Exercise 7.3

Find the integrals of the functions in Exercises 1 to 22:

1.

$\sin^2(2x + 5)$

Solution:

The given function can be rewritten as

$$\sin^2(2x+5)=\frac{1-\cos 2(2x+5)}{2}=\frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx=\int\frac{1-\cos(4x+10)}{2}dx$$

$$=\frac{1}{2}\int 1dx-\frac{1}{2}\int\cos(4x+10)\,dx$$

$$=\frac{1}{2}x-\frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right)+C`$$

$$=\frac{1}{2}x-\frac{1}{8}\sin(4x+10)+C$$

2.

sin3x.cos4x

Solution:

It is known that, $\sin A\cos B=\frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$

$$\therefore \int \sin 3x\cos 4x dx=\frac{1}{2}\int\{\sin(3x+4x)+\sin(3x-4x)\}dx$$

$$=\frac{1}{2}\int\{\sin 7x+\sin(-x)\}dx$$

$$=\frac{1}{2}\int\{\sin 7x-\sin x\}dx$$

$$=\frac{1}{2}\int\sin 7x dx-\frac{1}{2}\int\sin x dx$$

$$=\frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+C$$

$$=\frac{-\cos 7x}{14}+\frac{\cos x}{2}+C$$

where C is an arbitrary constant.

3.

cos 2x cos 4x cos 6x

Solution:

It is known that, $\cos A\cos B=\frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

$$\therefore \int\cos 2x(\cos 4x\cos 6x)\,dx=\int\cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos 2x\cos(-2x)\}\,dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos^2 2x\}\,dx$$

$$=\frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$=\frac{1}{4}\int(\cos 12x+\cos 8x+1+\cos 4x)\,dx$$

$$=\frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}+C\right]$$

where C is an arbitrary constant.

4.

Integrate $\sin^3(2x+1)$

Solution:

Let $I=\int\sin^3(2x+1)dx$

$$\Rightarrow\int\sin^3(2x+1)dx=\int\sin^2(2x+1).\sin(2x+1)\,dx$$

$$=\int\left(1-\cos^2(2x+1)\right)\sin(2x+1)\,dx$$

Let $\cos(2x+1)=t$

$$\Rightarrow -2\sin(2x+1)\,dx=dt$$

$$\Rightarrow \sin(2x+1)\,dx=\frac{-dt}{2}$$

$$\Rightarrow I=\frac{-1}{2}\int(1-t^2)\,dt$$

$$=\frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}+C$$

$$=\frac{-1}{2}\left\{\cos(2x+1)-\frac{\cos^3(2x+1)}{3}\right\}+C$$

$$=\frac{-\cos(2x+1)}{2}+\frac{\cos^3(2x+1)}{6}+C$$

where C is an arbitrary constant.

5.

Integrate $\sin^3 x \cos^3 x$

Solution:

Let $I = \int \sin^3 x \cos^3 x.dx$

$= \int \cos^3 x.\sin^2 x.\sin x.dx$

$= \int \cos^3 x\left(1-\cos^2 x\right)\sin x.dx$

Let cosx = t

$\Rightarrow$ -sinx.dx $= dt$

$\Rightarrow I = -\int t^3\left(1-t^2\right)dt$

$= -\int\left(t^3 - t^5\right)dt$

$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$

$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$

$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$

where C is an arbitrary constant.

6.

Integrate $\sin x \sin 2x \sin 3x$

Solution:

It is known that, $\sin A \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$

$\therefore \int \sin x \sin 2x \sin 3x dx = \int\left[\sin x.\frac{1}{2}\{\cos(2x-3x) - \cos(2x+3x)\}\right]dx$

$= \frac{1}{2}\int\left(\sin x \cos(-x) - \sin x \cos 5x\right)dx$

$= \frac{1}{2}\int\left(\sin \cos x - \sin x \cos 5x\right)dx$

$= \frac{1}{2}\int \frac{\sin 2x}{2}dx - \frac{1}{2}\int \sin x \cos 5x dx$

$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int \frac{1}{2}\{\sin(x+5x) + \sin(x-5x)\}dx$

$= \frac{-\cos 2x}{8} - \frac{1}{4}\int\left(\sin 6x + \sin(-4x)\right)dx$

$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{6} + \frac{\cos 4x}{4}\right] + C$$

$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$

$$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$$

where C is an arbitrary constant.

