EXERCISE 1(A) PAGE: 4

1. Is zero a rational number? Can it be written in the form $\frac{p}{q}$, where p and q are integers and q≠0?

Solution:

Yes, zero is a rational number.

As it can be written in the form of $\frac{p}{q}$, where p and q are integers and q≠0 ⇒ 0 = $\frac{0}{1}$

2. Are the following statements true or false? Give reasons for your answers.

(i) Every whole number is a natural number.

(ii) Every whole number is a rational number.

(iii) Every integer is a rational number.

(iv) Every rational number is a whole number.

Solution:

(i) False

Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals)
i.e., Natural numbers= 1,2,3,4…
Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)
i.e., Whole numbers= 0,1,2,3…
Or, we can say that whole numbers have all the elements of natural numbers and zero.
∴ Every natural number is a whole number, however, every whole number is not a natural number.

(ii) True

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)
i.e., Whole numbers= 0,1,2,3…
Rational numbers- All numbers in the form $\frac{p}{q}$, where p and q are integers and q≠0.
i.e., Rational numbers= $0, \frac{19}{30}, 2, \frac{9}{-3}, \frac{-12}{7}$…
∴ Every whole number is a rational number, however, every rational number is not a whole number.

(iii) True

Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.
i.e., integers= {…-4,-3,-2,-1,0,1,2,3,4…}
Rational numbers- All numbers in the form $\frac{p}{q}$, where p and q are integers and q≠0.
i.e., Rational numbers= $0, \frac{19}{30}, 2, \frac{9}{-3}, \frac{-12}{7}$…
∴ Every integer is a rational number, however, every rational number is not an integer.

(iv) False

Rational numbers- All numbers in the form p/q, where p and q are integers and q≠0.
i.e., Rational numbers= 0,19/30,2, 9/(-3), (-12)/7…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals) i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers includes whole numbers as well as negative numbers.

∴ Every whole numbers are rational, however, every rational numbers are not whole numbers.

3. Arrange $\frac{-5}{9}, \frac{7}{12}, \frac{-2}{3}$ and $\frac{11}{18}$ in the ascending order of their magnitudes. Also, find the difference between the largest and the smallest of these rational numbers. Express this difference as a decimal fraction correct to one decimal place.

Solution:

Consider the given numbers: $-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3}$ and $\frac{11}{18}$

The L.C.M of 9,12, and 18 is 36

Thus the given numbers are:

$$-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3} \text{ and } \frac{11}{18} = -\frac{5\times 4}{9\times 4}, \frac{7\times 3}{12\times 3}, -\frac{2\times 12}{3\times 12} \text{ and } \frac{11\times 2}{18\times 2}$$
$$= -\frac{20}{36}, \frac{21}{36}, -\frac{24}{36} \text{ and } \frac{22}{36}$$

Thus the numbers in ascending order are shown below:

$$-\frac{24}{36}, -\frac{20}{36}, \frac{21}{36} \text{ and } \frac{22}{36}$$

Thus the given numbers in ascending order are shown below:

$$-\frac{2}{3}, -\frac{5}{9}, \frac{7}{12} \text{ and } \frac{11}{18}$$

We need to find the difference between the largest and smallest of the above numbers.

$$\text{Thus, difference} = \frac{11}{18} - \left(-\frac{2}{3}\right)$$
$$= \frac{11}{18} + \frac{2}{3}$$
$$= \frac{11}{18} + \frac{2\text{x}6}{3\text{x}6}$$
$$= \frac{11}{18} + \frac{12}{18}$$
$$= \frac{11+12}{18}$$
$$= \frac{23}{18}$$

We need to express this fraction as a decimal, correct to one decimal place.

Thus, we have $\frac{23}{18} = 1.2\bar{7} \approx 1.3.$

4. Arrange $\frac{5}{8}, \frac{-3}{16}, \frac{-1}{4}$ and $\frac{17}{32}$ in the descending order of their magnitudes. Also,

find the sum of the lowest and the largest of these rational numbers. Express the result obtained as a decimal fraction correct to two decimal places.

Solution:

Consider the given numbers: $\frac{5}{8}$, $-\frac{3}{16}$, $-\frac{1}{4}$ and $\frac{17}{32}$.

The LCM of 8, 16, 4 and 32 is 32.

Thus, the given numbers are given below:

$$\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4} \text{ and } \frac{17}{32} = \frac{5\times4}{8\times4}, -\frac{3\times2}{16\times2}, -\frac{1\times8}{4\times8} \text{ and } \frac{17\times1}{32\times1}$$

$$= \frac{20}{32}, -\frac{6}{32}, -\frac{8}{32} \text{ and } \frac{17}{32}$$

Thus, the numbers in descending order are shown below:

$\frac{20}{32}$, $\frac{17}{32}$, $-\frac{6}{32}$ and $-\frac{8}{32}$.

Thus, the given numbers in descending order are listed below:

$\frac{5}{8}, \frac{17}{32}, -\frac{3}{16}$ and $-\frac{1}{4}$.

We need to find the sum of the

largest and the smallest of the above numbers.

$$\text{Thus, sum } = \frac{5}{8} + \left(-\frac{1}{4}\right)$$

$$= \frac{5}{8} - \frac{1}{4}$$

$$= \frac{5}{8} - \frac{1\times2}{4\times2}$$

$$= \frac{5}{8} - \frac{2}{8}$$

$$= \frac{3}{8}$$

We need to express this fraction as a decimal,

correct to two decimal places.

Thus, we have $\frac{3}{8} = 0.375 \approx 0.38$.

5. Without doing any actual division, find which of the following rational numbers have terminating decimal representation:

(i) $\frac{7}{16}$

(ii) $\frac{23}{125}$

(iii) $\frac{9}{14}$

(iv) $\frac{32}{45}$

(v) $\frac{43}{50}$

(vi) $\frac{17}{40}$

(vii) $\frac{61}{75}$

(viii) $\frac{123}{250}$

Solution:

(i)

Given number is $\frac{7}{16}$

Since $16 = 2 \times 2 \times 2 \times 2 = 2^4 = 2^4 \times 5^0$

i.e. 16 can be expressed as $2^m \times 5^n$

$\therefore \frac{7}{16}$ is convertible into the terminating decimal.

(ii)

Given number is $\frac{23}{125}$

Since $125 = 5 \times 5 \times 5 = 5^3 = 2^0 \times 5^3$

i.e. 125 can be expressed as $2^m \times 5^n$

$\therefore \frac{23}{125}$ is convertible into the terminating decimal.

(iii)

Given number is $\frac{9}{14}$

Since $14 = 2 \times 7 = 2^1 \times 7^1$

i.e. 14 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{9}{14}$ is not convertible into the terminating decimal.

(iv)

Given number is $\frac{32}{45}$

Since $45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

i.e. 45 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{32}{45}$ is not convertible into the terminating decimal.

(v)

Given number is $\frac{43}{50}$

Since $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$

i.e. 50 can be expressed as $2^m \times 5^n$

$\therefore \frac{43}{50}$ is convertible into the terminating decimal.

(vi)

Given number is $\frac{17}{40}$

Since $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

i.e. 40 can be expressed as $2^m \times 5^n$

$\therefore \frac{17}{40}$ is convertible into the terminating decimal.

(vii)

Given number is $\frac{61}{75}$

Since $75 = 3 \times 5 \times 5 = 3^1 \times 5^2$

i.e. 75 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{61}{75}$ is not convertible into the terminating decimal.

(viii)

Given number is $\frac{123}{250}$

Since $250 = 2 \times 5 \times 5 \times 5 = 2^1 \times 5^3$

i.e. 250 can be expressed as $2^m \times 5^n$

$\therefore \frac{123}{250}$ is convertible into the terminating decimal.

EXERCISE 1(B) PAGE: 13

1. State whether the following numbers are rational or not:

(i) $(2+\sqrt{2})^2$

(ii) $(3-\sqrt{3})^2$

(iii) $(5+\sqrt{5})(5-\sqrt{5})$

(iv) $(\sqrt{3}-\sqrt{2})^2$

(v) $\left(\frac{3}{2\sqrt{2}}\right)^2$

(vi) $\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2$

Solution:

(i)

$$(2+\sqrt{2})^2 = 2^2 + 2(2)(\sqrt{2}) + (\sqrt{2})^2$$
$$= 4 + 4\sqrt{2} + 2 = 6 + 4\sqrt{2}$$

∴, irrational

(ii)

$$(3-\sqrt{3})^2 = (3)^2 - 2(3)(\sqrt{3}) + (\sqrt{3})^2$$
$$= 9 - 6\sqrt{3} + 3$$
$$= 12 - 6\sqrt{3} = 6(2-\sqrt{3})$$

∴, irrational

(iii)

$$(5+\sqrt{5})(5-\sqrt{5}) = (5)^2 - (\sqrt{5})^2$$
$$= 25 - 5 = 20$$

∴, rational

(iv)

$$(\sqrt{3}-\sqrt{2})^2 = (\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2$$
$$= 3 - 2\sqrt{6} + 2 = 5 - 2\sqrt{6}$$

