Exercise 9.4 Page: 395

For each of the differential equations in Exercises 1 to 10, find the general solution:

1:
$$\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$$

Solution:

The given differential equation is:

$$\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx}=\left(\sec^2\frac{x}{2}-1\right)$$

Separating the variables, we get:

$$dy=\left(\sec^2\frac{x}{2}-1\right)dx$$

Now, integrating both sides of this equation, we get:

$$\int dy=\int\left(\sec^2\frac{x}{2}-1\right)dx=\int\sec^2\frac{x}{2}dx-\int dx$$

$$\Rightarrow y=2\tan\frac{x}{2}-x+C$$

This is the required general solution of the given differential equation.

2:

$$\frac{dy}{dx}=\sqrt{4-y^2}\left(-2<y<2\right)$$

Solution:

The given differential equation is:

$$\frac{dy}{dx}=\sqrt{4-y^2}$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{\sqrt{4-y^2}}=dx$$

Now, integrating both sides of this equation, we get:

$$\int\frac{dy}{\sqrt{4-y^2}}=\int dx$$

$$\Rightarrow \sin^{-1}\frac{y}{2}=x+c$$

$$\Rightarrow \frac{y}{2}=\sin\left(x+C\right)$$

$$\Rightarrow y=2\sin\left(x+C\right)$$

This is the required general solution of the given differential equation.

3:

$$\frac{dy}{dx}=+y=1\left(y\neq 1\right)$$

Solution:

The given differential equation is:

$$\frac{dy}{dx}=+y=1\left(y\neq 1\right)$$

$$\Rightarrow dy+ydx=dx$$

$$\Rightarrow dy=\left(1-y\right)dx$$

Separating the variables, we get

$$\Rightarrow \frac{dy}{1-y}=dx$$

Now, integrating both sides, we get:

$$\int \frac{dy}{1-y} = \int dx$$
$$\Rightarrow \log(1-y) = x + \log C$$
$$\Rightarrow -\log C - \log(1-y) = x$$
$$\Rightarrow \log C(1-y) = -x$$
$$\Rightarrow C(1-y) = e^{-x}$$
$$\Rightarrow y = 1 - \frac{1}{C}e^{-x}$$
$$\Rightarrow y = 1 + Ae^{-x} \left(\text{Where } A = -\frac{1}{C} \right)$$

This is the required general solution of the given differential equation.

4:

$$\sec^2 c \tan ydx + \sec^2 y \tan xdy = 0$$

Solution:

The given differential equation is:

$$\sec^2 c \tan ydx + \sec^2 y \tan xdy = 0$$
$$\Rightarrow \frac{\sec^2 c \tan ydx + \sec^2 y \tan xdy = 0}{\tan x \tan y}$$
$$\Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$
$$\Rightarrow \frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrating both sides of this equation, we get:

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy \qquad \text{..... (1)}$$

Let tan x = t

$$\therefore \frac{d}{dx}(\tan x)=\frac{dt}{dx}$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\frac{\sec^2 x}{\tan x}dx = \int \frac{1}{t}dt$$

$$= \log t$$

$$= \log(\tan x)$$

Similarly, $\int \frac{\sec^2 x}{\tan x}dy = \log(\tan\ y)$

Substituting these values in equation (1), we get:
log(tan x) = − log(tan y) + log C

$$\Rightarrow \log(\tan\ x) = \log\left(\frac{C}{\tan y}\right)$$

$$\Rightarrow \tan x = \frac{C}{\tan\ y}$$

$$\Rightarrow \tan x \tan y = C$$

This is the required general solution of the given differential equation.

5:

$$\left(e^x + e^{-x}\right)dy - \left(e^x - e^{-x}\right)dx = 0$$

Solution:

The given differential equation is:

$$\left(e^x + e^{-x}\right)dy - \left(e^x - e^{-x}\right)dx = 0$$

$$\Rightarrow \left(e^x + e^{-x}\right)dy = \left(e^x - e^{-x}\right)dx$$

$$\Rightarrow dy = \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right]dx$$

Integrating both sides of this equation, we get:

$$\int dy = \int\left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right]dx + C$$

$$\Rightarrow y = \int\left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right]dx + C$$

Let (ex + e – x) = t.

