Miscellaneous Exercise

1:Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

Solution:

If $\vec{r}$ is a unit vector in the XY-plane, then $\vec{r}=\cos\theta\hat{i}+\sin\theta\hat{j}$.

Here, θ is the angle made by the unit vector with the positive direction of the x-axis.

Therefore, for $\theta=30^\circ$.

$$\vec{r}=\cos 30^o\,\hat{i}+\sin 30^o\,\hat{j}=\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}$$

Hence, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}$

2:

Find the scalar components and magnitude of the vector joining the points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$.

Solution:

The vector joining the points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ can be obtained by,

$\overrightarrow{PQ}=$ Position vector of Q-Position vector of P

$$=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$$

$$\left|\overrightarrow{PQ}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Hence, the scalar components and the magnitude of the vector joining the given points are respectively $\{(x_2-x_1),(y_2-y_1),(z_2-z_1)\}$ and $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

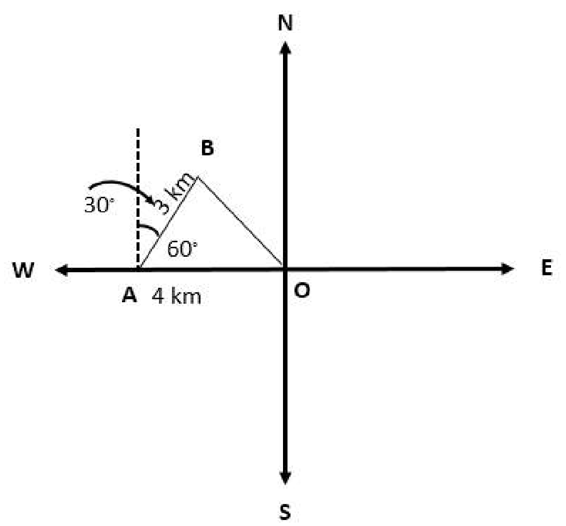
3:

A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Solution 3:

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:



Now, we have:

$$\vec{OA} = -4\hat{i}$$

$$\vec{AB} = \left|\vec{AB}\right|\cos 60^o\,\hat{i} + \left|\vec{AB}\right|\sin 60^o\,\hat{j}$$

$$= 3\times\frac{1}{2}\hat{i} + 3\times\frac{\sqrt{3}}{2}\hat{j}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

By the triangle law of vector addition, we have:

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$

$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Hence, the girl's displacement from her initial point of departure is

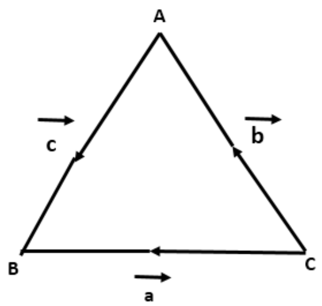
$$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

4:

If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Solution:

In ΔABC, let $\vec{CB} = \vec{a}$, $\vec{CA} = \vec{b}$, and $\vec{AB} = \vec{c}$ (as shown in the following figure).



Now, by the triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$.

It is clearly known that $|\vec{a}|$, $|\vec{b}|$ and $|\vec{c}|$ represent the sides of ΔABC.

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$

Hence, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$

5:

Find the value of x for which $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector

Solution:

$x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector if $\left|x\left(\hat{i} + \hat{j} + \hat{k}\right)\right| = 1$.

Now,

$$\left|x\left(\hat{i} + \hat{j} + \hat{k}\right)\right| = 1$$

$$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$$

$$\Rightarrow \sqrt{3x^2} = 1$$

$$\Rightarrow \sqrt{3}x = 1$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

Hence, the required value of x is $\pm\frac{1}{\sqrt{3}}$.

6:

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors

$$\boldsymbol{a} = 2\hat{i} + 3\hat{j} - k \text{ and } \boldsymbol{b} = \hat{i} - 2\hat{j} + \hat{k}$$

Solution:

We have,

Let $\vec{c}$ be the resultant of $\vec{a}, \vec{b} \text{ and } \vec{c}$.