∴, irrational

(v)

$$\left(\frac{3}{2\sqrt{2}}\right)^2 = \frac{(3)^2}{(2\sqrt{2})^2} = \frac{9}{4\times 2} = \frac{9}{8}$$

∴, rational

(vi)

$$\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2 = \frac{\left(\sqrt{7}\right)^2}{\left(6\sqrt{2}\right)^2} = \frac{7}{36\times 2} = \frac{7}{72}$$

$\therefore$, rational

2. Find the square of:

(i) $\left(\frac{3\sqrt{5}}{5}\right)^2$

(ii) $\sqrt{3}+\sqrt{2}$

(iii) $\sqrt{5}-2$

(iv) $3+2\sqrt{5}$

Solution:

(i)

$$\left(\frac{3\sqrt{5}}{5}\right)^2 = \frac{3^2\left(\sqrt{5}\right)^2}{5^2}$$
$$= \frac{9\times 5}{25}$$
$$= \frac{9}{5}$$
$$= 1\frac{4}{5}$$

(ii)

$$\left(\sqrt{3}+\sqrt{2}\right)^2 = \left(\sqrt{3}\right)^2 + 2\left(\sqrt{3}\right)\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2$$
$$= 3+2\sqrt{6}+2 = 5+2\sqrt{6}$$

(iii)

$$\left(\sqrt{5}-2\right)^2 = \left(\sqrt{5}\right)^2 - 2\left(\sqrt{5}\right)(2) + (2)^2$$
$$= 5-4\sqrt{5}+4$$
$$= 9-4\sqrt{5}$$

(iv)

$$\left(3+2\sqrt{5}\right)^2 = 3^2 + 2(3)\left(2\sqrt{5}\right) + \left(2\sqrt{5}\right)^2$$
$$= 9+12\sqrt{5}+20$$
$$= 29+12\sqrt{5}$$

3. State, in each case, whether true or false:

(i) $\sqrt{2}+\sqrt{3}=\sqrt{5}$

(ii) $2\sqrt{4}+2=6$

(iii) $3\sqrt{7}-2\sqrt{7}=\sqrt{7}$

(iv) $\frac{2}{7}$ **is an irrational number.**

(v) $\frac{5}{11}$ **is a rational number.**

(vi) **All rational numbers are real numbers.**

(vii) **All real numbers are rational numbers.**

(viii) **Some real numbers are rational numbers.**

Solution:

(i) False
(ii) True
(iii) True
(iv) False
(v) True
(vi) True
(vii) False
(viii) True

4.

Given Universal set is

$$\left\{-6,\ -5\frac{3}{4},\ -\sqrt{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ \sqrt{8},\ 3.01,\ \pi,\ 8.47\right\}$$

From the given set, find:

(i) **Set of Rational numbers**

(ii) **Set of irrational numbers**

(iii) **Set of integers**

(iv) **Set of non-negative integers**

Solution:

(i)

We need to find the set of rational numbers.

Rational numbers are numbers of the form $\frac{p}{q}$, where $q \neq 0$.

$$U=\left\{-6,\ -5\frac{3}{4},\ -\sqrt{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ \sqrt{8},\ 3.01,\ \pi,\ 8.47\right\}$$

Clearly, $-5\frac{3}{4}$, $-\frac{3}{5}$, $-\frac{3}{8}$, $\frac{4}{5}$ and $1\frac{2}{3}$ are of the form $\frac{p}{q}$.

Hence, they are rational numbers.

Since the set of integers is a subset of rational numbers,

−6, 0 and 1 are also rational numbers.

Thus, decimal numbers 3.01 and 8.47 are also rational numbers

because they are terminating decimals.

Hence, from the above set, the set of rational

numbers is Q, and $Q=\left\{-6,\ -5\frac{3}{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ 3.01,\ 8.47\right\}$

(ii)
We need to find the set of irrational numbers.
Irrational numbers are numbers which are not rational.
From the above subpart, the set of rational
numbers is Q,

and $Q=\left\{-6,\ -5\frac{3}{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ 3.01,\ 8.47\right\}$

Set of irrational numbers is the set of complement of
the rational numbers over real numbers.
Here the set of irrational numbers is $U - Q = \left\{\sqrt{8}, \pi\right\}$

(iii)
We need to find the set of integers.
Set of integers consists of zero, the natural numbers
and their additive inverses.
The set of integers is Z
$Z=\{\ldots -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ldots\}$
Here the set of integers is $U \cap Z = \left\{-6,\ \sqrt{4},\ 0,\ 1\right\}$.

(iv)
We need to find the set of non-negative integers.
Set of non-negative integers consists of zero and
the natural numbers.
The set of non-negative integers is Z^+ and
$Z^+=\{0, 1, 2, 3, \ldots\}$
Here the set of integers is $U \cap Z^+ = \{0, 1\}$

5. Use method of contradiction to show that $\sqrt{3}$ and $\sqrt{5}$ are irrational.

Solution:

Let us suppose that $\sqrt{3}$ and $\sqrt{5}$ are rational numbers

$\sqrt{3} = \frac{a}{b}$ and $\sqrt{5} = \frac{x}{y}$ (Where a, b $\in$7 and b, y $\neq$ 0 x , y)

Squaring both sides

$3 = \frac{a^2}{b^2}$ $\quad ,5 = \frac{x^2}{y^2}$

$\left.3b^2 = a^2 \quad ,5y^2 = x^2\right\} \quad \ldots(*)$

$\Rightarrow a^2$ and x^2 are odd as $3b^2$ and $5y^2$ are odd .
$\Rightarrow$ a and x are odd....(1)
Let a = 3c, x = 5z

$a^2 = 9c^2$, $x^2 = 25z^2$

$3b^2 = 9c^2$, $5y^2 = 25z^2$(From equation $(*)$)
$\Rightarrow b^2 = 3c^2$, $y^2 = 5z^2$
$\Rightarrow b^2$ and y^2 are odd as $3c^2$ and $5z^2$ are odd .
$\Rightarrow$ b and y are odd...(2)
From equation (1) and (2) we get a, b, x, y are odd integers.
i.e., a, b, and x, y have common factors 3 and 5 this contradicts our assumption that $\frac{a}{b}$ and $\frac{x}{y}$ are rational i.e, a, b and x, y do not have any common factors other than.
$\Rightarrow \frac{a}{b}$ and $\frac{x}{y}$ is not rational
$\Rightarrow \sqrt{3}$ and $\sqrt{5}$ are irrational.

6. Prove that each of the following numbers is irrational:

(i) $\sqrt{3}+\sqrt{2}$
(ii) $3-\sqrt{2}$
(iii) $\sqrt{5}-2$

Solution:

(i) $\sqrt{3}+\sqrt{2}$

Let $\sqrt{3}+\sqrt{2}$ be a rational number.
$\Rightarrow \sqrt{3}+\sqrt{2} = x$
Squaring on both the sides, we get

$$\left(\sqrt{3}+\sqrt{2}\right)^2 = x^2$$
$$\Rightarrow 3 + 2 + 2 \times \sqrt{3} \times \sqrt{2} = x^2$$
$$\Rightarrow x^2 - 5 = 2\sqrt{6}$$
$$\Rightarrow \sqrt{6} = \frac{x^2 - 5}{2}$$

Here, x is a rational number.
$\Rightarrow x^2$ is a rational number.
$\Rightarrow x^2 - 5$ is a rational number.
$\Rightarrow \frac{x^2 - 5}{2}$ is also a rational number.
$\Rightarrow \frac{x^2 - 5}{2} = \sqrt{6}$ is a rational number.
But $\sqrt{6}$ is an irrational number.
$\Rightarrow \frac{x^2 - 5}{2}$ is an irrational number.
$\Rightarrow x^2 - 5$ is an irrational number.
$\Rightarrow x^2$ is an irrational number.
$\Rightarrow$ x is an irrational number.

But we have assume that x is a rational number.
∴ we arrive at a contradiction.
So, our assumption that $\sqrt{3}+\sqrt{2}$ is a rational number is wrong.
∴ $\sqrt{3}+\sqrt{2}$ is an irrational number.

(ii) $3-\sqrt{2}$
Let $3-\sqrt{2}$ be a rational number.
$\Rightarrow 3-\sqrt{2} = x$
Squaring on both the sides, we get

$$\left(3-\sqrt{2}\right)^2 = x^2$$
$$\Rightarrow 9 + 2 - 2 \times 3 \times \sqrt{2} = x^2$$
$$\Rightarrow 11 - x^2 = 6\sqrt{2}$$
$$\Rightarrow \sqrt{2} = \frac{11-x^2}{6}$$

Here, x is a rational number.
$\Rightarrow x^2$ is a rational number.
$\Rightarrow 11 - x^2$ is a rational number.
$\Rightarrow \frac{11-x^2}{6}$ is also a rational number.
$\Rightarrow \sqrt{2} = \frac{11-x^2}{6}$ is a rational number.
But $\sqrt{2}$ is an irrational number.
$\Rightarrow \frac{11-x^2}{6} = \sqrt{2}$ is an irrational number.
$\Rightarrow 11 - x^2$ is an irrational number.
$\Rightarrow x^2$ is an irrational number.
$\Rightarrow$ x is an irrational number.
But we have assume that x is a rational number.
∴ we arrive at a contradiction.
So, our assumption that $3-\sqrt{2}$ is a rational number is wrong.
∴ $3-\sqrt{2}$ is an irrational number.