Differentiating both sides with respect to x, we get:

$$\frac{d}{dx}\left(e^{x}+e^{-x}\right)=\frac{dt}{dx}$$

$$\Rightarrow e^{x}-e^{-x}=\frac{dt}{dx}$$

$$\Rightarrow\left(e^{x}-e^{-x}\right)dx=dt$$

Substituting this value in equation (1), we get:

$$y=\int\frac{1}{t}+C$$

$$\Rightarrow y=\log(t)+C$$

$$\Rightarrow y=\log\left(e^{x}+e^{-x}\right)+C$$

This is the required general solution of the given differential equation.

6:

$$\frac{dy}{dx}=\left(1+x^{2}\right)\left(1+y^{2}\right)$$

Solution:

The given differential equation is:

$$\frac{dy}{dx}=\left(1+x^{2}\right)\left(1+y^{2}\right)$$

$$\Rightarrow\frac{dy}{1+y^{2}}=\left(1+x^{2}\right)dx$$

Integrating both sides of this equation, we get:

$$\int\frac{dy}{1+y^{2}}=\int\left(1+x^{2}\right)dx$$

$$\Rightarrow\tan^{-1}y=\int dx+\int x^{2}dx$$

$$\Rightarrow\tan^{-1}y=x+\frac{x^{3}}{3}+C$$

This is the required general solution of the given differential equation.

7:

$$y\log y\,dx-x\,dy=0$$

Solution:

The given differential equation is:

$$y\log y dx - x dy = 0$$

$$\Rightarrow y\log y dx = x dy$$

$$\Rightarrow \frac{dy}{y\log y} = \frac{dx}{x}$$

Integrating both sides, we get:

$$\Rightarrow \int \frac{dy}{y\log y} = \int \frac{dx}{x} \quad \ldots\ldots (1)$$

Let log y = t

$$\therefore \frac{d}{dx}(\log y) = \frac{dt}{dy}$$

$$= \frac{1}{y} = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} dy = dt$$

Substituting this value in equation (1), we get:

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \log t = \log x + \log c$$

$$\Rightarrow \log(\log y) = \log Cx$$

$$\Rightarrow \log y = Cx$$

$$\Rightarrow y = e^{cx}$$

This is the required general solution of the given differential equation.

8:

$$x^5 \frac{dy}{dx} = -y^5$$

Solution:

The given differential equation is:

$$x^5 \frac{dy}{dx} = -y^5$$

$$\Rightarrow \frac{dy}{y^5} = -\frac{dx}{x^5}$$

$$\Rightarrow \frac{dx}{x^5} + \frac{dy}{y^5} = 0$$

Integrating both sides, we get:

$$\int \frac{dx}{x^5} + \int \frac{dy}{y^5} = k \qquad \text{(Where k is any constant)}$$

$$\Rightarrow \int x^{-5} dx + \int y^{-5} dy = k$$

$$\Rightarrow \frac{x^{-4}}{-4} + \frac{y^{-4}}{-4} = k$$

$$\Rightarrow x^{-4} + y^{-4} = -4k$$

$$\Rightarrow x^{-4} + y^{-4} = C \quad (c = -4k)$$

This is the required general solution of the given differential equation.

9:

$$\frac{dy}{dx} = \sin^{-1} x$$

Solution:

The given differential equation is:

$$\frac{dy}{dx} = \sin^{-1} x$$

$$\Rightarrow dy = \sin^{-1} x dx$$

Integrating both sides, we get:

$$\int dy = \int \sin^{-1} x dx$$

$$\Rightarrow y = \int \left(\sin^{-1} x.1\right) dx$$

$$\Rightarrow y = \sin^{-1} x.\int (1) dx - \int \left[\left(\frac{d}{dx}\left(\sin^{-1} x\right) \int (1) dx\right)\right] dx$$

$$\Rightarrow y = \sin^{-1} x.x - \int \left(\frac{1}{\sqrt{1-x^2}} x\right) dx$$

$$\Rightarrow y = \sin^{-1} x + \int \frac{-x}{\sqrt{1-x^2}} dx \qquad \ldots\ldots (1)$$

