

EXERCISE 3(A)

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1. Find the amount and the compound interest on Rs. 12,000 in 3 years at 5%; interest being compounded annually.

Solution:Given: $P = \text{Rs}12,000$; $n=3\text{years}$ and $r=5\%$

$$\begin{aligned}\text{Amount} &= P \left(1 + \frac{r}{100}\right)^n \\ &= 12000 \left(1 + \frac{5}{100}\right)^3 \\ &= 12000 \left(\frac{21}{20}\right)^3 \\ &= \text{Rs}13,891.50 .\end{aligned}$$

$$\begin{aligned}\therefore \text{C.I.} &= \text{Rs}13,891.50 - \text{Rs}12,000 \\ &= \text{Rs}1,891.50\end{aligned}$$

2. Calculate the amount, if Rs. 15000 is lent at compound interest for 2 years and the rates for the successive years are 8% p.a. and 10% p.a. respectively.

Solution:Given: $P = \text{Rs}15,000$; $n=2\text{years}$; $r_1 = 8\%$ and $r_2 = 10\%$

$$\begin{aligned}\text{Amount} &= P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \\ &= 15,000 \left(1 + \frac{8}{100}\right) \left(1 + \frac{10}{100}\right) \\ &= 15,000 \left(\frac{27}{25}\right) \left(\frac{11}{10}\right) \\ &= \text{Rs}17,820\end{aligned}$$

3. Calculate the compound interest accrued on Rs.6000 in 3 years, compounded yearly, if the rates for the successive years are 5%, 8% and 10% respectively.

Solution:

Given: $P = \text{Rs}6,000$; $n = 3 \text{ years}$; $r_1 = 5\%$; $r_2 = 8\%$ and $r_3 = 10\%$

$$\begin{aligned} \text{Amount} &= P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \left(1 + \frac{r_3}{100}\right) \\ &= 6,000 \left(1 + \frac{5}{100}\right) \left(1 + \frac{8}{100}\right) \left(1 + \frac{10}{100}\right) \\ &= 6000 \left(\frac{21}{20}\right) \left(\frac{27}{25}\right) \left(\frac{11}{10}\right) \\ &= \text{Rs}7,484.40 \\ \therefore \text{C.I.} &= \text{Rs}7,484.40 - \text{Rs}6,000 \\ &= \text{Rs}1,484.40 \end{aligned}$$

4. What sum of money will amount to Rs. 5445 in 2 years at 10% per annum compound interest?

Solution:

Given : Amount = Rs5,445; $n = 2 \text{ years}$ and $r = 10\%$

$$\begin{aligned} \therefore A &= P \left(1 + \frac{r}{100}\right)^n \\ \Rightarrow 5,445 &= P \left(1 + \frac{10}{100}\right)^2 \\ \Rightarrow 5,445 &= P \left(\frac{11}{10}\right)^2 \\ \Rightarrow P &= \frac{5,445 \left(\frac{10}{11}\right)^2}{1} \\ &= \text{Rs}4,500 \end{aligned}$$

5. On what sum of money will the compound interest for 2 years at 5 per cent per annum amount to Rs.768.75?

Solution:

Given : C.I. = Rs768.75; $n = 2 \text{ years}$ and $r = 5\%$

$$\begin{aligned}
 \therefore A &= P \left(1 + \frac{r}{100}\right)^n \\
 \Rightarrow A &= P \left(1 + \frac{5}{100}\right)^2 \\
 \Rightarrow A &= P \left(\frac{21}{20}\right)^2 = \frac{441}{400}P \\
 \therefore A - P &= C.I \\
 \Rightarrow \frac{441}{400}P - P &= \text{Rs}768.75 \\
 \Rightarrow \frac{41}{400}P &= \text{Rs}768.75 \\
 \Rightarrow P &= \text{Rs} \frac{768.75 \times 400}{41} = \text{Rs}7,500 \\
 \Rightarrow &\text{Rs. 7500}
 \end{aligned}$$

6. Find the sum on which the compound interest for 3 years at 10% per annum amounts to Rs. 1655.

Solution:

Given : C.I.= Rs1,655; n= 3years and r = 10%

$$\begin{aligned}
 \therefore A &= P \left(1 + \frac{r}{100}\right)^n \\
 \Rightarrow A &= P \left(1 + \frac{10}{100}\right)^3 \\
 \Rightarrow A &= P \left(\frac{11}{10}\right)^3 = \frac{1,331}{1,000}P \\
 \therefore A - P &= C.I \\
 \Rightarrow \frac{1,331}{1,000}P - P &= \text{Rs}1,655 \\
 \Rightarrow \frac{331}{1,000}P &= \text{Rs}1,655 \\
 \Rightarrow P &= \text{Rs} \frac{1,655 \times 1,000}{331} = \text{Rs}5,000 \\
 \Rightarrow &\text{Rs. 5000}
 \end{aligned}$$

7. What principal will amount to Rs. 9856 in two years, if the rates of interest for successive years are 10% and 12% respectively?

Solution:

Given : Amount = Rs 9,856; $n=2$ years; $r_1 = 10\%$ and $r_2 = 12\%$

$$\begin{aligned}\therefore \text{Amount} &= P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \\ \Rightarrow 9,856 &= P \left(1 + \frac{10}{100}\right) \left(1 + \frac{12}{100}\right) \\ \Rightarrow 9,856 &= P \left(\frac{11}{10}\right) \left(\frac{28}{25}\right) \\ \Rightarrow P &= \text{Rs } \frac{9,856 \times 10 \times 25}{11 \times 28} = \text{Rs } 8,000 \\ &= \text{Rs } 8000\end{aligned}$$

8. On a certain sum, the compound interest in 2 years amounts to Rs.4240. If the rates of interest for successive years are 10% and 15% respectively, find the sum.

Solution:

$$\begin{aligned}A &= P \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \\ \Rightarrow (P + 4240) &= P \left(1 + \frac{10}{100}\right) \left(1 + \frac{15}{100}\right) \\ \Rightarrow (P + 4240) &= P (1.265) \\ \Rightarrow P &= 16000 \\ \text{The sum is Rs.16,000}\end{aligned}$$

9. At what rate per cent per annum will Rs.6000 amount to Rs.6615 in 2 years when interest is compounded annually?

Solution:

$$\begin{aligned}A &= P \left(1 + \frac{r}{100}\right)^n \\ \Rightarrow 6,615 &= 6,000 \left(1 + \frac{r}{100}\right)^2 \\ \Rightarrow \left(1 + \frac{r}{100}\right)^2 &= \frac{6,615}{6,000} \\ \Rightarrow 1 + \frac{r}{100} &= \frac{21}{20} \\ \Rightarrow r &= 5\%\end{aligned}$$

At 5% per annum the sum of Rs.6,000 amounts to Rs.6,615 in 2 years when the interest is compounded annually.