(iii) $\sqrt{5}-2$
Let $\sqrt{5}-2$ be a rational number.
$\Rightarrow \sqrt{5}-2 = x$
Squaring on both the sides, we get

$$\left(\sqrt{5} - 2\right)^2 = x^2$$
$$\Rightarrow 5 + 4 - 2 \times 2 \times \sqrt{5} = x^2$$
$$\Rightarrow 9 - x^2 = 4\sqrt{5}$$
$$\Rightarrow \sqrt{5} = \frac{9 - x^2}{4}$$

Here, x is a rational number.
⇒ x^2 is a rational number.
⇒ $9 - x^2$ is a rational number.
⇒ $\frac{9 - x^2}{4}$ is also a rational number.
⇒ $\sqrt{2} = \frac{11 - x^2}{6}$ is a rational number.
But $\sqrt{2}$ is an irrational number.
⇒ $\sqrt{5} = \frac{9 - x^2}{4}$ is an irrational number.
⇒ $9 - x^2$ is an irrational number.
⇒ x^2 is an irrational number.
⇒ x is an irrational number.
But we have assume that x is a rational number.
∴ we arrive at a contradiction.
So, our assumption that $\sqrt{5} - 2$ is a rational number is wrong.
∴ $\sqrt{5} - 2$ is an irrational number.

7. Write a pair of irrational numbers whose sum is irrational.
Solution:

$\sqrt{3} + 5$ and $\sqrt{5} - 3$
are irrational numbers whose sum is irrational.
$$\left(\sqrt{3} + 5\right) + \left(\sqrt{5} - 3\right) = \sqrt{3} + \sqrt{5} + 5 - 3 = \sqrt{3} + \sqrt{5} + 2$$
Here, the resultant is irrational.

8. Write a pair of irrational numbers whose sum is rational.
Solution:

$\sqrt{3} + 5$ and $4 - \sqrt{3}$
are two irrational numbers whose sum is rational.
$$\left(\sqrt{3} + 5\right) + \left(4 - \sqrt{3}\right) = \sqrt{3} + 5 + 4 - \sqrt{3} = 9$$
Here, the resultant is rational.

9. Write a pair of irrational numbers whose difference is irrational.
Solution:

$\sqrt{3}+2$ and $\sqrt{2}-3$
are two irrational numbers whose difference is irrational.
$(\sqrt{3}+2)-(\sqrt{2}-3)=\sqrt{3}+2-\sqrt{2}+3=\sqrt{3}-\sqrt{2}+5$
Here, the resultant is irrational.

10.Write a pair of irrational numbers whose difference is rational.
Solution:
$\sqrt{5}-3$ and $\sqrt{5}+3$
are irrational numbers whose difference is rational.
$(\sqrt{5}-3)-(\sqrt{5}+3)=\sqrt{5}-3-\sqrt{5}-3=-6$
Here, the resultant is rational.

11.Write a pair of irrational numbers whose product is irrational.
Solution:
Consider two irrational numbers $(5+\sqrt{2})$ *and* $(\sqrt{5}-2)$
Thus, the product, $(5+\sqrt{2})\times(\sqrt{5}-2)=5\sqrt{5}-10+\sqrt{10}-2\sqrt{2}$ *is irrational*.

12.Write a pair of irrational numbers whose product is rational.
Solution:
Consider two irrational numbers $(2\sqrt{3}-3\sqrt{2})$ *and* $(2\sqrt{3}+3\sqrt{2})$
Thus, the product, $(3\sqrt{2}-2\sqrt{3})\times(3\sqrt{2}+2\sqrt{3})=(3\sqrt{2})^2-(2\sqrt{3})^2=18-12=6$
Here, the resultant is rational.

13.Write in ascending order:
(i) $\boldsymbol{3\sqrt{5}\ and\ 4\sqrt{3}}$
(ii) $\boldsymbol{2\sqrt[3]{5}\ and\ 3\sqrt[3]{2}}$
(iii) $\boldsymbol{6\sqrt{5}, 7\sqrt{3}\ and\ 8\sqrt{2}}$
Solution:
(i)
$3\sqrt{5}=\sqrt{3^2\times5}=\sqrt{45}, 4\sqrt{3}=\sqrt{4^2\times3}=\sqrt{48}$
We know that, 45 < 48
$\therefore\ \sqrt{45}<\sqrt{48}\Rightarrow 3\sqrt{5}<4\sqrt{3}$

(ii)
$2\sqrt[3]{5}=\sqrt[3]{2^3\times5}=\sqrt[3]{40}, 3\sqrt[3]{2}=\sqrt[3]{3^3\times2}=\sqrt[3]{54}$
We know that,40 < 54
$\Rightarrow \sqrt[3]{40}<\sqrt[3]{54}$
$\Rightarrow 2\sqrt[3]{5}<3\sqrt[3]{2}$

(iii)

$$6\sqrt{5} = \sqrt{6^2 \times 5} = \sqrt{180}$$

We know that, 128 < 147 < 180

$$\therefore \sqrt{128} < \sqrt{147} < \sqrt{180}$$

$$\Rightarrow 8\sqrt{2} < 7\sqrt{3} < 6\sqrt{5}$$

14. Write in descending order:

$$7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{147}$$

$$8\sqrt{2} = \sqrt{8^2 \times 2} = \sqrt{128}$$

(i) $2\sqrt[4]{6}$ ***and*** $3\sqrt[4]{2}$

(ii) $7\sqrt{3}$ ***and*** $3\sqrt{7}$

Solution:

(i)

$$2\sqrt[4]{6} = \sqrt[4]{2^4 \times 6} = \sqrt[4]{96}$$

$$3\sqrt[4]{2} = \sqrt[4]{3^4 \times 2} = \sqrt[4]{162}$$

We know that 162 > 96

$$\Rightarrow \sqrt[4]{162} > \sqrt[4]{96}$$

$$\Rightarrow 3\sqrt[4]{2} > 2\sqrt[4]{6}$$

(ii)

$$7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{141}$$

$$3\sqrt{7} = \sqrt{3^2 \times 7} = \sqrt{63}$$

We know that 141 > 63

$$\Rightarrow \sqrt{141} > \sqrt{63}$$

$$\Rightarrow 7\sqrt{3} > 3\sqrt{7}$$

15. Compare:

(i) $\sqrt[6]{15}$ ***and*** $\sqrt[4]{12}$

(ii) $\sqrt{24}$ ***and*** $\sqrt[3]{35}$

Solution:

(i) $\sqrt[6]{15} = (15)^{\frac{1}{6}}$ and $\sqrt[4]{12} = (12)^{\frac{1}{4}}$

To make the powers $\frac{1}{6}$ and $\frac{1}{4}$ same,

We find the L.C.M. of 6, 4 is 12

$$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$$

and

$$\frac{1}{4}\times\frac{3}{3}=\frac{3}{12}$$

$$\Rightarrow \sqrt[6]{15}=(15)^{\frac{1}{6}}=(15)^{\frac{2}{12}}=\left(15^2\right)^{\frac{1}{12}}=(225)^{\frac{1}{12}}$$

$$\text{and}\sqrt[4]{12}=(12)^{\frac{1}{4}}=(12)^{\frac{3}{12}}=\left(12^3\right)^{\frac{1}{12}}=(1728)^{\frac{1}{12}}$$

$$\Rightarrow 1272>225$$

$$\Rightarrow (1728)^{\frac{1}{12}}>(225)^{\frac{1}{12}}$$

$$\Rightarrow \sqrt[4]{12}>\sqrt[6]{15}$$

(ii) $\sqrt{24}=(24)^{\frac{1}{2}}$ and $\sqrt[3]{35}=(35)^{\frac{1}{3}}$

To make the powers $\frac{1}{2}$ and $\frac{1}{3}$ same,

L.C.M. of 2 and 3 is 6.

$$\frac{1}{2}\times\frac{3}{3}=\frac{3}{6},\ \frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$$

$$\Rightarrow (24)^{\frac{1}{2}}=(24)^{\frac{3}{6}}=\left(24^3\right)^{\frac{1}{6}}=(13824)^{\frac{1}{6}}$$

$$(35)^{\frac{1}{3}}=(35)^{\frac{2}{6}}=\left(35^2\right)^{\frac{1}{6}}=(1225)^{\frac{1}{6}}$$

$$\Rightarrow 13824>1225$$

$$\Rightarrow (13824)^{\frac{1}{6}}>\sqrt[3]{35}$$

$$\Rightarrow \sqrt{24}>\sqrt[3]{35}$$

16. Insert two irrational numbers between 5 and 6.

Solution:

Here, we write 5 and 6 as square root.

We know that $5=\sqrt{25}$ *and* $6=\sqrt{36}$.