Let $1-x^2=t$

$$\Rightarrow \frac{d}{dx}\left(1-x^2\right)=\frac{dt}{dx}$$

$$\Rightarrow -2x=\frac{dt}{dx}$$

$$\Rightarrow xdx=-\frac{1}{2}dt$$

Substituting this value in equation (1), we get:

$$y=x\sin^{-1}x+\int\frac{1}{2\sqrt{t}}dt$$

$$\Rightarrow y=x\sin^{-1}x+\frac{1}{2}\int\left(t\right)^{\frac{1}{2}dt}$$

$$\Rightarrow y=x\sin^{-1}x+\frac{1}{2}.\frac{t^{\frac{1}{2}}}{1}+C$$

$$\Rightarrow y=x\sin^{-1}x+\sqrt{t}+C$$

$$\Rightarrow y=x\sin^{-1}x+\sqrt{1-x^2}+C$$

This is the required general solution of the given differential equation.

10:

$$e^x\tan ydx+\left(1-e^x\right)\sec^2 ydy=0$$

Solution:

The given differential equation is:

$$e^x\tan ydx+\left(1-e^x\right)\sec^2 ydy=0$$

$$\left(1-e^x\right)\sec^2 ydy=-e^x\tan ydx$$

Separating the variables, we get:

$$\frac{\sec^2 y}{\tan y}dy=\frac{-e^x}{1-e^x}dx$$

Integrating both sides, we get:

$$\int\frac{\sec^2 y}{\tan y}dy=\int\frac{-e^x}{1-e^x}dx \qquad \ldots\ldots (1)$$

Let tan y = u

$$\Rightarrow \frac{d}{dy}(\tan\ y)=\frac{du}{dy}$$

$$\Rightarrow \sec^2\ y=\frac{du}{dy}$$

$$\Rightarrow \sec^2\ ydy=du$$

$$\therefore \int\frac{\sec^2\ y}{\tan y}dy=\int\frac{du}{u}=\log u=\log(\tan y)$$

Now, $1-e^x=t$

$$\therefore \frac{d}{dx}\left(1-e^x\right)=\frac{dt}{dx}$$

$$\Rightarrow -e^x=\frac{du}{dx}$$

$$\Rightarrow -e^x dx=dt$$

$$\Rightarrow \int\frac{-e^x}{1-e^x}dx=\int\frac{dt}{t}=\log t=\log\left(1-e^x\right)$$

Substituting the values of $\int\frac{\sec^2\ y}{\tan y}dy$ and $\int\frac{-e^x}{1-e^x}dx$ in equation (1), we get

$$\Rightarrow \log(\tan y)=\log\left(1-e^x\right)+\log C$$

$$\Rightarrow \log(\tan y)=\log\left[C\left(1-e^x\right)\right]$$

$$\Rightarrow \tan y=C\left(1-e^x\right)$$

This is the required general solution of the given differential equation.

11:

$$\left(x^3+x^2+x+1\right)\frac{dy}{dx}=2x^2+x; y=1$$

Solution:

The given differential equation is:

$$\left(x^3+x^2+x+1\right)\frac{dy}{dx}=2x^2+x; y=1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + x}{\left(x^3 + x^2 + x + 1\right)}$$

$$\Rightarrow dy = \frac{2x^2 + x}{(x+1)\left(x^2+1\right)}dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{2x^2 + x}{(x+1)\left(x^2+1\right)}dx \qquad \text{.... (1)}$$

Let $\frac{2x^2 + x}{(x+1)\left(x^2+1\right)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$ .... (2)

$$\Rightarrow \frac{2x^2 + x}{(x+1)\left(x^2+1\right)} = \frac{Ax^2 + A + (Bx+C)(x+1)}{(x+1)\left(x^2+1\right)}$$

$$\Rightarrow 2x^2 + x = Ax^2 + A + bx^2 + Bx + Cx + C$$

$$\Rightarrow 2x^2 + x = (A+B)x^2 + (B+C)x + (A+C)$$

Comparing the coefficients of x^2 and x, we get:

$A + B = 2$

$B + C = 1$

$A + C = 0$

Solving these equations, we get:

$$A = \frac{1}{2}, b = \frac{3}{2} \text{ and } C = \frac{-1}{2}$$

Substituting the values of A, B and C in equation (2), we get:

$$\frac{2x^2 + x}{(x+1)\left(x^2+1\right)} = \frac{1}{2}\frac{1}{(x+1)} + \frac{1}{2}\frac{(3x-1)}{\left(x^2+1\right)}$$