- 10. At what rate per cent compound interest, does a sum of money become 1.44 times of itself in 2 years?**

Solution:

Let Principal = Rs y
Then Amount = Rs 1.44y
n = 2 years

$$\begin{aligned}A &= P \left(1 + \frac{r}{100}\right)^n \\ \therefore 1.44y &= y \left(1 + \frac{r}{100}\right)^2 \\ \Rightarrow \frac{1.44y}{y} &= \left(1 + \frac{r}{100}\right)^2 \\ \Rightarrow \frac{36}{25} &= \left(1 + \frac{r}{100}\right)^2 \\ \Rightarrow \left(\frac{6}{5}\right)^2 &= \left(1 + \frac{r}{100}\right)^2 \\ \text{On comparing,} \\ \frac{6}{5} &= 1 + \frac{r}{100} \\ \text{On solving, we get} \\ r &= 20\%\end{aligned}$$

- 11. At what rate per cent will a sum of Rs.4000 yield Rs. 1324 as compound interest in 3 years?**

Solution:

Given: $P = \text{Rs. } 4,000$, $\text{C.I.} = \text{Rs. } 1,324$ and $n = 3$ years

Now, $A = P + I$

$$\Rightarrow A = \text{Rs. } (4,000 + 1,324) = \text{Rs. } 5,324$$

$$A = P \left(1 + \frac{r}{100} \right)^3$$

$$\Rightarrow 5324 = 4000 \left(1 + \frac{r}{100} \right)^3$$

$$\Rightarrow \frac{5324}{4000} = \left(1 + \frac{r}{100} \right)^3$$

$$\Rightarrow \frac{1331}{1000} = \left(1 + \frac{r}{100} \right)^3$$

$$\Rightarrow \left(1 + \frac{r}{100} \right)^3 = \frac{1331}{1000} = \left(\frac{11}{10} \right)^3$$

$$\Rightarrow 1 + \frac{r}{100} = \frac{11}{10}$$

$$\Rightarrow \frac{r}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\Rightarrow r = \frac{100}{100} = 10\%$$

Thus, the rate of interest is 10%.

- 12. A person invests Rs. 5000 for three years at a certain rate of interest compounded annually. At the end of two years this sum amounts to Rs. 6272. Calculate:**

- (i) The rate of interest per annum
- (ii) The amount at the end of the third year.

Solution:

Given: $P = \text{Rs. } 5,000$; $A = \text{Rs. } 6,272$ and $n = 2$ years

- (i)

$$\therefore A = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 6,272 = 5,000 \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \frac{6,272}{5,000} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \frac{784}{625} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \left(\frac{28}{25}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

On comparing

$$\frac{28}{25} = 1 + \frac{r}{100}$$

On solving, we get

$$r = 12\%$$

(ii) Amount at the third year

$$= 5,000 \left(1 + \frac{12}{100}\right)^3$$

$$= 5,000 \left(\frac{28}{25}\right)^3$$

$$= \text{Rs}7,024.64$$

13. In how many years will Rs7000 amount to Rs. 9317 at 10 per cent per annum compound interest?

Solution:

Given : $P = \text{Rs}7,000$; $A = \text{Rs}9,317$ and $r = 10\%$

$$\therefore A = P \left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 9,317 = 7,000 \left(1 + \frac{10}{100}\right)^n$$

$$\Rightarrow \frac{9,317}{7,000} = \left(\frac{11}{10}\right)^n$$

$$\Rightarrow \frac{1,331}{1,000} = \left(\frac{11}{10}\right)^n$$

$$\Rightarrow \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$$

On comparing

$$n = 3 \text{ years}$$

14. Find the time in years, in which Rs. 4000 will produce Rs. 630.50 as compound interest at 5% p.a. interest being compounded annually.

Solution:

Given : P=Rs4,000; C.I.=Rs630.50 and r=5%

$$\therefore \text{C.I.} = P \left[\left(1 + \frac{r}{100}\right)^n - 1 \right]$$

$$\Rightarrow 630.50 = 4,000 \left[\left(1 + \frac{5}{100}\right)^n - 1 \right]$$

$$\Rightarrow \frac{630.50}{4,000} = \left[\left(\frac{21}{20}\right)^n - 1 \right]$$

$$\Rightarrow \frac{1,261}{8,000} = \left(\frac{21}{20}\right)^n - 1$$

$$\Rightarrow \frac{1,261}{8,000} + 1 = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \frac{9,261}{8,000} = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n$$

On comparing

$$n = 3 \text{ years}$$

15. Divide Rs.28730 between A and B so that when their shares are

lent out at 10 per cent compound interest compounded per year, the amount that A receives in 3 years is the same as what B receives in 5 years.

Solution:

Let share of A = Rs y

share of B = Rs (28,730 - y)

rate of interest = 10%

According to question

Amount of A in 3 years = Amount of B in 5 years

$$\Rightarrow y \left(1 + \frac{10}{100}\right)^3 = (28,730 - y) \left(1 + \frac{10}{100}\right)^5$$

$$\Rightarrow y = (28,730 - y) \left(1 + \frac{10}{100}\right)^2$$

$$\Rightarrow y = (28,730 - y) \left(\frac{121}{100}\right)$$

$$\Rightarrow 100y = 121(28,730 - y)$$

$$\Rightarrow 100y + 121y = 121 \times 28,730$$

$$\Rightarrow 221y = 121 \times 28,730$$

$$\Rightarrow y = \frac{121 \times 28,730}{221} = \text{Rs } 15,730$$

\therefore , Share of A = Rs 15,730

Share of B = Rs 28,730 - Rs 15,730
= Rs 13,000

16. A sum of Rs. 44,200 is divided between John and Smith, 12 years and 14 years old respectively, in such a way that if their portions be invested at 10 per cent per annum compound interest, they will receive equal amounts on reaching 16 years of age.

- (i) What is the share of each out of Rs. 44,200?
(ii) What will each receive when 16 years old?

Solution:

- (i) Let share of John = Rs y

Share of Smith = Rs (44,200 - y)

Rate of interest = 10%

According to question

Amount of John in 4 years = Amount of Smith in 2 years

$$\Rightarrow y \left(1 + \frac{10}{100}\right)^4 = (44,200 - y) \left(1 + \frac{10}{100}\right)^2$$

$$\Rightarrow y \left(1 + \frac{10}{100}\right)^2 = (44,200 - y)$$

$$\Rightarrow y \left(\frac{11}{10}\right)^2 = (44,200 - y)$$

$$\Rightarrow 121y = 100(44,200 - y)$$

$$\Rightarrow 121y = 100 \times 44,200 - 100y$$

$$\Rightarrow 121y + 100y = 100 \times 44,200$$

$$\Rightarrow 221y = 100 \times 44,200$$

$$\Rightarrow y = \frac{100 \times 44,200}{221} = \text{Rs} 20,000$$

\therefore , Share of John = Rs 20,000

Share of Smith = Rs 44,200 - Rs 20,000
= Rs 24,200

(ii) Amount that each will receive

$$= 20,000 \left(1 + \frac{10}{100}\right)^4$$

$$= 20,000 \left(\frac{11}{10}\right)^4$$

$$= \text{Rs} 29,282$$

17. The simple interest on a certain sum of money at 10% per annum is Rs. 6000 in 2 years. Find:

(i) The sum.

(ii) The amount due at the end of 3 years and at the same rate of interest compounded annually.

(iii) The compound interest earned in 3 years.