Thus consider the numbers,

$$\sqrt{25}<\sqrt{26}<\sqrt{27}<\sqrt{28}<\sqrt{29}<\sqrt{30}<\sqrt{31}<\sqrt{32}<\sqrt{33}<\sqrt{34}<\sqrt{35}<\sqrt{36}$$

Therefore, any two irrational numbers between 5 and 6 is $\sqrt{27}$ *and* $\sqrt{28}$

17.Insert five irrational numbers between $2\sqrt{5}$ and $3\sqrt{3}$.

Solution:

We know that $2\sqrt{5}=\sqrt{4\times 5}=\sqrt{20}$ *and* $3\sqrt{3}=\sqrt{27}$

Thus, we have, $\sqrt{20}<\sqrt{21}<\sqrt{22}<\sqrt{23}<\sqrt{24}<\sqrt{25}<\sqrt{26}<\sqrt{27}$

So any five irrational numbers between $2\sqrt{5}$ *and* $3\sqrt{3}$ *are:*

$\sqrt{21},\sqrt{22},\sqrt{23},\sqrt{24}$ *and* $\sqrt{26}$

18.Write two rational numbers between $\sqrt{2}$ *and* $\sqrt{3}$.

Solution:

Let us take any two rational numbers between 2 and 3 which are perfect squares.
For example, let us consider 2.25 and 2.56.
Now, we have,

$\sqrt{2.25}=1.5$ *and* $\sqrt{2.56}=1.6$

$$\sqrt{2}<\sqrt{2.25}<\sqrt{2.56}<\sqrt{3}$$

$$\Rightarrow\sqrt{2}<1.5<1.6<\sqrt{3}$$

$$\Rightarrow\sqrt{2}<\frac{15}{10}<\frac{16}{10}<\sqrt{3}$$

$$\Rightarrow\sqrt{2}<\frac{3}{2}<\frac{8}{5}<\sqrt{3}$$

Therefore any two rational numbers between $\sqrt{2}$ *and* $\sqrt{3}$ *are:* $\frac{3}{2}$ *and* $\frac{8}{5}$

19.Write three rational numbers between $\sqrt{3}$ *and* $\sqrt{5}$.

Solution:

Let us take any two rational numbers between 3 and 5 which are perfect squares.
For example, let us consider, 3.24, 3.61, 4, 4.41 and 4.84
Now,

$\sqrt{3.24}=1.8, \sqrt{3.61}=1.9, \sqrt{4}=2, \sqrt{4.41}=2.1$ *and* $\sqrt{4.84}=2.2$

Thus we have,

$$\sqrt{3}<\sqrt{3.24}<\sqrt{3.61}<\sqrt{4}<\sqrt{4.41}<\sqrt{4.84}<\sqrt{5}$$

$$\Rightarrow\sqrt{3}<1.8<1.9<2<2.1<2.2<\sqrt{5}$$

$$\Rightarrow\sqrt{3}<\frac{18}{10}<\frac{19}{10}<2<\frac{21}{10}<\frac{22}{10}<\sqrt{5}$$

$$\Rightarrow\sqrt{3}<\frac{9}{5}<\frac{19}{10}<2<\frac{21}{10}<\frac{11}{5}<\sqrt{5}$$

Therefore, any three rational numbers between $\sqrt{3}$ *and* $\sqrt{5}$ *are:*

$\frac{9}{5},\frac{19}{10}$ *and* $\frac{21}{10}$

20.Simplify each of the following:

(i) $\sqrt[5]{16} \times \sqrt[5]{2}$

(ii) $\dfrac{\sqrt[4]{243}}{\sqrt[4]{3}}$

(iii) $(3+\sqrt{2})(4+\sqrt{7})$

(iv) $(\sqrt{3}-\sqrt{2})^2$

Solution:

(i)

$$\sqrt[5]{16} \times \sqrt[5]{2}$$
$$= 16^{\frac{1}{5}} \times 2^{\frac{1}{5}}$$
$$= 2^{4\times\frac{1}{5}} \times 2^{\frac{1}{5}}$$
$$= 2^{\frac{4}{5}} \times 2^{\frac{1}{5}}$$
$$= 2^{\frac{4}{5}+\frac{1}{5}}$$
$$= 2^{\frac{5}{5}}$$
$$= 2^1$$
$$= 2$$

(ii)

$$\frac{\sqrt[4]{243}}{\sqrt[4]{3}}$$
$$= \frac{\sqrt[4]{3^5}}{\sqrt[4]{3}}$$
$$= \frac{3^{5\times\frac{1}{4}}}{3^{\frac{1}{4}}}$$
$$= \frac{3^{\frac{5}{4}}}{3^{\frac{1}{4}}}$$
$$= 3^{\frac{5}{4}-\frac{1}{4}}$$
$$= 3^{\frac{4}{4}}$$
$$= 3$$

(iii)

$$\left(3+\sqrt{2}\right)\left(4+\sqrt{7}\right)$$
$$= 3\times 4+3\times\sqrt{7}+4\times\sqrt{2}+\sqrt{2}\times\sqrt{7}$$
$$= 12+3\sqrt{7}+4\sqrt{2}+\sqrt{14}$$

(iv)

$$\left(\sqrt{3}-\sqrt{2}\right)^2$$
$$= \left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\times\sqrt{3}\times\sqrt{2}$$
$$= 3+2-2\sqrt{6}$$
$$= 5-2\sqrt{6}$$

EXERCISE 1(C) PAGE: 21

1. State, with reason, which of the following are surds and which are not:

(i) $\sqrt{180}$

(ii) $\sqrt[4]{27}$

(iii) $\sqrt[5]{128}$

(iv) $\sqrt[3]{64}$

(v) $\sqrt[3]{23}.\sqrt[3]{40}$

(vi) $\sqrt[\square]{-125}$

(vii) $\sqrt{\pi}$

(viii) $\sqrt{3+\sqrt{2}}$

Solution:

(i)

$\sqrt{180} = \sqrt{2\times2\times5\times3\times3} = 6\sqrt{5}$

Which is irrational.

$\therefore$, $\sqrt{180}$ is a surd

(ii)

$\sqrt[4]{27} = \sqrt[4]{3\times3\times3}$

Which is irrational.

$\therefore$, $\sqrt[4]{27}$ is a surd

(iii)

$\sqrt[5]{128} = \sqrt[5]{2\times2\times2\times2\times2\times2\times2} = 2\sqrt[5]{4}$

Which is irrational.

$\therefore$, $\sqrt[5]{128}$ is a surd

(iv)

$\sqrt[3]{64} = \sqrt[3]{2\times2\times2\times2\times2\times2} = 4$

Which is rational.

$\therefore$, $\sqrt[3]{64}$ is not a surd

(v)

$\sqrt[3]{25}.\sqrt[3]{40} = \sqrt[3]{5\times5\times2\times2\times2\times5} = 2\times5 = 10$

Which is rational.

$\therefore$, $\sqrt[3]{23}.\sqrt[3]{40}$ is not a surd

(vi)

$\sqrt[3]{-125} = \sqrt[3]{-5 \times -5 \times -5}$

=-5

Which is rational.

∴, $\sqrt[3]{-125}$ is not a surd

(vii)

$\sqrt{\pi}$ is not a surd as π is irrational.

(viii) $\sqrt{3+\sqrt{2}}$ is not a surd as $3+\sqrt{2}$ is irrational.

2. Write the lowest rationalizing factor of:

(i) $5\sqrt{2}$
(ii) $\sqrt{24}$
(iii) $\sqrt{5}-3$
(iv) $7-\sqrt{7}$
(v) $\sqrt{18}-\sqrt{50}$
(vi) $\sqrt{5}-\sqrt{2}$
(vii) $\sqrt{13}+3$
(viii) $15-3\sqrt{2}$
(ix) $3\sqrt{2}+2\sqrt{3}$

Solution:

(i)

$5\sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$

which is rational

∴ lowest rationalizing factor is $\sqrt{2}$

(ii)

$\sqrt{24} = \sqrt{2 \times 2 \times 2 \times 3} = 2\sqrt{6}$

∴ lowest rationalizing factor is $\sqrt{6}$

(iii)

$(\sqrt{5}-3)(\sqrt{5}+3) = (\sqrt{5})^2 - (3)^2 = 5 - 9 = -4$

∴ lowest rationalizing factor is $(\sqrt{5}+3)$

(iv)

$7-\sqrt{7}$

$(7-\sqrt{7})(7+\sqrt{7}) = 49 - 7 = 42$

∴ lowest rationalizing factor is $(7+\sqrt{7})$

(v)

$$\sqrt{18}-\sqrt{50}$$

$$\sqrt{18}-\sqrt{50}=\sqrt{2\times3\times3}-\sqrt{5\times5\times2}$$

$$=3\sqrt{2}-5\sqrt{2}=-2\sqrt{2}$$

$\therefore$ lowest rationalizing factor is $\sqrt{2}$

(vi) $\sqrt{5}-\sqrt{2}$

$$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})=(\sqrt{5})^2-(\sqrt{2})^2=3$$

