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1. Let f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} be given by
f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down *gof*.

Solution:

Given function, f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} be given by

f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}

Find *gof*.

At f(1) = 2 and g(2) = 3, *gof* is

gof(1) = g(f(1)) = g(2) = 3

At f(3) = 5 and g(5) = 1, *gof* is

gof(3) = g(f(3)) = g(5) = 1

At f(4) = 1 and g(1) = 3, *gof* is

gof(4) = g(f(4)) = g(1) = 3

Therefore, *gof* = {(1,3), (3,1), (4,3)}

2. Let f, g and h be functions from R to R. Show that
(f + g) oh = foh + goh
(f . g) oh = (foh) . (goh)

Solution:

LHS = *(f + g) oh*

= *(f+g)(h(x))*

= *f(h(x)) + g(h(x))*

= *foh + goh*

= RHS

Again,

LHS = *(f . g) oh*

= f.g(h(x))

= f(h(x)) . g(h(x))

= (foh) . (goh)

= RHS

3. Find gof and fog, if

(i) f(x) = | x | and g(x) = | 5x – 2 |

(ii) f(x) = $8x^3$ and g(x) = $x^{1/3}$.

Solution:

(i) f(x) = | x | and g(x) = | 5x – 2 |

gof =(gof)(x) = g(f(x) = g(|x|) = |5 |x| - 2|

fog = (fog)(x) = f(g(x)) = f(| 5x – 2 |) = ||5x – 2|| = | 5x – 2 |

(ii) f(x) = $8x^3$ and g(x) = $x^{1/3}$ **.**

gof =(gof)(x) = g(f(x) = g($8x^3$) = $(8x^3)^{1/3}$ = 2x

fog = (fog)(x) = f(g(x)) = f($x^{1/3}$) = $8(x^{1/3})^3$ = 8x

4. If f(x) = $\frac{(4x+3)}{(6x-4)}$, x ≠ 2/3, Show that *fof*(x) = x, for all x ≠ 2/3. What is the inverse of f.

Solution:

f(x) = $\frac{(4x+3)}{(6x-4)}$, x ≠ 2/3,

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16}$$

$$= \frac{34x}{34}$$

$$= x$$

Therefore, *fof*(x) = x for all x ≠ 2/3.

Again, fof = I

The inverse of the given function, f is f.

5. State with reason whether following functions have inverse
(i) f : {1, 2, 3, 4} → {10} with
f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) g : {5, 6, 7, 8} → {1, 2, 3, 4} with
g = {(5, 4), (6, 3), (7, 4), (8, 2)}

(iii) h : {2, 3, 4, 5} → {7, 9, 11, 13} with
h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Solution:

(i) f : {1, 2, 3, 4} → {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

f has many-one function like f(1) = f(2) = f(3) = f(4) = 10, therefore f has no inverse.

(ii) g : {5, 6, 7, 8} → {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

g has many-one function like g(5) = g(7) = 4, therefore g has no inverse.

(iii) h : {2, 3, 4, 5} → {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

All elements have different images under h. So h is one-one onto function, therefore, h has an inverse.

6. Show that f : [–1, 1] → R, given by f (x) = x/(x+2) is one-one. Find the inverse of the function f : [–1, 1] → Range f.

(Hint: For y ∈ Range f, y = f(x) = x/(x+2), for some x in [–1, 1], i.e., x = 2y/(1-y).

Solution:
Given function: (x) = x/(x+2)
Let x, y ∈ [–1, 1]
Let f(x) = f(y)

x/(x+2) = y/(y+2)
xy + 2x = xy + 2y
x = y
f is one-one.

Again,

Since f : [–1, 1] → Range f is onto

say, y = x/(x+2)

yx + 2y = x

x(1 - y) = 2y

or x = 2y/(1-y)

x = f^{-1} (y) = 2y/(1-y); y not equal to 1

f is onto function, and f^{-1}(x) = 2x/(1-x).

7. Consider f : R → R given by f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

Solution:
Consider f : R → R given by f(x) = 4x + 3

Say, x, y ∈ R

Let f(x) = f(y) then
4x + 3 = 4y + 3
x = y
f is one-one function.

Let y ∈ Range of f

y = 4x + 3

or x = (y-3)/4

Here, f((y-3)/4) = 4((y-3)/4) + 3 = y

This implies f(x) = y

So f is onto

Therefore, f is invertible.

Inverse of f is x = f^{-1} (y) = (y-3)/4.

8. Consider f : $R_+ \rightarrow [4, \infty)$ given by f(x) = x^2 + 4. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

Solution:

Consider f : $R_+ \rightarrow [4, \infty)$ given by f(x) = x^2 + 4

Let x, y ∈ R → [4, ∞) then

f(x) = x^2 + 4 and

f(y) = y^2 + 4

if f(x) = f(y) then $x^2 + 4 = y^2 + 4$

or x = y

f is one-one.