Therefore, equation (1) becomes:

$$\int dy = \frac{1}{2}\int \frac{1}{x+1}dx + \int \frac{3x-1}{x^2+1}dx$$

$$\Rightarrow y = \frac{1}{2}\log(x+1) + \frac{3}{2}\int \frac{x}{x^2+1}dx - \frac{1}{2}\int \frac{x}{x^2+1}dx$$

$$\Rightarrow y = \frac{1}{2}\log(x+1)\frac{3}{4}\int \frac{2x}{x^2+1}dx - \frac{1}{2}\tan^{-1}x + C$$

$$\Rightarrow y = \frac{1}{2}\log(x+1)\frac{3}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$$

$$\Rightarrow y = \frac{1}{4}\left[2\log(x+1) + 3\log(x^2+1)\right] - \frac{1}{2}\tan^{-1}x + C$$

$$\Rightarrow y = \frac{1}{4}\left[(x^2+1)(x^2+1)\right] - \frac{1}{2}\tan^{-1}x + C \qquad \text{.....(3)}$$

Now, y = 1 when x = 0

$$\Rightarrow 1 = \frac{1}{4}\log(1) - \frac{1}{2}\tan^{-1}0 + C$$

$$\Rightarrow 1 = \frac{1}{4}\times 0 - \frac{1}{2}\times 0 + C$$

$$\Rightarrow C = 1$$

Substituting C = 1 in equation (3), we get:

$$y = \frac{1}{4}\left[\log(x^2+1)^2(x^2+1)^3\right] - \frac{1}{2}\tan^{-1}x + 1$$

12:

$$x(x^2-1)\frac{dy}{dx} = 1; y = 0 \text{ when } x = 2$$

Solution:

$$x(x^2-1)\frac{dy}{dx} = 1$$

$$\Rightarrow dy = \frac{dx}{x(x^2-1)}$$

$$\Rightarrow dy = \frac{1}{x(x-1)(x+1)}dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{1}{x(x-1)(x+1)} dx \quad \ldots..(1)$$

Let $\frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$ ….(2)

$$\Rightarrow \frac{1}{x(x-1)(x+1)} = \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)}$$

$$= \frac{(A+B+C)x^2 + (B-C)x - A}{x(x-1)(x+1)}$$

Comparing the coefficients of x^2, x and constant, we get:

A = -1

B – C = 0

A + B + C = 0

Solving these equations, we get B = $\frac{1}{2}$ and C = $\frac{1}{2}$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{1}{x(x-1)(x+1)} = \frac{-1}{x} + \frac{1}{x(x-1)} + \frac{1}{2(x+1)}$$

Therefore, equation (1) becomes:

$$\int dy = -\int \frac{1}{x} dx + \frac{1}{2}\int \frac{1}{x-1} dx + \frac{1}{2}\int \frac{1}{x+1} dx$$

$$\Rightarrow y = -\log x + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + \log k$$

Now,

y = 0 when = 2

$$0 = -\log 2 + \frac{1}{2}\log(2+1) + \frac{1}{2}\log(2-1) + \log C$$

$$0 = -\log 2 + \frac{\log 3}{2} + \log C$$

$$\log 1 = \log\left(\frac{\sqrt{3}C}{2}\right)$$

$$\frac{\sqrt{3}}{2}C = 1$$

$$C = \frac{2}{\sqrt{3}}$$

$$y = -\log x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) + \log\frac{2}{\sqrt{3}}$$

$$y = \log\left(\frac{2\sqrt{(x+1)(x-1)}}{\sqrt{3}x}\right)$$

$$y = \log\left(\frac{\sqrt{4(x^2-1)}}{\sqrt{3x^2}}\right)$$

$$y = \frac{1}{2}\log\left[\frac{4(x^2-1)}{3x^2}\right]$$

13:

$$\cos\left(\frac{dy}{dx}\right) = a\,(a \in R); 1 \text{ when } x = 0$$

Solution:

$$\cos\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a dx$$

Integrating both sides, we get:

$$\int dy = \cos^{-1} a \int dx dx$$

$$\Rightarrow y = \cos^{-1} a.x + C$$

$$\Rightarrow y = x\cos^{-1} a + C \qquad \text{....(1)}$$

Now, y =1 when x = 0

$$\Rightarrow 1 = 0.\cos^{-1} a + C$$

$$\Rightarrow C = 1$$

Substituting C = 1 in equation (1), we get

$$y = x\cos^{-1}a + 1$$

$$\Rightarrow \frac{y-1}{x} = \cos^{-1}a$$

$$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$$

14:

$\frac{dy}{dx} = y\tan x, y = 1$ when x = 0

Solution:

$$\frac{dy}{dx} = y\tan x$$

$$\Rightarrow \frac{dy}{dx} = \tan\ xdx$$

integrating both sides, we get:

$$\int\frac{dy}{y} = -\int\tan\ xdx$$

$$\Rightarrow \log y = \log(\sec x) + \log C$$

$$\Rightarrow \log\ y = \log(C\sec x)$$

$$\Rightarrow y = C\sec x \qquad (1)$$

Now y = 1 when x = 0

$$\Rightarrow 1 = C \times \sec 0$$

$$\Rightarrow 1 = C \times 1$$

$$\Rightarrow C = 1$$

Substituting C = 1 in equation (1), we get

y = sec x

15:

Find the equation of a curve passing through the point (0, 0) and whose differential equation is

$$y' = e^x \sin x$$

Solution:

The differential equation of the curve is:

$$y' = e^x \sin x$$

$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x$$

Intergrating both sides, we get:

$$\int dy = \int e^x \sin x dx \qquad \text{..... (1)}$$

Let $I = \int e^x \sin x dx$

$$\Rightarrow I = \sin x \int e^x dx - \int \left[\frac{d}{dx}(\sin x) . \int e^x dx \right] dx$$

$$\Rightarrow I = \sin x . e^x - \int \cos x . e^x dx$$

$$\Rightarrow I = \sin x . e^x - \left[\cos x . \int \left(\frac{d}{dx}(\cos x) . \int e^x dx \right) dx \right]$$

$$\Rightarrow I = \sin x . e^x - \left[\cos x e^x - \int (-\sin x) e^x dx \right]$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - 1$$

$$\Rightarrow 2I = e^x (\sin x - \cos x)$$

$$\Rightarrow I = \frac{e^x (\sin x - \cos x)}{2}$$

Substituting this value in equation (1), we get

$$y = \frac{e^x (\sin x - \cos x)}{2} + C \qquad \text{..... (2)}$$

Now, the curve passes through point (0, 0)

$$\therefore \ 0 = \frac{e^0 (\sin 0 - \cos 0)}{2} + C$$

$$\Rightarrow 0 = \frac{1(0 - 1)}{2} + c$$

$$\Rightarrow C = \frac{1}{2}$$

Substituting $C = \frac{1}{2}$ in equation (2), we get:

$$y=\frac{e^x\left(\sin x-\cos x\right)}{2}+\frac{1}{2}$$

$$\Rightarrow 2y=e^x\left(\sin x-\cos x\right)+1$$

$$\Rightarrow 2y-1=e^x\left(\sin x-\cos x\right)$$

Hence, the required equation of the curve is $2y-1=e^x\left(\sin x-\cos x\right)$

16:

For the differential equation $xy=\frac{dy}{dx}=\left(x+2\right)\left(y+2\right)$ find the solution curve passing through the
point (1, –1).

Solution:

The differential equation of the given curve is:

$$xy=\frac{dy}{dx}=\left(x+2\right)\left(y+2\right)$$

$$\Rightarrow\left(\frac{y}{y+2}\right)dy=\left(\frac{x+2}{x}\right)dx$$

Integrating both sides, we get:

$$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dx$$

$$\Rightarrow\int dy-2\int\frac{1}{y+2}dy=\int dx+2\int\frac{1}{x}dx$$

$$\Rightarrow y-2\log\left(y+2\right)=x+2\log x+C$$

$$\Rightarrow y-x-C=\log x^2+\log\left(y+2\right)^2$$

$$\Rightarrow y-x-C=\log\left[x^2\left(y+2\right)^2\right] \qquad \text{.....(1)}$$

Now, the curve passes through point (1, -1)

$$\Rightarrow -1-1-C=\log\left[\left(1\right)^2\left(-1+2\right)^2\right]$$

$$\Rightarrow -2-C=\log 1=0$$

$$\Rightarrow C=-2$$

Substituting C = -2 in equation (1), we get:

$$y - x + 2 = \log\left[x^2(y+2)^2\right]$$

This is the required solution of the given curve.