$\therefore$ lowest rationalizing factor is $\sqrt{5}+\sqrt{2}$

(vii)

$$\sqrt{13}+3$$

$$(\sqrt{13}+3)(\sqrt{13}-3)=(\sqrt{13})^2-3^2=13-9=4$$

$\therefore$ lowest rationalizing factor is $\sqrt{13}-3$

(viii)

$$15-3\sqrt{2}$$

$$15-3\sqrt{2}=3(5-\sqrt{2})$$

$$=3(5-\sqrt{2})(5+\sqrt{2})$$

$$=3\times\left[5^2-(\sqrt{2})^2\right]$$

$$=3\times[25-2]$$

$$=3\times23$$

$$=69$$

$\therefore$ lowest rationalizing factor is $5+\sqrt{2}$

(ix) $3\sqrt{2}+2\sqrt{3}$

$$3\sqrt{2}+2\sqrt{3}=(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})$$

$$=(3\sqrt{2})^2-(2\sqrt{3})^2$$

$$=9\times2-4\times3$$

$$=18-12$$

$$=6$$

$\therefore$ lowest rationalizing factor is $3\sqrt{2}-2\sqrt{3}$

3. Rationalize the denominators of:

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{2\sqrt{3}}{\sqrt{5}}$

(iii) $\frac{1}{\sqrt{3}-\sqrt{2}}$

(iv) $\frac{3}{\sqrt{5}+\sqrt{2}}$

(v) $\frac{2-\sqrt{3}}{2+\sqrt{3}}$

(vi) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(vii) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(viii) $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

(ix) $\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

Solution:

(i)

$$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

(ii)

$$\frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2}{5}\sqrt{15}$$

(iii)

$$\frac{1}{\sqrt{3}-\sqrt{2}} \times \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right) = \frac{\sqrt{3+\sqrt{2}}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} = \frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$= \sqrt{3}+\sqrt{2}$$

(iv)

$$\frac{3}{\sqrt{5}+\sqrt{2}} \times \left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right) = \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2} = \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$= \sqrt{5}-\sqrt{2}$$

(v)

$$\frac{2-\sqrt{3}}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = \frac{\left(2-\sqrt{3}\right)^2}{(2)^2-\left(\sqrt{3}\right)^2} = \frac{4+3-4\sqrt{3}}{4-3}$$

$$= \frac{7-4\sqrt{3}}{1} = 7-4\sqrt{3}$$

(vi)

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}=\frac{3+1+2\sqrt{3}}{3-1}=\frac{4+2\sqrt{3}}{2}$$

$$=\frac{\cancel{2}\left(2+\sqrt{3}\right)}{\cancel{2}}=2+\sqrt{3}$$

(vii)

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{3+2-2\sqrt{6}}{3-2}$$

$$=5-2\sqrt{6}$$

(viii)

$$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$

$$=\frac{6+5-2\sqrt{30}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}=\frac{11-2\sqrt{30}}{6-5}=11-2\sqrt{30}$$

(ix)

$$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}\times\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}=\frac{\left(2\sqrt{5}+3\sqrt{2}\right)^2}{\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$=\frac{4\times5+9\times2+12\sqrt{10}}{20-18}$$

$$=\frac{20+18+12\sqrt{10}}{2}=\frac{38+12\sqrt{10}}{2}=\frac{\cancel{2}\left(19+6\sqrt{10}\right)}{\cancel{2}}$$

$$=19+6\sqrt{10}$$

4. Find the values of 'a' and 'b' in each of the following:

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}}=a+b\sqrt{3}$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}}=a\sqrt{3}+b\sqrt{2}$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}}=a+b\sqrt{2}$

Solution:

(i)

$$\frac{2+\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=a+b\sqrt{3}$$

$$\frac{\left(2+\sqrt{3}\right)^2}{(2)^2-\left(\sqrt{3}\right)^2}=a+b\sqrt{3}$$
$$\frac{4+3+4\sqrt{3}}{4-3}=a+b\sqrt{3}$$
$$7+4\sqrt{3}=a+b\sqrt{3}$$
$$a=7,\ b=4$$

(ii)

$$\frac{\sqrt{7}-2}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}=a\sqrt{7}+b$$
$$\frac{\left(\sqrt{7}-2\right)^2}{\left(\sqrt{7}\right)^2-(2)^2}=a\sqrt{7}+b$$
$$\frac{7+4-4\sqrt{7}}{7-4}=a\sqrt{7}+b$$
$$\frac{11-4\sqrt{7}}{3}=a\sqrt{7}+b$$
$$a=\frac{-4}{3},\ b=\frac{11}{3}$$

(iii)

$$\frac{3}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=a\sqrt{3}-b\sqrt{2}$$
$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=a\sqrt{3}-b\sqrt{2}$$
$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{3-2}=a\sqrt{3}-b\sqrt{2}$$
$$\left(3\sqrt{3}+3\sqrt{2}\right)=a\sqrt{3}-b\sqrt{2}$$
$$\Rightarrow a=3, b=-3$$

(iv)

$$\frac{5+3\sqrt{2}}{5-3\sqrt{2}}\times\frac{5+3\sqrt{2}}{5+3\sqrt{2}}=a+b\sqrt{2}$$

$$\frac{\left(5+3\sqrt{2}\right)^2}{(5)^2-\left(3\sqrt{2}\right)^2}=a+b\sqrt{2}$$

$$\frac{25+18+30\sqrt{2}}{25-18}=a+b\sqrt{2}$$

$$\frac{43+30\sqrt{2}}{7}=a+b\sqrt{2}$$

$$a=\frac{43}{7}, \quad b=\frac{30}{7}$$

5. Simplify:

(i) $\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$

(ii) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

Solution:

(i)

$$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$$

$$\frac{22\left(2\sqrt{3}-1\right)+17\left(2\sqrt{3}+1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}=\frac{44\sqrt{3}-22+34\sqrt{3}+17}{\left(2\sqrt{3}\right)^2-1}$$

$$=\frac{78\sqrt{3}-5}{12-1}=\frac{78\sqrt{3}-5}{11}$$

(ii)

$$\frac{\sqrt{2}}{\sqrt{6}-2}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right)-\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{12}+2-\sqrt{18}+\sqrt{6}}{6-2}=\frac{2\sqrt{3}+2-3\sqrt{2}+\sqrt{6}}{4}$$

6. If $x=\frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $y=\frac{\sqrt{5}+2}{\sqrt{5}-2}$; Find:

(i) x^2
(ii) y^2
(iii) xy
(iv) $x^2+y^2=xy$

Solution:

(i)

$$x^2 = \left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)^2 = \frac{5+4-4\sqrt{5}}{5+4+4\sqrt{5}} = \frac{9-4\sqrt{5}}{9+4\sqrt{5}}$$

$$= \frac{9-4\sqrt{5}}{9+4\sqrt{5}} \times \left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right) = \frac{\left(9-4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}$$

$$= \frac{81+80-72\sqrt{5}}{81-80} = 161-72\sqrt{5}$$

(ii)

$$y^2 = \left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right)^2 = \frac{5+4+4\sqrt{5}}{5+4-4\sqrt{5}} = \frac{9+4\sqrt{5}}{9-4\sqrt{5}}$$

$$= \frac{9+4\sqrt{5}}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} = \frac{\left(9+4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2} = \frac{81+80+72\sqrt{5}}{81-80}$$

$$= 161+72\sqrt{5}$$

(iii)

$$xy = \frac{\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right)}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)} = 1$$

(iv) $x^2 + y^2 + xy = 161 - 72\sqrt{5} + 161 + 72\sqrt{5} + 1$

$= 322 + 1 = 323$

7. If $m=\frac{1}{3-2\sqrt{2}}$ and $n=\frac{1}{3+2\sqrt{2}}$, find:

(i) m^2

(ii) n^2

(iii) mn

Solution:

$$\text{(i) } m = \frac{1}{3-2\sqrt{2}}$$

$$= \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$

$$= \frac{3+2\sqrt{2}}{9-8}$$

$$= 3+2\sqrt{2}$$

$$\Rightarrow m^2 = (3+2\sqrt{2})^2$$

$$= (3)^2 + 2\times3\times2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 + 12\sqrt{2} + 8$$

$$= 17 + 12\sqrt{2}$$

$$\text{(ii) } n = \frac{1}{3+2\sqrt{2}}$$

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$

$$= \frac{3+2\sqrt{2}}{9-8}$$

$$= 3-2\sqrt{2}$$

$$\Rightarrow n^2 = (3-2\sqrt{2})^2$$

$$= (3)^2 - 2\times3\times2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 - 12\sqrt{2} + 8$$

$$= 17 - 12\sqrt{2}$$

$$\text{(iii) } mn = (3+2\sqrt{2})(3-2\sqrt{2}) = (3)^2 - (2\sqrt{2})^2 = 9-8 = 1$$

8. If $x = 2\sqrt{3} + 2\sqrt{2}$, find:

(i) $\frac{1}{x}$

(ii) $x + \frac{1}{x}$

(iii) $\left(x + \frac{1}{x}\right)^2$

Solution:

(i)
$$\frac{1}{x} = \frac{1}{2\sqrt{3}+2\sqrt{2}} \times \frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-2\sqrt{2}} = \frac{2\sqrt{3}-2\sqrt{2}}{12-8}$$
$$= \frac{\cancel{2}\left(\sqrt{3}-\sqrt{2}\right)}{\cancel{4}2} = \frac{\sqrt{3}-\sqrt{2}}{2}$$

(ii)
$$x + \frac{1}{x} = 2\sqrt{3}+2\sqrt{2}+\frac{\sqrt{3}-\sqrt{2}}{2}$$
$$= 2\left(\sqrt{3}+\sqrt{2}\right)+\frac{\left(\sqrt{3}-\sqrt{2}\right)}{2}$$
$$= \frac{4\left(\sqrt{3}+\sqrt{2}\right)+\left(\sqrt{3}-\sqrt{2}\right)}{2}$$
$$= \frac{4\sqrt{3}+4\sqrt{2}+\sqrt{3}-\sqrt{2}}{2}$$
$$= \frac{5\sqrt{3}+3\sqrt{2}}{2}$$

(iii)
$$\left(x+\frac{1}{x}\right)^2 = \left(\frac{5\sqrt{3}+3\sqrt{2}}{2}\right)^2 = \frac{75+18+30\sqrt{6}}{4}$$
$$= \frac{93+30\sqrt{6}}{4}$$

9. If $x = 1 - \sqrt{2}$, find the value of $\left(x + \frac{1}{x}\right)^3$

Solution:

Given that $x = 1 - \sqrt{2}$

We need to find the value of $\left(x - \frac{1}{x}\right)^3$.

