Odisha Board Class 11 Maths Sample Paper

Class: XI
Subject: Mathematics

Total: 100 Marks

General Instructions:

1. All questions are compulsory in Group A, which are very short answer type questions. All questions in the group are to be answered in one word, one sentences are as per exact requirement of the question.
2. Group – B contain 5 questions and each question have 5 bits, out of which only 3 bits are to be answered.
3. Group- C contains 5 questions and each question have 2 or 3 bits, out of which only 1 bit is to be answered. Each bit carries 6 marks.

GROUP -A

$\mathbf{10 \times 1 = 10}$

1. Find the Roster Form of the given set $\{x: x \in Z \text{ and } |x| \leq 2\}$ is
2. If $sin\ x = \frac{1}{2}$, Find the value of $\cot x + \tan x$
3. Find the value of $cos\ a\ cos\ 2a\ cos\ 4a \ldots .\ cos\ (2^n - 1\ a)$ using induction method
4. Solve for x: $27 \times 48 = 6^x$
5. Solve for x: $1 \leq |x - 2| \leq 3$
6. Find the number of terms in the expansion of $(7x + 2y)^9$
7. Find the 18th term of the sequence determined by $T_n = \frac{n(n-2)}{n+3}$
8. Find the gradient of a line who is perpendicular to another line having inclination is 60^O

9. Evaluate: $\lim_{x \to 1} 3x^2 + 4x + 5$
10. There are 3 red balls, 5 blue balls and 7 green balls in a box. One ball is drawn out randomly. What is the probability that the ball is not red?

Group – B

Total Marks: 60

11. Answer any 3 questions $\mathbf{3 \times 4 = 12}$

a) For any two sets A and B prove using properties of sets that:

(i) (A∪B) – (A∩B) = (A-B) ∪ (B-A)
(ii) (A∩B) ∪ (A-B) = A
(iii) (A∪B) – A = B-A

b) The functions of f is given as:
$f(x) = x^2 + kx + 6$
where f (3) = 0. The value of k is?

c) If tan α = -2, find the values of the remaining trigonometric functions of α.

d) If $2\sin^2 x - 5\ sin\ x\ cos\ x + 7\cos^2 x = 1$, find the possible values of $tan\ x$.

e) Using the principle of Mathematical Induction, prove that:
$3.2^2 + 3^2.2^3 + 3^3.2^4 + \ldots + 3^n.2^{n+1} = \frac{12}{5}(6^n - 1), n \in N.$

12. Answer any 3 questions $\mathbf{3 \times 4 = 12}$

a) Divide 17 into two parts, such that the sum of their squares is 145.

b) Express the given complex number in the form of $(a + ib)$: $(12i + 5)(3i - 2)$

c) The sum of two consecutive even integers is minimum -282. What are the lowest possible values of the integers?

d) Write the Solution Set for the inequality:
$\frac{7(x-12)}{5} \leq \frac{5(2x-7)}{8}$

e) In a coin tossing experiment, a coin is tossed 4 times. What is the probability of getting?
(i) Only one Tail.
(ii) At least One Tail.
(iii) No Tail.

13. Answer any 3 questions $\mathbf{3 \times 4 = 12}$

a) A student takes an examination, which comprises of 2 tests. Probability of the student passing both the examination is 0.45 and the probability of passing the any one of the examinations is 0.6. If the student passes Test 1 with a probability of 0.7, what is the probability of passing the Test 2?

b) Using binomial theorem, evaluate $(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$. Hence show that the value of $(\sqrt{3} + 1)^5$ lies between 152 and 153.

c) Find the coefficient of x^{11} in $\left(2x^2 + x - 3\right)^6$.

d) The fourth term of an A.P. is 2. The sum of the first 6 terms is 0. Find the sum of its first 18 terms.

e) In a G.P, the ratio of the sum of first 3 terms to the first 6 terms is 351: 343. Find the common ratio of the progression.

14. Answer any 3 questions $\mathbf{3 \times 4 = 12}$

a) Find the equation of the line passes through (1,-1) and is perpendicular to the line joining (3,8) and (2,-4).

b) In what ratio is the line joining the points (2,3) and (4, -5) divided by the line joining the points (6,8) and (-3, -2)?

c) The equation of a hyperbola is $\frac{x^2}{9} + \frac{y^2}{25} = 1$, find the length of latus rectum and eccentricity.

d) For what values of p and q will the line joining points A (3, 2, 5) and B (p, 5, 0) be parallel to the line joining points C (1, 3, q) and D (6, 4, -1)?

e) Find the locus of the point which is equidistant from the points A (0, 2, 3) and (2, -2, 1).

15. Answer any 3 questions $3 \times 4 = 12$

a) Evaluate: $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n}$

b) Differentiate: $\frac{3x - 2}{5x^2 + 7}$

c) Consider the following statement:
If you will not study, then you will fail.
Identify the necessary and sufficient conditions.

d) The following is the distribution of marks obtained by the students of a class in a Mathematics test:

Marks Obtained	Number of students
More than 60	0
More than 50	3
More than 40	10
More than 30	20
More than 20	40
More than 10	56
More than 0	60

e) In a game of dice, what is the probability of getting 3 exactly twice in 9 throws of a dice?

