

Exercise 6.4

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1. Using differentials, find the approximate value of each of the following up to 3 places of decimal:

- (i) $\sqrt{25.3}$
- (ii) $\sqrt{49.5}$
- (iii) $\sqrt{0.6}$
- (iv) $(0.009)^{\frac{1}{3}}$
- (v) $(0.999)^{\frac{1}{10}}$
- (vi) $(15)^{\frac{1}{4}}$
- (vii) $(26)^{\frac{1}{3}}$
- (viii) $(255)^{\frac{1}{4}}$
- (ix) $(82)^{\frac{1}{4}}$
- (x) $(401)^{\frac{1}{2}}$
- (xi) $(0.0037)^{\frac{1}{2}}$
- (xii) $(26.57)^{\frac{1}{3}}$
- (xiii) $(81.5)^{\frac{1}{4}}$
- (xiv) $(3.968)^{\frac{3}{2}}$
- (xv) $(32.15)^{\frac{1}{5}}$

Solution:

(i) $\sqrt{25.3}$

Consider, $y = \sqrt{x}$ (1) and then

$$y + \Delta y = \sqrt{x + \Delta x}$$

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow dy = \frac{dx}{2\sqrt{x}} \dots\dots\dots(2)$$

Now, given expression can be written as,

$$\sqrt{25.3} = \sqrt{25+0.3}$$

Here, $x = 25$ and $\Delta x = 0.3$, then $\Delta y = \sqrt{x+\Delta x} - \sqrt{x}$

$$= \sqrt{25.3} - \sqrt{25} = \sqrt{25.3} - 5$$

$$\Rightarrow \sqrt{25.3} = \Delta y + 5$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$$dy = \frac{0.3}{2\sqrt{25}}$$

$$= 0.03$$

Hence, approximately value of $\sqrt{25.3}$ is $5 + 0.03 = 5.03$.

(ii) $\sqrt{49.5}$

Consider, $y = \sqrt{x} \dots\dots\dots(1)$

On differentiating equation (1) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow dy = \frac{dx}{2\sqrt{x}} \dots\dots\dots(2)$$

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{49.5} = \sqrt{49 + 0.5}$$

Here, $x = 49$ and $\Delta x = 0.5$, then

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \sqrt{49.5} - \sqrt{49} = \sqrt{49.5} - 7$$

$$\Rightarrow \sqrt{49.5} = \Delta y + 7$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\text{From equation (2), } dy = \frac{0.5}{2\sqrt{49}}$$

$$= 0.0357$$

So, approximately value of $\sqrt{49.5}$ is $7 + 0.0357 = 7.0357$.

(iii) $\sqrt{0.6}$

Consider, $y = \sqrt{x}$ (1)

On differentiating equation (1) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$dy = \frac{dx}{2\sqrt{x}} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{0.6} = \sqrt{0.64 - 0.04}$$

Here, $x = 0.64$ and $\Delta x = -0.04$, then

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \sqrt{0.6} - \sqrt{0.64} = \sqrt{0.6} - 0.8$$

$$\sqrt{0.6} = \Delta y + 0.8$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2), $dy = \frac{-0.04}{2\sqrt{0.64}} = -0.025$

Therefore, approximately value of $\sqrt{0.6}$ is $0.8 - 0.025 = 0.775$.

(iv) $(0.009)^{\frac{1}{3}}$

Consider, $y = \sqrt[3]{x}$ (1)

On differentiating equation (1) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

$$dy = \frac{dx}{3x^{\frac{2}{3}}} = \frac{dx}{3\left(x^{\frac{1}{3}}\right)^2}$$

.....(2)

Now, from equation (1), $y + \Delta y = \sqrt[3]{x + \Delta x}$

$$= (0.009)^{\frac{1}{3}} = (0.008 + 0.001)^{\frac{1}{3}}$$

Here, $x = 0.008$ and $\Delta x = 0.001$,

then $\Delta y = \sqrt[3]{x + \Delta x} - \sqrt[3]{x}$

$$= (0.009)^{\frac{1}{3}} - (0.008)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - 0.2$$

$$(0.009)^{\frac{1}{3}} = \Delta y + 0.2$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation

$$dy = \frac{0.001}{3 \left((0.008)^{\frac{1}{3}} \right)^2}$$

(2),

$$= \frac{0.001}{3 \times 0.04} = 0.0083$$

Therefore, approximately value of $(0.009)^{\frac{1}{3}}$ is $0.2 + 0.0083 = 0.2083$.

