Exercise 5.4

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Differentiate the functions with respect to x in Exercise 1 to 10.

1. $\frac{e^x}{\sin x}$

Solution: Let $y=\frac{e^x}{\sin x}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=\frac{\sin x\frac{d}{dx}e^x-e^x\frac{d}{dx}\sin x}{\sin^2 x}$$

[Using quotient rule]

$$=\frac{\sin x e^x-e^x\cos x}{\sin^2 x}$$

$$=e^x\frac{(\sin x-\cos x)}{\sin^2 x}$$

2. $e^{\sin^{-1}x}$

Solution: Let $y=e^{\sin^{-1}x}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=e^{\sin^{-1}x}.\frac{d}{dx}\sin^{-1}x$$

$$=e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}$$

$$\left[\because \frac{d}{dx}e^{f(x)}=e^{f(x)}\frac{d}{dx}f(x)\right]$$

3. e^{x^3}

Solution: Let $y=e^{x^3}$ = $e^{\left(x^3\right)}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=e^{\left(x^3\right)}\frac{d}{dx}x^3$$

= $e^{\left(x^3\right)}.3x^2=3x^2.e^{\left(x^3\right)}$

$$\left[\because \frac{d}{dx}e^{f(x)}=e^{f(x)}\frac{d}{dx}f(x)\right]$$

4. $\sin\left(\tan^{-1}e^{-x}\right)$

Solution: Let $y=\sin\left(\tan^{-1}e^{-x}\right)$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx}=\cos\left(\tan^{-1}e^{-x}\right)\frac{d}{dx}\left(\tan^{-1}e^{-x}\right)$$

$$\left[\because \frac{d}{dx}\sin f(x)=\cos f(x)\frac{d}{dx}f(x)\right]$$

= $\cos\left(\tan^{-1}e^{-x}\right)\frac{1}{1+\left(e^{-x}\right)^2}\frac{d}{dx}e^{-x}$

$$\left[\because \frac{d}{dx}\tan^{-1}f(x)=\frac{1}{\left(f(x)\right)^2}\frac{d}{dx}f(x)\right]$$

= $\cos\left(\tan^{-1}e^{-x}\right)\frac{1}{1+e^{-2x}}e^{-x}\frac{d}{dx}(-x)$

= $-\frac{e^{-x}\cos\left(\tan^{-1}e^{-x}\right)}{1+e^{-2x}}$

5. $\log\left(\cos e^{x}\right)$

Solution: Let $y=\log\left(\cos e^{x}\right)$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{\cos e^x}\frac{d}{dx}\left(\cos e^x\right) \quad \left[\because \frac{d}{dx}\log f(x) = \frac{1}{f(x)}\frac{d}{dx}f(x)\right]$$

$$= \frac{1}{\cos e^x}\left(-\sin e^x\right)\frac{d}{dx}e^x \quad \left[\because \frac{d}{dx}\cos f(x) = -\sin f(x)\frac{d}{dx}f(x)\right]$$

$$= -\left(\tan e^x\right)e^x = -e^x\left(\tan e^x\right)$$

6. $e^x + e^{x^2} + \ldots\ldots + e^{x^5}$

Solution: Let $y = e^x + e^{x^2} + \ldots.. + e^{x^5}$

Define the given function for 5 terms,

Let us say, y = $e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}e^x + \frac{d}{dx}e^{x^2} + \frac{d}{dx}e^{x^3} + \frac{d}{dx}e^{x^4} + \frac{d}{dx}e^{x^5}$$

$$= e^x + e^{x^2}\frac{d}{dx}x^2 + e^{x^3}\frac{d}{dx}x^3 + e^{x^4}\frac{d}{dx}x^4 + e^{x^5}\frac{d}{dx}x^5$$

$$= e^x + e^{x^2}.2x + e^{x^3}.3x^2 + e^{x^4}.4x^3 + e^{x^5}.5x^4$$

$$= e^x + 2x e^{x^2} + 3x^2 e^{x^3} + 4x^3.e^{x^4} + 5x^4.e^{x^5}$$

7. $\sqrt{e^{\sqrt{x}}}, x > 0$

Solution: Let $y = \sqrt{e^{\sqrt{x}}}$

or y = $\left(e^{\sqrt{x}}\right)^{\frac{1}{2}}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}\left(e^{\sqrt{x}}\right)^{\frac{-1}{2}}\frac{d}{dx}e^{\sqrt{x}}$$

$$\left[\because \frac{d}{dx}\left(f(x)\right)^n = n\left(f(x)\right)^{n-1}\frac{d}{dx}f(x)\right]$$

$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}e^{\sqrt{x}}\frac{d}{dx}\sqrt{x}$

$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}e^{\sqrt{x}}\frac{1}{2\sqrt{x}}$

$= \frac{e^{\sqrt{x}}}{4\sqrt{x}\sqrt{e^{\sqrt{x}}}}$

8. $\log(\log x), x > 1$

Solution: Let $y = \log(\log x)$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{\log x}\frac{d}{dx}(\log x)$$

$= \frac{1}{\log x}.\frac{1}{x} = \frac{1}{x\log x}$

9. $\frac{\cos x}{\log x}, x > 0$

Solution: Let $y = \frac{\cos x}{\log x}$

Differentiate the functions with respect to x, we get

$$\frac{dy}{dx} = \frac{\log x\frac{d}{dx}(\cos x) - \cos x\frac{d}{dx}(\log x)}{(\log x)^2}$$ [By quotient rule]

$= \frac{\log x(-\sin x) - \cos x.\frac{1}{x}}{(\log x)^2}$

= $\dfrac{-\left(\sin x \log x+\dfrac{\cos x}{x}\right)}{(\log x)^{2}}$

= $\dfrac{-(x \sin x \log x+\cos x)}{x(\log x)^{2}}$

10. $\cos\left(\log x+e^{x}\right), x>0$

Solution: Let $y=\cos\left(\log x+e^{x}\right)$

Differentiate the functions with respect to x, we get

$\dfrac{dy}{dx}=-\sin\left(\log x+e^{x}\right)\dfrac{d}{dx}\left(\log x+e^{x}\right)$

= $-\sin\left(\log x+e^{x}\right).\left(\dfrac{1}{x}+e^{x}\right)$

= $\left(\dfrac{1}{x}+e^{x}\right)\sin\left(\log x+e^{x}\right)$