GROUP-C

Total Marks: 30

16. Answer any 1 question $6 \times 1 = 6$

a) Find the area enclosed by the ellipse $\frac{x^2}{\left(\frac{2}{3}\right)^2} + \frac{y^2}{\left(\frac{3}{2}\right)^2} = 1$

b) Find all the points of discontinuity of the function F defined by

$$f(x) = \begin{cases} x + 3; if\ x \geq 1 \\ x^2 - 5; if\ x < 1 \end{cases}$$

17. Answer any 1 question $6 \times 1 = 6$

a) Find the area of the triangle whose vertices are $(2,4), (-5,1), and\ (5,3)$

b) . $If\ A = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$ Then show that $A^2 - 3A + 4I = 0$. Using the results calculate A^4

18. Answer any 1 question $\mathbf{6 \times 1 = 6}$

a) Evaluate $\int \frac{2x-4}{(x+1)^2(x+2)}$

b) Find the domain for which the function $f(x) = x^2 - 11$

$and\ g = x + 9\ are\ equal$.

19. Answer any 1 question $\mathbf{6 \times 1 = 6}$

a) Find the vector joining points $R(5,3,2) and\ S(-3,-1,-2)$ directed from R to S.

b) Find the angle between two planes $3x + 6y + 2z = 7\ \acute{and}\ 2x + 2y + 2\acute{z} = 5$.

20. Answer any 1 question $\mathbf{6 \times 1 = 6}$

a) Show that $tan^{-1}\frac{3}{5} + tan^{-1}\frac{1}{2} = tan^{-1}\ \frac{11}{7}$

b) A nutritionist wishes to mix two types of food in such a way that the content of the mixture contains at least 28 parts equal units of both Calcium and Vitamin C. Food I contains 4 units of Vit. C and 7 units of Ca. Food II contains 7 units of Vit. C and 2 units of Ca. If food I cost Rs. 60/kg and Food II cost Rs. 90/kg then formulate the minimum cost of a mixture that can be formed by the nutritionist.

Answers & Explanations
GROUP-A

1. Solution:

It is found that x is an integer satisfying $|x| \le 2$.

Thus, $|x| = 0, 1, 2$

$\Rightarrow x = 0, +1, + 2$

So, values of x = $\{-2, -1, 0, 1, 2\}$.

2. Solution:

Given $sin\ x = \frac{1}{2}$

$\Rightarrow cosec\ x = \frac{1}{\sin x} = \frac{1}{\frac{1}{2}} = 2$

$And\ cosec^2\ x = (2)^2 = 4$

We know, $1 + \cot^2 x = cosec^2\ x$ [Trigonometric Identity]

$\Rightarrow 1 + \cot^2 x = 4$

$\Rightarrow \cot^2 x = 4 - 1 = 3$

$\Rightarrow \cot^2 x = 3$

$\Rightarrow \cot x = \sqrt{3}$

And, $\tan x = \frac{1}{\cot x} = \frac{1}{\sqrt{3}}$

Thus, cot x + tan x = $\sqrt{3} + \frac{1}{\sqrt{3}} = \frac{(\sqrt{3}\times\sqrt{3})+1}{\sqrt{3}} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

3. Solution:

Let $P\ (n) = cos\ a\ cos\ 2a\ cos\ 4a\ \ldots.\ cos\ (2^{n-1}a)$.

When, $P\ (1) = cos\ a = \frac{2\ sin\ a\ cos\ a}{2\ \sin a}$

$= \frac{sin\ 2a}{2\ \sin a} = \frac{\sin^1 2a}{2\ \sin^1 a}$

In terms of n, $\frac{\sin^n 2a}{2\ \sin^n a}$

4. Solution:

Given equation, $27 \times 48 = 6^x$
$\Rightarrow (3)^3 \times (3 \times 16) = 6^x$
$\Rightarrow (3)^3 \times 3 \times (2)^4 = 6^x$
$\Rightarrow 6^x = 3^4 \times 2^4$
$\Rightarrow 6^x = 6^4$
$\Rightarrow x = 4$

5. Solution:
We know,
$a \leq |x - c| \leq b$
$\Leftrightarrow x \in [-b + c, -a + c] \cup [a + c, b + c]$
$Thus, 1 \leq |x - 2| \leq 3$
$\Leftrightarrow x \in [-3 + 2, -1 + 2] \cup [1 + 2, 3 + 2]$
$\Leftrightarrow x \in [-1,1] \cup [3,5]$.

6. Solution:
As the no. of terms in the expansion of $(x + a)^n$ is $(n + 1)$,
Thus, the no. of terms in the expansion of $(7x + 2y)^9$ is $(9 + 1) = 10$.

7. Solution:
Given, $T_n = \frac{n(n-2)}{n+3}$
Putting $n = 18$, $T_{18} = \frac{18(18-2)}{18+3} = \frac{18*16}{21} = \frac{96}{7}$

8. Solution:
Let, m_1 be the gradient of the second line, where,
$$m_1 = tan\,60° = \sqrt{3}$$

And let the gradient of the second line perpendicular to the line having gradient m_1 be m_2.
Thus, $m_2 = -\frac{1}{m_1} = -\frac{1}{\sqrt{3}} = -\sqrt{\frac{1}{3}}$

9. Solution:
Given, $\lim_{x \to 1} 3x^2 + 4x + 5$
$= 3(1)^2 + 4(1) + 5 = 12.$

10. Solution:
Number of red balls = 3
Number of blue balls = 5
Number of green balls = 7
Thus, Total number of balls = 3 + 5 + 7 = 15
Let the event of drawing one red ball be R.
Thus, Probability of drawing a red ball = P(R) = $\frac{Number\ of\ red\ balls}{Total\ number\ of\ balls} = \frac{3}{15} = \frac{1}{5}$
Thus, the Probability of drawing a ball that is not red = $P'(R) = 1 - P(R)$ = $1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5}$

