

Exercise 5.2

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Differentiate the functions with respect to x in Exercise 1 to 8.

1. $\sin(x^2 + 5)$

Solution: Let $y = \sin(x^2 + 5)$

Apply derivative both the sides with respect to x .

$$\frac{dy}{dx} = \cos(x^2 + 5) \frac{d}{dx}(x^2 + 5)$$

$$= \cos(x^2 + 5)(2x + 0)$$

$$= 2x \cos(x^2 + 5)$$

2. $\cos(\sin x)$

Solution: Let $y = \cos(\sin x)$

Apply derivative both the sides with respect to x .

$$\frac{dy}{dx} = -\sin(\sin x) \frac{d}{dx} \sin x$$

$$= -\sin(\sin x) \cos x$$

3. $\sin(ax + b)$

Solution: Let $y = \sin(ax + b)$

Apply derivative both the sides with respect to x .

$$\frac{dy}{dx} = \cos(ax + b) \frac{d}{dx}(ax + b)$$

$$= \cos(ax + b)(a + 0) = a \cos(ax + b)$$

4. $\sec(\tan \sqrt{x})$

Solution: Let $y = \sec(\tan \sqrt{x})$

Apply derivative both the sides with respect to x.

$$\frac{dy}{dx} = \sec(\tan \sqrt{x}) \tan(\tan \sqrt{x}) \sec^2 \sqrt{x} \frac{d}{dx} \sqrt{x}$$

$$= \sec(\tan \sqrt{x}) \tan(\tan \sqrt{x}) \sec^2 \sqrt{x} \cdot \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= \sec(\tan \sqrt{x}) \tan(\tan \sqrt{x}) \sec^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

5. $\frac{\sin(ax+b)}{\cos(cx+d)}$

$$y = \frac{\sin(ax+b)}{\cos(cx+d)}$$

Solution: Let
Using quotient rule,

$$\frac{dy}{dx} = \frac{\cos(cx+d) \frac{d}{dx} \sin(ax+b) - \sin(ax+b) \frac{d}{dx} \cos(cx+d)}{\cos^2(cx+d)}$$

$$= \frac{\cos(cx+d) \cos(ax+b) \frac{d}{dx}(ax+b) - \sin(ax+b) \{-\sin(cx+d)\} \frac{d}{dx}(cx+d)}{\cos^2(cx+d)}$$

$$= \frac{\cos(cx+d) \cos(ax+b)(a) + \sin(ax+b) \sin(cx+d)(c)}{\cos^2(cx+d)}$$

6. $\cos x^3 \sin^2(x^5)$

Solution: Let $y = \cos x^3 \cdot \sin^2(x^5)$

Apply derivative both the sides with respect to x.

$$\frac{dy}{dx} = \cos x^3 \frac{d}{dx} \sin^2(x^5) + \sin^2(x^5) \frac{d}{dx} \cos x^3$$

$$\begin{aligned}
 &= \cos x^3 \cdot 2 \sin(x^5) \frac{d}{dx} \sin(x^5) + \sin^2(x^5) (-\sin x^3) \frac{d}{dx} x^3 \\
 &= \cos x^3 \cdot 2 \sin(x^5) \cos(x^5) (5x^4) - \sin^2(x^5) \sin x^3 \cdot 3x^2 \\
 &= 10x^4 \cos x^3 \sin(x^5) \cos(x^5) - 3x^2 \sin^2(x^5) \sin x^3
 \end{aligned}$$

7. $2\sqrt{\cot(x^2)}$

Solution: Let $y = 2\sqrt{\cos(x^2)}$

Apply derivative both the sides with respect to x.

$$\begin{aligned}
 \frac{dy}{dx} &= 2 \cdot \frac{1}{2} \{\cot(x^2)\}^{\frac{-1}{2}} \cdot \frac{d}{dx} \cot(x^2) \\
 &= \frac{1}{\sqrt{\cot(x^2)}} \cdot \{-\operatorname{cosec}(x^2)\} \frac{d}{dx} x^2 \\
 &= \frac{1}{\sqrt{\cot(x^2)}} \cdot \{-\operatorname{cosec}(x^2)\} (2x) \\
 &= \frac{-2x \operatorname{cosec}(x^2)}{\sqrt{\cot(x^2)}}
 \end{aligned}$$

8. $\cos(\sqrt{x})$

Solution: Let $y = \cos(\sqrt{x})$

Apply derivative both the sides with respect to x.

$$\begin{aligned}
 \frac{dy}{dx} &= -\sin \sqrt{x} \frac{d}{dx} \sqrt{x} \\
 &= -\sin \sqrt{x} \cdot \frac{1}{2} (x)^{\frac{-1}{2}} = \frac{-\sin \sqrt{x}}{2\sqrt{x}}
 \end{aligned}$$

9. Prove that the function f given by $f(x) = |x-1|, x \in \mathbb{R}$ is not differentiable at $x = 1$.

Solution: Given function: $f(x) = |x-1|$

$$f(1) = |1-1| = 0$$

Right hand limit: $f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$

$$= \lim_{h \rightarrow 0} \frac{|1+h-1| - 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{|h|}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

and Left hand limit:

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{|1-h-1| - 0}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{|-h|}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h} = -1$$

Right hand limit $\neq$ Left hand limit

Therefore, $f(x)$ is not differentiable at $x = 1$.

10. Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1$ and $x = 2$

Solution: Given function is

$$f(x) = [x], 0 < x < 3$$

Right hand limit:

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{|1+h|-1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1-1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

and Left hand limit

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{|1-h|-1}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{0-1}{-h} = \infty$$

Right hand limit $\neq$ Left hand limit

Therefore, $f(x) = [x]$ is not differentiable at $x = 1$.

In same way, $f(x) = [x]$ is not differentiable at $x = 2$.