Exercise 1.1 Page No: 5

1. Determine whether each of the following relations are reflexive, symmetric and transitive:
(i) Relation R in the set A = {1, 2, 3,…….., 13, 14} defined as
R = {(x, y) : 3x – y = 0}

(ii) Relation R in the set N of natural numbers defined as
R = {(x, y) : y = x + 5 and x < 4}

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as
R = {(x, y) : y is divisible by x}

(iv) Relation R in the set Z of all integers defined as
R = {(x, y) : x – y is an integer}

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) R = {(x, y) : x and y work at the same place}
(b) R = {(x, y) : x and y live in the same locality}
(c) R = {(x, y) : x is exactly 7 cm taller than y}
(d) R = {(x, y) : x is wife of y}
(e) R = {(x, y) : x is father of y}

Solution:
(i)R = {(x, y) : 3x - y = 0}

A = {1, 2, 3, 4, 5, 6, ……13, 14}

Therefore, **R = {(1, 3), (2, 6), (3, 9), (4, 12)}** …(1)

As per reflexive property: (x, x) ∈ R, then R is reflexive)
Since there is no such pair, so R is not reflexive.

As per symmetric property: (x, y) ∈ R and (y, x) ∈ R, then R is symmetric.
Since there is no such pair, R is not symmetric

As per transitive property: If (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R. Thus R is transitive.

From (1), (1, 3) ∈ R and (3, 9) ∈ R but (1, 9) ∉ R, R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(ii) R = {(x, y) : y = x + 5 and x < 4} in set N of natural numbers.

Values of x are 1, 2, and 3

So, **R = {(1, 6), (2, 7), (3, 8)}**

As per reflexive property: (x, x) ∈ R, then R is reflexive)

Since there is not such pair, R is not reflexive.

As per symmetric property: (x, y) ∈ R and (y, x) ∈ R, then R is symmetric.

Since there is no such pair, so R is not symmetric

As per transitive property: If (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R. Thus R is transitive.

Since there is no such pair, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(iii) R = {(x, y) : y is divisible by x} in A = {1, 2, 3, 4, 5, 6}

From above we have,

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}

As per reflexive property: (x, x) ∈ R, then R is reflexive.

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6) ∈ R . Therefore, R is reflexive.

As per symmetric property: (x, y) ∈ R and (y, x) ∈ R, then R is symmetric.

(1, 2) ∈ R but (2, 1) ∉ R. So R is not symmetric.

As per transitive property: If (x, y) ∈ R and (y, z) ∈ R, then (x, z) ∈ R. Thus R is transitive.

Also (1, 4) ∈ R and (4, 4) ∈ R and (1, 4) ∈ R, So R is transitive.

Therefore, R is reflexive and transitive but nor symmetric.

(iv) R = {(x, y) : x - y is an integer} in set Z of all integers.

Now, (x, x), say (1, 1) = x – y = 1 – 1 = 0 $\in$ Z => R is reflexive.

(x, y) $\in$ R and (y, x) $\in$ R, i.e.,
x – y and y – x are integers => R is symmetric.

(x, y) $\in$ R and (y, z) $\in$ R, then (x, z) $\in$ R i.e.,

x – y and y – z and x - z are integers.

(x, z) $\in$ R => R is transitive

Therefore, R is reflexive, symmetric and transitive.

(v)
(a) R = {(x, y) : x and y work at the same place}
For reflexive: x and x can work at same place
(x, x) $\in$ R
R is reflexive.

For symmetric: x and y work at same place so y and x also work at same place.
(x, y) $\in$ R and (y, x) $\in$ R
R is symmetric.

For transitive: x and y work at same place and y and z work at same place, then x and z also work at same place.

(x, y) $\in$ R and (y, z) $\in$ R then (x, z) $\in$ R
R is transitive

Therefore, R is reflexive, symmetric and transitive.

(b) R = {(x, y) : x and y live in the same locality}

(x, x) $\in$ R => R is reflexive.

(x, y) $\in$ R and (y, x) $\in$ R => R is symmetric.

Again,
(x, y) $\in$ R and (y, z) $\in$ R then (x, z) $\in$ R => R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(c) R = {(x, y) : x is exactly 7 cm taller than y}

x can not be taller than x, so R is not reflexive.

x is taller than y then y can not be taller than x, so R is not symmetric.