GROUP-B

11.

a) Solution:

(i) We have, $(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)'$ $[Since, X - Y = X \cap Y']$

$= (A \cup B) \cap (A' \cup B')$ $[Since, (A \cap B)' = (A' \cup B')]$

$= X \cap (A' \cup B')$ $\{where, X = (A \cup B)\}$

$= (X \cap A') \cup (X \cap B') = (B \cap A') \cup (A \cap B')$ [Since, $X \cap A' = (A \cup B) \cap A'$

$= (A \cap A') \cup (B \cap A')$

$= \emptyset \cup (B \cap A') = B \cap A'$ $Similarly, X \cap B' = A \cap B']$

$= (A \cap B') \cup (B \cap A')$

$= (A - B) \cup (B - A)$ $[Since, A - B = A \cap B' \: and, B - A = B \cap A']$

(ii) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$

$= X \cup (A \cap B')$ $\{where\ X = A \cap B\}$

$= (X \cup A) \cap (X \cup B') = A \cap (A \cup B')$ $[Since, X \cup A = (A \cap B) \cup A$

$= A\ (Since, A \cap B \subset A)$

$X \cup B' = (A \cap B) \cup B'$

$= (A \cup B') \cap (B \cup B')$

$= (A \cup B') \cap U = A \cup B']$

$= A$ $(Since, A \subset A \cup B')$

(iii) $(A \cup B) - A = (A \cup B) \cap A'$ $[Since, X - Y = X \cap Y']$

$= (A \cap A') \cup (B \cap A')$

$= \emptyset \cup (B \cap A')$ $[Since, (A \cap A') = \emptyset]$

$= B - A\ [Since, B - A = B \cap A']$

b) Solution:

Given $f(x) = x^2 + kx + 6$

$f(3) = 0$

$\Rightarrow f(3) = 3^2 + k.3 + 6 = 0$

$\Rightarrow 9 + 3k + 6 = 0$

$\Rightarrow 15 + 3k = 0$

$\Rightarrow 3k = -15$

$\Rightarrow k = -\frac{15}{3}$

$\Rightarrow k = -5$

c) Solution:

Given, tan α = -2 which is –ve, therefore α lies in second or fourth quadrant.

Also $\sec^2 \alpha = 1 + \tan^2 \alpha = 1 + (-2)^2$

$= 1 + 4 = 5$

$\Rightarrow sec\ \alpha = +\sqrt{5}$

Thus, two cases arise.

CASE – 1:

When α lies in 2nd quadrant, $\sec\alpha$ $is - ve$.

$Hence, \sec\alpha = -\sqrt{5}$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{5}}$

$\sin\alpha = \frac{\sin\alpha\cos\alpha}{\cos\alpha} = \tan\alpha\cos\alpha = -2\left(-\frac{1}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}}$

$\Rightarrow cosec\ \alpha = \frac{\sqrt{5}}{2}$

$Also \tan\alpha = -2 \Rightarrow \cot\alpha = -\frac{1}{2}$

CASE – 2:

When α lies in 4th quadrant, $sec\ \alpha\ is + ve$.

$Hence, \sec\alpha = \sqrt{5}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{5}}$

$\sin\alpha = \frac{\sin\alpha\cos\alpha}{\cos\alpha} = \tan\alpha\cos\alpha = 2\left(-\frac{1}{\sqrt{5}}\right) = -\frac{2}{\sqrt{5}}$

$\Rightarrow cosec\ \alpha = -\frac{\sqrt{5}}{2}$

$Also \tan\alpha = -2 \Rightarrow \cot\alpha = -\frac{1}{2}$

d) Solution:

$2\sin^2 x - 5\sin x\cos x + 7\cos^2 x = 1$

$= \sin^2 x + \cos^2 x$

$\Rightarrow 2\sin^2 x - 5\, sin\, x\, cos\, x + 7\cos^2 x - \sin^2 x - \cos^2 x = 0$

$\Rightarrow \sin^2 x - 5\, sin\, x\, cos\, x + 6\cos^2 x = 0$

$\Rightarrow \sin^2 x - 2\, sin\, x\, cos\, x - 3\, sin\, x\, cos\, x + 6\cos^2 x = 0$

$\Rightarrow (sin\, x - 2\, cos\, x)\,(sin\, x - 3\, cos\, x) = 0$

Either — Or,

$(sin\, x - 2\, cos\, x) = 0$ — $(sin\, x - 3\, cos\, x) = 0$

$\Rightarrow sin\, x = 2\, cos\, x$ — $\Rightarrow sin\, x = 3cos\, x$

$\Rightarrow \frac{sin\, x}{\cos x} = 2$ — $\Rightarrow \frac{sin\, x}{\cos x} = 3$

$\Rightarrow tan\, x = 2.$ — $\Rightarrow tan\, x = 3.$

e) Solution:

Let

P(n) $= 3.2^2 + 3^2.2^3 + 3^3.2^4 + \ldots + 3^n.2^{n+1} = \frac{12}{5}(6^n - 1)$

$P(1) \Rightarrow 3.2^2 = \frac{12}{5}(6^1 - 1)$

$\Rightarrow 3.4 = \frac{12}{5}(5)$

$\Rightarrow 12 = 12$

⇒ P (1) is true.

