EXERCISE 5(A) PAGE: 68

Factorise by taking out the common factors:

1. $2(2x - 5y)(3x + 4y) - 6(2x - 5y)(x - y)$

Solution:

Taking $(2x - 5y)$ common from both terms

$= (2x - 5y)[2(3x + 4y) - 6(x - y)]$

$=(2x - 5y)(6x + 8y - 6x + 6y)$

$=(2x - 5y)(8y + 6y)$

$=(2x - 5y)(14y)$

$=(2x - 5y)14y$

2. $xy(3x^2 - 2y^2) - yz(2y^2 - 3x^2) + zx(15x^2 - 10y^2)$

Solution:

$xy(3x^2 - 2y^2) - yz(2y^2 - 3x^2) + zx(15x^2 - 10y^2)$

$= xy(3x^2 - 2y^2) + yz(3x^2 - 2y^2) + zx(15x^2 - 10y^2)$

$= xy(3x^2 - 2y^2) + yz(3x^2 - 2y^2) + 5zx(3x^2 - 2y^2)$

$= (3x^2 - 2y^2)[xy + yz + 5zx]$

3. $ab(a^2 + b^2 - c^2) - bc(c^2 - a^2 - b^2) + ca(a^2 + b^2 - c^2)$

Solution:

$ab(a^2 + b^2 - c^2) - bc(c^2 - a^2 - b^2) + ca(a^2 + b^2 - c^2)$

$= ab(a^2 + b^2 - c^2) + bc(a^2 + b^2 - c^2) + ca(a^2 + b^2 - c^2)$

$= (a^2 + b^2 - c^2)[ab + bc + ca]$

4. $2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a)$

Solution:

$2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a)$

$= 2x(a - b) + 15y(a - b) - 8z(a - b)$

$= (a - b)[2x + 15y - 8z]$

Factorise by grouping method:

5. $a^3 + a - 3a^2 - 3$

Solution:

$a^3 + a - 3a^2 - 3 = a(a^2 + 1) - 3(a^2 + 1)$

$= (a^2 + 1)(a - 3)$

6. $16(a + b)^2 - 4a - 4b$

Solution:

$16(a + b)^2 - 4a - 4b = 16(a + b)^2 - 4(a + b)$

$= 4(a + b)[4(a + b) - 1]$

$= 4(a + b)(4a + 4b - 1)$

7. a^4-2a^3-4a+8

Solution:

$$a^4 - 2a^3 - 4a + 8 = a^3(a-2) - 4(a-2)$$
$$= (a^3 - 4)(a-2)$$

8. $ab - 2b + a^2 - 2a$

Solution:

$$ab - 2b + a^2 - 2a = b(a-2) + a(a-2)$$
$$= (a+b)(a-2)$$

9. $ab(x^2+1) + x(a^2+b^2)$

Solution:

$$ab(x^2+1) + x(a^2+b^2) = abx^2 + ab + a^2x + b^2x$$
$$= ax(bx+a) + b(bx+a)$$
$$= (ax+b)(bx+a)$$

10. $a^2 + b - ab - a$

Solution:

$$a^2 + b - ab - a = a^2 - a + b - ab$$
$$= a(a-1) + b(1-a)$$
$$= a(a-1) - b(a-1)$$
$$= (a-1)(a-b)$$

11. $(ax+by)^2 + (bx-ay)^2$

Solution:

$$(ax+by)^2 + (bx-ay)^2 = a^2x^2 + b^2y^2 + 2axby + b^2x^2 + a^2y^2 - 2bxay$$
$$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$$
$$= x^2(a^2+b^2) + y^2(a^2+b^2)$$
$$= (x^2+y^2)(a^2+b^2)$$

12. $a^2x^2 + (ax^2+1)x + a$

Solution:

$$a^2x^2 + (ax^2 + 1)x + a = a^2x^2 + a + (ax^2 + 1)x$$
$$= a(ax^2 + 1) + x(ax^2 + 1)$$
$$= (a + x)(ax^2 + 1)$$