17:

Find the equation of a curve passing through the point (0, –2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point.

Solution:

Let x and y be the x-coordinate and y-coordinate of the curve respectively.

We know that the slope of a tangent to the curve in the coordinate axis is given by the $\frac{dy}{dx}$

According to the given information, we get:

$$y\frac{dy}{dx} = x$$

$$\Rightarrow ydy = xdx$$

Integrating both sides, we get:

$$\int ydy = \int xdx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\Rightarrow y^2 - x^2 = 2C \qquad \text{..... (1)}$$

Now, the curve passes through point (0, –2).

$\therefore (-2)^2 - 02 = 2C$

$\Rightarrow 2C = 4$

Substituting 2C = 4 in equation (1), we get: $y^2 - x^2 = 4$

This is the required equation of the curve.

18:

At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (–4, –3). Find the equation of the curve given that it passes through (–2, 1).

Solution:

It is given that (x, y) is the point of contact of the curve and its tangent.

The slope (m_1) of the line segment joining (x, y) and (–4, –3) is $\frac{y+3}{x+4}$

We know that the slope of the tangent to the curve is given by the relation, $\frac{dy}{dx}$

∴ Slope (m_2) of tangent = $\frac{dy}{dx}$

According to the given information: **19:**
$m_2 = 2m_1$

$$\Rightarrow \frac{dy}{dx} = \frac{2(y+3)}{x+4}$$

$$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$$

Integrating both sides, we get:

$$\int \frac{dy}{y+3} = 2\int \frac{dx}{x+4}$$

$$\Rightarrow \log(y+3) = 2\log(x+4) + \log C$$

$$\Rightarrow \log(y+3)\log C(x+4)^2$$

$$\Rightarrow y+3 = C(x+4)^2 \quad \text{….. (1\_}$$

This is the general equation of the curve.
It is given that it passes through point (–2, 1).

$$\Rightarrow 1+3 = C(-2+4)^2$$

$$\Rightarrow 4 = 4C$$

$$\Rightarrow C = 1$$

Substituting C = 1 in equation (1), we get:
$y + 3 = (x + 4)^2$
This is the required equation of the curve.

19:
The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Solution:
Let the rate of change of the volume of the balloon be k (where k is a constant).

$$\Rightarrow \frac{dy}{dx} = k$$

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k \qquad \left[\text{Volume of sphere} = \frac{4}{3}\pi r^3\right]$$

$$\Rightarrow \frac{4}{3}\pi.3r^2.\frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 dr = k\,dt$$

Integrating both sides, we get:

$$\Rightarrow 4\pi \frac{r^3}{3} = kt + C$$

$$\Rightarrow 4\pi r^3 = 3(kt + C) \qquad \text{..... (1)}$$

Now, at t = 0, r = 3

$$\Rightarrow 4\pi \times 33 = 3(k \times 0 + C)$$

$$\Rightarrow 108\pi = 3C$$

$$\Rightarrow C = 36\pi$$

At t = 3, r = 6:

$$\Rightarrow 4\pi \times 6^3 = 3(k \times 3 + C)$$

$$\Rightarrow 864\pi = 3(3k + 36\pi)$$

$$\Rightarrow 3k = -288\pi - 36\pi = 252\pi$$

$$\Rightarrow k = 84\pi$$

Substituting the values of k and C in equation (1), we get:

$$4\pi r^3 = 3[84\pi t + 36\pi]$$

$$\Rightarrow 4\pi r^3 = 4\pi(63t + 27)$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after t seconds is $(63t + 27)^{\frac{1}{3}}$

20:

In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 doubles itself in 10 years (loge 2 = 0.6931).

Solution:

Let p, t, and r represent the principal, time, and rate of interest respectively.

