

Exercise 5.3

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Find $\frac{dy}{dx}$ in the following Exercise 1 to 15.

1. $2x + 3y = \sin x$

Solution: Given function is $2x + 3y = \sin x$
Derivate function with respect to x, we have

$$\frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx} \sin x$$

$$2 + 3 \frac{dy}{dx} = \cos x$$

$$3 \frac{dy}{dx} = \cos x - 2$$

$$\frac{dy}{dx} = \frac{\cos x - 2}{3}$$

2. $2x + 3y = \sin y$

Solution: Given function is $2x + 3y = \sin y$
Derivate function with respect to x, we have

$$\frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx} \sin y$$

$$2 + 3 \frac{dy}{dx} = \cos y \frac{dy}{dx}$$

$$-\frac{dy}{dx}(\cos y - 3) = -2$$

$$\frac{dy}{dx} = \frac{2}{\cos y - 3}$$

3. $ax + by^2 = \cos y$

Solution: Given function is $ax + by^2 = \cos y$
Derivate function with respect to x, we have

$$\frac{d}{dx}(ax) + \frac{d}{dx}(by^2) = \frac{d}{dx} \cos y$$

$$a + b \cdot 2y \frac{dy}{dx} = -\sin y \frac{dy}{dx}$$

$$2by \frac{dy}{dx} + \sin y \frac{dy}{dx} = -a$$

$$-\frac{dy}{dx}(2by + \sin y) = -a$$

$$\frac{dy}{dx} = \frac{-a}{2by + \sin y}$$

4. $xy + y^2 = \tan x + y$

Solution: Given function is $xy + y^2 = \tan x + y$

Derivate function with respect to x, we have

$$\frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx} \tan x + \frac{d}{dx} y$$

$$x \frac{d}{dx} y + y \frac{d}{dx} x + 2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

[Solving first term using Product Rule]

$$x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$x \frac{dy}{dx} + 2y \frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x - y$$

$$(x + 2y - 1) \frac{dy}{dx} = \sec^2 x - y$$

$$\frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$$

5. $x^2 + xy + y^2 = 100$

Solution: Given function is $x^2 + xy + y^2 = 100$

Derivate function with respect to x, we have

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}y^2 = \frac{d}{dx}100$$

$$2x + \left(x \frac{d}{dx}y + y \frac{d}{dx}x \right) + 2y \frac{dy}{dx} = 0$$

$$2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$$

$$(x + 2y) \frac{dy}{dx} = -2x - y$$

6. $x^3 + x^2y + xy^2 + y^3 = 81$

Solution: Given function is $x^3 + x^2y + xy^2 + y^3 = 81$

Derivate function with respect to x, we have

$$\frac{d}{dx}x^3 + \frac{d}{dx}x^2y + \frac{d}{dx}xy^2 + \frac{d}{dx}y^3 = \frac{d}{dx}81$$

$$3x^2 + \left(x^2 \frac{dy}{dx} + y \cdot \frac{d}{dx}x^2 \right) + x \frac{d}{dx}y^2 + y^2 \frac{d}{dx}x + 3y^2 \frac{dy}{dx} = 0 \quad \text{(using product rule)}$$

$$3x^2 + x^2 \frac{dy}{dx} + y \cdot 2x + x \cdot 2y \frac{dy}{dx} + y^2 \cdot 1 + 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x^2 + 2xy + 3y^2) = -3x^2 - 2xy - y^2$$

$$\frac{dy}{dx} = \frac{(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$$

7. $\sin^2 y + \cos xy = \pi$

Solution: Given function is $\sin^2 y + \cos xy = \pi$

Derivate function with respect to x, we have

$$\frac{d}{dx}(\sin y)^2 + \frac{d}{dx} \cos xy = \frac{d}{dx}(\pi)$$

$$2 \sin y \frac{d}{dx} \sin y - \sin xy \frac{d}{dx}(xy) = 0$$

$$2 \sin y \cos y \frac{dy}{dx} - \sin xy \left(x \frac{dy}{dx} + y \cdot 1 \right) = 0$$