13. $(2a - b)^2 - 10a + 5b$

Solution:

$$(2a - b)^2 - 10a + 5b = (2a - b)^2 - 5(2a - b)$$
$$= (2a - b)(2a - b - 5)$$

14. $a(a - 4) - a + 4$

Solution:

$$a(a - 4) - a + 4 = a(a - 4) - 1(a - 4)$$
$$= (a - 4)(a - 1)$$

15. $y^2 - (a + b)y + ab$

Solution:

$$y^2 - (a + b)y + ab = y^2 - ay - by + ab$$
$$= y(y - a) - b(y - a)$$
$$= (y - a)(y - b)$$

16. $a^2 + \frac{1}{a^2} - 2 - 3a + \frac{3}{a}$

Solution:

$$a^2 + \frac{1}{a^2} - 2 - 3a + \frac{3}{a} = \left(a - \frac{1}{a}\right)^2 - 3\left(a - \frac{1}{a}\right)$$
$$= \left(a - \frac{1}{a}\right)\left[\left(a - \frac{1}{a}\right) - 3\right]$$
$$= \left(a - \frac{1}{a}\right)\left[a - \frac{1}{a} - 3\right]$$

17. $x^2+y^2+x+y+2xy$

Solution:

$x^2+y^2+x+y+2xy = (x^2 + y^2 + 2xy) + (x + y)$

[As $(x + y)^2 = x^2 + 2xy + y^2$]

$=(x + y)^2 + (x + y)$

$=(x + y)(x + y + 1)$

18. $a^2 + 4b^2 - 3a + 6b - 4ab$

Solution:

$a^2 + 4b^2 - 3a + 6b - 4ab = a^2 + 4b^2 - 4ab - 3a + 6b$
$= a^2 + (2b)^2 - 2 \times a \times (2b) - 3(a - 2b)$
[As $(a - b)^2 = a^2 - 2ab + b^2$]
$=(a - 2b)^2 - 3(a - 2b)$
$=(a - 2b)[(a - 2b)- 3]$
$=(a - 2b)(a - 2b - 3)$

19. $m (x - 3y)^2 + n (3y-x) + 5x - 15y$

Solution:

$m (x - 3y)^2 + n (3y-x) + 5x - 15y$
$= m (x - 3y)^2 - n (x - 3y) + 5(x - 3y)$
[Taking (x - 3y) common from all the three terms]
$=(x - 3y) [m(x - 3y) - n + 5]$
$=(x - 3y)(mx - 3my - n + 5)$

20. $x(6x - 5y) - 4(6x - 5y)^2$

Solution:

$x(6x - 5y) - 4(6x - 5y)^2$
$=(6x - 5y)[x - 4(6x - 5y)]$
[Taking (6x - 5y) common from the three terms]
$= (6x - 5y)(x - 24x + 20y)$
$= (6x - 5y)(-23x + 20y)$
$= (6x - 5y)(20y - 23x)$

EXERCISE 5(B)

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Factorise:

1. $a^2 + 10a + 24$

Solution:

$$a^2 + 10a + 24 = a^2 + 6a + 4a + 24$$
$$= a(a + 6) + 4(a + 6)$$
$$= (a + 6)(a + 4)$$

2. $a^2 - 3a - 40$

Solution:

$$a^2 - 3a - 40 = a^2 - 8a + 5a - 40$$
$$= a(a - 8) + 5(a - 8)$$
$$= (a - 8)(a + 5)$$

3. $1 - 2a - 3a^2$

Solution:

$$1 - 2a - 3a^2 = 1 - 3a + a - 3a^2$$
$$= 1(1 - 3a) + a(1 - 3a)$$
$$= (1 + a)(1 - 3a)$$

4. $x^2 - 3ax - 88a^2$

Solution:

$$x^2 - 3ax - 88a^2 = x^2 - 11ax + 8ax - 88a^2$$
$$= x(x - 11a) + 8a(x - 11a)$$
$$= (x + 8a)(x - 11a)$$