(v) $(0.999)^{\frac{1}{10}}$

Consider, $y = x^{\frac{1}{10}}$ (1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{10} x^{-\frac{9}{10}} = \frac{1}{10x^{\frac{9}{10}}}$$

$$dy = \frac{dx}{10 \left(x^{\frac{1}{10}} \right)^9} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt[10]{x + \Delta x}$

$$= (0.999)^{\frac{1}{10}} = (1 - 0.001)^{\frac{1}{10}} \text{(3)}$$

Here $x = 1$ and $\Delta x = -0.001$

$$\text{Then } \Delta y = (x + \Delta x)^{\frac{1}{10}} - x^{\frac{1}{10}}$$

$$= (0.999)^{\frac{1}{10}} - 1$$

$$\Rightarrow (0.999)^{\frac{1}{10}} = 1 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$$\frac{dy}{dx} = \frac{-0.001}{10 \left(1^{\frac{1}{10}}\right)^9} = -0.0001$$

Therefore, approximate value of $(0.999)^{\frac{1}{10}}$ is $1 - 0.0001 = 0.9999$.

(vi) $(15)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ (1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{4} x^{-\frac{3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dx}{4 \left(x^{\frac{1}{4}}\right)^3} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt[4]{x + \Delta x}$

$$= (15)^{\frac{1}{4}} = (16-1)^{\frac{1}{4}} \text{(3)}$$

Here $x=16$ and $\Delta x=-1$

$$\text{Then } \Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$$

$$= (15)^{\frac{1}{4}} - 16^{\frac{1}{4}} = (15)^{\frac{1}{4}} - 2$$

$$\Rightarrow (15)^{\frac{1}{4}} = 2 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$dy = \frac{-1}{4 \left(16^{\frac{1}{4}}\right)^3} = \frac{-1}{32}$$

From equation (2),

Therefore, approximate value of $(15)^{\frac{1}{4}}$ is $2 - \frac{1}{32} = \frac{63}{32} = 1.96875$.

(vii) $(26)^{\frac{1}{3}}$

Consider, $y = \sqrt[3]{x}$ (1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

$$dy = \frac{dx}{3x^{\frac{2}{3}}} = \frac{dx}{3 \left(x^{\frac{1}{3}}\right)^2} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt[3]{x + \Delta x}$

$$= (26)^{\frac{1}{3}} = (27 - 1)^{\frac{1}{3}}$$

Here, $x = 27$ and $\Delta x = -1$,

$$\text{then } \Delta y = \sqrt[3]{x + \Delta x} - \sqrt[3]{x} = (26)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26)^{\frac{1}{3}} - 3$$

$$(26)^{\frac{1}{3}} = \Delta y + 3$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$$\frac{dy}{dx} = \frac{-1}{3 \left((27)^{\frac{1}{3}} \right)^2}$$

$$= \frac{-1}{27}$$

Therefore, approximately value of $(26)^{\frac{1}{3}}$ is $3 - \frac{1}{27} = \frac{80}{27} = 2.9629$.

(viii) $(255)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ (1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{4} x^{-\frac{3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\frac{dy}{dx} = \frac{dx}{4 \left(x^{\frac{3}{4}} \right)} \dots\dots\dots(2)$$

Now, from equation (1), $y + \Delta y = \sqrt[4]{x + \Delta x}$

$$= (15)^{\frac{1}{4}} = (256 - 1)^{\frac{1}{4}} \dots\dots\dots(3)$$

Here $x = 256$ and $\Delta x = -1$

Then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

$$= (255)^{\frac{1}{4}} - (256)^{\frac{1}{4}} = (255)^{\frac{1}{4}} - 4$$

$$(255)^{\frac{1}{4}} = 4 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$dy = \frac{-1}{4 \left(256^{\frac{1}{4}} \right)^3} = \frac{-1}{256}$$

From equation (2),

Therefore, approximate value of $(255)^{\frac{1}{4}}$ is $4 - \frac{1}{256} = \frac{1023}{256} = 3.9961$.