Let P (m) be true,
i.e. $3.2^2 + 3^2.2^3 + 3^3.2^4 + \ldots + 3^m.2^{m+1} = \frac{12}{5}(6^m - 1)$
For P (m+1):
$3.2^2 + 3^2.2^3 + 3^3.2^4 + \ldots + 3^{m+1}.2^{(m+1)+1} = \frac{12}{5}(6^{m+1} - 1)$
$\Rightarrow (3.2^2 + 3^2.2^3 + 3^3.2^4 + \ldots + 3^m + 1.2^m + 1) + 3^m + 1.2^m + 2$
$= \frac{12}{5}(6^m - 1) + 3^m + 1.2^m + 1.2$
$= \frac{2}{5}.6.6^m - \frac{12}{5} + (3.2)^m + 1.2$
$= \frac{2}{5}.6^{m+1} - \frac{12}{5} + 2.6^{m+1}$

$= \left(\frac{2}{5} + 2\right).6^{m+1} - \frac{12}{5}$-
$= 12.6^{m+1} - \frac{12}{5}$-
$= \frac{12}{5}(6^{m+1} - 1)$
⇒ P (m+1) is true.
Hence by the principle of mathematical Induction, P(n) is true for all n ∈ N.

12.
a) Solution:
Let one part be x.
Thus, the second part = 17-x.
Given, Sum of the squares of the two parts = 145
$\Rightarrow x^2 + (17 - x)^2 = 145$
$\Rightarrow x^2 + 289 - 34x + x^2 = 145$
$\Rightarrow 2x^2 - 34x + 289 = 145$
$\Rightarrow 2x^2 - 34x + 289 - 145 = 0$
$\Rightarrow 2x^2 - 34x + 144 = 0$
$\Rightarrow x^2 - 17x + 72 = 0$
$\Rightarrow x^2 - 9x - 8x + 72 = 0$
$\Rightarrow x(x - 9) - 8(x - 9) = 0$
$\Rightarrow (x - 9)(x - 8) = 0$
$\Rightarrow x = 8,9.$

b) Solution:

Given, $(12i + 5)(3i - 2) = (12i + 5)(3i) - (12i + 5)2$
$= 36i^2 + 15i - 24i - 10$
$= 36(-1) - 9i - 10$ $[\because i^2 = -1]$
$= -36 - 9i - 10$
$= -46 - 9i$
$= -(46 + 9i)$

c) Solution:
Let x be the smaller of the two consecutive even positive integers. Then, the other integer is x+2.

their sum ≥ -282,
i.e. $x + (x + 2) \geq -282$
$\Rightarrow\ 2x + 2 \geq -282$
$\Rightarrow\ 2x \geq -282 - 2$
$\Rightarrow\ 2x \geq -284$
$\Rightarrow\ x \geq -284/2$
$\Rightarrow\ x \geq\ -142,\qquad [Since, x\ is\ an\ even\ integer.]$
∴ The smallest values of the consecutive even integers are -142 and -140.
The required pair of even integers = (-142, -140).

d) Solution:

$$\frac{7(x-12)}{5} \leq \frac{5(2x-7)}{8}$$

$\Rightarrow 8\ \times\ 7(x-12) \leq 5 \times 5(2x-7)$
$\Rightarrow 56(x-12) \leq 25(2x-7)$
$\Rightarrow 56x - 672 \leq 50x - 175$
$\Rightarrow 56x - 50x \leq 672 - 175$
$\Rightarrow 6x \leq 497$
$\Rightarrow x \leq \frac{497}{6}$
$\Rightarrow x \leq 82\frac{5}{6}$

Thus, the solution of the given inequality = All real numbers $\leq 82\frac{5}{6}$ = $\left(-\infty, 82\frac{5}{6}\right]$.

e) Solution:

In a coin tossing game,
Possible outcomes from one coin = Head (H) and Tail (T).
The coin is tossed 4 times.
Thus, Number of possible outcomes = 2^4 =16
When a coin is tossed three times, the sample space is = (HHHH), (HHHT), (HHTH), (HTHH), (THHH), (HHTT), (HTTH), (TTHH), (THHT), (HTTT), (TTTH), (TTHT), (THTT), (THTH), (HTHT), (TTTT).

(i) Let the event of getting Only One Tail be T.

Thus, Probability P(T) = $\frac{4}{16} = \frac{1}{4}$

(ii) Let the event of getting at least One Tail be A

Thus, Probability P(A) = $\frac{15}{16}$

(iii) Let the event of getting No Tail be N

Thus, Probability P(N) = $\frac{1}{16}$

13.
a) Solution:
Let the event of passing Test 1 be T.
Thus, Probability of passing Test 1 P(T) = 0.7

Let the event of passing Test 1 be V.
Thus, Probability of passing Test 1 P(V)

So, the event of passing both the examination be (T and V)
Thus, Probability of passing both the examination P(T and V) = 0.45

