Exercise 7.ρ1 Page: 3τ7

1:

$$\int_0^{\frac{\pi}{2}} \cos^2 x dx$$

Solution:

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2 \left(\frac{\pi}{2} - x\right) dx \qquad \left(\int_0^0 f(x) dx = \int_0^0 f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1.dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

2:

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

Solution:

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

Let $I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \qquad ...(1)$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx \qquad \left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad .... (2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

3:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} xdx}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}\,dx$$

Solution:

Let $I=\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} xdx}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}\,dx \qquad ...(1)$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1.dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

4:

$$\int_0^{\frac{\pi}{2}}\frac{\cos^5 xdx}{\sin^5 x+\cos^5 x}dx$$

Solution:

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\cos^5 xdx}{\sin^5 x+\cos^5 x}dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^5 x+\cos^5 x}{\sin^5 x+\cos^5 x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1.dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

5:

$$\int_{-5}^{5} |x+2| dx$$

Solution:

Let $I = \int_{-5}^{5} |x+2| dx$

It can be seen that (x + 2) ≤ 0 on [−5, −2] and (x + 2) ≥ 0 on [−2, 5].

$$\therefore I = \int_{-5}^{-2} -(x+2)\, dx + \int_{-2}^{5} (x+2)\, dx$$

$$I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$$

$$= -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$$

$$= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4$$

$$= 29$$

6:

$$\int_{2}^{8} |x-5| dx$$

Solution:

Let $I = \int_{2}^{8} |x-5| dx$

It can be seen that (x − 5) ≤ 0 on [2, 5] and (x − 5) ≥ 0 on [5, 8].

$$I = \int_{2}^{5} -(x-5)\, dx + \int_{2}^{8} (x-5)\, dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= -\left[\frac{x^2}{2} - 5x\right]_{2}^{5} + \left[\frac{x^2}{2} - 5x\right]_{5}^{8}$$

$$= -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right]$$

$$= 9$$

7:

$$\int_0^1 x(1-x)^n\,dx$$

Solution:

Let $I=\int_0^1 x(1-x)^n\,dx$

$\therefore I=\int_0^1 (1-x)\left(1-(1-x)\right)^n dx \qquad \left(\int_1^a f(x)dx=\int_0^a f(a-x)dx\right)$

$=\int_0^1 (1-x)(x)^n\,dx$

$=\int_0^1 \left(x^n-x^{n+1}\right)dx$

$=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$

$=\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$

$=\frac{(n+2)-(n+1)}{(n+1)(n+2)}$

$=\frac{1}{(n+1)(n+2)}$

8:

$$\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$$

Solution:

Let $I=\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx \qquad \ldots(1)$

$\therefore I=\int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan x}\right\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log 2dx-\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - I \qquad [From(1)]$$

$$\Rightarrow 2I = [x \log 2]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I = \frac{\pi}{4} \log 2$$

$$\Rightarrow I = \frac{\pi}{8} \log 2$$

9:

$$\int_0^2 x\sqrt{2-x}dx$$

Solution 9:

Let $I = \int_0^2 x\sqrt{2-x}dx$

$$I = \int_0^2 (2-x)\sqrt{x}dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx$$

$$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$

$$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4 \times 2\sqrt{2}}{3} - \frac{2}{5} \times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

10:

$$\int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x)dx$$

Solution:

Let $I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log(2\sin x\cos x)\}\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log\sin x-\log\cos x-\log 2\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\sin x-\log\cos x-\log 2\}\,dx \qquad ...(1)$

It is known that, $\left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\cos x-\log\sin x-\log 2\}\,dx \qquad ...(2)$

Adding (1) and (2), we obtain

$2I=\int_0^{\frac{\pi}{2}}(-\log 2-\log 2)\,dx$

$\Rightarrow 2I=-2\log 2\int_0^{\frac{\pi}{2}}1.dx$

$\Rightarrow I=-\log 2\left[\frac{\pi}{2}\right]$

$\Rightarrow I=\frac{\pi}{2}(-\log 2)$

$\Rightarrow I=\frac{\pi}{2}\left[\log\frac{1}{2}\right]$

$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$

11:

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^2 x dx$

Solution:

Let $I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^2 x dx$

As $\sin^2(-x)=(\sin(-x))^2=(-\sin x)^2=\sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)\,dx=2\int_0^a f(x)\,dx$

$I=2\int_0^{\frac{\pi}{2}}\sin^2 x dx$

$=2\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}\,dx$

$$= \int_0^{\frac{\pi}{2}} (1 - \cos 2x)\, dx$$

$$= \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

12:

$$\int_0^{\pi} \frac{x\,dx}{1 + \sin x}$$

Solution:

Let $I = \int_0^{\pi} \frac{x\,dx}{1 + \sin x}$ ...(1)

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin(\pi - x)} dx \qquad \left(\int_0^{\pi} f(x)\,dx = \int_0^{\pi} f(a - x)\,dx \right)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin x} dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \left\{ \sec^2 x - \tan x \sec x \right\} dx$$

$$\Rightarrow 2I = \pi [2]$$

$$\Rightarrow I = \pi$$

13:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$$

Solution:

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$ ...(1)

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if f(x) is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0$$

14:

$$\int_0^{2\pi} \cos^5 x dx$$

Solution:

Let $I = \int_0^{2\pi} \cos^5 x dx \qquad ...(1)$

$\cos^5(2\pi - x) = \cos^5 x$

It is known that,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)$$
$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2\int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \qquad [\cos^5(\pi - x) = -\cos^5 x]$$

15:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \qquad ...(1)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

16:

$$\int_0^{\pi} \log(1 + \cos x) dx$$

Solution:

Let $I=\int_0^{\pi}\log\left(1+\cos x\right)dx \qquad ...(1)$

$\Rightarrow I=\int_0^{\pi}\log\left(1+\cos\left(\pi-x\right)\right)dx \qquad \left(\int_0^a f\left(x\right)dx=\int_0^a f\left(a-x\right)dx\right)$

$\Rightarrow I=\int_0^{\pi}\log\left(1-\cos x\right)dx \qquad ...(2)$

Adding (1) and (2), we obtain

$2I=\int_0^{\pi}\left\{\log\left(1-\cos x\right)+\log\left(1-\cos x\right)\right\}dx$

$\Rightarrow 2I=\int_0^{\pi}\log\left(1-\cos^2 x\right)dx$

$\Rightarrow 2I=\int_0^{\pi}\log\sin^2 xdx$

$\Rightarrow 2I=2\int_0^{\pi}\log\sin xdx$

$\Rightarrow I=\int_0^{\pi}\log\sin xdx \qquad ...(3)$

sin (π − x) = sin x

$\therefore I=2\int_0^{\frac{\pi}{2}}\log\sin x\,dx \qquad ...(4)$

$\Rightarrow I=2\int_0^{\frac{\pi}{2}}\log\sin\left(\frac{\pi}{2}-x\right)dx=2\int_0^{\frac{\pi}{2}}\log\cos x\,dx \qquad ...(5)$

Adding (4) and (5), we obtain

$2I=2\int_0^{\frac{\pi}{2}}(\log\sin x+\log\cos x)\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\left(\log\sin x+\log\cos x+\log 2-\log 2\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\left(\log 2\sin x\cos x-\log 2\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\sin 2xdx-\int_0^{\frac{\pi}{2}}\log 2dx$

Let 2x = t ⇒ 2dx = dt

When x = 0, t = 0 and when x = π/2, t = π

$\therefore I=\frac{1}{2}\int_0^{\pi}\log\sin t\,dt-\frac{\pi}{2}\log 2$

$\Rightarrow I=\frac{I}{2}-\frac{\pi}{2}\log 2$

$\Rightarrow \frac{I}{2}=-\frac{\pi}{2}\log 2$

$\Rightarrow I=-\pi\log 2$

17:

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

Solution:

Let $I=\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(1)

It is known that, $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$I=\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(2)

Adding (1) and (2), we obtain

$$2I=\int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I=\int_0^a 1.dx$$

$$\Rightarrow 2I=[x]_0^a$$

$$\Rightarrow 2I=a$$

$$\Rightarrow I=\frac{a}{2}$$

18:

$$\int_0^4 |x-1|dx$$

Solution:

$$I=\int_0^4 |x-1|dx$$

It can be seen that, $(x-1)\le 0$ when $0\le x\le 1$ and $(x-1)\ge 0$ when $1\le x\le 4$

$I=\int_0^1 |x-1|dx+\int_1^4 |x-1|dx$ $\left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$

$$I=\int_0^1 -(x-1)dx+\int_0^4 (x-1)dx$$

$$=\left[x-\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{2}-x\right]_1^4$$

$$=1-\frac{1}{2}+\frac{(4)^2}{2}-4-\frac{1}{2}+1$$

$$=1-\frac{1}{2}+8-4-\frac{1}{2}+1$$

$$=5$$

19:

Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$, if *f* and *g* are defined as f(x) = f(a – x) and g (x) + g (a – x) = 4

Solution:

Let $\int_0^a f(x)g(x)dx \qquad ...(1)$

$\Rightarrow \int_0^a f(a-x)g(a-x)dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow \int_0^a f(x)g(a-x)dx \qquad ...(2)$

Adding (1) and (2), we obtain

$2I = \int_0^a \{f(x)g(x) + f(x)g(a-x)\}dx$

$\Rightarrow 2I = \int_0^a f(x)\{g(x)+g(a-x)\}dx$

$\Rightarrow 2I = \int_0^a f(x)\times 4dx \qquad [g(x)+g(a-x)=4]$

$\Rightarrow I = 2\int_0^a f(x)dx$

Chose the correct answer in Exercises 20 and 21.

20:

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3 + x\cos x + \tan^5 x + 1\right)dx$ is

A. 0
B. 2
C. π
D. 1

Solution:

Let $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3 + x\cos x + \tan^5 x + 1\right)dx$

$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos xdx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 xdx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1.dx$

It is known that if f (x) is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

if f (x) is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

and $I = 0+0+0+2\int_0^{\frac{\pi}{2}} 1dx$

$= 2[x]_0^{\frac{\pi}{2}}$

$= \frac{2\pi}{2}$

$= \pi$

Hence, the correct Answer is C.

21:

The value of $\int_0^{\frac{\pi}{2}} \left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is

A. 2

B. $\frac{3}{4}$

C. 0

D. –2

Solution:

Let I = $\int_0^{\frac{\pi}{2}} \left(\frac{4+3\sin x}{4+3\cos x}\right) dx \qquad \ldots(1)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right] dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left\{\log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right\} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \left(\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1\, dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0\, dx$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is C.