Then,

$$\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{i} + (3+2)\hat{j} + (-1+1)\hat{k} = 3\hat{i} + \hat{j}$$

$$\therefore |\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{\left(3\hat{i} + \hat{j}\right)}{\sqrt{10}}$$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is

$$\pm 5.\hat{c} = \pm 5.\frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{j}\right) = \pm\frac{3\sqrt{10}\hat{i}}{2} \pm \frac{\sqrt{10}}{2}\hat{j}$$

7:

If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$,$\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ *and* $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$,find a unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

Solution:

We have,

$\vec{a}=\hat{i}+\hat{j}+\hat{k}$,$\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ *and* $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$

$2\vec{a}-\vec{b}+3\vec{c}= 2\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(2\hat{i}-\hat{j}+3\hat{k}\right)+3\left(\hat{i}-2\hat{j}+\hat{k}\right)$

$=2\hat{i}+2\hat{j}+2\hat{k}-2\hat{i}+j-3\hat{k}+3\hat{i}-6\hat{j}+3\hat{k}$

$=3\hat{i}-3\hat{j}+2\hat{k}$

$\left|2\vec{a}-\vec{b}+3\vec{c}\right|=\sqrt{3^2+(-3)^2+2^2}=\sqrt{9+9+4}=\sqrt{22}$

Hence, the unit vector along $2\vec{a}-\vec{b}+3\vec{c}$ is

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{\left|2\vec{a}-\vec{b}+3\vec{c}\right|}=\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}=\frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}.$$

8:

Show that the points A $(1,\ -2,\ -8)$, B $(5,\ 0,\ -2)$ and C $(11,\ 3,\ 7)$ **are collinear, and find the ratio in which B divides AC.**

Solution:

The given points are A $(1,\ -2,\ -8)$, B $(5,\ 0,\ -2)$ and C $(11,\ 3,\ 7)$

$\therefore\ \vec{AB}=(5-1)\hat{i}+(0+2)\hat{j}+(-2+8)\hat{k}=4\hat{i}+2\hat{j}+6\hat{k}$

$\vec{BC}=(11-5)\hat{i}+(3-0)\hat{j}+(7+2)\hat{k}=6\hat{i}+3\hat{j}+9\hat{k}$

$\vec{AC}=(11-1)\hat{i}+(3+2)\hat{j}+(7+8)\hat{k}=10\hat{i}+5\hat{j}+15\hat{k}$

$\left|\vec{AB}\right|=\sqrt{4^2+2^2+6^2}=\sqrt{16+4+36}=\sqrt{56}=2\sqrt{14}$

$\left|\vec{BC}\right|=\sqrt{6^2+3^2+9^2}=\sqrt{36+9+81}=\sqrt{126}=3\sqrt{14}$

$\left|\vec{AC}\right|=\sqrt{10^2+5^2+15^2}=\sqrt{100+25+225}=\sqrt{350}=5\sqrt{14}$

$\therefore\ \left|\vec{AC}\right|=\left|\vec{AB}\right|+\left|\vec{BC}\right|$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda:1$. Then, we have:

$$\vec{OB}=\frac{\lambda\ \vec{OC}+\vec{AA}}{(\lambda+1)}$$

$$\Rightarrow 5\hat{i}-2\hat{k}=\frac{\lambda\left(11\hat{i}+3\hat{j}+7\hat{k}\right)+\left(\hat{i}-2\hat{j}-8\hat{k}\right)}{\lambda+1}$$

$$\Rightarrow(\lambda+1)\left(5\hat{i}-2\hat{k}\right)=11\lambda\hat{i}+3\lambda\hat{j}+7\lambda\hat{k}+\hat{i}-2\hat{j}-8\hat{k}$$

$$\Rightarrow 5(\lambda+1)\hat{i}-2(\lambda+1)\hat{k}=(11\lambda+1)\hat{i}+(3\lambda-2)\hat{j}+(7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1)=11\lambda+1$$
$$\Rightarrow 5\lambda+1=11\lambda+1$$
$$\Rightarrow 6\lambda=4$$
$$\Rightarrow \lambda=\frac{4}{6}=\frac{2}{3}$$

Hence, point B divides AC in the ratio $2:3$.

9:

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}\text{-}3\vec{b}\right)$ externally in the ratio $1:2$. Also, show that P is the midpoint of the line segment RQ.

Solution:

It is given that $\overrightarrow{OP}=2\vec{a}+\vec{b},\overrightarrow{OQ}=\vec{a}\text{-}3\vec{b}$.