Solution:

(i) $I = \text{Rs. } 6000$, $T = 2$ years and $R = 10\%$

$$\therefore P = \frac{I \times 100}{R \times T} = \frac{6000 \times 100}{10 \times 2} = \text{Rs. } 30,000$$

(ii) $P = \text{Rs. } 30,000$, $n = 3$ years and $r = 10\%$

$$\begin{aligned} A &= P \left(1 + \frac{r}{100} \right)^n \\ &= 30000 \left(1 + \frac{10}{100} \right)^3 \\ &= 30000 \left(\frac{11}{10} \right)^3 \\ &= 30000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \\ &= \text{Rs. } 39,930 \end{aligned}$$

(iii) C.I. earned in 3 years $= A - P = \text{Rs. } (39,930 - 30,000)$
 $= \text{Rs. } 9,930$

18. Find the difference between compound interest and simple interest on Rs. 8000 in 2 years and at 5% per annum.

Solution:

Given: $P = \text{Rs. } 8000$, $R = 5\%$, $T = 2$ years

For simple interest,

$$\begin{aligned}\text{S.I.} &= \frac{P \times R \times T}{100} \\ &= \frac{8,000 \times 5 \times 2}{100} \\ &= \text{Rs. } 800\end{aligned}$$

For compound interest,

$$\begin{aligned}A &= P \left(1 + \frac{r}{100}\right)^n \\ A &= 8,000 \left(1 + \frac{5}{100}\right)^2 \\ &= 8,000 \times \frac{21}{20} \times \frac{21}{20} \\ &= \text{Rs. } 8,820\end{aligned}$$

$$\begin{aligned}\text{C.I.} &= A - P \\ &= \text{Rs. } (8,820 - 8,000) \\ &= \text{Rs. } 820\end{aligned}$$

Now, $\text{C.I.} - \text{S.I.} = \text{Rs. } (820 - 800) = \text{Rs. } 20$

Thus, the difference between the compound interest and the simple interest is Rs. 20.

EXERCISE 3(B)

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1. The difference between simple interest and compound interest on a certain sum is Rs. 5440 for 2 years at 8 percent per annum. Find the sum.

Solution:

Let principal (P) = x

R = 8%

T = 2 years

$$SI = \frac{x \times 8 \times 2}{100} = \frac{4x}{25}$$

$$\begin{aligned} CI &= A - P = x \left(1 + \frac{8}{100} \right)^2 - x \\ &= x \left[\left(1 + \frac{2}{25} \right)^2 - 1 \right] \\ &= x \left[\left(\frac{27}{25} \right)^2 - 1 \right] \\ &= \frac{104}{625}x \end{aligned}$$

Given, CI = SI = 54.40

$$\frac{104x}{625} - \frac{4x}{25} = \text{Rs. } 54.40$$

$$x \left(\frac{104}{625} - \frac{4}{25} \times \frac{25}{25} \right) = 54.40$$

$$x \left(\frac{4}{625} \right) = 54.40$$

$$x = \frac{54.40 \times 625}{4}$$

$$x = \text{Rs. } 8500$$

Thus, principal sum = Rs. 8500

2. A sum of money, invested at compound interest, amounts to Rs.19360

in 2 years and to Rs.23425.60 in 4 years. Find the rate per cent and the original sum of money.

Solution:

(for 2 years) $A = \text{Rs. } 19360$

$T = 2 \text{ years}$

Let $P = X$

$$X \left(1 + \frac{R}{100} \right)^2 = 19360 \quad \dots(1)$$

$A \text{ (for 4 years)} = \text{Rs. } 23425.60$

$$X \left(1 + \frac{R}{100} \right)^4 = 23425.60 \quad \dots(2)$$

$(2) \div (1)$

$$\left(1 + \frac{R}{100} \right)^2 = \frac{23425.60}{19360}$$

$$\left(1 + \frac{R}{100} \right)^2 = \frac{2342560}{1936000}$$

$$\left(1 + \frac{R}{100} \right)^2 = \frac{14641}{12100}$$

$$\left(1 + \frac{R}{100} \right)^2 = \left(\frac{121}{110} \right)^2$$

$$1 + \frac{R}{100} = \frac{121}{110}$$

$$\frac{R}{100} = \frac{121}{110} - 1$$

$$R = 10\%$$

$$\text{Form (1)} \quad X \left(1 + \frac{10}{100} \right)^2 = 19360$$

$$X = \frac{19360 \times 10 \times 10}{11 \times 11}$$

$$X = \text{Rs. } 16000$$

Thus, sum = Rs. 16000

3. A sum of money lent out at C.I at a certain rate per annum becomes

three times of itself in 8 years. Find in how many years will the money become twenty-seven times of itself at the same rate of interest p.a.

Solution:

Let principal = x , $A = 3x$, $T = 8$ years, $R = ?$

Case I,

$$\begin{aligned} A &= P \left(1 + \frac{R}{100} \right)^T \\ 3x &= x \left(1 + \frac{R}{100} \right)^8 \\ 3^{1/8} &= 1 + \frac{R}{100} \quad \dots(1) \end{aligned}$$

Case II,

$P = x, A = 27x, T = ?$

$$\begin{aligned} 27x &= x \left(1 + \frac{R}{100} \right)^T \\ 27^{1/T} &= 1 + \frac{R}{100} \quad \dots(2) \end{aligned}$$

From (1) and (2) $3^{1/8} = 27^{1/T}$

$$3^{1/8} = 3^{3/T} = 3^{1/T}$$

$$T = 24$$

Time = 24 years.

4. On what sum of money will compound interest (payable annually) for 2 years be the same as simple interest on Rs. 9430 for 10 years, both at the rate of 5 percent per annum?

Solution:

$$P = \text{Rs. } 9430$$

$$R = 5\%$$

$$R = 10 \text{ years}$$

$$SI = \frac{9430 \times 5 \times 10}{100} = \text{Rs. } 4715$$

$$\text{Let sum} = x$$

$$CI = 4715, T = 2 \text{ years, } R = 5\%$$

$$CI = AP$$

$$4715 = x \left(1 + \frac{R}{100} \right)^T - x$$

$$4715 = x \left(1 + \frac{5}{100} \right)^2 - x$$

$$4715 = x \left[\left(\frac{21}{20} \right)^2 - 1 \right]$$

$$4715 = x \left(\frac{441 - 400}{400} \right)$$

$$x = \frac{4715 \times 400}{41} = \text{Rs. } 46,000$$

Thus principal from = Rs. 46,000

5. Kamal and Anand each lent the same sum of money for 2 years at 5% at simple interest and compound interest respectively. Anand received Rs.15 more than Kamal. Find the amount of money lent by each and the interest received.

Solution:

Let principal = Rs. 100, $R = 5\%$ $T = 2$ years

$$\text{For Kamal, } SI = \frac{100 \times 5 \times 2}{100} = \text{Rs. } 10$$

$$\text{For Anand, } A = P \left(1 + \frac{R}{100} \right)^T$$

$$\begin{aligned}
 &= 100 \left(1 + \frac{5}{100} \right)^2 \\
 &= 100 \times \frac{21}{20} \times \frac{21}{20} \\
 &= \frac{441}{4}
 \end{aligned}$$

$$CI = \frac{441}{4} - 100 = \frac{41}{4}$$

$$\begin{aligned}
 \text{Difference of CI and SI} &= \frac{41}{4} - 10 \\
 &= \frac{41 - 40}{4} \\
 &= \text{Rs. } \frac{1}{4}
 \end{aligned}$$

When difference is Rs. $\frac{1}{4}$, then principal = Rs. 100

If difference is 1, principal = 100×4

If difference is Rs. 15, principal = $100 \times 4 \times 15 = \text{Rs. } 6000$

$$\text{For Kamal, interest} = \frac{6000 \times 5 \times 2}{100} = \text{Rs. } 600$$

$$\begin{aligned}
 \text{For Anand, interest} &= 6000 \left(1 + \frac{5}{100} \right)^2 - 6000 \\
 &= 6000 \left(\left(\frac{21}{20} \right)^2 - 1 \right) \\
 &= 6000 \left[\frac{441}{400} - 1 \right] \\
 &= 6000 \times \frac{41}{400} \\
 &= \text{Rs. } 615
 \end{aligned}$$

6. Simple interest on a sum of money for 2 years at 4% is Rs. 450. Find

compound interest on the same sum and at the same rate for 2 years.