Since $x = 1 - \sqrt{2}$, we have

$$\frac{1}{x} = \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{1^2 - \left(\sqrt{2}\right)^2} \qquad \left[\text{Since } (a-b)(a+b) = a^2 - b^2\right]$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{1 - 2}$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{-1}$$

$$\Rightarrow \frac{1}{x} = -\left(1 + \sqrt{2}\right) \ldots (1)$$

Thus, $\left(x - \frac{1}{x}\right) = \left(1 - \sqrt{2}\right) - \left(-\left(1 + \sqrt{2}\right)\right)$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 1 - \sqrt{2} + 1 + \sqrt{2}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 2^3$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 8$$

10.If $x = 5 - 2\sqrt{6}$, find: $x^2 + \frac{1}{x^2}$

Solution:

Given $x = 5 - 2\sqrt{6}$

We need to find $x^2 + \frac{1}{x^2}$:

Since $x = 5 - 2\sqrt{6}$, we have

$$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{\left(5 - 2\sqrt{6}\right)\left(5 + 2\sqrt{6}\right)}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{5^2 - \left(2\sqrt{6}\right)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - 24}$$

$$\Rightarrow \frac{1}{x} = 5 + 2\sqrt{6} \ldots (1)$$

Thus, $\left(x - \frac{1}{x}\right) = \left(5 - 2\sqrt{6}\right) - \left(5 + 2\sqrt{6}\right)$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 5 - 2\sqrt{6} - 5 - 2\sqrt{6}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = -4\sqrt{6} \ldots (2)$$

Now consider $\left(x - \frac{1}{x}\right)^2$:

Thus

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2x \times \frac{1}{x} \quad \left[\text{since } (a-b)^2 = a^2 - 2ab + b^2\right]$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = x^2 + \frac{1}{x^2} \ldots (3)$$

Thus, from equations (2) and (3), we have

$$x^2 + \frac{1}{x^2} = \left(-4\sqrt{6}\right)^2 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 96 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 98$$

11. Show that:

$$\frac{1}{3-2\sqrt{2}}-\frac{1}{2\sqrt{2}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5$$

Solution:

$$\text{L.H.S.}=\frac{1}{3-2\sqrt{2}}-\frac{1}{2\sqrt{2}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$

$$-\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$=\frac{3+\sqrt{8}}{(3)^2-(\sqrt{8})^2}-\frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2}+\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}-\frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2}+\frac{\sqrt{5}+2}{(\sqrt{5})^2-(2)^2}$$

$$=\frac{3+\sqrt{8}}{9-8}-\frac{\sqrt{8}+\sqrt{7}}{8-7}+\frac{\sqrt{7}+\sqrt{6}}{7-6}-\frac{\sqrt{6}+\sqrt{5}}{6-5}+\frac{\sqrt{5}+2}{5-4}$$

$$=3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$=3+2$$

$$=5$$

$$=\text{R.H.S.}$$

12. Rationalize the denominator of:

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

Solution:

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

$$= \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)+1} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{\left(\sqrt{3}-\sqrt{2}\right)-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}-\sqrt{2}\right)^2-(1)^2}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}\right)^2-2\sqrt{6}+\left(\sqrt{2}\right)^2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{3-2\sqrt{6}+2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{4-2\sqrt{6}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{2\left(2-\sqrt{6}\right)}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{2\left(2-\sqrt{6}\right)} \times \frac{2+\sqrt{6}}{2+\sqrt{6}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+\sqrt{18}-\sqrt{12}-\sqrt{6}}{2\left[(2)^2-\left(\sqrt{6}\right)^2\right]}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{2(4-6)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{2(-2)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{-4}$$

$$= \frac{1}{4}\left(2+\sqrt{6}-\sqrt{2}\right)$$

13. If $\sqrt{2} = 1.4$ and $\sqrt{3} = 1.7$, find the value of each of the following, correct to one decimal place:

(i) $\frac{1}{\sqrt{3}-\sqrt{2}}$

(ii) $\frac{1}{\sqrt{3}+\sqrt{2}}$

(iii) $\frac{2-\sqrt{3}}{\sqrt{3}}$

Solution:

(i)

$$\sqrt{2} = 1.4 \text{ and } \sqrt{3} = 1.7$$

$$\frac{1}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$= \sqrt{3}+\sqrt{2}$$

$$= 1.7 + 1.4$$

$$= 3.1$$

(ii)

$$\sqrt{2} = 1.4 \text{ and } \sqrt{3} = 1.7$$

$$\text{(ii)} \quad \frac{1}{3+2\sqrt{2}}$$

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3-2\sqrt{2}}{9-8}$$

$$= 3-2\sqrt{2}$$

$$= 3-2(1.4)$$

$$= 3-2.8$$

$$= 0.2$$

(iii)

$$\sqrt{2} = 1.4 \text{ and } \sqrt{3} = 1.7$$

$$\frac{2-\sqrt{3}}{\sqrt{3}}$$
$$\frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$\frac{2\sqrt{3}-3}{3} = \frac{(2 \times 1.7)-3}{3}$$
$$\frac{3.4-3}{3} = \frac{0.4}{3} = 0.1$$

14. Evaluate:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}}$$

Solution:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}}$$
$$= \frac{4-\sqrt{5}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$$
$$= \frac{\left(4-\sqrt{5}\right)^2}{4^2-\left(\sqrt{5}\right)^2} + \frac{\left(4+\sqrt{5}\right)^2}{4^2-\left(\sqrt{5}\right)^2}$$
$$= \frac{16+5-8\sqrt{5}}{16-5} + \frac{16+5+8\sqrt{5}}{16-5}$$
$$= \frac{21-8\sqrt{5}}{11} + \frac{21+8\sqrt{5}}{11}$$
$$= \frac{21-8\sqrt{5}+21+8\sqrt{5}}{11}$$
$$= \frac{42}{11}$$
$$= 3\frac{9}{11}$$

15. If $\frac{2+\sqrt{5}}{2-\sqrt{5}} = x$ and $\frac{2-\sqrt{5}}{2+\sqrt{5}} = y$; find the value of x^2-y^2.

Solution:

$$x = \frac{2+\sqrt{5}}{2-\sqrt{5}} \qquad y = \frac{2-\sqrt{5}}{2+\sqrt{5}}$$

$$= \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} \qquad = \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{\left(2+\sqrt{5}\right)^2}{2^2-\left(\sqrt{5}\right)^2} \qquad = \frac{\left(2-\sqrt{5}\right)^2}{2^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{4+4\sqrt{5}+5}{4-5} \qquad = \frac{4-4\sqrt{5}+5}{4-5}$$

$$= \frac{9+4\sqrt{5}}{-1} \qquad = \frac{9-4\sqrt{5}}{-1}$$

$$= -9-4\sqrt{5} \qquad = -9+4\sqrt{5}$$

$$\therefore\ x^2-y^2 = \left(-9-4\sqrt{5}\right)^2-\left(-9+4\sqrt{5}\right)^2$$

$$= 81+72\sqrt{5}+80-\left(81-72\sqrt{5}+80\right)$$

$$= 81+72\sqrt{5}+80-81+72\sqrt{5}-80$$

$$= 144\sqrt{5}$$

EXERCISE 1(D)

1. **Simplify:**

$$\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{72}+2\sqrt{162}}$$

Solution:

$$\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{72}-2\sqrt{162}}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{4\times 18}-2\sqrt{9\times 18}}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}+(3\times 2\sqrt{18})-(2\times 3\sqrt{18})}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}+6\sqrt{18}-6\sqrt{18}}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}}=\frac{1}{5}$$

2. **Simplify:**

$$\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\div\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}$$

Solution:

$$\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\div\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}$$

$$=\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\times\frac{\sqrt{x^2+y^2}+y}{\sqrt{x^2-y^2}+x}$$

$$=\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\times\frac{\sqrt{x^2+y^2}+y}{x+\sqrt{x^2-y^2}}$$

$Using\ the\ identity$

$(a+b)(a-b)=a^2-b^2$

$we\ get,$

$$=\frac{(\sqrt{x^2+y^2})^2-y^2}{x^2-(\sqrt{x^2-y^2})^2}$$

$$=\frac{x^2+y^2-y^2}{x^2-x^2+y^2}$$

$$=\frac{x^2}{y^2}$$

3. **Evaluate, correct to one place of decimal. The expression $\frac{5}{\sqrt{20}-\sqrt{10}}$, if $\sqrt{5}$=2.2 and $\sqrt{10}$=3.2.**

Solution:

$$\begin{aligned}\frac{5}{\sqrt{20}-\sqrt{10}} &= \frac{5}{\sqrt{4\times 5}-\sqrt{10}} \\ &= \frac{5}{\sqrt{4\times 5}-\sqrt{10}} \\ &= \frac{5}{2\sqrt{5}-\sqrt{10}} \\ &= \frac{5}{(2\times 2.2)-3.2} \\ &= \frac{5}{4.4-3.2} \\ &= \frac{5}{1.2} \\ &= 4.2\end{aligned}$$

[Note: In textual answer, the value of $\sqrt{20}$ has been directly taken, which is 4.5. Hence the answer 3.8.]

