Exercise 7.7

1: Integrate

$\sqrt{4-x^2}$

Solution:

Let $I=\int\sqrt{4-x^2}dx=\int\sqrt{(2)^2-(x)^2}dx$

It is known that,

$\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\therefore I=\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}+C$

$=\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

Where C is an arbitrary constant.

2:

Integrate $\sqrt{1-4x^2}$

Solution:

Let $I=\int\sqrt{1-4x^2}dx=\int\sqrt{(1)^2-(2x)^2}dx$

Let $2x=t\Rightarrow 2dx=dt$

$\therefore I=\frac{1}{2}\int\sqrt{(1)^2-(t)^2}$

It is known that,

$\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\Rightarrow I=\frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+C$

$=\frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+C$

$=\frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$

$=\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$

Where C is an arbitrary constant.

3: Integrate

$$\sqrt{x^2+4x+6}$$

Solution:

Let $I=\int\sqrt{x^2+4x+6}dx$

$=\int\sqrt{x^2+4x+4+2}dx$

$=\int\sqrt{\left(x^2+4x+4\right)+2}dx$

$=\int\sqrt{\left(x+2\right)^2+\left(\sqrt{2}\right)^2}dx$

It is known that,

$\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+C$

$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

$=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

Where C is an arbitrary constant.

4:

Integrate $\sqrt{x^2+4x+1}$

Solution:

Let $I=\int\sqrt{x^2+4x+1}dx$

$=\int\sqrt{\left(x^2+4x+4\right)-3}dx$

$=\int\sqrt{\left(x+2\right)^2-\left(\sqrt{3}\right)^2}dx$

It is known that,

$\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|+C$

$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$

Where C is an arbitrary constant.

5:

Integrate $\sqrt{1-4x-x^2}$

Solution:

Let $I=\int\sqrt{1-4x-x^2}\,dx$

$=\int\sqrt{1-\left(x^2+4x+4-4\right)}dx$

$=\int\sqrt{1+4-\left(x+2\right)^2}\,dx$

$=\int\sqrt{\left(\sqrt{5}\right)^2-\left(x+2\right)^2}\,dx$

It is known that,

$\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\therefore I=\frac{\left(x+2\right)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

Where C is an arbitrary constant.

6:

Integrate $\sqrt{x^2+4x-5}$

Solution:

Let $I=\int\sqrt{x^2+4x-5}dx$

$=\int\sqrt{\left(x^2+4x+4\right)-9}dx$

$=\int\sqrt{\left(x+2\right)^2-\left(3\right)^2}\,dx$

It is known that, $\int\sqrt{x^2-a^2}\ dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$\therefore I=\frac{\left(x+2\right)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|\left(x+2\right)+\sqrt{x^2+4x-5}\right|+C$

Where C is an arbitrary constant.

7:

Integrate $\sqrt{1+3x-x^2}$

Solution:

Let $I=\int\sqrt{1+3x-x^2}\,dx$

$=\int\sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}dx$

$=\int\sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx$

$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\, dx$$

It is known that,

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

$$\therefore I = \frac{x - \frac{3}{2}}{2}\sqrt{1 + 3x - x^2} + \frac{13}{4 \times 2}\sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$$

$$= \frac{2x - 3}{4}\sqrt{1 + 3x - x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x - 3}{\sqrt{13}}\right) + C$$

Where C is an arbitrary constant.

8:

Integrate $\sqrt{x^2 + 3x}$

Solution:

Let $I = \int \sqrt{x^2 + 3x}\, dx$

$$= \int \sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}}\, dx$$

$$= \int \sqrt{\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{2}\right)^2}\, dx$$

It is known that,

$$\sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$$

$$\therefore I = \frac{\left(x + \frac{3}{2}\right)}{2}\sqrt{x^2 + 3x} - \frac{\frac{9}{4}}{2}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right| + C$$

$$= \frac{(2x + 3)}{4}\sqrt{x^2 + 3x} - \frac{9}{8}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right| + C$$

Where C is an arbitrary constant.

9:

Integrate $\sqrt{1 + \frac{x^2}{9}}$

Solution:

Let $I=\int\sqrt{1+\frac{x^2}{9}}dx=\frac{1}{3}\int\sqrt{9+x^2}dx=\frac{1}{3}\int\sqrt{(3)^2+x^2}dx$

It is known that,

$\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$

$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$

Where C is an arbitrary constant.

10:

$\int\sqrt{1+x^2}$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}}+C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{2}{3}}+C$

D. $\frac{x^3}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

Solution:

It is known that,

$\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+C$

$\therefore\int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

Hence, the correct Answer is A.

11:

$\int\sqrt{x^2-8x+7}dx$ is equal to

A. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Solution:

Let $I=\int\sqrt{x^2-8x+7}dx$

$=\int\sqrt{\left(x^2-8x+16\right)-9}dx$

$=\int\sqrt{(x-4)^2-(3)^2}dx$

It is known that, $\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\int\sqrt{x^2-8x+7}\right|+C$

Hence, the correct Answer is D.