So, the event of passing any one of the examinations be (T or V).
Thus, Probability of passing both the examination $P(T\ or\ V) = 0.6$
We know,
$P(T\ or\ V) = P(T) + P(V) - P(T\ and\ V)$
$\Rightarrow 0.6 = 0.7 + P(V) - 0.45$
$\Rightarrow P(V) = 0.6 - 0.7 + 0.45 \Rightarrow P(V) = 0.35$

b) Solution:
$(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$
$= (5C_0(\sqrt{3})^5 + 5C_1(\sqrt{3})^4 + 5C_2(\sqrt{3})^3 + 5C_3(\sqrt{3})^2 + 5C_4(\sqrt{3})^1 + 5C_5) - (5C_0(\sqrt{3})^5 - 5C_1(\sqrt{3})^4 +$
$5C_2(\sqrt{3})^3 - 5C_3(\sqrt{3})^2 + 5C4(\sqrt{3})1 - 5C5)$
$= 2(5C_1(\sqrt{3})^4 + 5C_3(\sqrt{3})^2 + 5C_5)$
$= 2(5(9) + 10.3 + 1)$
$= 2(4^5 + 30 + 1) = 2(7^6) = 15^2.$
$\Rightarrow (\sqrt{3} + 1)^5 = 15^2 + (\sqrt{3} - 1)^5 \ \ \ldots..(i)$
But, we know that
$\sqrt{3} = 1.732$
$\Rightarrow 0 < \sqrt{3} - 1 < 1$
$\Rightarrow 0 < (\sqrt{3} - 1)^5 < 1$
Therefore, from (i), $(\sqrt{3} + 1)^5 = 152 + a$ positive real no. less than 1.
$\Rightarrow (\sqrt{3} + 1)^5$ lies between 152 and 153.

c) Solution:
$(2x^2 + x - 3)^6 = ((x-1)(2x+3))^6 = (x-1)^6\,(2x+3)^6$
$= (6C_0\,x^6 - 6C_1\,x^5 + 6C_2\,x^4 - \ldots.)(6C_0\,(2x)^6 + 6C_1\,(2x)^5 \times 3 + 6C_2\,(2x)^4 \times 32 - \ldots.)$

Thus, the term containing x^{11} in $(2x^2 + x - 3)^6$
$= 6C_0\,x^6\,.6C_1\,(2x)^5 \times 3 - 6C_1\,x^5.\,6C_0\,(2x)^6$
$= 6C_0.\,6C_1.\,2^5.\,3x^{11} - 6C_1.\,6C_0.\,2^6x^{11}$
$= (1.6.32.3 - 6.1.64)x^{11} = 6(96 - 64)x^{11}$
$= 192\,x^{11}.$
Thus, the coefficient of x^{11} in $(2x^2 + x - 3)^6 = 192.$

d) Solution:
Let the first term be a,
The fourth term be a_4,
Sum of the first 6 terms be S_6,
Sum of the first 18 terms be S_{18}.

Given,
$a_4 = 2$.
$\Rightarrow a + 3d = 2 \quad \ldots(i)$
and, $S_6 = 0$.
$\Rightarrow \frac{n}{2}(2a + 5d) = 0$
$\Rightarrow 2a + 5d = 0 \quad \ldots(ii)$
$(i) \times 2 \quad 2a + 6d = 4 \quad \ldots(iii)$
$(iii) - (ii) \quad \therefore d = 4$
Putting d=4 in (i),
$\Rightarrow a + 3 \times 4 = 2$
$\Rightarrow a + 12 = 2 \Rightarrow a = 2 - 12$
$\Rightarrow a = -10$
$\therefore a_{18} = a + 17\,d = -10 + 17 \times 4$
$= -10 + 68 = 58$

$\therefore$ Sum of first 18 terms = $S_{18} = \frac{18}{2}(-10 + 58)$
$= 9(48) = 432$

e) Solution:

Let a be the first term and r be the common ratio of the given G.P.
Then, Sum of the first 3 terms= $S_3 = \left(\frac{a(r^3-1)}{r-1}\right)$
And, Sum of the first 3 terms= $S_6 = \left(\frac{a(r^6-1)}{r-1}\right)$
Given,
$$\frac{\left(\frac{a(r^3-1)}{r-1}\right)}{\left(\frac{a(r^3-1)}{r-1}\right)} = \frac{351}{343}$$

$\Rightarrow \frac{r^3-1}{r^6-1} = \frac{351}{343}$
$\Rightarrow \frac{r^3-1}{(r^3)^2-(1)^2} = \frac{351}{343}$
$\Rightarrow \frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{351}{343} \quad \Rightarrow \frac{1}{r^3+1} = \frac{343}{351}$
$\Rightarrow r^3 + 1 = \frac{351}{343}$
$\Rightarrow r^3 = \frac{351}{343} - 1 = \frac{351-343}{343} = \frac{8}{343}$
$\Rightarrow \sqrt[3]{r^3} = \sqrt[3]{\frac{8}{343}}$
$\Rightarrow r = \frac{2}{7}$ (Ans.)

14.
a) Solution:
Slope of the line joining (3,8) and (2,-4) (m_1) = $\frac{-4-8}{2-3}$
$= \frac{-12}{-1}$
$= 12.$
$\therefore$Slope of the line perpendicular to line joining (3,8) and (2,-4) (m_2) = $-\frac{1}{m1} = -\frac{1}{12}$

∴ Equation of the line: $y-(-1)=-\frac{1}{12}(x-1)$

$$\Rightarrow 12(y+1)=-x+1$$
$$\Rightarrow 12y+12=-x+1$$
$$\Rightarrow x+12y+12-1=0$$
$$\Rightarrow x+12y+11=0$$
$$(Ans.)$$

b) Solution:

The equation of the straight line joining the q points (6,8) and (-3,-2) is

$$y-8=\frac{-2-8}{-3-6}(x-6)$$

$$\Rightarrow y-8=\frac{10}{9}(x-6)$$

$$\Rightarrow 9y-72=10x-60$$

$$\Rightarrow 10x-9y+12=0$$

Let the line joining the points (6,8) and (-3,-2), i.e. the line (i) divide the line segment joining the points (2,3) and (4,-5) at the point P in the ratio k:1, then the coordinates of the point P are,

$$\left(\frac{k.4\ +\ 1.2}{k+1},\frac{k(-5)+\ 1.3}{k+1}\right)$$
$$i.e.\ \left(\frac{4k\ +\ 2}{k+1},\frac{-5k\ +\ 3}{k+1}\right)$$