Now y = f(x) = x^2 + 4 or x = $\sqrt{y-4}$ as x > 0

f($\sqrt{y-4}$)= ($\sqrt{y-4}$)^2 + 4 = y

f(x) = y

f is onto function.

Therefore, f is invertible and Inverse of f is f^{-1} (y) = $\sqrt{y-4}$.

9. Consider f : $R_+ \to [-5, \infty)$ given by f (x) = $9x^2 + 6x - 5$. Show that f is invertible with

$$f^{-1}(y) = \left(\frac{(\sqrt{y+6})-1}{3}\right)$$

Solution:

Consider f : $R_+ \to [-5, \infty)$ given by f (x) = $9x^2 + 6x - 5$

Consider f : $R_+ \to [4, \infty)$ given by f(x) = $x^2 + 4$

Let x, y $\in$ R $\to$ [-5, ∞) then

f(x) = $9x^2 + 6x - 5$ and

f(y) = $9y^2 + 6y - 5$

if f(x) = f(y) then $9x^2 + 6x - 5 = 9y^2 + 6y - 5$

$9(x^2 - y^2) + 6(x - y) = 0$

$9\{(x-y)(x+y)\} + 6(x - y) = 0$

$(x - y)(9)(x+y) + 6) = 0$

either x – y = 0 or 9(x+ y) + 6 = 0

Say x – y = 0, then x = y. So f is one-one.

Now, y = f(x) = $9x^2 + 6x - 5$

Solving this quadratic equation, we have

$x = \frac{-6\pm6\sqrt{y+6}}{18}$ $or\ x = \frac{\sqrt{y+6}-1}{3}$

So, $f(x) = f(\frac{\sqrt{y+6}-1}{3}) = 9(\frac{\sqrt{y+6}-1}{3})^2 + 6(\frac{\sqrt{y+6}-1}{3}) - 5$

$= y + 7 - 2\sqrt{y+6} + 2\sqrt{y+6} - 2 - 5 = y$

f(x) = y, therefore, f is onto.

f(x) is invertible and $f^{-1}(x) = \frac{\sqrt{y+6}-1}{3}$.

10. Let f : X → Y be an invertible function. Show that f has unique inverse.

(Hint: suppose g_1 and g_2 are two inverses of f. Then for all y ∈ Y, $fog_1(y) = 1_Y(y) = fog_2(y)$. Use one-one ness of f)

Solution:

Given, f : X → Y be an invertible function. And g_1 and g_2 are two inverses of f.

For all y ∈ Y, we get

$fog_1(y) = 1_Y(y) = fog_2(y)$

$f(g_1(y)) = f(g_2(y))$

$g_1(y) = g_2(y)$

$g_1 = g_2$

Hence f has unique inverse.

11. Consider f : {1, 2, 3} → {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

Solution:

Consider f : {1, 2, 3} → {a, b, c} given by f(1) = a, f(2) = b and f(3) = c

So f = {(a, 1), (b, 2), (c, 3)}

Hence f^{-1} (a) = 1, f^{-1} (b) = 2 and f^{-1} (c) = 3

Now, f^{-1} = {(a, 1), (b, 2), (c, 3)}

Therefore, inverse of f^{-1} = $(f^{-1})^{-1}$ = {(1, a), (2, b), (3, c)} = f

Hence $(f^{-1})^{-1} = f$.

13. If f: R → R be given by f(x) =$(3 - x^3)^{\frac{1}{3}}$, then *fof*(x) is

(A) $x^{1/3}$ (B) x^3 (C) x (D) $(3 - x^3)$

Solution:

f: R → R be given by f(x) =$(3-x^3)^{\frac{1}{3}}$, then

fof(x) = f(f(x)

$$= f\left((3-x^3)^{\frac{1}{3}}\right)$$

$$= \left[3-\left((3-x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$$

$$= \left[3-(3-x^3)\right]^{\frac{1}{3}}$$

$$= (x^3)^{\frac{1}{3}} = x$$

Option (C) is correct.

14. Let f : R – { -4/3 } → R be a function defined as f(x) = $\frac{4x}{3x+4}$. The inverse of f is the map g : Range f → R – { -4/3 } given by

(A) g(y) = 3y/(3-4y) **(B) g(y) = 4y/(4-3y)**

(C) g(y) = 4y/(3-4y) **(D) g(y) = 3y/(4-3y)**

Solution:

Let f : R – { -4/3 } → R be a function defined as f(x) = $\frac{4x}{3x+4}$. And Range f → R – { -4/3 }

y = f(x) = $\frac{4x}{3x+4}$

y(3x + 4) = 4x

3xy + 4y = 4x

x(3y – 4) = -4y

x = 4y/(4-3y)

Therefore, f^{-1} (y) = g(y) = 4y/(4-3y). Option (B) is the correct answer.