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1:

A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of (i) 5 successes? (ii) at least 5 success? (iii) at most 5 successes?

Solution:

The repeated tosses of a die are Bernoulli trials. Let X denote the number of successes of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is, $p=\frac{3}{6}=\frac{1}{2}$

$\therefore q=1-p=\frac{1}{2}$

X has a binominal distribution.

Therefore, $\mathrm{P}\left(\mathrm{X}=\mathrm{x}\right)={}^{n}C_{n-1}q^{n-x}p^{x}$, where n = 0, 1, 2 ….n

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6-x}.\left(\frac{1}{2}\right)^{x}$

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6}$

(i) P (5 success)

$={}^{6}C_{5}\left(\frac{1}{2}\right)^{6}$

$=6.\frac{1}{64}$

$=\frac{3}{32}$

(ii) P (at least 5 success) $=\mathrm{P}\left(\mathrm{X}\geq 5\right)$

$=\mathrm{P}\left(\mathrm{X}=5\right)+\mathrm{P}\left(\mathrm{X}=6\right)$

$={}^{6}C_{5}\left(\frac{1}{2}\right)^{6}+{}^{6}C_{6}\left(\frac{1}{2}\right)^{6}$

$=6.\frac{1}{64}+6.\frac{1}{64}$

$=\frac{7}{64}$

(iii) P (at most 5 success) $=\mathrm{P}\left(\mathrm{X}\leq 5\right)$

$=1-\mathrm{P}\left(\mathrm{X}>5\right)$

$=1-\mathrm{P}\left(\mathrm{X}=6\right)$

$=1-{}^{6}C_{6}\left(\frac{1}{2}\right)^{6}$

$=1-\frac{1}{64}$

$=\frac{63}{64}$

2:

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two success.

Solution:

The repeated tosses of a pair of dice are Bernoulli trials. Let X denote the number of times of getting doublets in an experiment of throwing two dice simultaneously four times.
Probability of getting doublets in a single throw of the pair of dice is

$$p=\frac{6}{36}=\frac{1}{6}$$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

Clearly, X has the binomial distribution with $\mathrm{n}=4,\ \mathrm{p}=\frac{1}{6},$ and $\mathrm{q}=\frac{5}{6}$

$$\therefore \mathrm{P}(\mathrm{X}=\mathrm{x})={}^{n}C_{x}q^{n-x}p^{x},\ \text{where } \mathrm{x}=0,\ 1,\ 2,\ 3\ldots\mathrm{n}$$

$$={}^{4}C_{x}\left(\frac{5}{6}\right)^{4-x}.\left(\frac{1}{6}\right)^{x}$$

$$={}^{6}C_{x}.\frac{5^{4-x}}{6^{4}}$$

5%

$$\therefore\ \mathrm{P}\,(2\text{ successes})\ =\ \mathrm{P}(\mathrm{X}=2)$$

$$={}^{4}C_{2}.\frac{5^{4-2}}{6^{4}}$$

$$=6.\frac{25}{1296}$$

$$=\frac{25}{216}$$

3:

There are defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution:

Let X denote the number of defective items in a sample of 10 items drawn successively.
Since the drawing is done with replacement, the trials are Bernoulli trials.

$$\Rightarrow \mathrm{p}=\frac{5}{100}=\frac{1}{20}$$

$\mathrm{P}(\mathrm{X}=\mathrm{x})={}^{n}C_{x}q^{n-x}p^{x}$, where x=0, 1, 2...n

$$\therefore q=1-\frac{1}{20}=\frac{19}{20}$$

X has a binomial distribution with $\mathrm{n}=10,\ \mathrm{p}=\frac{1}{20}$

$$={}^{10}C_{x}\left(\frac{19}{20}\right)^{10-x}.\left(\frac{1}{20}\right)^{x}$$

P (not more than 1 defective item) $=\ \mathrm{P}(\mathrm{X}\leq 1)$

$= P(X=0)+P(X=1)$

$= {}^{10}C_0\left(\frac{19}{20}\right)^{10}.\left(\frac{1}{20}\right)^{0} + {}^{10}C_1\left(\frac{19}{20}\right)^{9}.\left(\frac{1}{20}\right)^{1}$