Again, x is 7 cm taller than y and y is 7 cm taller than z, then x can not be 7 cm taller than z, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(d) R = {(x, y) : x is wife of y}

x is not wife of x, so R is not reflexive.

x is wife of y but y is not wife of x, so R is not symmetric.

Again, x is wife of y and y is wife of z then x can not be wife of z, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(e) R = {(x, y) : x is father of y}

x is not father of x, so R is not reflexive.

x is father of y but y is not father of x, so R is not symmetric.

Again, x is father of y and y is father of z then x cannot be father of z, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

2. Show that the relation R in the set R of real numbers, defined as R = {(a, b) : a ≤ b^2} is neither reflexive nor symmetric nor transitive.

Solution:
R = {(a, b) : a ≤ b^2} , Relation R is defined as the set of real numbers.

(a, a) ∈ R then a ≤ a^2 , which is false. R is not reflexive.

(a, b)=(b, a) ∈ R then a ≤ b^2 and b ≤ a^2, it is false statement. R is not symmetric.

Now, a ≤ b^2 and b ≤ c^2,then a ≤ c^2 , which is false. R is not transitive

Therefore, R is neither reflexive, nor symmetric and nor transitive.

3. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

Solution: R = {(a, b) : b = a + 1}

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

When b = a, a = a + 1: which is false, So R is not reflexive.

If (a, b) = (b,a), then b = a+1 and a =b+1: Which is false, so R is not symmetric.

Now, if (a, b), (b,c) and (a, c) belongs to R then
b = a+1 and c =b+1 which implies c = a + 2: Which is false, so R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

4. Show that the relation R in R defined as R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric.

Solution:
a ≤ a: which is true, (a, a) ∈ R, So R is reflexive.

a ≤ b but b ≤ a (false): (a, b) ∈ R but (b, a) ∉ R, So R is not symmetric.

Again, a ≤ b and b ≤ c then a ≤ c : (a, b) ∈ R and (b, c) and (a, c) ∈ R, So R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

5. Check whether the relation R in R defined by R = {(a, b) : $a \leq b^3$} is reflexive, symmetric or transitive.

Solution: R = {(a, b) : $a \leq b^3$}

$a \leq a^3$: which is true, (a, a) ∉ R, So R is not reflexive.

$a \leq b^3$ but $b \leq a^3$ (false): (a, b) ∈ R but (b, a) ∉ R, So R is not symmetric.

Again, $a \leq b^3$ and $b \leq c^3$ then $a \leq c^3$ (false) : (a, b) ∈ R and (b, c) ∈ R and (a, c) ∉ R, So R is transitive.

Therefore, R is neither reflexive, nor transitive and nor symmetric.

6. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Solution:

R = {(1, 2), (2, 1)}

(x, x) ∉ R. R is not reflexive.

(1, 2) ∈ R and (2,1) ∈ R. R is symmetric.

Again, (x, y) ∈ R and (y, z) ∈ R then (x, z) does not imply to R. R is not transitive.

Therefore, R is symmetric but neither reflexive nor transitive.

7. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

Solution:

Books x and x have same number of pages. (x, x) ∈ R. R is reflexive.

If (x, y) ∈ R and (y, x) ∈ R, so R is symmetric.
Because, Books x and y have same number of pages and Books y and x have same number of pages.

Again, (x, y) ∈ R and (y, z) ∈ R and (x, z) ∈ R. R is transitive.

Therefore, R is an equivalence relation.

8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

Solution:

A = {1, 2, 3, 4, 5} and R = {(a, b) : |a – b| is even}

We get, R = {(1, 3), (1, 5), (3, 5), (2, 4)}

For (a, a), |a – b| = |a – a| = 0 is even. Therfore, R is reflexive.

If |a – b| is even, then |b – a| is also even. R is symmetric.

Again, if |a – b| and |b – c| is even then |a – c| is also even. R is transitive.

Therefore, R is an equivalence relation.

(b) We have to show that, Elements of {1, 3, 5} are related to each other.

|1 – 3| = 2
|3 – 5| = 2
|1 - 5| = 4
All are even numbers.

Elements of {1, 3, 5} are related to each other.