Solution:

$$SI = \text{Rs. } 450$$

$$R = 4\%$$

$$R = 2 \text{ years}$$

$$P = ?$$

$$P = \frac{SI \times 100}{R \times T} = \frac{450 \times 100}{4 \times 2} = \text{Rs. } 5625$$

$$\text{Now, } P = 5625, R = 4\%, T = 2 \text{ years}$$

$$A = 5625 \left(1 + \frac{4}{100} \right)^2 = 5625 \left(\frac{26}{25} \right)^2$$
$$= \frac{3802500}{625} = \text{Rs. } 6084$$

$$CI = A - P = 6084 - 5625$$

$$= \text{Rs. } 459$$

7. Simple interest on a certain sum of money for 4 years at 4% per annum exceeds the compound interest on the same sum for 3 years at 5 percent per annum by Rs.228. Find the sum.

Solution:

$$\text{Let principal (P), } R = 4\%, T = 4 \text{ years}$$

$$SI = \frac{P \times 4 \times 4}{100} = \frac{4P}{25}$$

$$CI = P \left(1 + \frac{5}{100} \right)^3 - P = P \left[\left(\frac{21}{20} \right)^3 - 1 \right] = P \left(\frac{9261}{8000} - 1 \right)$$
$$= \frac{1261}{8000} P$$

$$\text{Given } SI - CI = \text{Rs. } 228$$

$$\begin{aligned}\frac{4P}{25} - \frac{1261}{8000}P &= 228 \\ \frac{4 \times 320P - 1261P}{8000} &= 228 \\ 19P &= 228 \times 8000 \\ P &= \frac{228 \times 8000}{19} = \text{Rs. } 96000\end{aligned}$$

Thus, Principal = Rs. 96000

8. Compound interest on a certain sum of money at 5% per annum for two years is Rs.246. Calculate simple interest on the same sum for 3 years at 6% per annum.

Solution:

CI = Rs. 246, R = 5%, T = 2 years

$$CI = A - P$$

$$246 = P \left(1 + \frac{5}{100} \right)^2 - P$$

$$246 = P \left[\left(\frac{21}{20} \right)^2 - 1 \right]$$

$$246 = P \frac{61}{400}$$

$$\begin{aligned}P &= \frac{246 \times 400}{61} \\ &= \text{Rs. } 2400\end{aligned}$$

Now, P = Rs. 2400, R = 6%, T = 3 years

$$\begin{aligned}SI &= \frac{2400 \times 6 \times 3}{100} \\ &= \text{Rs. } 432\end{aligned}$$

9. A certain sum of money amounts to Rs. 23400 in 3 years at 10% per annum simple interest. Find the amount of the same sum in 2 years and at 10% p.a. compound interest

Solution:

Let the sum (principle) = x

Given Amount = 23400, R = 10% and T = 3 years

$$\Rightarrow \text{interest } I = \frac{x \times 10 \times 3}{100} = \frac{3x}{10}$$

Amount = Principle + Interest

$$23400 = x + \frac{3x}{10}$$

$$x = 18000$$

Principle = 18000

Now,

Principle = ₹18000, r = 10% and n = 2 years

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$$A = 18000 \left(1 + \frac{10}{100} \right)^2$$

$$A = 18000 \left(\frac{11}{10} \right)^2$$

$$A = 18000 \left(\frac{121}{100} \right)$$

$$A = 21780$$

The amount of the same sum in 2 years and at 10% p.a. compound interest is 21780.

10. Mohit borrowed a certain sum at 5% per annum compound interest and cleared this loan by paying Rs. 12600 at the end of the first year and Rs. 17640 at the end of the second year. Find the sum borrowed.

Solution:

For the payment of Rs. 12,600 at the end of first year:

A = Rs. 12,600; n = 1 year and r = 5%

$$\text{Now, } A = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 12,600 = P \left(1 + \frac{5}{100} \right)^1$$

$$\Rightarrow 12,600 = P \left(\frac{21}{20} \right)$$

$$\Rightarrow P = \frac{20}{21} \times 12,600 = \text{Rs. } 12,000$$

For the payment of Rs. 17,640 at the end of second year:

A = Rs. 17,640; n = 2 years and r = 5%

$$\text{Now, } A = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 17,640 = P \left(1 + \frac{5}{100} \right)^2$$

$$\Rightarrow 17,640 = P \left(\frac{21}{20} \right)^2$$

$$\Rightarrow P = \frac{20}{21} \times \frac{20}{21} \times 17,640 = \text{Rs. } 16,000$$

\therefore Sum borrowed = Rs. (12,000 + 16,000) = Rs. 28,000

EXERCISE 3(C)

PAGE: 50

1. If the interest is compounded half-yearly, calculate the amount when principal is Rs.7400; the rate of interest is 5% per annum and the duration is one year.

Solution:

Given: $P = \text{Rs}7,400$; $r = 5\%$ p.a. and $n = 1$ year

Since the interest is compounded half-yearly,

$$\begin{aligned} \text{Then } A &= P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} \\ &= 7,400 \left(1 + \frac{5}{2 \times 100} \right)^{1 \times 2} \\ &= 7,400 \left(\frac{41}{40} \right)^2 \\ &= \text{Rs}7,774.63 \end{aligned}$$

2. Find the difference between the compound interest compounded yearly and half-yearly on Rs.10000 for 18 months at 10% per annum.

Solution:

(i) When interest is compounded yearly

Given: $P = \text{Rs}10,000$; $n = 18 \text{ months} = 1\frac{1}{2} \text{ year}$ and $r = 10\%$ p.a.

For 1 year

$$A = P \left(1 + \frac{r}{100} \right)^n = 10,000 \left(1 + \frac{10}{100} \right)^1 = 10,000 \left(\frac{11}{10} \right)^1 = \text{Rs}11,000$$

For $1\frac{1}{2}$ year

$P = \text{Rs}11,000$; $n = 1\frac{1}{2}$ year and $r = 10\%$

$$\begin{aligned} A &= P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = 11,000 \left(1 + \frac{10}{2 \times 100} \right)^{\frac{1}{2} \times 2} = 11,000 \left(\frac{21}{20} \right)^1 \\ &= \text{Rs}11,550 \end{aligned}$$

$$\therefore \text{C.I.} = \text{Rs}11,550 - \text{Rs}10,000 = \text{Rs}1,550$$

(ii) When interest is compounded half-yearly

$P = \text{Rs}10,000$; $n = 1\frac{1}{2}$ year and $r = 10\%$ p.a.

$$A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = 10,000 \left(1 + \frac{10}{2 \times 100} \right)^{\frac{3}{2} \times 2}$$
$$= 10,000 \left(\frac{21}{20} \right)^3$$

$$= \text{Rs}11,576.25$$

$$\therefore \text{C.I.} = \text{Rs}11,576.25 - \text{Rs}10,000 = \text{Rs}1,576.25$$

$$\therefore \text{Difference between both C.I.} = \text{Rs}1,576.25 - \text{Rs}1,550$$
$$= \text{Rs}26.25$$

3. A man borrowed Rs.16000 for 3 years under the following terms:
20% simple interest for the first 2 years.
20% C.I. for the remaining one year on the amount due after 2 years,
interest being compounded half-yearly.
Find the total amount to be paid at the end of three years.

