Exercise 7.9

1:

$\int_{-1}^{1}(x+1)\,dx$

Solution:

Let $I=\int_{-1}^{1}(x+1)\,dx$

$\int(x+1)\,dx=\frac{x^2}{2}+x=F(x)$

By second fundamental theorem of calculus, we obtain

$I=F(1)-F(-1)$

$=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)$

$=\frac{1}{2}+1-\frac{1}{2}+1$

$=2$

2:

$\int_{2}^{3}\frac{1}{x}dx$

Solution:

Let $I=\int_{2}^{3}\frac{1}{x}dx$

$\int\frac{1}{x}dx=\log|x|=F(x)$

By second fundamental theorem of calculus, we obtain

$I=F(3)-F(2)$

$=\log|3|-\log|2|=\log\frac{3}{2}$

3:

$\int_{1}^{2}\left(4x^3-5x^2+6x+9\right)dx$

Solution:

Let $I=\int_{1}^{2}\left(4x^3-5x^2+6x+9\right)dx$

$\int\left(4x^3-5x^2+6x+9\right)dx=4\left(\frac{x^4}{4}\right)-5\left(\frac{x^3}{3}\right)+6\left(\frac{x^2}{2}\right)+9(x)$

$=x^4-\frac{5x^3}{3}+3x^2+9x=F(x)$

By second fundamental theorem of calculus, we obtain

I = F (2) – F (1)

$$I=\left\{2^4-\frac{5.(2)^3}{3}+3(2)^2+9(2)\right\}-\left\{(1)^4-\frac{5(1)^3}{3}+3(1)^2+9(1)\right\}$$

$$=\left(16-\frac{40}{3}+12+18\right)-\left(1-\frac{5}{3}+3+9\right)$$

$$=16-\frac{40}{3}+12+18-1+\frac{5}{3}-3-9$$

$$=33-\frac{35}{3}$$

$$=\frac{99-35}{3}$$

$$=\frac{64}{3}$$

4:

$$\int_0^{\frac{x}{4}}\sin 2x dx$$

Solution:

Let $I=\int_0^{\frac{x}{4}}\sin 2x dx$

$$\int \sin 2x dx=\left(\frac{-\cos 2x}{2}\right)=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=F\left(\frac{\pi}{4}\right)-F(0)$$

$$=-\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right)-\cos 0\right]$$

$$=-\frac{1}{2}[0-1]$$

$$=\frac{1}{2}$$

5:

$$\int_0^{\frac{\pi}{2}}\cos 2x dx$$

Solution:

Let $I=\int_0^{\frac{\pi}{2}}\cos 2x dx$

$$\int \cos 2x dx = \left(\frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{2}\right) - F(0)$$

$$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$$

$$= \frac{1}{2}[\sin \pi - \sin 0]$$

$$= \frac{1}{2}[0-0] = 0$$

6:

$$\int_4^5 e^x dx$$

Solution:

Let $I = \int_4^5 e^x dx$

$$\int e^x dx = e^x = F(x)$$

By second fundamental theorem of calculus, we obtain

I = F (5) – F (4)

$$= e^5 - e^4$$

$$= e^4(e-1)$$

7:

$$\int_0^{\frac{\pi}{4}} \tan x dx$$

Solution:

Let $I = \int_0^{\frac{\pi}{4}} \tan x dx$

$$\int \tan x dx = -\log|\cos x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$$

$$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$$

$= -\log(2)^{-\frac{1}{2}}$

$= \frac{1}{2}\log 2$

8:

$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x\, dx$

Solution:

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x\, dx$

$\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$

$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$

$= \log\left|\sqrt{2}-1\right| - \log\left|2-\sqrt{3}\right|$

$= \log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$

9:

$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Solution:

Let $I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = F(x)$

By second fundamental theorem of calculus, we obtain

I = F (1) – F (0)

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= \frac{\pi}{2} - 0$

$= \frac{\pi}{2}$

10:

$$\int_0^1 \frac{dx}{1+x^2}$$

Solution:

Let $I = \int_0^1 \frac{dx}{1+x^2}$

$$\int \frac{dx}{1+x^2} = \tan^{-1} = F(x)$$

By second fundamental theorem of calculus, we obtain

I = F (1) – F (0)

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4}$$

11:

$$\int_2^3 \frac{dx}{x^2-1}$$

Solution 11:

Let $I = \int_2^3 \frac{dx}{x^2-1}$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = F(x)$$

By second fundamental theorem of calculus, we obtain

I =F (3) -F (2)

$$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right]$$

$$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$

$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

12:

$$\int_0^{\frac{\pi}{2}} \cos^2 x dx$$

Solution:

Let $I=\int_0^{\frac{\pi}{2}}\cos^2 x dx$

$$\int \cos^2 x dx=\int\left(\frac{1+\cos 2x}{2}\right)dx=\frac{x}{2}+\frac{\sin 2x}{4}=\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=\left[F\left(\frac{\pi}{2}\right)-F(0)\right]$$

$$=\frac{1}{2}\left[\left(\frac{\pi}{2}+\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$

$$=\frac{\pi}{4}$$

13:

$$\int_2^3 \frac{xdx}{x^2+1}$$

Solution:

Let $I=\int_2^3 \frac{x}{x^2+1}dx$

$$\int\frac{x}{x^2+1}dx=\frac{1}{2}\int\frac{2x}{x^2+1}dx=\frac{1}{2}\log\left(1+x^2\right)=F(x)$$

By second fundamental theorem of calculus, we obtain

I = F (3) – F (2)

$$=\frac{1}{2}\left[\log\left(1+(3)^2\right)-\log\left(1+(2)^2\right)\right]$$

$$=\frac{1}{2}\left[\log(10)-\log(5)\right]$$

$$=\frac{1}{2}\log\left(\frac{10}{5}\right)=\frac{1}{2}\log 2$$

14:

$$\int_0^1 \frac{2x+3}{5x^2+1}dx$$

Solution:

Let $I=\int_0^1 \frac{2x+3}{5x^2+1}dx$

$$\int \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$
$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$
$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$
$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$
$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}.\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$
$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)x$$
$$= F(x)$$

By second fundamental theorem of calculus, we obtain

I = F (1) – F (0)

$$= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5})\right\} - \left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\}$$
$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

15:

$$\int_0^1 xe^{x^2}dx$$

Solution:

Let $I = \int_0^1 xe^{x^2}dx$

Put $x^2 = t \Rightarrow 2xdx = dt$

As $x \to 0, t \to 0$ *and as* $x \to 1, t \to 1,$

$$\therefore I = \frac{1}{2}\int_0^1 e^t dt$$
$$\frac{1}{2}\int e^t dt = \frac{1}{2}e^t = F(t)$$

By second fundamental theorem of calculus, we obtain

I = F (1) – F (0)

$$= \frac{1}{2}e - \frac{1}{2}e^0$$
$$= \frac{1}{2}(e-1)$$

16:

$$\int_0^1 \frac{5x^2}{x^2+4x+3}$$

Solution:

Let $I=\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$

Dividing $5x^2$ by x^2+4x+3, we obtain

$$I=\int_1^2\left\{5-\frac{20x+15}{x^2+4x+3}\right\}dx$$

$$=\int_1^2 5dx-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$

$$=[5x]_1^2-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$

$I = 5 - I_1$, where $I_1=\int_1^2\frac{20x+15}{x^2+4x+3}dx$ … (1)

Consider

Let $20x+15=A\frac{d}{dx}\left(x^2+4x+3\right)+B$

$$=2Ax+(4A+B)$$

Equating the coefficients of x and constant term, we obtain

A = 10 and B = −25

Let x^2+4x+3 = t

$\Rightarrow (2x+4)dx=dt$

$$\Rightarrow I_1=10\int\frac{dt}{t}-25\int\frac{dx}{(x+2)^2-1^2}$$

$$=10\log t-25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2-1}\right)\right]$$

$$=\left[10\log\left(x^2+4x+3\right)\right]_1^2-25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$

$$=[10\log 15-10\log 8]-25\left[\frac{1}{2}\log\frac{3}{5}-\frac{1}{2}\log\frac{2}{4}\right]$$

$$=[10\log(5\times3)-10\log(4\times2)]-\frac{25}{2}[\log 3-\log 5-\log 2+\log 4]$$

$$=[10\log 5+10\log 3-10\log 4-10\log 2]-\frac{25}{2}[\log 3-\log 5-\log 2+\log 4]$$