Since P lies on (i), we get $10.\frac{4k+2}{k+1}-9,\frac{-5k+3}{k+1}+12=0$

$$\Rightarrow\ 40k\ +\ 20\ +\ 45k-27\ +\ 12k\ +12\ =0$$
$$\Rightarrow\ 97k\ +\ 5\ =\ 0$$
$$\Rightarrow\ k\ =\ -\frac{5}{97}.$$

Hence, the required ratio is $-\frac{5}{97}$ i.e. 5:97 externally.

c) Solution:

Given, the equation of the hyperbola be

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$

Comparing with the general equation of hyperbola- $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we get,

$a^2=9\Rightarrow a=\sqrt{9}=3$,

and, $b^2=25\Rightarrow a=\sqrt{25}=5$,

∴ Latus Rectum $=\frac{2b^2}{a}=\frac{2\times25}{3}=\frac{50}{3}=16\frac{2}{3}$

$\therefore$ Eccentricity (e) $= \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{9+25}}{3} = \frac{\sqrt{34}}{3}$

d) Solution:

Direction ratios of the line AB are

<p-3, 5-2, 0-5> i.e. <p-3, 3, -5>

and the Direction ratios of the line CD are

<6-1, 4-3, -1-q> i.e. <5, 1, -1-q>

Now AB will be || to CD if,

$$\frac{p-3}{5} = \frac{3}{1} = -\frac{5}{-1-q}$$

$$\Rightarrow p - 3 = 15 \; and \; 3(1+q) = 5$$

$$\Rightarrow p = 18 \; and \; q = \frac{2}{3}.$$

e) Solution:

Let P(x,y,z) be any point which is equidistant from the points A (0, 2, 3) and (2, -2, 1). Then,

PA = PB

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow \sqrt{\{(x-0)^2 + (y-2)^2 + (z-3)^2\}} = \sqrt{\{(x-2)^2 + (y+2)^2 + (z-1)^2\}}$$

$$\Rightarrow 4x - 8y - 4z + 4 = 0$$

$$\Rightarrow x - 2y - z + 1 = 0.$$

Hence, the required locus is $x - 2y - z + 1 = 0$.

15.

a) Solution:

Given, $\lim\limits_{x \to a} \frac{x^m - a^m}{x^n - a^n}$

$$= \lim_{x \to a} \frac{x^m - a^m}{x-a} \times \frac{x-a}{x^n - a^n}$$

$$= \lim_{x \to a} \frac{x^m - a^m}{x-a} \div \frac{x^n - a^n}{x-a}$$

$$= \lim_{x \to a} \frac{x^m - a^m}{x-a} \div \lim \frac{x^n - a^n}{x-a}$$

$$= ma^{m-1} \div na^{n-1} = \frac{m}{a} a^{m-n}$$

b) Solution:

Let y = $\frac{3x-2}{5x^2+7}$

Differentiating w.r.t. x,

$$\frac{dy}{dx} = \frac{(5x^2+7).\frac{d}{dx}(3x-2)-(3x-2).\frac{d}{dx(5x^2+7)}}{(5x^2+7)^2}$$

$$= \frac{(5x^2+7).(3.1+0)-(3x-2).(5.2x+0)}{(5x^2+7)^2}$$

$$= \frac{15x^2+21-30x^2+20x}{(5x^2+7)^2}$$

$$= \frac{(-15x^2-20x-21)}{(5x^2+7)^2}$$

c) Solution:

The given statement can be written as p→q where,

p: You don't study.

q: You fail.

Now, p→q means "p is sufficient for q" . Hence, the sufficient condition is "You not studying", as it is sufficient to get you failed.

Also in p→q, we know that q is necessary for p. Hence necessary condition is "getting failed".

d) Solution:

Class width (c) = 10. Let Assumed Mean = 25.

The table is as follows.

Class	Class mark(x_i)	$ui = \frac{xi - A}{c}$	Frequency (f_i)	$f_i u_i$
0-10	5	-2	4	-8
10-20	15	-1	16	-16
20-30	25	0	20	0
30-40	35	1	10	10
40-50	45	2	7	14
50-60	55	3	3	9
Total			60	9

Hence, Mean = $A + \frac{c.\Sigma fiui}{\Sigma fi}$

$= 25 + 10.\left(\frac{9}{60}\right)$

$= 26.5$

e) Solution:

Let X be the Number of times getting 3

Die thrown is a Bernoulli trial.

So, X has a binomial distribution

$P(X = x) = nC_x\, q^{n-x}\, p^x$

Where, n = no. of times die is thrown = 9

P = Probability of getting 3 = $\frac{1}{6}$

q = 1- p = $1-\left(\frac{1}{6}\right)=\frac{5}{6}$

GROUP -C

16.

