Exercise 4.6 Page No: 136

Examine the consistency of the system of equations in Exercises 1 to 6.

1. x +2y = 2: and 2x + 3y = 3

Solution:
Given set of equations is : x +2y = 2: and 2x + 3y = 3

This set of equation can be written in the form of matrix as AX = B, where

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and

$X = \begin{bmatrix} x \\ y \end{bmatrix}$

So, AX = B is

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1 \neq 0$$

Inverse of matrix exists. So system of equations is consistent.

2. 2x – y = 5 and x + y = 4

Solution:
Given set of equations is : 2x – y = 5 and x + y = 4

This set of equation can be written in the form of matrix as AX = B, where

$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

So, AX = B is

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3 \neq 0$$

Inverse of matrix exists. So system of equations is consistent.

3. x + 3y and 2x + 6y = 8

Solution:
Given set of equations is : x + 3y and 2x + 6y = 8

This set of equation can be written in the form of matrix as AX = B.

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 0$$

$$\text{adj. } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

And

$$(\text{adj. } A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

The given equations are inconsistent.

4. x + y + z =1; 2x + 3y + 2z = 2 and ax + ay + 2az = 4

Solution:
Given set of equations is : x + y + z =1; 2x + 3y + 2z = 2 and ax + ay + 2az = 4

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix} = 1(6a-2a)-1(4a-2a)+1(2a-3a)$$

$$= 4a-2a-a=a \neq 0$$

System of equations is consistent.

5. 3x – y – 2z = 2; 2y – z = -1 and 3x – 5y = 3

Solution:

Given set of equations is : 3x – y – 2z = 2; 2y – z = -1 and 3x – 5y = 3

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

= 3(-5) +(3) -2(-6)

= 15 – 15

= 0

Now,

$$(\text{adj. } A) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\text{adj. A})B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

6. Given set of equations is :
5x – y + 4z = 5
2x + 3y + 5z = 2
5x – 2y + 6z = –1

Solution:
Given set of equations is: 5x – y + 4z = 5 ; 2x + 3y + 5z = 2; 5x – 2y + 6z = –1

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

= 5(18 + 10) + 1(12-25) + 4(-4 – 15)

= 140 – 13 -76

= 140- 89

= 51

≠ 0

System of equations is consistent.

Solve system of linear equations, using matrix method, in Exercises 7 to 14.

7. 5x + 2y = 4 and 7x + 3y = 5
Solution:

Given set of equations is : 5x + 2y = 4 and 7x + 3y = 5

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$$

And |A| = 1 ≠ 0
System is consistent.
Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

=> x = 2 and y = -3

8. 2x - y = -2 and 3x + 4y = 3

Solution:
Given set of equations is : 2x - y = -2 and 3x + 4y = 3
This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

|A| = 11 ≠ 0

System is consistent.
So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -5 \\ 12 \end{bmatrix}$$

Therefore, x = -5/11 and y = 12/11

9. 4x – 3y = 3 and 3x – 5y = 7

Solution:
Given set of equations is : 4x – 3y = 3 and 3x – 5y = 7
This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$$

And |A| = -20 + 9 = -11 ≠ 0
System is consistent.
So,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$= \frac{1}{-11}\begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 6 \\ 19 \end{bmatrix}$$

Therefore, x = 6/-11 and y = 19/-11

10. 5x + 2y = 3 and 3x + 2y = 5

Solution:
Given set of equations is : 5x + 2y = 3 and 3x + 2y = 5

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$$

And |A| = 4 ≠ 0

System is consistent.
So,

$$X = A^{-1}B \ = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Therefore, x = -1 and y = 4.

11. 2x + y + z = 1 and x – 2y – z = 3/2 and 3y – 5z = 9

Solution:
Given set of equations is : 2x + y + z = 1 and x – 2y – z = 3/2 and 3y – 5z = 9

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

$$= 34 \neq 0$$

System is consistent.
So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$$= \frac{1}{34}\begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix}$$

Therefore, x = 1, y = ½ and z = 3/2

12. x – y + z = 4 and 2x + y – 3z = 0 and x + y + z = 2

Solution:

Given set of equations is : x – y + z = 4 and 2x + y – 3z = 0 and x + y + z = 2
This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 10 \neq 0$
System is consistent.
So,

$$X = A^{-1}B \ = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Therefore, x = 2, y = -1 and z = 1

13.
2x + 3y + 3z = 5
x – 2y + z = – 4
3x – y – 2z = 3
Solution:

Given set of equations is :
2x + 3y + 3z = 5
x – 2y + z = – 4
3x – y – 2z = 3

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

And,

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

= 40 ≠ 0

System is consistent.
So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z = -1.

14.
x – y + 2z = 7
3x + 4y – 5z = – 5
2x – y + 3z = 12

Solution:
Given set of equations is :
x – y + 2z = 7
3x + 4y – 5z = – 5
2x – y + 3z = 12

This set of equation can be written in the form of matrix as AX = B

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

= 4 ≠ 0

System is consistent.

So,

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Therefore, x = 2, y = 1 and z = 3.

15. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations.
2x – 3y + 5z = 11
3x + 2y – 4z = – 5
x + y – 2z = – 3

Solution:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

= -1 ≠ 0; Inverse of matrix exists.

Find the inverse of matrix:
Cofactors of matrix:

$$A_{11} = 0, A_{12} = 2, A_{13} = 1$$

$$A_{21} = -1, A_{22} = -9, A_{23} = -5$$

$$A_{31} = 2, A_{32} = 23, A_{33} = 13$$

$$\text{adj. } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, matrix of equation can be written as:
AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

And, X = A^{-1} B

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z =3.

16. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 2 kg onion, 4 kg wheat and 2 kg rice is Rs. 90. The cost of 6 kg onion, 2 k wheat and 3 kg rice is Rs. 70. Find cost of each item per kg by matrix method.

Solution:
Let x, y and z be the per kg. prices of onion, wheat and rice respectively.
According to given statement, we have following equations,
4x + 3y + 2z = 60
2x + 4y + 6z = 90
6x + 2y + 3z = 70

The above system of equations can be written in the form of matrix as, AX = B

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$$

And

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix}$$

= 4(0) – 3(-30) + 2(-20)

= 50 ≠ 0

System is consistent, and $X = A^{-1} B$

First find invers of A.
Cofactors of all the elements of A are:

$$A_{11} = 0, A_{12} = 30, A_{13} = -20$$
$$A_{21} = -5, A_{22} = 0, A_{23} = 10$$
$$A_{31} = 10, A_{32} = -20, A_{33} = 10$$

$$\text{adj. } A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Again,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} -450+700 \\ 1800-1400 \\ -1200+900+700 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Therefore, x = 5, y = 8 and z = 8.
The cost of onion, wheat and rice per kg are Rs. 5, Rs, 8 and Rs. 8 respectively.