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Determine order and degree (if defined) of differential equations given in Exercise 1 to 10.

1. $\frac{d^4y}{dx^4} + \sin(y''') = 0$

Solution: $\frac{d^4y}{dx^4} + \sin(y''') = 0$

$\Rightarrow y'''' + \sin(y''') = 0$

The highest order derivative present in the differential equation is y''''. Therefore, its order is four.

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined.

2. y '+ 5y = 0

Solution: The given differential equation is:

y '+5y = 0

The highest order derivative present in the differential equation is y'. Therefore, its order is one.

It is a polynomial equation in y'. The highest power raised to y' is 1. Hence, its degree is one.

3: $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

Solution: $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

The highest order derivative present in the given differential equation is $\frac{d^2s}{dt^2}$. Therefore, its order is two.

It is a polynomial equation in $\frac{d^2s}{dt^2}$ and $\frac{ds}{dt}$. The power raised to $\frac{d^2s}{dt^2}$ is 1.

Hence, its degree is one.

4:

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

Solution: $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is 2.

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined.

5:

$$\left(\frac{d^2y}{dx^2}\right)^2 = \cos 3x + \sin 3x$$

Solution: $\left(\frac{d^2y}{dx^2}\right)^2 = \cos 3x + \sin 3x$

$$\Rightarrow \frac{d^2y}{dx^2} - \cos 3x + \sin 3x = 0$$

The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

It is a polynomial equation $\frac{d^2y}{dx^2}$ in and the power raised to $\frac{d^2y}{dx^2}$ is 1.

Hence, its degree is one.

6:

$$(y''')^2 + y'')^3 + (y')^4 + y^5 = 0$$

Solution : $(y''')^2+(y'')^3+(y')^4+y^5=0$

The highest order derivative present in the differential equation is y'''. Therefore, its order is three.

The given differential equation is a polynomial equation in y''', y'', and y'.

The highest power raised to y''' is 2. Hence, its degree is 2.

7: y''' + 2y'' + y' = 0

Solution: y''' + 2y'' + y ' = 0

The highest order derivative present in the differential equation is y'''. Therefore, its order is three.

It is a polynomial equation in y''', y'', and y'. The highest power raised to y'' is 1. Hence, its degree is 1.

8: y'+ y = e'

Solution: y' + y = e'

$\Rightarrow$ y' + y - e' = 0

The highest order derivative present in the differential equation is y'. Therefore, its order is one.

The given differential equation is a polynomial equation in y' and the highest power raised to y' is one. Hence, its degree is one.

9: $y'+(y')^2+2y=0$

Solution: : $y'+(y')^2+2y=0$

The highest order derivative present in the differential equation is y''. Therefore, its order is two.

The given differential equation is a polynomial equation in y '' and y' and the highest power raised to y'' is one.

Hence, its degree is one.

10: y'' + 2y' + sin y = 0

Solution: y'' + 2y' + sin y = 0

The highest order derivative present in the differential equation is y''. Therefore, its order is two.

This is a polynomial equation in y'' and y' and the highest power raised y'' to is one.

Hence, its degree is one.

11: The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0 \text{ is}$$

(A) 3

(B) 2

(C) 1

(D) Not defined

Solution $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$

The given differential equation is not a polynomial equation in its derivatives. Therefore, its degree is not defined.

Hence, the correct answer is D.

12: The order of the differential equation

$$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$$

(A) 2

(B) 1

(C) 0

(D) not defined

Solution: $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Therefore, its order is two.

Hence, the correct answer is A.