$= \left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^{9}.\left(\frac{1}{20}\right)$

$= \left(\frac{19}{20}\right)^{9}.\left[\frac{19}{20}+\frac{10}{20}\right]$

$= \left(\frac{19}{20}\right)^{9}.\left(\frac{29}{20}\right)$

$= \left(\frac{29}{20}\right).\left(\frac{19}{20}\right)^{9}$

4:
Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that
(i) all the five cards are spades?
(ii) only 3 cards are spades?
(iii) none is a spade?

Solution:
Let X represent the number of spade cards among the five cards drawn. Since the drawing of cards is with replacement. The trials are Bernoulli trials.
In a well shuffled deck of 52 cards, there are 13 spades cards.

$\Rightarrow p = \frac{13}{52} = \frac{1}{4}$

$\therefore\ q = 1 - \frac{1}{4} = \frac{3}{4}$

X has a binomial distribution with $n = 5$ *and* $p = \frac{1}{4}$

$P(X=x) = {}^{n}C_x q^{n-x} p^{x}$, where x=0, 1, 2...n

$= {}^{5}C_x\left(\frac{3}{4}\right)^{5-x}.\left(\frac{1}{4}\right)^{x}$

(i)P(all five cards are spades) $= P\ (X=5)$

$= {}^{5}C_5\left(\frac{3}{4}\right)^{0}.\left(\frac{1}{4}\right)^{5}$

$= 1.\frac{1}{1024}$

$= \frac{1}{1024}$

(ii) P (only 3 cards are spades) $=\text{P}\ (\text{X}=3)$

$$= {}^5C_3\left(\frac{3}{4}\right)^2.\left(\frac{1}{4}\right)^3$$

$$=10.\frac{9}{6}.\frac{1}{64}$$

$$=\frac{45}{512}$$

(iii) P (none is a spade) $=\text{P}\ (\text{X}=0)$

$$= {}^5C_0.\left(\frac{3}{4}\right)^5.\left(\frac{1}{4}\right)^0$$

$$=1.\frac{243}{1024}$$

$$=\frac{243}{1024}$$

5:
The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. What is the probability that out of 5 such bulbs
(i) none
(ii) not more than one
(iii) more than one
(iv) at least one
Will fuse after 150 days of use.

Solution:
Let X represents the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.
It is given that, $\text{p}=0.05$

$$\therefore \text{q}=1-\text{p}=1-0.05=0.95$$

X has a binominal distribution with $\text{n}=5$ and $\text{p}=0.05$

$\therefore$ P(X=x)=${}^nC_x q^{n-x}p^x$, where x=0, 1, 2...n

$$= {}^5C_x\,(0.95)^{5-x}.(0.05)^x$$

(i) $\text{P}(\text{none})=\text{P}(\text{X}=0)$

$$= {}^5C_0\,(0.95)^5.(0.05)^0$$

$$=1\times(0.95)^5$$

$=(0.95)^5$

(ii) P (not more than one)

$= \mathrm{P}(\mathrm{X}=0)+\mathrm{P}(\mathrm{X}=1)$

$= {}^5C_0 \times (0.95)^5.(0.05)^0 + {}^5C_1(0.95)^4.(0.05)^1$

$=1\times(0.95)^5+5\times(0.95)^4.(0.05)$ $= \mathrm{P}(\mathrm{X}\le 1)$

$=(0.95)^5+(0.25)(0.95)^4$

$=(0.95)^4+[0.95+0.25]$

$=(0.95)^4\times 1.2$

(iii) P (more than 1)

$=1-\mathrm{P}(\mathrm{X}\le 1)$

$=1-\mathrm{P}$ (not more than 1)

$=1-(0.95)^4\times 1.2\backslash$

$= \mathrm{P}(\mathrm{X}\le 1)$

(iv) P (at least one) $=\mathrm{P}(\mathrm{X}\ge 1)$

$=1-\mathrm{P}(\mathrm{X}<1)$

$=1-\mathrm{P}(\mathrm{X}=0)$

$=1-{}^5C_0(0.95)^5\times(0.05)^0$

$=1-1\times(0.95)^5$

$=1-(0.95)^5$

6:

A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Solution:

Let X denote the number of balls marked with the digit 0 among the 4 balls drawn.
Since the balls are drawn with replacement, the trials are Bernoulli trials.