4. **If x =$\sqrt{3}-\sqrt{2}$. Find the value of:**

(i) $x+\frac{1}{x}$

(ii) $x^2+\frac{1}{x^2}$

(iii) $x^3+\frac{1}{x^3}$

(iv) $x^3+\frac{1}{x^3}-3\left(x^2+\frac{1}{x^2}\right)+x+\frac{1}{x}$

Solution:

(i) $x+\frac{1}{x}$

$$(\sqrt{3}-\sqrt{2})+\frac{1}{(\sqrt{3}-\sqrt{2})}$$
$$=\frac{(\sqrt{3}-\sqrt{2})^2+1}{(\sqrt{3}-\sqrt{2})}$$
$$=\frac{3-2\sqrt{3}\sqrt{2}+2+1}{(\sqrt{3}-\sqrt{2})}$$
$$=\frac{6-2\sqrt{3}\sqrt{2}}{(\sqrt{3}-\sqrt{2})}$$
$$=\frac{6-2\sqrt{6}}{(\sqrt{3}-\sqrt{2})}\times\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$
$$=\frac{6\sqrt{3}-2\sqrt{6}\sqrt{3}+6\sqrt{2}-2\sqrt{6}\sqrt{2}}{1}$$
$$=6\sqrt{3}-2\sqrt{18}+6\sqrt{2}-2\sqrt{12}$$
$$=6\sqrt{3}-2\sqrt{9\times 2}+6\sqrt{2}-2\sqrt{4\times 3}$$
$$=6\sqrt{3}-2\times 3\sqrt{2}+6\sqrt{2}-2\times 2\sqrt{3}$$
$$=6\sqrt{3}-6\sqrt{2}+6\sqrt{2}-4\sqrt{3}$$
$$=6\sqrt{3}-4\sqrt{3}$$
$$=2\sqrt{3}$$

(ii) $x^2+\frac{1}{x^2}$

$$(\sqrt{3}-\sqrt{2})^2+\frac{1}{(\sqrt{3}-\sqrt{2})^2}$$
$$=(3-2\sqrt{3}\sqrt{2}+2)+\frac{1}{(3-2\sqrt{3}\sqrt{2}+2)}$$
$$=(5-2\sqrt{6})+\frac{1}{(5-2\sqrt{6})}$$
$$=\frac{25-10\sqrt{6}-10\sqrt{6}+4\times 6+1}{(5-2\sqrt{6})}$$
$$=\frac{25-20\sqrt{6}+25}{(5-2\sqrt{6})}$$
$$=\frac{50-20\sqrt{6}}{(5-2\sqrt{6})}$$
$$=\frac{10(5-2\sqrt{6})}{(5-2\sqrt{6})}$$
$$=10$$

(iii) $x^3+\frac{1}{x^3}$

$$(\sqrt{3}-\sqrt{2})^3+\frac{1}{(\sqrt{3}-\sqrt{2})^3}$$
$$We\ know\ that (a-b)^3=a^3-b^3-3ab(a-b)$$
$$(\sqrt{3}-\sqrt{2})^3=3\sqrt{3}-2\sqrt{2}-3\sqrt{3}\sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$$
$$=3\sqrt{3}-2\sqrt{2}-3\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)$$
$$=3\sqrt{3}-2\sqrt{2}-3\sqrt{18}+3\sqrt{12}$$
$$=3\sqrt{3}-2\sqrt{2}-3\sqrt{9\times 2}+3\sqrt{4\times 3}$$
$$=3\sqrt{3}-2\sqrt{2}-3\times 3\sqrt{2}+3\times 2\sqrt{3}$$
$$=3\sqrt{3}-2\sqrt{2}-9\sqrt{2}+6\sqrt{3}$$
$$=9\sqrt{3}-11\sqrt{2}$$
$$\therefore \left(\sqrt{3}-\sqrt{2}\right)^3+\frac{1}{\left(\sqrt{3}-\sqrt{2}\right)^3}=\left(9\sqrt{3}-11\sqrt{2}\right)+\frac{1}{(9\sqrt{3}-11\sqrt{2})}$$
$$Considering\ \frac{1}{(9\sqrt{3}-11\sqrt{2})}$$
$$\frac{1}{(9\sqrt{3}-11\sqrt{2})}\times\frac{(9\sqrt{3}+11\sqrt{2})}{(9\sqrt{3}+11\sqrt{2})}$$
$$=\frac{(9\sqrt{3}+11\sqrt{2})}{(81\times 3)-(121\times 2)}$$
$$=\frac{(9\sqrt{3}+11\sqrt{2})}{(243)-(242)}$$
$$=(9\sqrt{3}+11\sqrt{2})$$

$$Now, (9\sqrt{3}-11\sqrt{2})+\frac{1}{(9\sqrt{3}-11\sqrt{2})}=(9\sqrt{3}-11\sqrt{2})+(9\sqrt{3}+11\sqrt{2})$$
$$=9\sqrt{3}-11\sqrt{2}+9\sqrt{3}+11\sqrt{2}$$
$$=18\sqrt{3}$$

(iv) $\boldsymbol{x^3+\frac{1}{x^3}-3\left(x^2+\frac{1}{x^2}\right)+x+\frac{1}{x}}$

$$x^3+\frac{1}{x^3}-3(x^2+\frac{1}{x^2})+x+\frac{1}{x}$$
$$According\ to\ the\ solutions\ obtained\ in\ (i), (ii)\ and\ (iii),\ we\ get,$$
$$x^3+\frac{1}{x^3}-3(x^2+\frac{1}{x^2})+x+\frac{1}{x}=18\sqrt{3}-3(10)+2\sqrt{3}$$
$$=20\sqrt{3}-30$$
$$=10(2\sqrt{3}-3)$$

5. Show that:

(i) Negative of an irrational number is irrational.

Solution:

Let the irrational number be $\sqrt{2}$.

Considering the negative of $\sqrt{2}$, we get $-\sqrt{2}$

We know that $-\sqrt{2}$ is an irrational number.

Hence, negative of an irrational number is irrational.

(ii) The product of a non−zero rational number and an irrational number is an irrational number.

Solution:

Let the non−zero rational number be 3.

Let the irrational number be $\sqrt{5}$.

Then, according to the question,

$3 \times \sqrt{5} = 3\sqrt{5} = 3 \times 2.2 = 6.6$, which is irrational.

6. Draw a line segment of length $\sqrt{5}$ cm.

Solution:

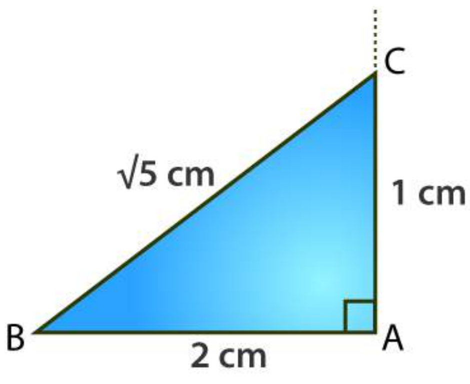
We know that, $\sqrt{5} = \sqrt{2^2 + 1^2}$

Which relates to: Hypotenuse= $\sqrt{\text{Side}1^2 + \text{Side}2^2}$ [Pythagoras theorem]

Hence, Considering Side 1=2 and Side 2 =1,

We get a right angled triangle such that:

$\angle A$=90°, AB=2cm and AC=1cm



7. Draw a line segment of length $\sqrt{3}$ cm.

Solution:

We know that, $\sqrt{3} = \sqrt{2^2 - 1^2}$

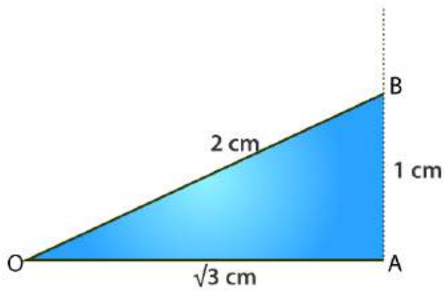
Which relates to: Hypotenuse= $\sqrt{\text{Side}1^2 + \text{Side}2^2}$ [Pythagoras theorem]