Solution:

For the first 2 years

$$\text{S.I.} = \frac{P \times N \times R}{100}$$
$$\Rightarrow \text{S.I.} = \frac{16,000 \times 2 \times 20}{100}$$
$$\Rightarrow \text{S.I.} = 6,400$$

$$\text{Amount} = \text{S.I.} + P$$

$$\Rightarrow \text{Amount} = 6,400 + 16,000$$

$$\Rightarrow \text{Amount} = 22,400$$

Amount in the account at the end of the two years is Rs.22,400.

For the remaining one year

$$\begin{aligned}
 A &= P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} \\
 \Rightarrow A &= 22,400 \left(1 + \frac{20}{200} \right)^2 \\
 \Rightarrow A &= 22,400 \left(\frac{11}{10} \right)^2 \\
 \Rightarrow A &= 27,104
 \end{aligned}$$

The total amount to be paid at the end of the three years is Rs.27,104.

4. What sum of money will amount to Rs. 27783 in one and a half years at 10% per annum compounded half yearly?

Solution:

$$\begin{aligned}
 A &= P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} \\
 \Rightarrow 27,783 &= P \left(1 + \frac{10}{200} \right)^{3 \times 2} \\
 \Rightarrow 27,783 &= P \left(\frac{21}{20} \right)^3 \\
 \Rightarrow P &= 27,783 \left(\frac{20}{21} \right)^3 \\
 \Rightarrow P &= 24,000
 \end{aligned}$$

The sum of Rs.24,000 amount Rs.27,783 in one and a half years at 10% per annum compounded half yearly.

5. Ashok invests a certain sum of money at 20% per annum, compounded yearly. Geeta invests an equal amount of money at the same rate of interest per annum compounded half-yearly. If Geeta gets Rs.33 more than Ashok in 18 Months, calculate the money invested.

Solution:

(i) For Ashok(interest is compounded yearly)

Let $P = \text{Rs } y$; $n = 18 \text{ months} = 1\frac{1}{2} \text{ year}$ and $r = 20\% \text{ p.a.}$

For 1 year

$$A = P \left(1 + \frac{r}{100} \right)^n = y \left(1 + \frac{20}{100} \right)^1 = \left(\frac{6}{5} \right) y$$

For 1/2 year

$$P = \text{Rs} \left(\frac{6}{5} \right) y$$

$n = \frac{1}{2}$ year and $r = 20\%$

$$A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = \text{Rs} \left(\frac{6}{5} \right) y \left(1 + \frac{20}{2 \times 100} \right)^{\frac{1}{2} \times 2} = \text{Rs} \left(\frac{66}{50} \right) y$$

(ii) For Geeta (interest is compounded half-yearly)

$$P = \text{Rs } y; n = 1\frac{1}{2} \text{ year and } r = 20\% \text{ p.a.}$$

$$A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = y \left(1 + \frac{20}{2 \times 100} \right)^{\frac{3}{2} \times 2} = y \left(\frac{11}{10} \right)^3$$

$$= \text{Rs} \left(\frac{1,331}{1,000} \right) y$$

According to question

$$\therefore \left(\frac{1,331}{1,000} \right) y - \left(\frac{66}{50} \right) y = \text{Rs } 33$$

$$\Rightarrow \left(\frac{11}{1,000} \right) y = \text{Rs } 33$$

$$\Rightarrow y = \text{Rs} \frac{33 \times 1,000}{11} = \text{Rs } 3,000$$

\therefore Money invested by each person = Rs 3,000

6. At what rate of interest per annum will a sum of Rs.62500 earn a compound interest of Rs.5100 in one year? The interest is to be compounded half-yearly.

Solution:

$$C.I = P \left[\left(1 + \frac{r}{2 \times 100} \right)^{2 \times n} - 1 \right]$$

$$\Rightarrow 5,100 = 62,500 \left[\left(1 + \frac{r}{200} \right)^2 - 1 \right]$$

$$\Rightarrow \left(1 + \frac{r}{200} \right)^2 = \frac{67,600}{62,500}$$

$$\Rightarrow 1 + \frac{r}{200} = \frac{260}{250}$$

$$\Rightarrow r = 8$$

The rate of interest is 8%.

7. In what time will Rs.1500 yield Rs.496.50 as compound interest at 20% per year compounded half yearly?

Solution:

Given: $P = \text{Rs}1,500$; $\text{C.I.} = \text{Rs}496.50$ and $r = 20\%$

Since interest is compounded semi-annually

$$\text{Then } \text{C.I.} = P \left[\left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} - 1 \right]$$

$$\Rightarrow 496.50 = 1,500 \left[\left(1 + \frac{20}{2 \times 100} \right)^{n \times 2} - 1 \right]$$

$$\Rightarrow \frac{496.50}{1,500} = \left(\frac{11}{10} \right)^{2n} - 1$$

$$\Rightarrow \frac{331}{1,000} + 1 = \left(\frac{11}{10} \right)^{2n}$$

$$\Rightarrow \frac{1,331}{1,000} = \left(\frac{11}{10} \right)^{2n}$$

$$\Rightarrow \left(\frac{11}{10} \right)^3 = \left(\frac{11}{10} \right)^{2n}$$

On comparing, we get

$$2n = 3 \Rightarrow n = 1\frac{1}{2} \text{ years}$$

8. Calculate the C.I. on Rs.3500 at 6% per annum for 3 years, the interest being compounded half-yearly.

Do not use mathematical tables. Use the necessary information from the following:

$$(1.06)^3 = 1.191016;$$

$$(1.03)^3 = 1.092727;$$

$$(1.06)^6 = 1.418519;$$

$$(1.03)^6 = 1.194052;$$

Solution:

Given: $P = \text{Rs} 3,500$; $r = 6\%$ and $n = 3 \text{ years}$

Since interest is being compounded half-yearly

$$\begin{aligned} \text{C.I.} &= P \left[\left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} - 1 \right] \\ \text{Then} \\ &= 3,500 \left[\left(1 + \frac{6}{2 \times 100} \right)^{3 \times 2} - 1 \right] \\ &= 3,500 \left[\left(\frac{103}{100} \right)^6 - 1 \right] \\ &= 3,500 \left[(1.03)^6 - 1 \right] \\ &= 3,500 [1.194052 - 1] \\ &= 3,500 \times 0.194052 \\ &= \text{Rs}679.18 \end{aligned}$$

9. Find the difference between compound interest and simple interest on Rs.12000 and in $1\frac{1}{2}$ years at 10% p.a. compounded yearly.

Solution:

Given: $P = \text{Rs}12,000$; $n = 1\frac{1}{2}$ years and $r = 10\%$

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{12,000 \times 10 \times \frac{3}{2}}{100} = \text{Rs}1,800$$

To calculate C.I.