$$=\left[10+\frac{25}{2}\right]\log 5+\left[-10-\frac{25}{2}\right]\log 4+\left[10-\frac{25}{2}\right]\log 3+\left[-10+\frac{25}{2}\right]\log 2$$

$$= \frac{45}{2}\log 5 = \frac{45}{2}\log 4 - \frac{5}{2}\log 3 + \frac{5}{2}\log 2$$

$$= \frac{45}{2}\log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}$$

Substituting the value of I_1 in (1), we obtain

$$I = 5 - \left[\frac{45}{2}\log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}\right]$$

$$= 5 - \frac{5}{2}\left[9\log\frac{5}{4} - \log\frac{3}{2}\right]$$

17:

$$\int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$$

Solution:

Let $I = \int_0^{\frac{\pi}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$

$$\int\left(2\sec^2 x + x^3 + 2\right)dx = 2\tan x + \frac{x^4}{4} + 2x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= \left\{\left(2\tan\frac{\pi}{4} + \frac{1}{4}\left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)\right) - (2\tan 0 + 0 + 0)\right\}$$

$$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2}$$

$$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

18:

$$\int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$$

Solution:

Let $I = \int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$

$$= -\int_0^{\pi}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)dx$$

$$= -\int_0^{\pi}\cos x\,dx$$

$-\int_0^{\pi} \cos x\,dx = -\sin x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(\pi) - F(0)$

$= -\sin\pi + \sin 0$

= 0

19:

$\int_0^2 \frac{6x+3}{x^2+4}dx$

Solution:

Let $I = \int_0^2 \frac{6x+3}{x^2+4}dx$

$\int \frac{6x+3}{x^2+4}dx = 3\int \frac{2x+1}{x^2+4}dx$

$= 3\int \frac{2x}{x^2+4}dx + 3\int \frac{1}{x^2+4}dx$

$= 3\log(x^2+4) + \frac{3}{2}\tan^{-1}\frac{x}{2} = F(x)$

By second fundamental theorem of calculus, we obtain

I = F(2) – F(0)

$= \left\{3\log(2^2+4) + \frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log(0+4) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$

$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$

$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$

$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$

$= 3\log 2 + \frac{3\pi}{8}$

20:

$\int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

Solution:

Let $I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

$$\int_0^1\left(xe^x+\sin\frac{\pi x}{4}\right)dx = x\int e^x dx-\int\left\{\left(\frac{d}{dx}x\right)\int e^x dx\right\}dx+\left\{\frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right\}$$

$$= xe^x-\int e^x dx-\frac{4}{\pi}\cos\pi\frac{x}{4}$$

$$= xe^x-e^x-\frac{4}{\pi}\cos\pi\frac{x}{4}$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

I = F (1) – F (0)

$$=\left(1.e^1-e^1-\frac{4}{\pi}\cos\frac{\pi}{4}\right)-\left(0.e^0-e^0-\frac{4}{\pi}\cos 0\right)$$

$$=e-e-\frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right)+1+\frac{4}{\pi}$$

$$=1+\frac{4}{\pi}-\frac{2\sqrt{2}}{\pi}$$

Chose the correct answer in Exercises 21 and 22.

21:

$$\int_1^{\sqrt{3}}\frac{dx}{1+x^2}$$

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$ equals

Solution:

$$\int\frac{dx}{1+x^2}=\tan^{-1}x=F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_1^{\sqrt{3}}\frac{dx}{1+x^2}=F\left(\sqrt{3}\right)-F(1)$$

$$=\tan^{-1}\sqrt{3}-\tan^{-1}1$$

$$=\frac{\pi}{3}-\frac{\pi}{4}$$

$$=\frac{\pi}{12}$$

Hence, the correct Answer is D.

22:

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{4}$ equals

Solution:

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2+(3x)^2}$$

Put 3x=t $\Rightarrow$ 3dx=dt

$$\therefore \int \frac{dx}{(2)^2+(3x)^2} = \frac{1}{3}\int \frac{dt}{(2)^2+t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3}{2}.\frac{2}{3}\right) - \frac{1}{6}\tan^{-1}0$$

$$= \frac{1}{6}\tan^{-1}1 - 0$$

$$= \frac{1}{6}\times\frac{\pi}{4}$$

$$= \frac{\pi}{24}$$

Hence, the correct Answer is C.