Similarly, |2 - 4| = 2 (even number), elements of (2, 4) are related to each other.

Hence no element of {1, 3, 5} is related to any element of {2, 4}.

9. Show that each of the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by
(i) R = {(a, b) : |a – b| is a multiple of 4}
(ii) R = {(a, b) : a = b}
is an equivalence relation. Find the set of all elements related to 1 in each case.

Solution:

(i) A = {x ∈ Z : 0 ≤ x ≤ 12}
So, A = {0, 1, 2, 3, ………, 12}

Now R = {(a, b) : |a – b| is a multiple of 4}

R = {(4, 0), (0, 4), (5, 1), (1, 5), (6, 2), (2, 6), ..., (12, 9), (9, 12),...., (8, 0), (0, 8), ..., (8, 4), (4, 8),....., (12, 12)}

Here, (x, x) = |4-4| = |8-8|= |12-12| = 0 : multiple of 4.

R is reflexive.

|a – b| and |b – a| are multiple of 4. (a, b) ∈ R and (b, a) ∈ R.

R is symmetric.

And |a – b| and |b – c| then |a - c| are multiple of 4. (a, b) ∈ R and (b, c) ∈ R and (a, c) ∈ R
R is transitive.

Hence R is an equivalence relation.

(ii) Here, (a, a) = a = a.

(a, a) ∈ R . So R is reflexive.

a = b and b = a. (a, b) ∈ R and (b, a) ∈ R.

R is symmetric.

And a = b and b = c then a = c. (a, b) ∈ R and (b, c) ∈ R and (a, c) ∈ R
R is transitive.

Hence R is an equivalence relation.

Now set of all elements related to 1 in each case is

(i) Required set = {1, 5, 9}
(ii) Required set = {1}

10. Give an example of a relation. Which is
(i) Symmetric but neither reflexive nor transitive.
(ii) Transitive but neither reflexive nor symmetric.
(iii) Reflexive and symmetric but not transitive.
(iv) Reflexive and transitive but not symmetric.
(v) Symmetric and transitive but not reflexive.

Solution:
(i)Consider a relation R = {(1, 2), (2, 1)} in the set {1, 2, 3}

(x, x) ∉ R. R is not reflexive.

(1, 2) ∈ R and (2,1) ∈ R. R is symmetric.

Again, (x, y) ∈ R and (y, z) ∈ R then (x, z) does not imply to R. R is not transitive.

Therefore, R is symmetric but neither reflexive nor transitive.

(ii) Relation R = {(a, b): a > b}

a > a (false statement).
Also a > b but b > a (false statement) and
If a > b but b > c, this implies a > c

Therefore, R is transitive, but neither reflexive nor symmetric.

(iii) R = {a, b): a is friend of b}

a is friend of a. R is reflexive.

Also a is friend of b and b is friend of a. R is symmetric.

Also if a is friend of b and b is friend of c then a cannot be friend of c. R is not transitive.

Therefore, R is reflexive and symmetric but not transitive.

(iv) Say R is defined in R as R = {(a, b) : a ≤ b}

a ≤ a: which is true, (a, a) ∈ R, So R is reflexive.

a ≤ b but b ≤ a (false): (a, b) ∈ R but (b, a) ∉ R, So R is not symmetric.

Again, a ≤ b and b ≤ c then a ≤ c : (a, b) ∈ R and (b, c) and (a, c) ∈ R, So R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

(v)R = {(a, b): a is sister of b} (suppose a and b are female)

a is not sister of a. R is not reflexive.

a is sister of b and b is sister of a. R is symmetric.

Again, a is sister of b and b is sister of c then a is sister of c.

Therefore, R is symmetric and transitive but not reflexive.

11. Show that the relation R in the set A of points in a plane given by R = {(P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Solution: R = {(P, Q): distance of the point P from the origin is the same as the distance of the point Q from the origin}

Say "O" is origin Point.
Since the distance of the point P from the origin is always the same as the distance of the same point P from the origin.
OP = OP

So (P, P) R. R is reflexive.

Distance of the point P from the origin is the same as the distance of the point Q from the origin

OP = OQ then OQ = OP
R is symmetric.

Also OP = OQ and OQ = OR then OP = OR. R is transitive.