(ix) $(82)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ (1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{4} x^{-\frac{3}{4}}$$

$$= \frac{1}{4x^{\frac{3}{4}}}$$

$$dy = \frac{dx}{4 \left(x^{\frac{1}{4}} \right)^3} \dots\dots\dots(2)$$

Now, from equation (1), $y + \Delta y = \sqrt[4]{x + \Delta x} = (82)^{\frac{1}{4}} = (81+1)^{\frac{1}{4}} \dots\dots\dots(3)$

Here $x = 81$ and $\Delta x = 1$

then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

$$= (82)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (82)^{\frac{1}{4}} - 3$$

$$(82)^{\frac{1}{4}} = 3 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$dy = \frac{1}{4 \left(81^{\frac{1}{4}} \right)^3} = \frac{1}{108}$$

From equation (2),

Therefore, approximate value of $(82)^{\frac{1}{4}}$ is $3 + \frac{1}{108} = \frac{325}{108} = 3.0092$.

(x) $\sqrt{401}$

Consider, $y = \sqrt{x}$ (1)

On differentiating equation (1) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$dy = \frac{dx}{2\sqrt{x}} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{401} = \sqrt{400 + 1}$$

Here, $x = 400$ and $\Delta x = 1$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$$= \sqrt{401} - \sqrt{400} = \sqrt{401} - 20$$

$$\Rightarrow \sqrt{401} = \Delta y + 20$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\text{From equation (2), } dy = \frac{1}{2\sqrt{400}} = \frac{1}{40}$$

Therefore, approximately value of $\sqrt{401}$ is $20 + \frac{1}{40} = \frac{801}{40} = 20.025$.

(xi) $\sqrt{0.0037}$

Consider, $y = \sqrt{x}$ (1)

On differentiating equation (1) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$dy = \frac{dx}{2\sqrt{x}} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$$= \sqrt{0.0037} = \sqrt{0.0036 + 0.0001}$$

Here, $x = 0.0036$ and $\Delta x = 0.0001$, then

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

$$= \sqrt{0.0037} - \sqrt{0.0036} = \sqrt{0.0037} - 0.06$$

$$\sqrt{0.0037} = \Delta y + 0.06$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$\text{From equation (2), } dy = \frac{0.0001}{2\sqrt{0.0036}}$$

$$= \frac{0.0001}{0.12}$$

Therefore, approximately value of $\sqrt{0.0037}$ is $0.06 + \frac{0.0001}{0.12} = 0.060833$.

(xi) $(26.57)^{\frac{1}{3}}$

Consider, $y = \sqrt[3]{x}$ (1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$$

$$dy = \frac{dx}{3x^{\frac{2}{3}}} = \frac{dx}{3\left(x^{\frac{1}{3}}\right)^2} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (26.57)^{\frac{1}{3}} = (27 - 0.43)^{\frac{1}{3}}$$

Here, $x = 27$ and $\Delta x = -0.43$,

then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$$= (26.57)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - 3$$

$$(26.57)^{\frac{1}{3}} = \Delta y + 3$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$dy = \frac{-0.43}{3\left((27)^{\frac{1}{3}}\right)^2}$$

From equation (2),

$$= \frac{-0.43}{27} = 0.0159$$

Therefore, approximately value of $(26.57)^{\frac{1}{3}}$ is $3 - 0.0159 = 2.9841$.

(xii) $(81.5)^{\frac{1}{4}}$

Consider, $y = x^{\frac{1}{4}}$ (1)

On differentiating equation (1) w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{4} x^{-\frac{3}{4}} = \frac{1}{4x^{\frac{3}{4}}}$$

$$dy = \frac{dx}{4 \left(x^{\frac{1}{4}} \right)^3} \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt[4]{x + \Delta x}$

$$= (81.5)^{\frac{1}{4}} = (81 + 0.5)^{\frac{1}{4}} \text{(3)}$$

Here $x = 81$ and $\Delta x = 0.5$

Then $\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}}$

$$= (81.5)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (81.5)^{\frac{1}{4}} - 3$$

$$(81.5)^{\frac{1}{4}} = 3 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

$$dy = \frac{0.5}{4 \left(81^{\frac{1}{4}} \right)^3} = \frac{0.5}{108}$$

From equation (2),

$$= 0.00462$$

Therefore, approximate value of $(82)^{\frac{1}{4}}$ is $3 + 0.00462 = 3.00462$.