7.

Integrate $\sin 4x \sin 8x$

Solution:

It is known that, $\sin A \sin B = \frac{1}{2}\cos(A-B) - \cos(A+B)$

$$\therefore \int \sin 4x \sin 8x dx = \int\left\{\frac{1}{2}\cos(4x-8x) - \cos(4x+8x)\right\}dx$$

$$= \frac{1}{2}\int(\cos(-4x) - \cos 12x)dx$$

$$= \frac{1}{2}\int(\cos 4x - \cos 12x)dx$$

$$= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right] + \text{C}$$

where C is an arbitrary constant.

8.

Integrate $\frac{1-\cos x}{1+\cos x}$

Solution:

Consider,

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \qquad \left[2\sin^2\frac{x}{2} = 1-\cos x \text{ and } 2\cos^2\frac{x}{2} = 1+\cos x\right]$$

$$= \tan^2 \frac{x}{2}$$

$$= \left(\sec^2 \frac{x}{2} - 1\right)$$

$$\therefore \int \frac{1-\cos x}{1+\cos x} dx = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C$$

$$= 2\tan \frac{x}{2} - x + C$$

where C is an arbitrary constant.

9.

Integrate $\frac{\cos x}{1+\cos x}$

Solution:

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \qquad \left[\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \text{ and } \cos x = 2\cos^2 \frac{x}{2} - 1 \right]$$

$$= \frac{1}{2}\left[1 - \tan^2 \frac{x}{2} \right]$$

$$\therefore \int \frac{\cos x}{1+\cos x} dx = \frac{1}{2} \int \left(1 - \tan^2 \frac{x}{2} \right) dx$$

$$= \frac{1}{2} \int \left(1 - \sec^2 \frac{x}{2} + 1 \right) dx$$

$$= \frac{1}{2} \int \left(2 - \sec^2 \frac{x}{2} \right) dx$$

$$= \frac{1}{2} \left[2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}} \right] + C$$

$$= x - \tan \frac{x}{2} + C$$

where C is an arbitrary constant.

10.

Integrate $\sin^4 x$

Solution:

Consider $\sin^4 x = \sin^2 x \sin^2 x$

$$=\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$=\frac{1}{4}(1-\cos 2x)^2$$

$$=\frac{1}{4}\left[1+\cos^2 2x-2\cos 2x\right]$$

$$=\frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right)-2\cos 2x\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$=\frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$\therefore \int \sin^4 x dx=\frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]dx$$

$$=\frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right)-\frac{2\sin 2x}{2}\right]+C$$

$$=\frac{1}{8}\left[3x+\frac{\sin 4x}{4}-2\sin 2x\right]+C$$

$$=\frac{3x}{8}-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$$

where C is an arbitrary constant.

11.

Integrate $\cos^4 2x$

Solution:

$$\cos^4 2x=\left(\cos^2 2x\right)^2$$

$$=\left(\frac{1+\cos 4x}{2}\right)^2$$

$$=\frac{1}{4}\left[1+\cos^2 4x+2\cos 4x\right]$$

$$=\frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$

$$=\frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$

$$\therefore \int \cos^4 2x dx=\int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$

$$=\frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$

where C is an arbitrary constant.

12.