$$\sin 2y \frac{dy}{dx} - x \sin xy \frac{dy}{dx} - y \sin xy = 0$$

$$(\sin 2y - x \sin xy) \frac{dy}{dx} = y \sin xy$$

$$\frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x \sin xy}$$

8. $\sin^2 x + \cos^2 y = 1$

Solution: Given function is $\sin^2 x + \cos^2 x = 1$

Derivate function with respect to x, we have

$$\frac{d}{dx}(\sin x)^2 + \frac{d}{dx}(\cos x)^2 = \frac{d}{dx}(1)$$

$$2 \sin x \frac{d}{dx} \sin x + 2 \cos x \frac{d}{dx} \cos x = 0$$

$$2 \sin x \cos x + 2 \cos x \left(-\sin x \frac{dx}{dx} \right) = 0$$

$$\sin 2x - \sin 2x = 0$$

$$-\sin 2y \frac{dy}{dx} = -\sin 2x$$

$$\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Solution: Given function is

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Step 1: Simplify the given function,

Put $x = \tan \theta$, we have

$$y = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$= \sin^{-1}(\sin 2\theta) = 2\theta$$

Result in terms of x, we get

$$y = 2 \tan^{-1} x$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

10. $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Solution: Given function is

$$y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Step 1: Simplify the given function,

$$y = \tan^{-1} \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right)$$

$$y = \tan^{-1}(\tan 3\theta) = 3\theta$$

Result in terms of x, we get

$$y = 3 \tan^{-1} x$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 3 \cdot \frac{1}{1+x^2} = \frac{3}{1+x^2}$$

11. $y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right), 0 < x < 1$

Solution: Given function is

$$y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right), 0 < x < 1$$

Step 1: Simplify the given function,

Put $x = \tan \theta$

$$y = \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$= \cos^{-1}(\cos 2\theta) = 2\theta = 2 \tan^{-1} x$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

12. $y = \sin^{-1} \left(\frac{1-x^2}{1+x^2} \right), 0 < x < 1$

$$y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right), 0 < x < 1$$

Solution: Given function is

Step 1: Simplify the given function,

Put $x = \tan \theta$

$$\begin{aligned}
 y &= \sin^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) \\
 &= \sin^{-1} (\cos 2\theta) \\
 &= \sin^{-1} \sin \left(\frac{\pi}{2} - 2\theta \right) = \frac{\pi}{2} - 2\theta \\
 &= \frac{\pi}{2} - 2 \tan^{-1} x
 \end{aligned}$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 0 - 2 \cdot \frac{1}{1+x^2} = \frac{-2}{1+x^2} \quad (\text{Derivative of a constant is always revert a value zero})$$

13. $y = \cos^{-1} \left(\frac{2x}{1+x^2} \right), -1 < x < 1$

$$y = \cos^{-1} \left(\frac{2x}{1+x^2} \right), -1 < x < 1$$

Solution: Given function is

Step 1: Simplify the given function,

Put $x = \tan \theta$

$$\begin{aligned}
 y &= \cos^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \\
 &= \cos^{-1} (\cos 2\theta) \\
 &= \cos^{-1} \cos \left(\frac{\pi}{2} - 2\theta \right) = \frac{\pi}{2} - 2\theta \\
 &= \frac{\pi}{2} - 2 \tan^{-1} x
 \end{aligned}$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 0 - 2 \cdot \frac{1}{1+x^2} = \frac{-2}{1+x^2} \quad (\text{Derivative of a constant is zero})$$

14. $y = \sin^{-1}(2x\sqrt{1-x^2}), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Solution: Given function is $y = \sin^{-1}(2x\sqrt{1-x^2}), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Step 1: Simplify the given function,

Put $x = \sin \theta$

$$y = \sin^{-1}(2 \sin \theta \sqrt{1 - \sin^2 \theta})$$

$$= \sin^{-1}(2 \sin \theta \sqrt{\cos^2 \theta})$$

$$= \sin^{-1}(2 \sin \theta \cos \theta)$$

$$= \sin^{-1}(\sin 2\theta) = 2\theta = 2 \sin^{-1} x$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 2 \cdot \frac{1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

15. $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

Solution: Given function is $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

Step 1: Simplify the given function,

Put $x = \cos \theta$

$$y = y = \sec^{-1}\left(\frac{1}{2\cos^2 \theta - 1}\right)$$

$$= \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$$

$$= \sec^{-1}(\sec 2\theta)$$

$$= 2\theta = 2\cos^{-1}x$$

Step 2: Derivative the function

$$\frac{dy}{dx} = 2 \cdot \frac{-1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$$