Hypotenuse^2–$\text{Side}1^2$ = $\text{Side}2^2$

Hence, Considering Hypotenuse =2cm and Side 1 =1 cm,

We get a right angled triangle OAB such that:

$\angle O$=90°, OB=2cm and AB=1cm



8. Draw a line segment of length $\sqrt{8}$ cm.

Solution:

We know that, $\sqrt{8} = \sqrt{3^2 - 1^2}$

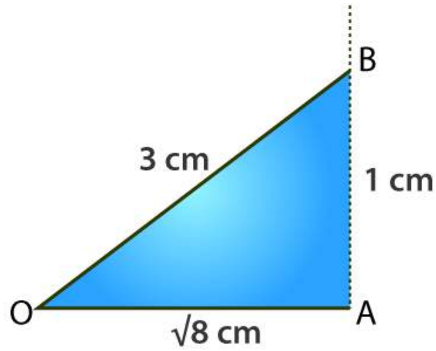
Which relates to: Hypotenuse= $\sqrt{\text{Side}1^2 + \text{Side}2^2}$ [Pythagoras theorem]

Hypotenuse^2–$\text{Side}1^2$ = $\text{Side}2^2$

Hence, Considering Hypotenuse =3cm and Side 1 =1 cm,

We get a right angled triangle OAB such that:

$\angle A$=90°, OB=3cm and AB=1cm



9. Show that:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} + \frac{2}{5+\sqrt{3}} + \frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{2}{5-\sqrt{3}} = \frac{52}{11}$$

Solution:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}=\frac{52}{11}$$

Here,

$$Considering\ \frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$\Rightarrow \frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}=\frac{(4-\sqrt{5})^2}{16-5}=\frac{(4-\sqrt{5})^2}{11}$$

$$Now,\ Considering\ \frac{2}{5+\sqrt{3}}$$

$$\Rightarrow \frac{2}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}=\frac{10-2\sqrt{3}}{25-3}=\frac{10-2\sqrt{3}}{22}$$

$$Now,\ Considering\ \frac{4+\sqrt{5}}{4-\sqrt{5}}$$

$$\Rightarrow \frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}=\frac{(4+\sqrt{5})^2}{16-5}=\frac{(4+\sqrt{5})^2}{11}$$

$$Now,\ Considering\ \frac{2}{5-\sqrt{3}}$$

$$\Rightarrow \frac{2}{5-\sqrt{3}}\times\frac{5+\sqrt{3}}{5+\sqrt{3}}=\frac{10+2\sqrt{3}}{25-3}=\frac{10+2\sqrt{3}}{22}$$

$$\therefore \frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}$$

$$=\frac{(4-\sqrt{5})^2}{11}+\frac{10-2\sqrt{3}}{22}+\frac{(4+\sqrt{5})^2}{11}+\frac{10+2\sqrt{3}}{22}$$

$$=\frac{(4-\sqrt{5})^2}{11}+\frac{5-\sqrt{3}}{11}+\frac{(4+\sqrt{5})^2}{11}+\frac{5+\sqrt{3}}{11}$$

$$=\frac{16-8\sqrt{5}+5+5-\sqrt{3}+16+8\sqrt{5}+5+5+\sqrt{3}}{11}$$

$$=\frac{52}{11}$$

Hence proved

10. Show that:

(i) $x^3+\frac{1}{x^3}=52$, if $x=2+\sqrt{3}$

(ii) $x^2+\frac{1}{x^2}=34$, if $x=3+2\sqrt{2}$

(iii) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}=11$

Solution:

(i)

$$We\ know\ that,\ (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$x^3 + \frac{1}{x^3} = (2+\sqrt{3})^3 + \frac{1}{(2+\sqrt{3})^3}$$

$$Here,\ taking\ (2+\sqrt{3})^3$$

$$\Rightarrow (2+\sqrt{3})^3 = 2^3 + \sqrt{3}^3 + 3\times 2\times\sqrt{3}(2+\sqrt{3})$$

$$= 8 + 3\sqrt{3} + 6\sqrt{3}(2+\sqrt{3})$$

$$= 8 + 3\sqrt{3} + 12\sqrt{3} + 18$$

$$= 26 + 15\sqrt{3}$$

$$Now,\ (2+\sqrt{3})^3 + \frac{1}{(2+\sqrt{3})^3} = 26 + 15\sqrt{3} + \frac{1}{26+15\sqrt{3}}$$

$$Taking\ \frac{1}{26+15\sqrt{3}},$$

$$\Rightarrow \frac{1}{26+15\sqrt{3}} \times \frac{26-15\sqrt{3}}{26-15\sqrt{3}} = \frac{26-15\sqrt{3}}{676-675} = 26-15\sqrt{3}$$

$$= 26+15\sqrt{3}+26-15\sqrt{3} = 52$$

Hence proved

(ii)

$$We\ know\ that,\ (a+b)^2 = a^2 + b^2 + 2ab$$

$$x^2 + \frac{1}{x^2} = (3+2\sqrt{2})^2 + \frac{1}{(3+2\sqrt{2})^2}$$

$$= (9+12\sqrt{2}+8) + \frac{1}{(9+12\sqrt{2}+8)}$$

$$= (17+12\sqrt{2}) + \frac{1}{(17+12\sqrt{2})}$$

$$Taking\ \frac{1}{(17+12\sqrt{2})}\ we\ get:$$

$$\frac{1}{(17+12\sqrt{2})} \times \frac{(17-12\sqrt{2})}{(17-12\sqrt{2})} = \frac{(17-12\sqrt{2})}{289-288} = 17-12\sqrt{2}$$

$$\therefore (17+12\sqrt{2}) + \frac{1}{(17+12\sqrt{2})} = 17+12\sqrt{2}+17-12\sqrt{2} = 34$$

Hence proved

(iii)

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$First, taking\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}},$$

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{(3\sqrt{2}-2\sqrt{3})^2}{18-12}=\frac{18-12\sqrt{6}+12}{6}$$

$$=\frac{6(3-2\sqrt{6}+2)}{6}=5-2\sqrt{6}$$

$$Now, taking\ \frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}},$$

$$\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{6+2\sqrt{6}}{3-2}=6+2\sqrt{6}$$

$$\therefore\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}=5-2\sqrt{6}+6+2\sqrt{6}=11$$

Hence proved

11. Show that x is irrational if:

(i) $x^2=6$

(ii) $x^2=0.009$

(iii) $x^2=27$

Solution:

(i) $x^2=6$

$\Rightarrow x=\sqrt{6}$=2.449⋯ which is irrational.

(ii) $x^2=0.009$

$\Rightarrow x=\sqrt{0.009}$=0.0948⋯ which is irrational.

(iii) **$x^2=27$**

$\Rightarrow x=\sqrt{27}$=5.1961⋯ which is irrational.

12. Show that x is rational if:

(i) $x^2=16$

(ii) **x^2=0.0004**

(iii) **$x^2=1\frac{7}{9}$**

Solution:

(i) **x^2=16**

$\Rightarrow x = \sqrt{16}$=4, which is rational.

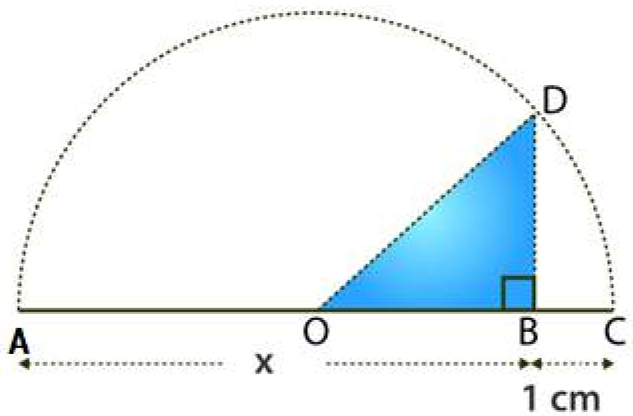
(ii) **x^2=0.0004**

$\Rightarrow x = \sqrt{0.0004}$=0.02, which is rational.

(iii) **$x^2=1\frac{7}{9}$**

$\Rightarrow x = \sqrt{\mathbf{1}\frac{7}{9}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$, which is rational.

13. Using the following figure, show that BD=$\sqrt{x}$.



Solution:

Let AB=x

BC=1

AC=x+1

Here, AC is diameter and O is the center

OA= OC = OD = radius = $\frac{x+1}{2}$

And OB = OC − BC = $\frac{x+1}{2} - 1 = \frac{x-1}{2}$

Now, using Pythagoras theorem,

$OD^2 = OB^2 + BD^2$

$$\left(\frac{x+1}{2}\right)^2 = \left(\frac{x-1}{2}\right)^2 + BD^2$$

$$\Rightarrow BD^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$$

$$\Rightarrow \frac{x^2+2x+1-x^2+2x-1}{4}$$

$$\Rightarrow \frac{4x}{4} = x$$

$$\therefore, BD = \sqrt{x}$$

Hence proved