Integrate $\frac{\sin^2 x}{1+\cos x}$

Solution:

$$\frac{\sin^2 x}{1+\cos x}=\frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}} \qquad \left[\sin x=2\sin\frac{x}{2}\cos\frac{x}{2};\cos x=2\cos^2\frac{x}{2}-1\right]$$

$$=\frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$=2\sin^2\frac{x}{2}$$

$$=1-\cos x$$

$$\therefore \int\frac{\sin^2 x}{1+\cos x}dx=\int(1-\cos x)dx$$

$$=x-\sin x+C$$

where C is an arbitrary constant.

13.

Integrate $\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$

Solution:

Consider,

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2a}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \qquad \left[\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$= 2\left[\cos(x)+\cos\alpha\right]$$

$$= 2\cos x + 2\cos\alpha$$

$$\therefore \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx = \int 2\cos x + 2\cos\alpha$$

$$= 2[\sin x + x\cos\alpha] + C$$

where C is an arbitrary constant.

14.

Integrate $\frac{\cos x - \sin x}{1+\sin 2x}$

Solution:

$$\frac{\cos x - \sin x}{1+\sin 2x} = \frac{\cos x - \sin x}{\left(\sin^2 x + \cos^2 x\right) + 2\sin x\cos x}$$

$\left[\sin^2 x + \cos^2 x = 1; \sin 2x = 2\sin x\cos x\right]$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Let $\sin x + \cos x = t$

$$\therefore (\cos x - \sin x)dx = dt$$

$$\Rightarrow \int \frac{\cos x - \sin x}{1+\sin 2x}dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2}dx$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2}dt$$

$$= -t^{-1} + C$$

$$= -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$

where C is an arbitrary constant.

15.

Integrate $\tan^3 2x \sec 2x$

Solution:

$$\tan^3 2x\sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right)\tan 2x \sec 2x$$
$$= \sec^2 2x.\tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x\, dx = \int \sec^2 2x.\tan 2x \sec 2x\, dx - \int \tan 2x \sec 2x dx$$
$$= \int \sec^2 2x.\tan 2x \sec 2x\, dx - \frac{\sec 2x}{2} + C$$

$Let \sec 2x = t$

$\therefore 2\sec 2x \tan 2x dx = dt$

$$\therefore \int \tan^3 2x \sec 2x dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{\sec^3 2x}{6} - \frac{\sec 2x}{2} + C$$

where C is an arbitrary constant.

16.

Integrate $\tan^4 x$

Solution:

$\tan^4 x$

$= \tan^2 x.\tan^2 x$

$= \left(\sec^2 x - 1\right)\tan^2 x$

$= \sec^2 x \tan^2 x - \tan^2 x$

$= \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$

$= \sec^2 x \tan^2 x - \sec^2 x + 1$

$$\therefore \int \tan^4 x dx = \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx + \int 1 dx$$
$$= \int \sec^2 x \tan^2 x dx - \tan x + x + C \qquad ...(1)$$

Consider $\int \sec^2 x \tan^2 x dx$

$Let \tan x = 1 \Rightarrow \sec^2 x dx = dt$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

where C is an arbitrary constant.

17.

Integrate $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Solution:

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \tan x \sec x + \cot x \cos ecx$$

$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \cos ecx) dx$$

$$= \sec x - \text{cosec} x + C$$

where C is an arbitrary constant.

18.

Integrate $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Solution:

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad [\cos 2x = 1 - 2\sin^2 x]$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + C$$

where C is an arbitrary constant.

19.

Integrate $\dfrac{1}{\sin x \cos^3 x}$

Solution:

$$\frac{1}{\sin x\cos^3 x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}$$
$$=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\cos x}$$
$$=\tan x\sec^2 x+\frac{\frac{1/\cos^2 x}{\sin x\cos x}}{\cos^2 x}$$
$$=\tan x\sec^2 x+\frac{\sec^2 x}{\tan x}$$
$$\therefore\int\frac{1}{\sin x\cos^3 x}dx=\int\tan x\sec^2 xdx+\int\frac{\sec^2 x}{\tan x}dx$$
$$Let\ \tan x=1\Rightarrow\sec^2 xdx=dt$$
$$\Rightarrow\int\frac{1}{\sin x\cos^3 x}dx=\int tdt+\int\frac{1}{t}dt$$
$$=\frac{t^2}{2}+\log|t|+C$$
$$=\frac{1}{2}\tan^2 x+\log|\tan x|+C$$

where C is an arbitrary constant.

