In a deck of 52 cards, there are 4 aces. A combinations of 5 cards have to be made in which there is exactly one ace.

Then, one ace can be selected in ${}^{4}C_{3}$ ways and the remaining 4 cards can be selected out of the 48 cards in ${}^{48}C_{4}$ ways.

Thus, by multiplication principle, required number of 5 card combinations

$$={}^{48}C_{4}\times{}^{4}C_{1}=\frac{48!}{4!44!}\times\frac{4!}{1!3!}$$

$$=\frac{48\times47\times46\times45}{4!\times3\times2\times1}\times4!$$

$$=778320$$

7:

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution:

Out of 17 players, 5 players are bowlers.

A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.

4 bowlers can be selected in ${}^{5}C_{4}$ ways and the remaining 7 players can be selected out of the 12 players in ${}^{12}C_{7}$ ways.

Thus, by multiplication principle, required number of ways of selecting cricket team

$$={}^{5}C_{4}\times{}^{12}C_{7}=\frac{5!}{4!1!}\times\frac{12!}{7!5!}=5!\times\frac{12\times11\times10\times9\times8}{5!\times4\times3\times2\times1}=3960$$

8:

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution:

There are 5 black and 6 red balls in the bag.

2 black balls can be selected out of 5 black balls in ${}^5C_2$ ways and 3 red balls can be selected out of 6 red balls in ${}^6C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls

$$= {}^5C_2 \times {}^6C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!} = \frac{5\times4}{2} \times \frac{6\times5\times4}{3\times2\times1} = 10\times20 = 200.$$

9:

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution:

There are 9 courses available out of which, 2 specific courses are compulsory for every student. Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^7C_3$ ways.

Thus, required number of ways of choosing the programme

$$= {}^7C_3 = \frac{7!}{3!4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!} = 35.$$