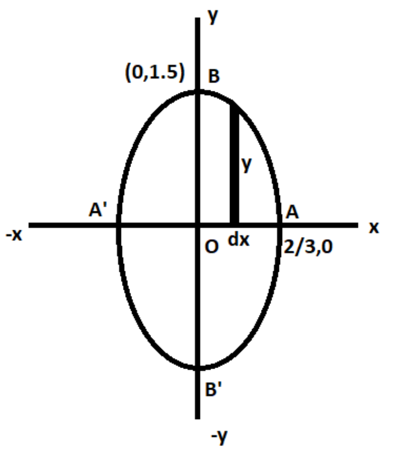
a) Solution:

Formula: $\frac{x^2}{(a)^2}+\frac{y^2}{(b)^2}=1\,, y=\pm\sqrt{a^2-x^2}$

$4\times\left[\text{area of the region ADBA is the first quadrant bounded by the curve, } x-\text{axis}, x=0 \text{ and } x=\frac{2}{3}\right].$

$=4\int_0^a y dx,\quad 4\int_0^{2/3}$ ydx (taking vertical step)

Now $\frac{x^2}{\left(\frac{2}{3}\right)^2}+\frac{y^2}{\left(\frac{3}{2}\right)^2}=1$; gives $y=\pm\sqrt{a^2-x^2}$, but as AOBA lies in first quadrant, y is taken positive. So, the required area is



$=\;4\int_0^{2/3}\frac{2}{3}\times\frac{3}{2}\times\sqrt{\frac{4}{9}-x^2dx}$

$=\;\int_0^{\frac{2}{3}}\sqrt{\frac{4}{9}-x^2dx}$

= $\frac{1}{3}\int\sqrt{4-9x^2}\,dx$

Substituting $x=\frac{2}{3}\sin U$

= $u=\frac{2}{3}\sin^{-1}\left(\frac{3x}{2}\right)$

= $dx=\frac{2\cos(u)}{3}du$

= $\int\frac{2\cos(u)\sqrt{4-4\sin^2u}\,du}{3}$

= $\frac{4}{3}\int\cos^2(u)du$

= $\frac{4}{3}\left[\frac{\cos(u)\sin(u)}{2}+\frac{1}{2}\int du\right]$

= $\frac{4}{3}\left[\frac{\cos(u)\sin(u)}{2}+\frac{1}{2}u\right]$

$= \frac{2}{3}\cos(u)\sin(u) + \frac{2}{3}u; \quad u = \sin^{-1}\left(\frac{3x}{2}\right)$

$= \frac{2\sin^{-1}\left(\frac{3x}{2}\right)}{3} + x\sqrt{1 - \frac{9x^2}{4}}dx$

$= \frac{2\sin^{-1}\left(\frac{3x}{2}\right)}{9} + \frac{x\sqrt{1-\frac{9x^2}{4}}}{3}; \quad \text{put } x = \frac{2}{3}$

$= \frac{2}{9}\sin^{-1}(1)$

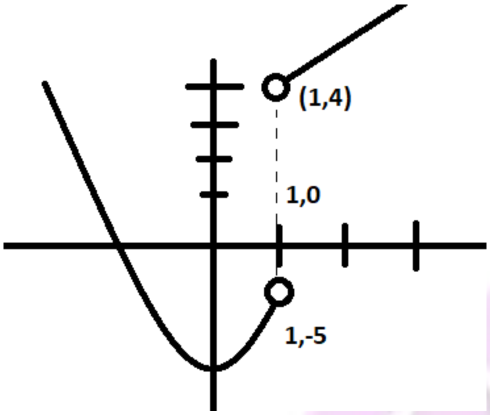
$= \frac{2}{9} \times \frac{\pi}{2};$

Answer $\frac{\pi}{9}$ is the area of the the ellipse $\frac{x^2}{\left(\frac{2}{3}\right)^2} + \frac{y^2}{\left(\frac{3}{2}\right)^2} = 1$

b) Solution:

The right- hand limit of f(x); $\lim_{x\to1^+} f(x) = \lim_{x\to1^+}(x+3) = 4$

The left- hand limit of f(x); $\lim_{x\to1^-} f(x) = \lim_{x\to1^-}(x^2-5) = -4$



Answer

Thus, the left and right hand at x=1, doesn't coincides. Hence x=1 is the point of discontinuity of F.

17.

a) Solution:

The area of the triangle is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 4 & 1 \\ -5 & 1 & 1 \\ 5 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(1-3) - 4(-5-5) + 1(-15-5)]$$

$$= \frac{1}{2}[2 \times (-2) - 4 \times (-10) + 1 \times (-20)]$$

$$= \frac{1}{2}[16] = 8 \text{ units}^2$$

b) Solution:

We have $A^2 = \begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix}\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ -6 & -1 \end{vmatrix}$

$-3A = -3\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} = -\begin{vmatrix} 6 & 3 \\ -6 & 3 \end{vmatrix}$

and $4I = 4\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

therefore

$= A - 3A + 4I = \begin{vmatrix} 2 & 3 \\ -6 & -1 \end{vmatrix} - \begin{vmatrix} 6 & 3 \\ -6 & 3 \end{vmatrix} + \begin{vmatrix} 4 & 0 \\ 0 & 4 \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$

Now using A^2; We calculate A^4

$A^2 . A^2 = A^4 = \begin{vmatrix} 2 & 3 \\ -6 & -1 \end{vmatrix}\begin{vmatrix} 2 & 3 \\ -6 & -1 \end{vmatrix} = \begin{vmatrix} -14 & 3 \\ -6 & -7 \end{vmatrix}$

Answer

$A^2 = \begin{vmatrix} -14 & 3 \\ -6 & -7 \end{vmatrix}$

18.

a) Solution:

Using Partial Fraction Method

$= \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)}$

therefore

$2x - 4 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$

$= A(x^2 + 3x + 2) + B(x+2) + C(x^2 + 2x + 1)$

$A + C = 0$ ____________________(1)

$3A + B + 2C = 2$ ______________(2)