It is given that point R divides a line segment joining two points P and Q externally in the ratio $1:2$. Then, on using the section formula, we get:

$$\overrightarrow{OR}=\frac{2\left(2\vec{a}+\vec{b}\right)-\left(\vec{a}-3\vec{b}\right)}{2-1}$$
$$=\frac{4\vec{a}+2\vec{b}-\vec{a}+3\vec{b}}{1}$$
$$=3\vec{a}+5\vec{b}$$

Therefore, the position vector of point R is $3\vec{a}+5\vec{b}$

Position vector of the mid-point of RQ = $\dfrac{\overrightarrow{OQ}+\overrightarrow{OR}}{2}$

$$=\frac{\left(\vec{a}\text{-}3\vec{b}\right)+\left(3\vec{a}\text{+}5\vec{b}\right)}{2}$$
$$=2\vec{a}+\vec{b}$$
$$=\overrightarrow{OP}$$

Hence, P is the mid-point of the line segment RQ.

10:

The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$.
Find the unit vector parallel to its diagonal. Also, find its area.

Solution:
Adjacent sides of a parallelogram are given as: $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}-3\hat{k}$

Then, the diagonal of a parallelogram is given by $\vec{a}+\vec{b}$

$\vec{a}+\vec{b}=(2+1)\hat{i}+(-4-2)\hat{j}+(5-3)\hat{k}=3\hat{i}-6\hat{j}+2\hat{k}$

Thus, the unit vector parallel to the diagonal is

$$\frac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}.$$

$\therefore$ Area of parallelogram ABCD = $\left|\vec{a}+\vec{b}\right|$

$$\vec{a}+\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$$

$=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$

$=22\hat{i}+11\hat{j}$

$=11\left(2\hat{i}+\hat{j}\right)$

$\therefore \ \left|\vec{a}+\vec{b}\right|=11\sqrt{2^2+1^2}=11\sqrt{5}$

Hence, the area of the parallelogram is $11\sqrt{5}$ square units

11:
Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$.

Solution:
Let a vector be equally inclined to axes OX, OY, and OZ at angle α.
Then, the direction cosines of the vector are $\cos\alpha$, $\cos\alpha$, and $\cos\alpha$.

Now,

$\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$

$\Rightarrow 3\cos^2\alpha=1$

$\Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{3}}$.

12:

Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$ **and** $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ *and* $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$ **.Find a vector** $\vec{d}$ **which is perpendicular to both** $\vec{a}$ **and** $\vec{b}$ **, and** $\vec{c}.\vec{d}=15$

Solution:

Let $\vec{d}=d_1\hat{i}+d_2\hat{j}+d_3\hat{k}$

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\vec{d}.\vec{a}=0$

$\Rightarrow d_1+4d_2+2d_3=0 \qquad ...(i)$

$\vec{d}.\vec{b}=0$

$\Rightarrow 3d_1-2d_2+7d_3=0 \qquad ...(ii)$

Also, it is given that:

$\vec{c}.\vec{d}=15$

$\Rightarrow 2d_1-d_2+4d_3=15 \qquad ...(iii)$

On solving (i), (ii), and (iii), we get:

$$d_1=\frac{160}{3}, d_2=-\frac{5}{3} \, and \, d_3=-\frac{70}{3}$$

$$\therefore \vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}=\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$

13:

The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

Solution:

$$\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$$

Therefore, unit vector along $\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$ is given as:

$$\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

Scalar product of ($\hat{i}+\hat{j}+\hat{k}$) with this unit vector is 1.

$$\Rightarrow \hat{i}+\hat{j}+\hat{k}.\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow \frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow \sqrt{\lambda^2+4\lambda+44}=\lambda+6$$

$$\Rightarrow \lambda^2+4\lambda+44=(\lambda+6)^2$$

$$\Rightarrow \lambda^2+4\lambda+44=\lambda^2+12\lambda+36$$

$$\Rightarrow 8\lambda=8$$

$$\Rightarrow \lambda=1$$

Hence, the value of λ is 1.

14:

If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$.

Solution:

Since $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, we have

$\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$. It is given that: $|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a},\vec{b}$ and $\vec{c}$ at angles $\theta_1,\theta_2,$ and θ_3 respectively.

