Exercise 9.2 Page: 385

For each of the differential equations in Exercise 1 to 10. find the general solution:

1: $y = e^x + 1$ $: y'' - y' = 0$

Solution: $y = e^x + 1$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{dy}{dx} = \frac{d}{dx}(e^x + 1)$$

$$\Rightarrow y' = e^x \quad ...(1)$$

Now, differentiating equation (1) with respect to x, we get:

$$\frac{d}{dx}(y') = \frac{d}{dx}(e^x)$$

$$\Rightarrow y'' = e^x$$

Substituting the values of and in the given differential equation, we get the L.H.S. as:

$$y'' - y' = e^x = 0 = \text{RHS}$$

Thus, the given function is the solution of the corresponding differential equation.

2: $y = x^2 + 2x + C$ $: y' - 2x - 2 = 0$

Solution: $y = x^2 + 2x + C$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}(x^2 + 2x + C)$$

$$\Rightarrow y' = 2x + 2$$

Substituting the value of in the given differential equation, we get:

L.H.S.= $y' - 2x - 2 = 2x + 2 - 2x - 2 = 0$ = R.H.S

Hence, the given function is the solution of the corresponding differential equation.

3: y = cosx + C : y' + sinx = 0

Solution: : y = cosx + C
Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}(\cos x + C)$$

$$\Rightarrow y' = \sin x$$

Substituting the value of in the given differential equation, we get:
L.H.S. = y' + sinx = -sinx + sinx = 0 = R.H.S.
Hence, the given function is the solution of the corresponding differential equation.

4: $y = \sqrt{1+x^2}$: $y' = \frac{xy}{1+x^2}$

Solution : $y = \sqrt{1+x^2}$
Differentiating both sides of the equation with respect to x, we get:

$$y' = \frac{d}{dx}(\sqrt{1+x^2})$$

$$y' = \frac{1}{2\sqrt{1+x^2}}.\frac{d}{dx}(1+x^2)$$

$$y' = \frac{2}{2\sqrt{1+x^2}}$$

$$y' = \frac{x}{2\sqrt{1+x^2}}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \times \sqrt{1+x^2}$$

$$\Rightarrow y' = \frac{x}{1+x^2}.y$$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

$\therefore$ L.H.S. = R.H.S.
Hence, the given function is the solution of the corresponding differential equation

5: y = Ax $\therefore$ xy’=y(x $\neq$ 0)

Solution: y = Ax
Differentiating both sides with respect to x, we get:

$$y' = \frac{d}{dx}(Ax)$$
$$\Rightarrow y' = A$$

Substituting the value of in the given differential equation, we get:
L.H.S. = xy'= xA = Ax = y = R.H.S.
Hence, the given function is the solution of the corresponding differential equation.

6: $y = x\sin x$ $: xy' = y + x\sqrt{x^2 - y^2}\,(x \neq 0 \text{ and } x > y) \text{or } x < -y)$

Solution : $y = x\sin x$
Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}(x\sin x)$$
$$\Rightarrow y' = \sin x.\frac{d}{dx}(x).\frac{d}{dx}(\sin x)$$
$$\Rightarrow y' = \sin x + x\cos x$$

Substituting the value of in the given differential equation, we get:
L.H.S.= $xy' = x(\sin x + x\cos x)$
= $x\sin x + x^2\cos x$
= $y + x^2.\sqrt{1-\sin^2 x}$
= $y + x^2\sqrt{1-\left(\frac{y}{x}\right)^2}$
= $y + x\sqrt{y^2 - x^2}$
= R.H.S
Hence, the given function is the solution of the corresponding differential equation.

7: $xy = \log y + C$ $: y' = \frac{y^2}{1-xy}\,(xy \neq 1)$

Solution : $xy = \log y + C$
Differentiating both sides of this equation with respect to x, we get:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$
$$\Rightarrow y\frac{d}{dx}(x) + x.\frac{d}{dx} = \frac{1}{y}\frac{dy}{dx}$$
$$\Rightarrow y + xy' = \frac{1}{y}y'$$
$$\Rightarrow y^2 + xy\,y' = y'$$

$\Rightarrow (xy-1)\,y' = -y^2$

$\Rightarrow y' = \dfrac{y^2}{1-xy}$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

8: $y-\cos y = x$ $\quad : (y\sin y+\cos y+x)y' = 1$

Solution: $y-\cos y = x$ $\quad \ldots(1)$

Differentiating both sides of the equation with respect to x, we get:

$\dfrac{dy}{dx} - \dfrac{d}{dx}(\cos y) = \dfrac{d}{dx}(x)$

$\Rightarrow y'\sin y.y' = 1$

$\Rightarrow y'(1+\sin y) = 1$

$\Rightarrow y' = \dfrac{1}{1+\sin y}$

Substituting the value of in equation (1), we get:

L.H.S.= $(y\sin y+\cos y+x)y'$

$= (y\sin y+\cos y+y-\cos y)\times\dfrac{1}{1+\sin y}$

$= y(1+\sin y).\dfrac{1}{1+\sin y}$

= y

= R.H.S

Hence, the given function is the solution of the corresponding differential equation.

9: $x+y = \tan^{-1} y$ $\quad : y^2y' + y^2 + 1 = 0$

Solution: $x+y = \tan^{-1} y$

Differentiating both sides of this equation with respect to x, we get:

$\dfrac{d}{dx}(x+y) = \dfrac{d}{dx}(\tan^{-1} y)$

$\Rightarrow 1+y' = \left[\dfrac{1}{1+y^2}\right]y'$

$\Rightarrow y'\left[\dfrac{1}{1+y^2}-1\right] = 1$

$$\Rightarrow y'\left[\frac{1-\left(1+y^2\right)}{1+y^2}\right]=1$$

$$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right]=1$$

$$\Rightarrow y'=\frac{(-1+y^2)}{y^2}$$

Substituting the value of in the given differential equation, we get:

L.H.S.= $y^2y'+y^2+1=y^2\left[\frac{-\left(1+y^2\right)}{y^2}\right]+y^2+1$

= $-1-y^2+y^2+1$

= 0

= R.H.S

Hence, the given function is the solution of the corresponding differential equation.

10: $y=\sqrt{a^2-x^2}\,x\in(-a,a)$ $\quad : x+y\frac{dy}{dx}=0\,(y\neq 0)$

Solution: $y=\sqrt{a^2-x^2}$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{a^2-x^2}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}.\frac{d}{dx}(a^2-x^2)$$

$$=\frac{1}{2\sqrt{a^2-x^2}}.(-2x)$$

$$=\frac{-x}{\sqrt{a^2-x^2}}$$

Substituting the value of $\frac{dy}{dx}$ in the given differential equation, we get:

L.H.S= $x+y\frac{dy}{dx}=x+\sqrt{a^2-x^2}\times\frac{-x}{\sqrt{a^2-x^2}}$

= x-x

= 0

= R.H.S

Hence, the given function is the solution of the corresponding differential equation.

11: The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0

(B) 2

(C) 3
(D) 4

Solution: We know that the number of constants in the general solution of a differential equation of order n is equal to its order.
Therefore, the number of constants in the general equation of fourth order differential equation is four.
Hence, the correct answer is D.

12: The numbers of arbitrary constants in the particular solution of a differential equation of third order are:
(A) 3
(B) 2
(C) 1
(D) 0

Solution: In a particular solution of a differential equation, there are no arbitrary constants.
Hence, the correct answer is D.