$2A + 2B + C = 0$ ______________(3)

Evaluating Eqn. 1, 2 and 3, we get

$2B + 4 = C$ ___________I

$A = -2(B + 2)$_______II

$C = 8 - 2A$__________III

Equating I, II, III we get $A = 8, B = -6, C = -8$

$\frac{8}{(x+1)} + \frac{-6}{(x+1)^2} + \frac{-8}{(x+2)}$

$8\int \frac{dx}{(x+1)} - 6\int \frac{dx}{(x+1)^2} - 8\int \frac{dx}{(x+2)}$

$8 \log|x+1| - \frac{3}{(x+1)^2} - 8\log|x+2| + C$

$8 \log\frac{x+1}{x+2} - \frac{3}{(x+1)} + C$

b) Solution:

$f(x) = g(x)$

$x^2 - 11 = x + 9$

$x^2 - x - 20 = 0$

$x^2 - 5x + 4x - 20 = 0$

$x(x-5) + 4(x-5) = 0$

$(x+4)(x-5) = 0$

Answer:

The domain for which the function $f(x) = g(x)$ is $\{-4,5\}$.

19.

a) Solution:

Since the vector is to be directed from, point R to S, clearly R is the initial point and S is the terminal point. Therefore, the required vector joining R and S is the vector $\underset{RS}{\rightarrow}$, given by

$\underset{PQ}{\rightarrow} = (-3 - -5)\hat{\imath} + (-1 - 3)\hat{\jmath} + (-2 - 2)\hat{k}$

$\underset{PQ}{\rightarrow} = 2i - 4j - 4k$

Answer

Hence the required vector is $2i - 4j - 4k$

b) Solution:

Comparing the given equation of the planes with the equations

$$A_1x + B_1y + C_1z + D_1 = 0 \text{ and } A_2x + B_2y + C_2z + D_2 = 0$$
$$A_1 = 2\ B_1 = 3\ C_1 = 4$$
$$A_2 = 4\ B_2 = 5\ C_2 = 2$$

$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A^2 + B^2 + C^2}\sqrt{A^2 + B^2 + C^2}}\right|$

$\cos\theta = \left|\frac{2 \times 4 + 3 \times 5 + 4 \times 2}{\sqrt{2^2 + 3^2 + 4^2}\sqrt{4^2 + 5^2 + 2^2}}\right|$

$\cos\theta = \left|\frac{2 \times 4 + 3 \times 5 + 4 \times 2}{\sqrt{2^2 + 3^2 + 4^2}\sqrt{4^2 + 5^2 + 2^2}}\right|$

$\cos\theta = \left|\frac{31}{\sqrt{29}\sqrt{45}}\right|$

$\theta = \cos^{-1}\left|\frac{31}{\sqrt{29}\sqrt{45}}\right|$

$\theta = 36.86^o$is the the angle between two planes

$2x + 3y + 4z = 7$ and $4x + 5y + 2z = 5$.

20.

a) Solution:

To prove that = $\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{2} = \tan^{-1}\frac{11}{7}$

L.H.S

$\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{\frac{1}{2}+\frac{3}{5}}{1-\frac{3}{5}+\frac{1}{2}} == \tan^{-1}\frac{\frac{11}{10}}{\frac{7}{10}} = \tan^{-1}\frac{11}{7}$

$= \tan^{-1}\frac{11}{7} = \text{R.H.S}$

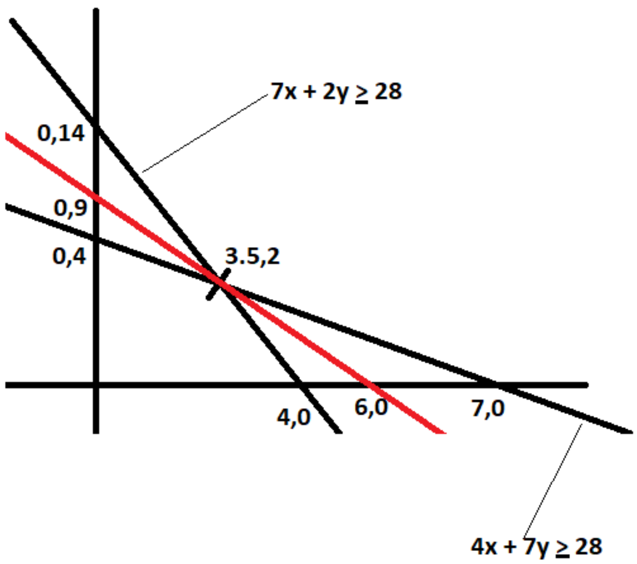
b) Solution:

The minimum value required for mixture that contains Vitamin C and Calcium:

$4x + 7y \geq 28$

$7x + 2y \geq 28$

Resource	FOOD		Requirement
	I	II	
Vitamin C	4	7	28
Calcium	7	2	28
Cost(Rs./kg)	60	90	



coordinates	
0,14	60x+90y = 60x0+90x14 = 1260
3.5,2	60x+90y = 60x3.5+90x2 = 390
7,0	60x+90y = 60x7+90x0 = 420

In the table we find the smallest value of Z is 390, at point (3.5,2)
Can we say that the minimum value of Z is 390? As, the region is unbounded.
Therefore, we have to draw the graph of inequality.

$60x + 90y < 390$

$2x + 3y < 13$

The minimum value of Z is 390 obtained by (3.5,2). Hence optimal mixing strategy for the nutritionist will be to mix 3.5kg of food I to 2 kg of food II.