It is given that the principal increases continuously at the rate of r% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{r}{100}\right)p$$

$$\Rightarrow \frac{dp}{p} = \left(\frac{r}{100}\right)dt$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{r}{100}\int dt$$

$$\Rightarrow \log p = \frac{rt}{100} + k$$

$$\Rightarrow p = e^{\frac{n}{100}+k} \qquad .....(1)$$

It is given that when t = 0, p = 100

$$\Rightarrow 100 = e^k \quad ..... (2)$$

Now, if t = 10, then p = 2 × 100 = 200

Therefore, equation (1) becomes:

$$200 = e^{\frac{n}{10}+k}$$

$$\Rightarrow 200 = e^{\frac{n}{10}+k}.e^k$$

$$\Rightarrow 200 = e^{\frac{n}{10}+k}.100 \qquad \text{From (2)}$$

$$\Rightarrow e^{\frac{r}{10}} = 2$$

$$\Rightarrow \frac{r}{10} = \log.2$$

$$\Rightarrow \frac{r}{10} = 0.6931$$

$$\Rightarrow r = 6.931$$

Hence, the value of r is 6.93%

21:

In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years $\left(e^{0.5} = 1.648\right)$.

Solution:

Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{d}{dt}\left(\frac{5}{100}\right)p$$

$$\Rightarrow \frac{dp}{dt} = \frac{p}{20}$$

$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{1}{20}\int dt$$

$$\Rightarrow \log p = \frac{t}{20} + c$$

$$\Rightarrow p = e^{\frac{1}{20}+c} \qquad .....(1)$$

Now, when t = 0, p = 100

$$\Rightarrow 1000 = e^{c} \qquad ....(2)$$

At t = 10, equation (1) becomes:

$$p = e^{\frac{1}{20}+c}$$

$$\Rightarrow p = e^{0.5} \times e^{C}$$

$$\Rightarrow P = 1.648 \times 1000$$

$$\Rightarrow P = 1648$$

Hence, after 10 years the amount will worth Rs. 1648.

22:

In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

Solution:

Let y be the number of bacteria at any instant t.

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\therefore \frac{dy}{dt} \infty y$$

$$\Rightarrow \frac{dy}{dt} = ky \text{ (Where k is a constant)}$$

$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get:

$$\int \frac{dy}{y} = k\int dt$$

$$\Rightarrow \log\ y = kt + C \quad \text{..... (1)}$$

Let y_0 be the number of bacteria at t = 0.

⇒ log y_0 = C

Substituting the value of C in equation (1), we get:

$$\Rightarrow \log\ y = kt + \log\ y_0$$

$$\Rightarrow \log y - \log y_0 = kt$$

$$\Rightarrow \log\left(\frac{y}{y_0}\right) = kt$$

$$\Rightarrow kt = \log\left(\frac{y}{y_0}\right) \qquad \text{....(2)}$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$$\Rightarrow y = \frac{110}{100} y_0$$

$$\Rightarrow \frac{y}{y_0} = \frac{11}{10} \quad \text{..... (3)}$$

Substituting this value in equation (2), we get:

$$k.2 = \log\left(\frac{11}{10}\right)$$

$$\Rightarrow k = \frac{1}{2}\log\left(\frac{11}{10}\right)$$

Therefore, equation (2) becomes:

$$\frac{1}{2}\log\left(\frac{11}{10}\right)t = \log\left(\frac{y}{y_0}\right)$$

$$\Rightarrow t = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \qquad \text{.....(4)}$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1 ⇒ y = y_0 at t = t_1

From equation (4), we get

$$t_1 = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ in hours the number of bacteria increases from 100000 to 200000.

23:

The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

A. $e^x + e^{-y} = C$

B. $e^x + e^y = C$

C. $e^{-x} + e^y = C$

D. $e^{-x} + e^{-y} = C$

Solution:

$$\frac{dy}{dx} = e^{x+y} = e^x.e^y$$

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

$$\Rightarrow e^{-y}dy = e^x dx$$

Intergrating both sides, we get:

$$\int e^{-y}dy = \int e^x dx$$

$$\Rightarrow -e^{-y} = e^x + k$$

$$\Rightarrow e^x + e^{-y} = -k$$

$$\Rightarrow e^x + e^{-y} = c \qquad (c = -k)$$

Hence, the correct answer is A.