Exercise 11.1 Page: 467

1:

If a line makes angles $90^o, 135^o, 45^o$ with x, y and z – axes respectively, find its direction cosines.

Solution:

Let direction cosines of the line be l, m, and n.

$l = \cos 90^o = 0$

$m = cos 135^o = -\frac{1}{\sqrt{2}}$

$n = \cos 45^o = \frac{1}{\sqrt{2}}$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}$, and $\frac{1}{\sqrt{2}}$.

2:

Find the direction cosines of a line which makes equal angles with the coordinates axes.

Solution:

Let the direction cosines of the line make an angle α with each of the coordinates axes.

$\therefore\ 1 = \cos\alpha,\ m = \cos\alpha,\ n = \cos\alpha$

$1^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = \frac{1}{3}$

$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$

Thus, the direction cosines of the line, which is equally inclined to the coordinate axes, are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$, and $\pm\frac{1}{\sqrt{3}}$.

3:

If a line has the direction ratios – 18, 12, -4, then what are its direction cosines?

Solution:

If a line has direction ratios – 18, 12, -4, then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$$

i.e., $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$

$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Thus, the direction cosines are $\frac{-9}{11}, \frac{6}{11},$ and $\frac{-2}{11}$.

4:

Show that the points (2, 3, 4), (-1, -2, 1), (5, 8, 7) are collinear.

Solution:

The given points are A (2, 3, 4), B (-1, -2, 1), and C (5, 8, 7).

It is known that the direction ratios of the line joining the points,

(x_1, y_1, z_1) and (x_2, y_2, z_2), are given by $x_2 - x_1$, $y_2 - y_1$, and $z_2 - z_1$.

The direction ratios of AB are (- 1, - 2), (-2, -3), and (1, -4) i.e., -3, -5, and -3.

The direction ratios of BC are (5 – (-1)), (8 – (-2)), and (7 – 1) i.e., 6, 10, and 6.

It can be seen that the direction ratios of BC are -2 times that AB i.e., they are proportional.

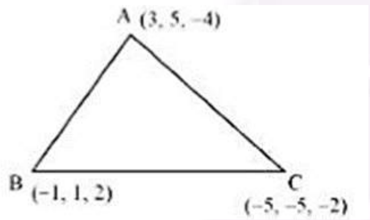
Therefore, AB is parallel to BC. Since point B is common to both AB and BC, points A, B, and C are collinear.

5:

Find the direction cosines of the sides of the triangle whose vertices are (3, 5, -4), (-1, 1, 2) and (-5, -5, -2)

Solution:

The vertices of ΔABC are A (3, 5, -4), B (-1, 1, 2), and C (-5, -5, -2).



The direction ratios of the side AB are (-1 -3), (1 -5), and (2 – (-4)) i.e., -4, -4, and 6.

Then, $\sqrt{(-4)^2 + (-4)^2 + (6)^2} = \sqrt{16 + 16 + 36}$

$$= \sqrt{68}$$

$$= 2\sqrt{17}$$

Therefore, the direction cosines of AB are

$$\frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{6}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}$$

$$\frac{-4}{2\sqrt{17}}, -\frac{4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}$$

$$\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

The direction ratios of BC are (-5 – (-1)), (-5 -1), and (-2 -2) i.e., -4, -6, and -4.
Therefore, the direction cosines of BC are

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

i.e., $\frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$

The direction ratios of CA are (-5 -3), (-5 -5), and (-2 – (-4)) i.e., -8, -10, and 2.
Therefore, the direction cosines of AC are

$$\frac{-8}{\sqrt{(-8)^2+(10)^2+(2)^2}}, \frac{-10}{\sqrt{(-8)^2+(10)^2+(2)^2}}, \frac{2}{\sqrt{(-8)^2+(10)^2+(2)^2}}$$

i.e., $\frac{-8}{2\sqrt{42}}, \frac{-10}{2\sqrt{42}}, \frac{2}{2\sqrt{42}}$