For 1 year

$P = \text{Rs}12,000$; $n = 1$ year and $r = 10\%$

$$A = P \left(1 + \frac{r}{100} \right)^n = 12,000 \left(1 + \frac{10}{100} \right)^1 = \text{Rs}13,200$$

For next $1/2$ year

$P = \text{Rs}13,200$; $n = 1/2$ year and $r = 10\%$

$$\begin{aligned} A &= P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = 13,200 \left(1 + \frac{10}{2 \times 100} \right)^{\frac{1}{2} \times 2} \\ &= 13,200 \left(\frac{21}{20} \right)^1 \\ &= \text{Rs}13,860 \end{aligned}$$

$$\therefore \text{C.I.} = \text{Rs}13,860 - \text{Rs}12,000 = \text{Rs}1,860$$

\therefore Difference between C.I. and S.I

$$= \text{Rs}1,860 - \text{Rs}1,800 = \text{Rs}60$$

10. Find the difference between compound interest and simple interest on Rs.12000 and in $1\frac{1}{2}$ years at 10% p.a. compounded half-yearly.

Solution:

Given: $P = \text{Rs}12,000$; $n = 1\frac{1}{2}$ years and $r =$

$$10\% \quad \text{S.I.} = \frac{P \times R \times T}{100} = \frac{12,000 \times 10 \times \frac{3}{2}}{100} = \text{Rs}1,800$$

To calculate C.I.(compounded half-yearly)

$P = \text{Rs}12,000$; $n = 1\frac{1}{2}$ years and $r = 10\%$

$$A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = 12,000 \left(1 + \frac{10}{2 \times 100} \right)^{\frac{3}{2} \times 2}$$

$$= 12,000 \left(\frac{21}{20} \right)^3$$

$$= \text{Rs}13,891.50$$

$$\therefore \text{C.I.} = \text{Rs}13,891.50 - \text{Rs}12,000 = \text{Rs}1,891.50$$

\therefore Difference between C.I. and S.I

$$= \text{Rs}1,891.50 - \text{Rs}1,800 = \text{Rs}91.50$$

EXERCISE 3(D)

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1. The cost of a machine is supposed to depreciate each year by 12% of its value at the beginning of the year. If the machine is valued at Rs.44000 at the beginning of 2008, find its value:
- (i) At the end of 2009.
- (ii) At the beginning of 2007.

Solution:

Cost of machine in 2008 = Rs44,000

Depreciation rate=12%

(i) \therefore Cost of machine at the end of 2009

$$\begin{aligned} &= P \left(1 - \frac{r}{100} \right)^n \\ &= 44,000 \left(1 - \frac{12}{100} \right)^2 \\ &= 44,000 \times \left(\frac{88}{100} \right)^2 = \text{Rs}34,073.60 \end{aligned}$$

(ii) Cost of machine at the beginning of 2007(P)

$$\begin{aligned} A &= P \left(1 - \frac{r}{100} \right)^n \\ \Rightarrow 44,000 &= P \left(1 - \frac{12}{100} \right)^1 \\ \Rightarrow 44,000 &= P \left(\frac{88}{100} \right)^1 \\ \Rightarrow P &= \frac{44,000 \times 100}{88} = \text{Rs}50,000 \end{aligned}$$

2. The value of an article decreased for two years at the rate of 10% per year and then in the third year it increased by 10%. Find the original value of the article, if its value at the end of 3 years is Rs.40095.

Solution:

Let x be the value of the article.

The value of an article decreases for two years at the rate of 10% per year.

The value of the article at the end of the 1st year is

$$X - 10\% \text{ of } x = 0.90x$$

The value of the article at the end of the 2nd year is

$$0.90x - 10\% \text{ of } (0.90x) = 0.81x$$

The value of the article increases in the 3rd year by 10%.

The value of the article at the end of 3rd year is

$$0.81x + 10\% \text{ of } (0.81x) = 0.891x$$

The value of the article at the end of 3 years is Rs.40,095.

$$0.891x = 40,095$$

$$\Rightarrow x = 45,000$$

The original value of the article is Rs.45,000.

3. According to a census taken towards the end of the year 2009, the population of a rural town was found to be 64000. The census authority also found that the population of this particular town had a growth of 5% per annum. In how many years after 2009 did the population of this town reach 74088?

Solution:

Population in 2009 (P) = 64,000

Let after n years its population be 74,088(A)

Growth rate= 5% per annum

$$\therefore A = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 74,088 = 64,000 \left(1 + \frac{5}{100} \right)^n$$

$$\Rightarrow \frac{74,088}{64,000} = \left(\frac{21}{20} \right)^n$$

$$\Rightarrow \frac{9,261}{8,000} = \left(\frac{21}{20} \right)^n$$

$$\Rightarrow \left(\frac{21}{20} \right)^3 = \left(\frac{21}{20} \right)^n$$

On comparing, we get

$$n = 3 \text{ years}$$

4. The population of a town decreased by 12% during 1998 and then increased by 8% during 1999. Find the population of the town, at the beginning of 1998, if at the end of 1999 its population was 285120

Solution:

Let the population in the beginning of 1998 = P

The population at the end of 1999 = 2,85,120(A)

$r_1 = -12\%$ and $r_2 = +8\%$

$$\therefore A = P \left(1 - \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right)$$

$$\Rightarrow 2,85,120 = P \left(1 - \frac{12}{100}\right) \left(1 + \frac{8}{100}\right)$$

$$\Rightarrow 2,85,120 = P \left(\frac{22}{25}\right) \left(\frac{27}{25}\right)$$

$$\Rightarrow P = \frac{2,85,120 \times 25 \times 25}{22 \times 27} = 3,00,000$$

5. A sum of money, invested at compound interest, amounts to Rs.16500 in 1 year and to Rs.19965 in 3 years. Find the rate per cent and the original sum of money invested.

Solution:

Let sum of money be Rs P and rate of interest = $r\%$

Money after 1 year = Rs16,500

Money after 3 years = Rs19,965

For 1 year

$$\therefore A = P \left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 16,500 = P \left(1 + \frac{r}{100}\right)^1 \text{ ----- (1)}$$

For 3 years

$$\therefore A = P \left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 19,965 = P \left(1 + \frac{r}{100}\right)^3 \text{ ----- (2)}$$

Divide (2) by (1)

$$\frac{19,965}{16,500} = \frac{P \left(1 + \frac{r}{100}\right)^3}{P \left(1 + \frac{r}{100}\right)^1}$$

$$\Rightarrow \frac{121}{100} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \left(\frac{11}{10}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

On comparing, we get

$$\frac{11}{10} = 1 + \frac{r}{100}$$

$$\Rightarrow r = 10\%$$

Put value of r in (1)

$$16,500 = P \left(1 + \frac{10}{100}\right)$$

$$\Rightarrow P = \frac{16,500 \times 10}{11} = \text{Rs } 15,000$$

6. The difference between C.I. and S.I. on Rs.7500 for two years is Rs. 12 at the same rate of interest per annum. Find the rate of interest.

Solution:

Given: $P = \text{Rs } 7,500$ and Time(n) = 2 years

Let rate of interest = $y\%$

$$\therefore \text{S.I.} = \frac{P \times R \times T}{100} = \frac{7,500 \times y \times 2}{100} = \text{Rs } 150y$$

$$\therefore \text{C.I.} = P \left(1 + \frac{r}{100}\right)^n - P = \text{Rs } 7,500 \left(1 + \frac{y}{100}\right)^2 - \text{Rs } 7,500$$

Given: C.I. -; S.I. = Rs 12

$$\Rightarrow 7,500 \left(1 + \frac{y}{100}\right)^2 - 7,500 - 150y = 12$$

$$\Rightarrow 7,500 \left(1 + \frac{y^2}{10000} + \frac{2y}{100}\right) - 7,500 - 150y = 12$$

$$\Rightarrow 7,500 + \frac{7,500y^2}{10000} + 150y - 7,500 - 150y = 12$$

$$\Rightarrow \frac{3y^2}{4} = 12$$

$$\Rightarrow y^2 = 16 \quad \Rightarrow y = 4\%$$

7. A sum of money lent out at C.I. at a certain rate per annum becomes three times of itself in 10 years. Find in how many years will the money become twenty-seven times of itself at the same rate of interest p.a.