20.

Integrate $\frac{\cos 2x}{(\cos x+\sin x)^2}$

Solution:

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$
$$\therefore\int\frac{\cos 2x}{(\cos x+\sin x)^2}dx=\int\frac{\cos 2x}{(1+\sin 2x)}dx$$
$$Let\ 1+\sin 2x=t$$
$$\Rightarrow 2\cos 2xdx=dt$$
$$\therefore\int\frac{\cos 2x}{(\cos x+\sin x)^2}dx=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+C$$
$$=\frac{1}{2}\log|1+\sin 2x|+C$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$
$$=\log\left|\sin x+\cos x\right|+C$$

where C is an arbitrary constant.

21.

Integrate $\sin^{-1}(\cos x)$

Solution:

$\sin^{-1}(\cos x)$

$Let\ \cos x = t$

$Then,\ \sin x=\sqrt{1-t^2}$

$\Rightarrow (-\sin x)dx = dt$

$dx=\frac{-dt}{\sin x}$

$dx=\frac{-dt}{\sqrt{1-t^2}}$

$\therefore \int \sin^{-1}(\cos x)dx=\int \sin^{-1}t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$

$$=-\int\frac{\sin^{-1}t}{\sqrt{1-t^2}}dt$$

$Let\ \sin^{-1}t=u$

$\Rightarrow \frac{1}{\sqrt{1-t^2}}dt=du$

$\therefore \int \sin^{-1}(\cos x)dx=-\int 4du$

$$=-\frac{u^2}{2}+C$$
$$=\frac{-\left(\sin^{1}t\right)^2}{2}+C$$
$$=\frac{-\left[\sin^{-1}(\cos x)\right]^2}{2}+C \quad ...(1)$$

It is known that,

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

$\therefore \sin^{-1}(\cos x)=\frac{\pi}{2}-\cos^{-1}(\cos x)=\left(\frac{\pi}{2}-x\right)$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x)dx = \frac{-\left[\frac{\pi}{2}-x\right]^2}{2}+C$$

$$= -\frac{1}{2}\left(\frac{\pi^2}{2}+x^2-\pi x\right)+C$$

$$= -\frac{\pi^2}{8}-\frac{x^2}{2}+\frac{1}{2}\pi x+C$$

$$= \frac{\pi x}{2}-\frac{x^2}{2}+\left(C-\frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2}-\frac{x^2}{2}+C$$

22.

Integrate $\frac{1}{\cos(x-a)\cos(x-b)}$

Solution:

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x-b)-(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\frac{\left[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)\right]}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-a)\right]$$

$$\Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left[-\log\left|\cos(x-b)\right|+\log\left|\cos(x-a)\right|\right]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

where C is an arbitrary constant.

Chose the correct answer in Exercises 23 and 24.

23.

$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

(A) tan x + cot x + C
(B) tan x + cosec x + C
(C) -tan x + cot x + C
(D) tan x + sec x + C

Solution:

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int \left(\sec^2 x - \cos ec^2 x \right) dx$$

$$= \tan x + \cot x + C$$

Hence, the correct Answer is A.

24.

$\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$ equals

(A) –cot (ex^x) + C
(B) tan (xe^x) + C
(C) tan (e^x) + C
(D) cot (e^x) + C

Solution:

$$\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$$

Let ex^x = t

$$\Rightarrow \left(e^x.x + e^x.1 \right) dx = dt$$

$$e^x (x+1) dx = dt$$

$$\therefore \int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t dt$$

$$= \tan t + C$$

$$= \tan \left(e^x.x \right) + C$$

Hence, the correct Answer is B.