5. $6a^2 - a - 15$

Solution:

$$6a^2 - a - 15 = 6a^2 - 10a + 9a - 15$$
$$= 2a(3a - 5) + 3(3a - 5)$$
$$= (2a + 3)(3a - 5)$$

6. $24a^3 + 37a^2 - 5a$

Solution:

$$24a^3 + 37a^2 - 5a = a(24a^2 + 37a - 5)$$
$$= a(24a^2 + 40a - 3a - 5)$$
$$= a \times [8a(3a + 5) - 1(3a + 5)]$$
$$= a[(8a - 1)(3a + 5)]$$
$$= a(8a - 1)(3a + 5)$$

7. $a(3a - 2) - 1$

Solution:

$$a(3a - 2) - 1 = 3a^2 - 2a - 1$$
$$= 3a^2 - 3a + a - 1$$
$$= 3a(a - 1) + 1(a - 1)$$
$$= (3a + 1)(a - 1)$$

8. $a^2b^2 + 8ab - 9$

Solution:

$$a^2b^2 + 8ab - 9 = a^2b^2 + 9ab - ab - 9$$
$$= ab(ab + 9) - 1(ab + 9)$$
$$= (ab + 9)(ab - 1)$$

9. $3 - a(4 + 7a)$

Solution:

$$3 - a(4 + 7a) = 3 - 4a - 7a^2$$
$$= 3 - 7a + 3a - 7a^2$$
$$= 1(3 - 7a) + a(3 - 7a)$$
$$= (3 - 7a)(a + 1)$$

10. $(2a + b)^2 - 6a - 3b - 4$

Solution:

$(2a+b)^2-6a-3b-4=(2a+b)^2-3(2a+b)-4$

Assume that 2a+b=x

Therefore,

$(2a+b)^2-6a-3b-4=x^2-3x-4$

$=x^2-4x+x-4$

$=1(x-4)+x(x-4)$

$=(x+1)(x-4)$

$=(2a+b+1)(2a+b-4)$

[resubstitute the value of x]

11. $1-2(a+b)-3(a+b)^2$

Solution:

Assume that a + b = x;

$1-2(a+b)-3(a+b)^2=1-2x-3x^2$

$=1-3x+x-3x^2$

$=1(1-3x)+x(1-3x)$

$=(1-3x)(1+x)$

$=(1-3(a+b))(1+(a+b))$

$=(1-3a-3b)(1+a+b)$

12. $3a^2-1-2a$

Solution:

$3a^2-1-2a=3a^2-2a-1$

$=3a^2-3a+a-1$

$=3a(a-1)+1(a-1)$

$=(3a+1)(a-1)$

13. $x^2+3x+2+ax+2a$

Solution:

$x^2+3x+2+ax+2a=x^2+2x+x+2+ax+2a$

$=x(x+2)+1(x+2)+a(x+2)$

$=(x+2)(x+a+1)$

14. $(3x-2y)^2+3(3x-2y)-10$

Solution:

Assume that $3x-2y=a$

Therefore,

$$(3x-2y)^2+3(3x-2y)-10=a^2+3a-10$$
$$=a^2+5a-2a-10$$
$$=a(a+5)-2(a+5)$$
$$=(a+5)(a-2)$$
$$=(3x-2y+5)(3x-2y-2)$$

15. $5-(3a^2-2a)(6-3a^2+2a)$

Solution:

$$5-(3a^2-2a)(6-3a^2+2a)=5-(3a^2-2a)\left[6-(3a^2-2a)\right]$$

Assume that $3a^2-2a=x$

Therefore,

$$5-(3a^2-2a)(6-3a^2+2a)=5-x(6-x)$$
$$=5-6x+x^2$$
$$=5-5x-x+x^2$$
$$=5(1-x)-x(1-x)$$
$$=(5-x)(1-x)$$
$$=(x-5)(x-1)$$
$$=(3a^2-2a-5)(3a^2-2a-1)$$
$$=(3a^2-5a+3a-5)(3a^2-3a+a-1)$$
$$=(a(3a-5)+1(3a-5))(3a(a-1)+1(a-1))$$
$$=(3a-5)(a+1)(3a+1)(a-1)$$