Solution:

Let Principal be Rs y and rate = $r\%$

According to 1st condition

Amount in 10 years = Rs 3y

$$\therefore A = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 3y = y \left(1 + \frac{r}{100} \right)^{10}$$

$$\Rightarrow 3 = \left(1 + \frac{r}{100} \right)^{10} \text{ ----- (1)}$$

According to 2nd condition

Let after n years amount will be Rs 27y

$$\therefore A = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 27y = y \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow (3)^3 = \left(1 + \frac{r}{100} \right)^n$$

Put value from first equation

$$\Rightarrow \left[\left(1 + \frac{r}{100} \right)^{10} \right]^3 = \left(1 + \frac{r}{100} \right)^n$$

On comparing, we get

$$n = 10 \times 3 = 30 \text{ years}$$

8. Mr. Sharma borrowed a certain sum of money at 10% per annum

compounded annually. If by paying Rs.19360 at the end of the second year and Rs. 31944 at the end of the third year he clears the debt; find the sum borrowed by him.

Solution:

At the end of the two years the amount is

$$A_1 = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow A_1 = P \left(1 + \frac{10}{100} \right)^2$$

Mr. Sharma paid Rs.19,360 at the end of the second year.

So for the third year the principal is $A_1 - 19,360$.

Also he cleared the debt by paying Rs.31,944 at the end of the third year.

$$A_2 = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 31,944 = \left(P \left(1 + \frac{10}{100} \right)^2 - 19,360 \right) \left(1 + \frac{10}{100} \right)^1$$

$$\Rightarrow 29,040 = \left(P \left(1 + \frac{10}{100} \right)^2 - 19,360 \right)$$

$$\Rightarrow P \left(1 + \frac{10}{100} \right)^2 = 48,400$$

$$\Rightarrow P = 40,000$$

Mr. Sharma borrowed Rs.40,000.

9. The difference between compound interest for a year payable half-yearly and simple interest on a certain sum of money lent out at 10% for a year is Rs.15. Find the sum of money lent out.

Solution:

Let sum of money be RS y

To calculate S.I.

$$S.I. = \frac{P \times R \times T}{100} = \frac{y \times 10 \times 1}{100} = \text{Rs } \frac{y}{10}$$

To calculate C.I.(compounded half-yearly)

$$\therefore C.I. = P \left[\left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} - 1 \right] = y \left[\left(1 + \frac{10}{2 \times 100} \right)^{1 \times 2} - 1 \right]$$

$$= y \left[\left(\frac{21}{20} \right)^2 - 1 \right] = \left(\frac{41}{400} \right) y$$

Given : C.I. - S.I. = Rs15

$$\Rightarrow \left(\frac{41}{400} \right) y - \frac{y}{10} = 15$$

$$\Rightarrow \frac{y}{400} = 15 \Rightarrow y = \text{Rs}6,000$$

10. The ages of Pramod and Rohit are 16 years and 18 years respectively. In what ratio must they invest money at 5% p.a. compounded yearly so that both get the same sum on attaining the age of 25 years?

Solution:

Let Rs.x and Rs.y be the money invested by Pramod and Rohit respectively such that they will get the same sum on attaining the age of 25 years.

Pramod will attain the age of 25 years after $25 - 16 = 9$ years

Rohit will attain the age of 25 years after $25 - 18 = 7$ years

$$x \left(1 + \frac{5}{100} \right)^9 = y \left(1 + \frac{5}{100} \right)^7$$

$$\Rightarrow \frac{x}{y} = \frac{1}{\left(1 + \frac{5}{100} \right)^2}$$

$$\Rightarrow \frac{x}{y} = \frac{400}{441}$$

Pramod and Rohit should invest in **400:441** ratio respectively such that they will get the same sum on attaining the age of 25 years.

EXERCISE 3(E)

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1. Simple interest on a sum of money for 2 years at 4% is Rs. 450. Find

compound interest on the same sum and at the same rate for 1 year, if the interest is reckoned half-yearly.

Solution:

1st case

Given: S.I. = Rs 450; Time = 2 years and Rate = 4%

$$\therefore \text{Principal} = \frac{I \times 100}{R \times T} = \frac{450 \times 100}{4 \times 2} = \text{Rs. } 5625$$

2nd case (compounded half-yearly)

P = Rs.5,625; n = 1 year and r = 4%

$$\therefore A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = 5,625 \left(1 + \frac{4}{2 \times 100} \right)^{1 \times 2}$$

$$= 5,625 \left(\frac{51}{50} \right)^2 = \text{Rs. } 5852.25$$

$$\therefore \text{C.I.} = 5,852.25 - 5,625 = \text{Rs. } 227.25$$

2. Find the compound interest to the nearest rupee on Rs.10800 for $2\frac{1}{2}$ years at 10% per annum.

Solution:

Given: P = Rs. 10,800; Time = $2\frac{1}{2}$ years and Rate = 10% p.a

For 2 years

$$A = P \left(1 + \frac{r}{100} \right)^n = 10,800 \left(1 + \frac{10}{100} \right)^2 = \text{Rs. } 13,068$$

For $\frac{1}{2}$ year

$$\therefore A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = 13,068 \left(1 + \frac{10}{2 \times 100} \right)^{\frac{1}{2} \times 2}$$

$$= 13,068 \times \frac{21}{20} = 13,721.40 = \text{Rs. } 13721 (\text{nearest rupee})$$

$$\therefore \text{Rs. } 13,721 - \text{Rs. } 10,800 = \text{Rs. } 2,921$$

3. The value of a machine, purchased two years ago, depreciates at the annual rate of 10%. If its present value is Rs. 97200, find:
- (i) Its value after 2 years.

(ii) Its value when it was purchased.

Solution:

(i) Present value of machine (P) = Rs.97,200

Depreciation rate = 10%

$$\begin{aligned}\therefore \text{Value of machine after 2 years} &= P \left(1 - \frac{r}{100}\right)^n \\ &= 97,200 \left(1 - \frac{10}{100}\right)^2 \\ &= 97,200 \left(\frac{9}{10}\right)^2\end{aligned}$$

$$= \text{Rs.}78732$$

(ii) Present value of machine (A) = Rs.97,200

Depreciation rate = 10% and time = 2 years

To calculate the cost 2 years ago

$$\begin{aligned}\therefore A &= P \left(1 - \frac{r}{100}\right)^n \\ \Rightarrow 97,200 &= P \left(1 - \frac{10}{100}\right)^2 \\ \Rightarrow 97,200 &= P \left(\frac{9}{10}\right)^2 \\ \Rightarrow P &= \text{Rs. } 97,200 \times \left(\frac{10}{9}\right)^2 = 1,20,000\end{aligned}$$

4. Anuj and Rajesh each lent the same sum of money for 2 years at 8% simple interest and compound interest respectively. Rajesh received Rs.64 more than Anuj. Find the money lent by each and interest received.