(xiv) $(3.968)^{\frac{3}{2}}$

Consider, $y = x^{\frac{3}{2}} = x^{1+\frac{1}{2}} = x\sqrt{x}$ (1)

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$$

$$dy = \frac{3\sqrt{x}}{2} dx \text{(2)}$$

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (3.968)^{\frac{3}{2}} = (4 - 0.032)^{\frac{3}{2}}$$

Here, $x = 4$ and $\Delta x = 0.032$, then $\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$

$$= (3.968)^{\frac{3}{2}} - (4)^{\frac{3}{2}} = (3.968)^{\frac{3}{2}} - 8$$

$$(3.968)^{\frac{3}{2}} = \Delta y + 8$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$$dy = \frac{3}{2}\sqrt{4}(-0.032) = -0.096$$

Therefore, approximately value of $(3.968)^{\frac{3}{2}}$ is $8 - 0.096 = 7.904$.

(xv) $(32.15)^{\frac{1}{3}}$

Consider, $y = x^{\frac{1}{3}}$ (1)

On differentiating equation (1) w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{5}x^{-\frac{4}{5}} = \frac{1}{5x^{\frac{4}{5}}}$$

$$dy = \frac{dx}{5\left(x^{\frac{1}{5}}\right)^4} \dots\dots\dots(2)$$

Now, from equation (1), $y + \Delta y = \sqrt{x + \Delta x}$

$$= (32.15)^{\frac{1}{5}} = (32 + 0.15)^{\frac{1}{5}} \dots\dots\dots(3)$$

Here $x = 32$ and $\Delta x = 0.15$

$$\text{Then } \Delta y = (x + \Delta x)^{\frac{1}{5}} - x^{\frac{1}{5}}$$

$$= (32.15)^{\frac{1}{5}} - (32)^{\frac{1}{5}} = (32.15)^{\frac{1}{5}} - 2$$

$$(32.15)^{\frac{1}{5}} = 2 + \Delta y$$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (2),

$$dy = \frac{0.15}{5\left(32^{\frac{1}{5}}\right)^4} = \frac{0.15}{80} = 0.001875$$

Therefore, approximate value of $(32.15)^{\frac{1}{5}}$ is $2 + 0.001875 = 2.001875$.

2. Find the approximate value of $f(2.01)$ where $f(x) = 4x^2 + 5x + 2$.

Solution: Consider, $f(x) = y = x^3 + 5x + 2 \dots\dots\dots(1)$

On differentiating equation (1) w.r.t. x , we get

$$f'(x) = \frac{dy}{dx} = 8x + 5$$

$$dy = (8x + 5) dx$$

$$= (8x + 5) \Delta x \dots\dots\dots(2)$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in equation (1),

$$y + \Delta y = f(x + \Delta x)$$

$$= f(2.01) = f(2 + 0.01) \dots\dots\dots(3)$$

Here, $x = 2$ and $\Delta x = 0.01$

From equation (3), $f(2.01) = y + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (1) and (2), we get

$$f(2.01) = (4x^2 + 5x + 2) + (8x + 5) \Delta x$$

Therefore,

$$f(2.01) = 4(4) + 5(2) + 2 + (8 \times 2 + 5)(0.01) = 28.21$$

Therefore, approximate value of $f(2.01)$ is 28.21.

3. Find the approximate value of $f(5.001)$ where $f(x) = x^3 - 7x^2 + 15$.

Solution: Consider, $f(x) = y = x^3 - 7x^2 + 15 \dots\dots\dots(1)$

On differentiating equation (1) w.r.t. x , we get

$$f'(x) = \frac{dy}{dx} = 3x^2 - 14x$$

or $dy = (3x^2 - 14x) dx = (3x^2 - 14x) \Delta x$ (2)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in equation (1),

$$y + \Delta y = f(x + \Delta x)$$

$$= f(5.001) = f(5 + 0.001) \text{(3)}$$

Here, $x = 5$ and $\Delta x = 0.001$

From equation (3), $f(5.001) = y + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (1) and (2),

$$f(5.001) = (x^3 - 7x^2 + 15) + (3x^2 - 14x) \Delta x$$

$$f(5.001) = 125 - 175 + 15 + (75 - 70)(0.001)$$

$$= -35 + 0.005 = -34.995$$

Therefore, approximate value of $f(5.001)$ is -34.995 .

4. Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 1%.