16. $\frac{1}{35} + \frac{12}{35} a + a^2$

Solution:

$$\frac{1}{35} + \frac{12}{35} a + a^2$$
$$= \frac{1}{35} [1 + 12a + 35a^2)]$$
$$= \frac{1}{35} [35a^2 + 12a + 1]$$
$$= \frac{1}{35} [35a^2 + 7a + 5a + 1]$$
$$= \frac{1}{35} [7a(5a + 1) + 1(5a + 1)]$$
$$= \frac{1}{35} (5a + 1)(7a + 1)$$

17. Factorise:

$(x^2 - 3x)(x^2 - 3x - 1) - 20$

Solution:

$(x^2 - 3x)(x^2 - 3x - 1) - 20$
$= (x^2 - 3x)[(x^2 - 3x) - 1] - 20$
$= a[a - 1] - 20$ ….(Taking $x^2 - 3x = a$)
$= a^2 - a - 20$
$= a^2 - 5a + 4a - 20$
$= a(a - 5) + 4(a - 5)$
$= (a - 5)(a + 4)$
$= (x^2 - 3x - 5)(x^2 - 3x + 4)$

18. For each trinomial (quadratic expression), given below, find whether it is factorisable or not. Factorise, if possible.

(i)

$$x^2 - 3x - 54$$

(ii)

$$2x^2 - 7x - 15$$

(iii)

$$2x^2 + 2x - 75$$

(iv)

$$3x^2 + 4x - 10$$

(v)

$$x(2x - 1) - 1$$

Solution:

(i) Given expression: $x^2 - 3x - 54$

Comparing with $ax^2 + bx + c$, we get $a = 1$, $b = -3$ and $c = -54$

$\therefore b^2 - 4ac = (-3)^2 - 4(1)(-54) = 9 + 216 = 225$, which is a perfect square.

$\therefore x^2 - 3x - 54$ is factorisable.

Now, $x^2 - 3x - 54 = x^2 - 9x + 6x - 54$

$= x(x - 9) + 6(x - 9)$

$= (x - 9)(x + 6)$

(ii) Given expression: $2x^2 - 7x - 15$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = -7$ and $c = -15$

$\therefore b^2 - 4ac = (-7)^2 - 4(2)(-15) = 49 + 120 = 169$, which is a perfect square.

$\therefore 2x^2 - 7x - 15$ is factorisable.

Now, $2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$

$= 2x(x - 5) + 3(x - 5)$

$= (2x + 3)(x - 5)$

(iii) Given expression: $2x^2 + 2x - 75$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = 2$ and $c = -75$

$\therefore b^2 - 4ac = (2)^2 - 4(2)(-75) = 4 + 600 = 604$, which is not a perfect square.

$\therefore 2x^2 + 2x - 75$ is not factorisable.

(iv) Given expression: $3x^2 + 4x - 10$

Comparing with $ax^2 + bx + c$, we get $a = 3$, $b = 4$ and $c = -10$

$\therefore b^2 - 4ac = (4)^2 - 4(3)(-10) = 16 + 120 = 136$, which is not a perfect square.

$\therefore 3x^2 + 4x - 10$ is not factorisable.

(v) Given expression: $x(2x - 1) - 1$

Now, $x(2x - 1) - 1 = 2x^2 - x - 1$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = -1$ and $c = -1$

$\therefore b^2 - 4ac = (-1)^2 - 4(2)(-1) = 1 + 8 = 9$, which is a perfect square.

$\therefore 2x^2 - x - 1$ is factorisable.