X has a binominal distribution with $\mathrm{n}=4$ and $p=\frac{1}{10}$

$\therefore q=1-p=1-\frac{1}{10}=\frac{9}{10}$

$\therefore$ P(X=x)=${}^nC_x q^{n-x}.p^x$, x=1, 2...n

$= {}^4C_x\left(\frac{9}{10}\right)^{4-x}.\left(\frac{1}{10}\right)^{x}$

P (none marked with 0) $= \mathrm{P}(\mathrm{X}=0)$

$= {}^4C_0\left(\frac{9}{10}\right)^{4}.\left(\frac{1}{10}\right)^{0}$

$= 1.\left(\frac{9}{10}\right)^{4}$

$= \left(\frac{9}{10}\right)^{4}$

7:

In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tail, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Solution:

Let X represent the number of correctly answered questions out of 20 questions.

The repeated tosses of a coin are Bernoulli trials. Since "head" on a coin represents the true answer and "tail" represents the false answer, the correctly answered questions are Bernoulli trials.

$\therefore\ p = \frac{1}{2}$

$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

X has a binominal distribution with $\mathrm{n}=20$ and $p = \frac{1}{2}$

$\therefore$ P(X=x)=${}^nC_x q^{n-x}.p^x$, *where* x=0,1, 2...n

$= {}^{20}C_x\left(\frac{9}{10}\right)^{20-x}.\left(\frac{1}{2}\right)^{x}$

$= {}^{20}C_x\left(\frac{1}{2}\right)^{20}$

P (at least 12 questions answered correctly) $= \mathrm{P}(\mathrm{X}\geq 12)$

$= \mathrm{P}(\mathrm{X}=12)+\mathrm{P}(\mathrm{X}=13)+\ldots+\mathrm{P}(\mathrm{X}=20)$

$= {}^{20}C_{12}\left(\frac{1}{2}\right)^{20} + {}^{20}C_{13}\left(\frac{1}{2}\right)^{20} + \ldots + {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}$

$= \left(\frac{1}{2}\right)^{20}.\left[{}^{20}C_{12} + {}^{20}C_{13} + \ldots + {}^{20}C_{20}\right]$

8:

Suppose X has a binominal distribution B$\left(6,\frac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

(Hint: $P(X=3)$ is the maximum among all $P(x_i)$, $x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution:

X is the random variable whose binomial distribution is B$\left(6,\frac{1}{2}\right)$.

Therefore, $n=6$ and $p=\frac{1}{2}$

$\therefore q=1-p=1-\frac{1}{2}=\frac{1}{2}$

Then, $P(X=x)={}^{n}C_{x}q^{n-x}p^{x}$

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6-x}\cdot\left(\frac{1}{2}\right)^{x}$

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6}$

It can be seen that P (X = x) will be maximum, if ${}^{6}C_{x}$ will be maximum.

Then, ${}^{6}C_{0}={}^{6}C_{6}=\frac{6!}{0!.6!}=1$

${}^{6}C_{1}={}^{6}C_{5}=\frac{6!}{1!.5!}=6$

${}^{6}C_{2}={}^{6}C_{4}=\frac{6!}{2!.4!}=15$

${}^{6}C_{3}=\frac{6!}{3!.3!}=20$

The value of ${}^{6}C_{3}$ is maximum. Therefore, for $x=3$, $P(X=x)$ is maximum.

Thus, $X=3$ is the most likely outcome.