Solution:

Let the sum of money lent by both Rs.y

For Anuj

P = Rs.y; rate = 8% and time = 2 years

$$\therefore \text{S.I.} = \frac{P \times R \times T}{100} = \frac{y \times 8 \times 2}{100} = \frac{4y}{25}$$

For Rajesh

P = Rs.y ; rate = 8% and time = 2 years

$$\therefore \text{C.I.} = P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right] = y \left[\left(1 + \frac{8}{100} \right)^2 - 1 \right] = \frac{104y}{625}$$

Given: C.I. = Rs.64

$$\Rightarrow \frac{104y}{625} - \frac{4y}{25} = 64$$

$$\Rightarrow \frac{4y}{625} = 64 \Rightarrow y = \frac{64 \times 625}{4} = \text{Rs.}10,000$$

$$\text{Interest received by Anuj} = \frac{4 \times 10,000}{25} = \text{Rs.}1600$$

$$\text{Interest received by Rajesh} = \frac{104 \times 10,000}{625} = \text{Rs.} 1664$$

5. Calculate the sum of money on which the compound interest (payable annually) for 2 years be four times the simple interest on Rs. 4715 for 5 years, both at the rate of 5 per cent per annum.

Solution:

Given: Principal = Rs.4,715; time = 5 years and rate = 5% p.a.

$$\therefore \text{S.I.} = \frac{P \times R \times T}{100} = \frac{4715 \times 5 \times 5}{100} = 1,178.75$$

Then C.I. = Rs.1,178.75 x 4 = Rs.4,715

Time = 2 years and rate = 5%

$$\therefore \text{C.I.} = P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right]$$

$$\Rightarrow 4,715 = P \left[\left(1 + \frac{5}{100} \right)^2 - 1 \right]$$

$$\Rightarrow 4,715 = P \left(\frac{41}{400} \right)$$

$$\Rightarrow P = \text{Rs.} \frac{4,715 \times 400}{41} = \text{Rs.} 46,000$$

6. A sum of money was invested for 3 years, interest being compounded annually. The rates for successive years were 10%, 15% and 18% respectively. If the compound interest for the second year amounted to Rs.4950, find the sum invested.

Solution:

Given: C.I. for the 2nd year = Rs.4,950 and rate = 15%

$$\text{Then, C.I.} = P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right]$$

$$\Rightarrow 4,950 = P \left[\left(1 + \frac{15}{100} \right)^1 - 1 \right]$$

$$\Rightarrow 4,950 = P \left(\frac{3}{20} \right)$$

$$\Rightarrow P = \frac{4,950 \times 20}{3}$$

$$\Rightarrow P = \text{Rs. } 33,000$$

Then amount at the end of 2nd year = Rs.33,000

For first 2 years

$$A = \text{Rs.} 33,000; r_1 = 10\%$$

$$\therefore A = P \left(1 + \frac{r_1}{100} \right)$$

$$\Rightarrow 33,000 = P \left(1 + \frac{10}{100} \right)$$

$$\Rightarrow 33,000 = P \left(\frac{11}{10} \right)$$

$$\Rightarrow P = \frac{33,000 \times 10}{11} = 30,000$$

The sum invested is Rs.30,000.

7. A sum of money is invested at 10% per annum compounded half-yearly. If the difference of amounts at the end of 6 months and 12 months is Rs.189, find the sum of money invested.

Solution:

Let the sum of money be Rs.y

And rate = 10% p.a. compounded half yearly

For first 6 months

$$\therefore A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = y \left(1 + \frac{10}{2 \times 100} \right)^{\frac{1}{2} \times 2} = \left(\frac{21}{20} \right) y$$

For first 12 months

$$\therefore A = P \left(1 + \frac{r}{2 \times 100} \right)^{n \times 2} = y \left(1 + \frac{10}{2 \times 100} \right)^{1 \times 2} = \left(\frac{441}{400} \right) y$$

Given: The difference between the above amounts = Rs.189

$$\Rightarrow \left(\frac{441}{400}\right)y - \left(\frac{21}{20}\right)y = 189$$

$$\Rightarrow \left(\frac{21}{400}\right)y = 189$$

$$\Rightarrow y = \frac{189 \times 400}{21}$$

$$y = 3600$$

8. Rohit borrows Rs.86000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of two years.

Solution:

P = Rs.86,000; time = 2 years and rate = 5% p.a.

To calculate S.I.

$$\therefore \text{S.I.} = \frac{P \times R \times T}{100} = \frac{86,000 \times 5 \times 2}{100} = \text{Rs. } 8,600$$

To calculate C.I.

$$\begin{aligned} \therefore \text{C.I.} &= P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right] \\ &= 86,000 \left[\left(1 + \frac{5}{100} \right)^2 - 1 \right] \\ &= 86,000 \left(\frac{41}{400} \right) = \text{Rs. } 8,815 \end{aligned}$$

$$\text{Profit} = \text{C.I.} - \text{S.I.} = \text{Rs. } 8,815 - \text{Rs. } 8,600 = \text{Rs. } 215$$

9. The simple interest on a certain sum of money for 3 years at 5% per annum is Rs.1200 Find the amount due and the compound interest on this sum of money at the same rate and after 2 years, interest is reckoned annually.

Solution:

Let Rs.x be the sum of money.

Rate = 5 % p.a. Simple interest = Rs.1,200, n = 3years.

$$\begin{aligned}1,200 &= \frac{x \times 5 \times 3}{100} \\ \Rightarrow x &= \frac{12,00,00}{15} \\ \Rightarrow x &= 8,000\end{aligned}$$

The amount due and the compound interest on this sum of money at the same rate and after 2 years.

P = Rs.8,000; rate = 5% p.a., n = 3 years

$$\begin{aligned}\therefore A &= P \left(1 + \frac{r}{100}\right)^n \\ \Rightarrow A &= 8,000 \left(1 + \frac{5}{100}\right)^2 \\ \Rightarrow A &= 8,000 (1.1025) \\ \Rightarrow A &= 8,820\end{aligned}$$

$$\begin{aligned}\text{C.I.} &= A - P \\ \Rightarrow \text{C.I.} &= 8,820 - 8,000 \\ \Rightarrow \text{C.I.} &= 820\end{aligned}$$

The amount due after 2 years is Rs.8,820 and the compound interest is Rs.820.

10. Nikita invests Rs.6000 for two years at a certain rate of interest compounded annually. At the end of the first year it amounts to Rs.6720. Calculate:

- (i) The rate of Interest.
- (ii) The amount at the end of the second year.

Solution:

Let x% be the rate of interest.

P = Rs.6,000, n = 2 years, A = Rs.6,720

i. For the first year

$$\begin{aligned}A &= P \left(1 + \frac{r}{100}\right)^n \\ \Rightarrow 6,720 &= 6,000 \left(1 + \frac{x}{100}\right)^1 \\ \Rightarrow 6,720 - 6,000 &= 60x \\ \Rightarrow x &= 12\end{aligned}$$

The rate of interest is $x\% = 12\%$.

ii. The amount at the end of the second year.

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow A = 6,000 \left(1 + \frac{12}{100} \right)^2$$

$$\Rightarrow A = 6,000 \left(\frac{112}{100} \right)^2$$

$$\Rightarrow A = 7,526.40$$

The amount at the end of the second year = Rs.7,526.40