Now, $2x^2 - x - 1 = 2x^2 - 2x + x - 1$

$= 2x(x - 1) + 1(x - 1)$

$= (2x + 1)(x - 1)$

19. Factorise:

(i)

$4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

(ii)

$7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

Solution:

(i) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$

$= 4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right)$

$= \left(\sqrt{3}x + 2\right)\left(4x - \sqrt{3}\right)$

(ii) $7\sqrt{2}x^2 - 10x - 4\sqrt{2} = 7\sqrt{2}x^2 - 14x + 4x - 4\sqrt{2}$

$= 7\sqrt{2}x\left(x - \sqrt{2}\right) + 4\left(x - \sqrt{2}\right)$

$= \left(x - \sqrt{2}\right)\left(7\sqrt{2}x + 4\right)$

20. Give possible expressions for the length and the breadth of the rectangle whose area is $12x^2 - 35x + 25$

Solution:

$12x^2 - 35x + 25$

$= 12x^2 - 20x - 15x + 25$

$= 4x(3x - 5) - 5(3x - 5)$

$= (3x - 5)(4x - 5)$

Thus,

Length = (3x - 5) and breadth = (4x - 5)

OR

Length = (4x - 5) and breadth = (3x - 5)

EXERCISE 5(C) PAGE: 72

Factorise:

1. $25a^2 - 9b^2$

Solution:

$$25a^2 - 9b^2 = (5a)^2 - (3b)^2$$
$$= (5a - 3b)(5a + 3b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

2. $a^2 - (2a + 3b)^2$

Solution:

$$a^2 - (2a + 3b)^2 = (a)^2 - (2a + 3b)^2$$
$$= (a - 2a - 3b)(a + 2a + 3b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$
$$= (-a - 3b)(3a + 3b)$$
$$= -3(a + 3b)(a + b)$$

3. $a^2 - 81(b - c)^2$

Solution:

$$a^2 - 81(b - c)^2 = (a)^2 - [9(b - c)]^2$$
$$= (a - (9b - 9c))(a + (9b - 9c)) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$
$$= (a - 9b + 9c)(a + 9b - 9c)$$

4. $25(2a - b)^2 - 81b^2$

Solution:

$$25(2a - b)^2 - 81b^2 = [5(2a - b)]^2 - (9b)^2$$
$$= [5(2a - b) - 9b][5(2a - b) + 9b]$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= [10a - 5b - 9b][10a - 5b + 9b]$$
$$= [10a - 14b][10a + 4b]$$
$$= 2 \times (5a - 7b) \times 2 \times (5a + 2b)$$
$$= 4(5a - 7b)(5a + 2b)$$

5. $50a^3 - 2a$

Solution:

$$50a^3 - 2a = 2a(25a^2 - 1)$$
$$= 2a\left[(5a)^2 - 1^2\right]$$
$$= 2a(5a + 1)(5a - 1) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

6. $4a^2b - 9b^3$

Solution:

$$4a^2b - 9b^3 = b\left(4a^2 - 9b^2\right)$$
$$= b\left[(2a)^2 - (3b)^2\right]$$
$$= b(2a - 3b)(2a + 3b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

7. $3a^5 - 108a^3$

Solution:

$$3a^5 - 108a^3 = 3a^3\left(a^2 - 36\right)$$
$$= 3a^3\left[(a)^2 - (6)^2\right]$$
$$= 3a^3(a - 6)(a + 6) \quad \because a^2 - b^2 = (a + b)(a - b)$$

8. $9(a - 2)^2 - 16(a + 2)^2$

Solution:

$$9(a - 2)^2 - 16(a + 2)^2 = \left[3(a - 2)\right]^2 - \left[4(a + 2)\right]^2$$
$$= \left[3(a - 2) - 4(a + 2)\right]\left[3(a - 2) + 4(a + 2)\right]$$
$$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$$
$$= \left[3a - 6 - 4a - 8\right]\left[3a - 6 + 4a + 8\right]$$
$$= (-a - 14)(7a + 2)$$
$$= -(a + 14)(7a + 2)$$