9:

On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution:

The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p=\frac{1}{3}$

$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{3}$

$\therefore P(X=x) = {}^{n}C_{x} q^{n-x} p^{x}$

$= {}^{5}C_{x} \left(\frac{2}{3}\right)^{5-x} . \left(\frac{1}{3}\right)^{x}$

P (guessing more than 4 correct answers) $= P(X \geq 4)$

$= P(X=4) + P(X=5)$

$= {}^{5}C_{4} \left(\frac{2}{3}\right) . \left(\frac{1}{3}\right)^{4} + {}^{5}C_{5} \left(\frac{1}{3}\right)^{5}$

$= 5 . \frac{2}{3} . \frac{1}{81} + 1 . \frac{1}{243}$

$= \frac{10}{243} + \frac{1}{243}$

$= \frac{11}{243}$

10:

A Person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will in a prize (a) at least once (B) exactly once (C) at least twice?

Solution:

Let X represents the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binominal distribution with $n = 50$ and $p = \frac{1}{100}$

$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$

$\therefore P(X=x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{50}C_{x} \left(\frac{99}{100}\right)^{50-x} . \left(\frac{1}{100}\right)^{x}$

(a) P (winning at least once) $= P(X \geq 1)$

$= 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - {}^{50}C_{0} \left(\frac{99}{100}\right)^{50}$

$= 1 - 1 . \left(\frac{99}{100}\right)^{50}$

$$=1-\left(\frac{99}{100}\right)^{50}$$

(b) P (Winning exactly once) $= P(X=1)$

$$= {}^{50}C_1\left(\frac{99}{100}\right)^{49}.\left(\frac{1}{100}\right)^{1}$$

$$=50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$$

$$=\frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P (at least twice) $= P(X\geq 2)$

$$=1-P(X<2)$$

$$=1-P(X\leq 1)$$

$$=1-\left[P(X=0)+P(X=1)\right]$$

$$=\left[1-P(X=0)\right]-P(X=1)$$

$$=1-\left(\frac{99}{100}\right)^{50}-\frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

$$=1-\left(\frac{99}{100}\right)^{49}.\left[\frac{99}{100}+\frac{1}{2}\right]$$

$$=1-\left(\frac{99}{100}\right)^{49}.\left(\frac{149}{100}\right)$$

$$=1-\left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$$

11:
Find the probability of getting 5 exactly twice in 7 throws of a die.

Solution:
The repeated tossing of a die are Bernoulli trials. Let X represent the number of times of getting 5 in 7 throws of the die.

Probability of getting 5 in a single throw of the die, $p=\frac{1}{6}$

Clearly, X has the probability distribution with $n=7$ and $p=\frac{1}{6}$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

$\therefore P(X=x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{7}C_{x} \left(\frac{5}{6}\right)^{7-x} \cdot \left(\frac{1}{6}\right)^{x}$

P (getting 5 exactly twice) $= P(X=2)$

$= {}^{7}C_{2} \left(\frac{5}{6}\right)^{5} \cdot \left(\frac{1}{6}\right)^{2}$

$= 21\left(\frac{5}{6}\right)^{5} \cdot \frac{1}{36}$

$= = \left(\frac{7}{12}\right)\left(\frac{5}{6}\right)^{5}$

12:

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Solution:

The repeated tossing of the die are Bernoulli trials. Let X represent the number of times of getting sixes in 6 throws of the die.

Probability of getting six in a single throw of die, $p = \frac{1}{6}$

Clearly, X has a binomial distribution with n = 6.

$\therefore P(X=x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{6}C_{x} \left(\frac{5}{6}\right)^{6-x} \cdot \left(\frac{1}{6}\right)^{x}$

P (at most 2 sixes) $= P(X \le 2)$

$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

$= P(X=0) + P(X=1) + P(X=2)$

$= {}^{6}C_{0} \left(\frac{5}{6}\right)^{6} + {}^{6}C_{1} \cdot \left(\frac{5}{6}\right)^{5} \cdot \left(\frac{1}{6}\right) + {}^{6}C_{2} \left(\frac{5^{4}}{6}\right) \cdot \left(\frac{1}{6}\right)^{2}$