Solution:

Side of a cube is x meters then cube volume (V) is x^3 .

or $V = x^3$ (1)

On differentiating equation (1) w.r.t. x , we get

$$\frac{dV}{dx} = 3x^2 \text{(2)}$$

NCERT Solutions for Class 12 Maths Chapter 6 Application of Derivatives

According to the

statement, increase in side = $1\% = \frac{x}{100}$

$$\text{So, } \Delta x = \frac{x}{100} \dots\dots\dots(3)$$

Approximate change in volume, V, of cube = $\Delta V \sim dV$

$$= \frac{dV}{dx} dx$$

$$= \frac{dV}{dx} \Delta x$$

$$\sim 3x^2 \left(\frac{x}{100} \right) \sim \frac{3}{100} x^3 \sim 0.03x^3 \text{ cubic meters}$$

5. Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%.

Solution:

Side of a cube is x meters then Surface area of a cube is (S) = $6x^2$

$$S = 6x^2$$

On differentiating above equation w.r.t. x, we get

$$\frac{dS}{dx} = 12x$$

At per question, if decrease in side 1% is

$$= -1\% \text{ of } x$$

$$= -0.01x$$

$$\Delta x = -0.01x$$

Since approximate change in surface area = $\Delta S \sim dS = \frac{dS}{dx} dx$

We have,

$$= \frac{dS}{dx} \Delta x \sim 12x(-0.01x) \sim -0.12x^2 \text{ square meters (decreasing)}$$

6. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

Solution: Consider, r be the radius of the sphere and Δr be the error.
then, as per question, $r = 7$ m and $\Delta r = 0.02$ m

We know that, Volume of sphere (V) = $\frac{4}{3}\pi r^3$

$$\frac{dV}{dr} = \frac{4}{3}\pi \cdot 3r^2$$

Approximate error in calculating the volume = Approximate value of ΔV

$$dV = \frac{dV}{dr} (dr)$$

$$= \left(\frac{4}{3}\pi \cdot 3r^2 \right) dr$$

$$= 4\pi(7)^2(0.02)$$

$$= 3.92 \times \frac{22}{7}$$

$$= 12.32 \text{ m}^3$$

Therefore, the approximate error in calculating volume is 12.32 m^3 .

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

Solution: Consider, r be the radius of the sphere.

And, Surface area of the sphere (S) = $4\pi r^2$ (formula for SA)

$$\frac{dS}{dr} = 8\pi r$$

$$dS = 8\pi r \, dr$$

$$dS = 8\pi r \, \Delta r$$

$$dS = 8\pi(9)(0.3)$$

$$= 2.16\pi \text{ square meters}$$

8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is:

- (A) 47.66 (B) 57.66 (C) 67.66 (D) 77.66

Solution:

Option (D) is correct.

Explanation:

Consider, $f(x) = y = 3x^2 + 15x + 5$ (1)

On differentiating equation (1) w.r.t. x , we get

$$f'(x) = \frac{dy}{dx} = 6x + 15$$

$$\text{or } dy = (6x + 15) \, dx = (6x + 15) \, \Delta x \text{(2)}$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in equation (1),

$$y + \Delta y = f(x + \Delta x)$$

$$= f(3.02) = f(3 + 0.02) \text{(3)}$$

Here, $x = 3$ and $\Delta x = 0.02$

So,

From equation (3), $f(3.02) = y + \Delta y$

Since, Δx and Δy is approximately equal to dx and dy respectively.

From equation (1) and (2),

$$f(3.02) = (3x^2 + 15x + 5) + (6x + 15)\Delta x$$

$$f(3.02) = 3(9) + 15(3) + 5 + (6 \times 3 + 15)(0.02)$$

$$= 77 + 0.66 = 77.66$$

9. The approximate change in the volume of a cube of side x meters caused by increasing the side by 3% is:

- (A) $0.06 x^3 \text{ m}^3$ (B) $0.6 x^3 \text{ m}^3$ (C) $0.09 x^3 \text{ m}^3$ (D) $0.9 x^3 \text{ m}^3$

Solution: option (C) is correct.

Explanation:

We know that, Volume (V) = x^3 (1)

$$\frac{dV}{dx} = 3x^2 \text{(2)}$$

As there is increase in side = $3\% = \frac{3x}{100}$

We have, $\Delta x = \frac{3x}{100}$ (3)

Since approximate change in volume V of cube = $\Delta V \sim dV = \frac{dV}{dx} dx$

$$= \frac{dV}{dx} \Delta x$$

$$\sim 3x^2 \left(\frac{3x}{100} \right) \sim \frac{9}{100} x^3 \sim 0.09x^3 \text{ cubic meters}$$