Therefore, R is an equivalent relation.

12. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Solution:
Case I:
T_1, T_2 are triangle.

R = {(T_1, T_2): T_1 is similar to T_2}

Check for reflexive:

As We know that each triangle is similar to itself, so (T_1, T_1) ∈ R
R is reflexive.

Check for symmetric:

Also two triangles are similar, then T_1 is similar to T_2 and T_2 is similar to T_1, so (T_1, T_2) ∈ R and (T_2, T_1) ∈ R
R is symmetric.

Check for transitive:

Again, if then T_1 is similar to T_2 and T_2 is similar to T_3, then T_1 is similar to T_3 , so (T_1, T_2) ∈ R and (T_2, T_3) ∈ R and (T_1, T_3) ∈ R
R is transitive

Therefore, R is an equivalent relation.

Case 2: It is given that T_1, T_2 and T_3 are right angled triangles.

T_1 with sides 3, 4, 5
T_2 with sides 5, 12, 13 and
T_3 with sides 6, 8, 10

Since, two triangles are similar if corresponding sides are proportional.

Therefore, 3/6 = 4/8 = 5/10 = 1/2

Therefore, T_1 and T_3 are related.

13. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) :P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Solution:

Case I:
R = {(P_1, P_2) :P_1 and P_2 have same number of sides}
Check for reflexive:

P_1 and P_1 have same number of sides, So R is reflexive.

Check for symmetric:

P_1 and P_2 have same number of sides then P_2 and P_1 have same number of sides, so (P_1, P_2) ∈ R and (P_2, P_1) ∈ R
R is symmetric.

Check for transitive:

Again, P_1 and P_2 have same number of sides, and P_2 and P_3 have same number of sides, then also P_1 and P_3 have same number of sides .
So (P_1, P_2) ∈ R and (P_2, P_3) ∈ R and (P_1, P_3) ∈ R
R is transitive

Therefore, R is an equivalent relation.

Since 3, 4, 5 are the sides of a triangle, the triangle is right angled triangle. Therefore, the set A is the set of right angled triangle.

14. Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L_1, L_2) : L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

Solution:

L_1 is parallel to itself i.e., $(L_1, L_1) \in R$

R is reflexive

Now, let $(L_1, L_2) \in R$

L_1 is parallel to L_2 and L_2 is parallel to L_1

$(L_2, L_1) \in R$, Therefore, R is symmetric

Now, let (L_1, L_2), $(L_2, L_3) \in R$

L_1 is parallel to L_2. Also, L_2 is parallel to L_3

L_1 is parallel to L_3

Therefore, R is transitive

Hence, R is an equivalence relation.

Again, The set of all lines related to the line y = 2x + 4, is the set of all its parallel lines.
Slope of given line is m = 2.
As we know slope of all parallel lines are same.
Hence, the set of all related to y = 2x + 4 is y = 2x + k, where k ∈ R.

15. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.
(A) R is reflexive and symmetric but not transitive.
(B) R is reflexive and transitive but not symmetric.
(C) R is symmetric and transitive but not reflexive.
(D) R is an equivalence relation.

Solution:
Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}.

Step 1: (1, 1), (2, 2), (3, 3), (4, 4) ∈ R R. R is reflexive.

Step 2: (1, 2) ∈ R but (2, 1) ∉ R. R is not symmetric.

Step 3: Consider any set of points, (1, 3) ∈ R and (3, 2) ∈ R then (1, 2) ∈ R. So R is transitive.

Option (B) is correct.

16. Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}. Choose the correct answer.
(A) (2, 4) ∈ R (B) (3, 8) ∈ R (C) (6, 8) ∈ R (D) (8, 7) ∈ R

Solution: R = {(a, b) : a = b – 2, b > 6}

(A) Incorrect : Value of b = 4, not true.

(B) Incorrect : a = 3 and b = 8 > 6
a = b – 2 => 3 = 8 – 2 and 3 = 6, which is false.

(C) Correct: a = 6 and b = 8 > 6
a = b – 2 => 6 = 8 – 2 and 6 = 6, which is true.

(D) Incorrect : a = 8 and b = 7 > 6
a = b – 2 => 8 = 7 – 2 and 8 = 5, which is false.

Therefore, option (C) is correct.