Then, we have:

$$\cos\theta_1=\frac{(\vec{a}+\vec{b}+\vec{c}).\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}.\vec{a}+\vec{b}.\vec{a}+\vec{c}.\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$=\frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} \qquad [\vec{b}.\vec{a}=\vec{c}.\vec{a}=0]$$

$$=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_2=\frac{(\vec{a}+\vec{b}+\vec{c}).\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\frac{\vec{a}.\vec{b}+\vec{b}.\vec{b}+\vec{c}.\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}$$

$$=\frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|} \qquad [\vec{a}.\vec{b}=\vec{c}.\vec{b}=0]$$

$$=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\text{os}\,\theta_3=\frac{(\vec{a}+\vec{b}+\vec{c}).\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}=\frac{\vec{a}.\vec{c}+\vec{b}.\vec{c}+\vec{c}.\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$$

$$= \frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|} \qquad [\vec{a}.\vec{c} = \vec{b}.\vec{c} = 0]$$

$$= \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

Now as $|\vec{a}| = |\vec{b}| = |\vec{c}|$, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3$

$\therefore \theta_1 = \theta_2 = \theta_3$

Hence, the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}\, and\, \vec{c}$.

15:

Prove that, $(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$

Solution:

$(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$

$\Leftrightarrow \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2$ $\quad$ [Distributivity of scalar products over addition]

$\Leftrightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ $\quad$ $[\vec{a}.\vec{b} = \vec{b}.\vec{a}$ (Scalar product is commutative)$]$

$\Leftrightarrow 2\vec{a}.\vec{b} = 0$

$\Leftrightarrow \vec{a}.\vec{b} = 0$

$\therefore \vec{a}$ and $\vec{b}$ are prerendicular. $\quad$ $[\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$ (Given)$]$

16:

If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then only when

(A) $0 < \theta < \frac{\pi}{2}$ $\qquad$ (B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$ $\qquad$ (D) $0 \leq \theta \leq \pi$

Solution:

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that

$|\vec{a}|$ and $|\vec{b}|$ be positive

It is known that $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore \vec{a}.\vec{b} \geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \cos\theta \geq 0 \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{are positive}\right]$

$\Rightarrow 0 \leq \theta \leq \dfrac{\pi}{2}$

Hence, $\vec{a}.\vec{b} \geq 0$ when $0 \leq \theta \leq \dfrac{\pi}{2}$

The correct answer is B.

17:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta = \dfrac{\pi}{4}$ (B) $\theta = \dfrac{\pi}{3}$ (C) $\theta = \dfrac{\pi}{2}$ (D) $\theta = \dfrac{2\pi}{3}$

Solution:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ be the angle between them.

Then, $|\vec{a}| = |\vec{b}| = 1$

Now, $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}| = 1$.

$|\vec{a}+\vec{b}| = 1$

$\Rightarrow (\vec{a}+\vec{b})^2 = 1$

$\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 1$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b} = 1$

$\Rightarrow |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 1$

$\Rightarrow 1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 = 1$

$\Rightarrow 1 + 2.1.1\cos\theta + 1 = 1$

$\Rightarrow \cos\theta = -\dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{2\pi}{3}$

Hence, $\vec{a}+\vec{b}$ is a unit vector if $\theta = \dfrac{2\pi}{3}$

The correct answer is D.

18:

The value of $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{i}\times\hat{k}) + \hat{k}.(\hat{i}\times\hat{j})$ **is**

(A) 0 (B) -1 (C) 1 (D) 3

Solution:

$\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$

$=\hat{i}.\hat{i}+\hat{j}.\left(-\hat{j}\right)+\hat{k}.\hat{k}$

$=1-\hat{j}.\hat{j}+1$

$=1-1+1$

$=1$

The correct answer is C.

19:

If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$ then $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$ when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Solution:

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so That $|\vec{a}|$ and $\left|\vec{b}\right|$ are positive.

$\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$

$\Rightarrow|\vec{a}|\left|\vec{b}\right|\cos\theta=|\vec{a}|\left|\vec{b}\right|\sin\theta$

$\Rightarrow\cos\theta=\sin\theta \qquad \left[|\vec{a}| \text{ and} \left|\vec{b}\right| \text{ are positive}\right]$

$\Rightarrow\tan\theta=1$

$\Rightarrow\theta=\frac{\pi}{4}$

Hence, $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$ when θ is equal to $\frac{\pi}{4}$

The correct answer is B.