$= 1 \cdot \left(\frac{5}{6}\right)^{6} + 6 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{5} + 15 \cdot \frac{1}{36}\left(\frac{5}{6}\right)^{4}$

$= \left(\frac{5}{6}\right)^{6} + \left(\frac{5}{6}\right)^{5} + \frac{5}{12} \cdot \left(\frac{5}{6}\right)^{4}$

$= \left(\frac{5}{6}\right)^{4} \left[\left(\frac{5}{6}\right)^{2} + \left(\frac{5}{6}\right) + \left(\frac{5}{12}\right)\right]$

$= \left(\frac{5}{6}\right)^{4} \cdot \left[\frac{25}{36} + \frac{5}{6} + \frac{5}{12}\right]$

$= \left(\frac{5}{6}\right)^{4} \cdot \left[\frac{25+30+15}{36}\right]$

$$= \frac{70}{36}\left(\frac{5}{6}\right)^4$$
$$= \frac{35}{18}\left(\frac{5}{6}\right)^4$$

13:
It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

Solution:
The repeated selections of articles in a random sample space are Bernoulli trials. Let X denote the number of times of selecting defective articles in a random sample space of 12 articles.

Clearly, X has a binominal distribution with $n = 12$ *and* $p = 10\% = \frac{10}{100} = \frac{1}{10}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\therefore \text{P(X=x)} = {}^{n}\text{C}_{x} q^{n-x} p^{x} = {}^{12}\text{C}_{x}\left(\frac{9}{10}\right)^{12-x} . \left(\frac{1}{10}\right)^{x}$$

P (Selecting 9 defective articles) $= {}^{12}\text{C}_{9}\left(\frac{9}{10}\right)^{3} . \left(\frac{1}{10}\right)^{9}$

$$= 220. \frac{9^3}{10^3} . \frac{1}{10^9}$$
$$= \frac{22 \times 9^3}{10^{11}}$$

14:
In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(A) 10^{-1}

(B) $\left(\frac{1}{2}\right)^5$

(C) $\left(\frac{9}{10}\right)^5$

(D) $\frac{9}{10}$

Solution:
The repeated sections of defective bulbs from a box are Bernoulli trials. Let X denote the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb, $p = \frac{10}{100} = \frac{1}{10}$

$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

Clearly, X has binomial distribution with $n = 5$ and $p = \frac{1}{10}$

$\therefore P(X=x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{5}C_{x} \left(\frac{9}{10}\right)^{5-x} \cdot \left(\frac{1}{10}\right)^{x}$

P (none of the bulbs is defective) $= P(X=0)$

$= {}^{5}C_{0} \cdot \left(\frac{9}{10}\right)^{5}$

$= 1 \cdot \left(\frac{9}{10}\right)^{5}$

$= \left(\frac{9}{10}\right)^{5}$

The correct answer is C.

15:

The probability that a student is not a swimming is $\frac{1}{5}$. The probability that out of five students, four are swimmers is

(A) ${}^{5}C_{4} \left(\frac{4}{5}\right)^{4} \frac{1}{5}$ (B) $\left(\frac{4}{5}\right)^{4} \frac{1}{5}$

(C) ${}^{5}C_{1} \frac{1}{5} \left(\frac{4}{5}\right)^{4}$ (D) None of these

Solution:

The repeated selection of students who are swimmers are Bernoulli trials. Let X denote the number of students, out of 5 students, who are swimmers.

Probability of students who are not swimmers, $q = \frac{1}{5}$

$\therefore p = 1 - q = 1 - \frac{1}{5} = \frac{4}{5}$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{4}{5}$

$\therefore P(X=x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{5}C_{x} \cdot \left(\frac{1}{5}\right)^{5-x} \cdot \left(\frac{4}{5}\right)^{x}$

P (four students are swimmers) $= P(X=4) = {}^{5}C_{4} \cdot \left(\frac{1}{5}\right) \cdot \left(\frac{4}{5}\right)^{4}$

Therefore, the correct answer is A.