9. $a^4 - 1$

Solution:

$$a^4 - 1 = \left(a^2\right)^2 - (1)^2$$
$$= \left(a^2 + 1\right)\left(a^2 - 1\right) \left[\because a^2 - b^2 = (a + b)(a - b)\right]$$
$$= \left(a^2 + 1\right)\left((a)^2 - (1)^2\right)$$
$$= \left(a^2 + 1\right)(a + 1)(a - 1)$$

10. $a^3 + 2a^2 - a - 2$

Solution:

$$a^3 + 2a^2 - a - 2 = a^2(a + 2) - 1(a + 2)$$
$$= \left(a^2 - 1\right)(a + 2)$$
$$= (a + 1)(a - 1)(a + 2) \quad \left[\because a^2 - b^2 = (a + b)(a - b)\right]$$

11. $(a + b)^3 - a - b$

Solution:

$$(a+b)^3 - a - b = (a+b)^3 - (a+b)$$
$$= (a+b)\left[(a+b)^2 - 1\right]$$
$$= (a+b)\left[(a+b)^2 - 1^2\right]$$
$$= (a+b)\left((a+b)+1\right)\left((a+b)-1\right)$$
$$\left[\because a^2 - b^2 = (a+b)(a-b)\right]$$
$$= (a+b)(a+b+1)(a+b-1)$$

12. $a(a-1) - b(b-1)$

Solution:

$$a(a-1) - b(b-1) = a^2 - a - b^2 + b$$
$$= a^2 - b^2 - a + b$$
$$= (a+b)(a-b) - (a-b)$$
$$\left[\because a^2 - b^2 = (a+b)(a-b)\right]$$
$$= (a-b)\left[(a+b) - 1\right]$$
$$= (a-b)[a+b-1]$$

13. $4a^2 - \left(4b^2 + 4bc + c^2\right)$

Solution:

$$4a^2 - \left(4b^2 + 4bc + c^2\right) = (2a)^2 - (2b+c)^2$$
$$= \left[2a - (2b+c)\right]\left[2a + (2b+c)\right]$$
$$\left[\because a^2 - b^2 = (a+b)(a-b)\right]$$
$$= \left[2a - 2b - c\right]\left[2a + 2b + c\right]$$

14. $4a^2 - 49b^2 + 2a - 7b$

Solution:

$$4a^2 - 49b^2 + 2a - 7b = \left[(2a)^2 - (7b)^2\right] + \left[2a - 7b\right]$$
$$= \left[2a - 7b\right]\left[2a + 7b\right] + \left[2a - 7b\right]$$
$$\left[\because a^2 - b^2 = (a+b)(a-b)\right]$$
$$= \left[2a - 7b\right]\left[2a + 7b + 1\right]$$

15. $9a^2 + 3a - 8b - 64b^2$

Solution:

$$9a^2 + 3a - 8b - 64b^2 = 9a^2 - 64b^2 + 3a - 8b$$
$$= (3a)^2 - (8b)^2 + 3a - 8b$$
$$= (3a - 8b)(3a + 8b) + (3a - 8b)$$
$$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$$
$$= (3a - 8b)(3a + 8b + 1)$$

16. $4a^2 - 12a + 9 - 49b^2$

Solution:

$$4a^2 - 12a + 9 - 49b^2 = (2a)^2 - 12a + (3)^2 - 49b^2$$
$$= (2a - 3)^2 - 49b^2$$
$$= (2a - 3)^2 - (7b)^2$$
$$= (2a - 3 - 7b)(2a - 3 + 7b)$$
$$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$$

17. $4xy - x^2 - 4y^2 + z^2$

Solution:

$$4xy - x^2 - 4y^2 + z^2 = z^2 - (x^2 + 4y^2 - 4xy)$$
$$= z^2 - (x - 2y)^2$$
$$= \left[z - (x - 2y)\right]\left[z + (x - 2y)\right]$$
$$\left[\because a^2 - b^2 = (a + b)(a - b)\right]$$
$$= \left[z - x + 2y\right]\left[z + x - 2y\right]$$

18. $a^2 + b^2 - c^2 - d^2 + 2ab - 2cd$

Solution:

$$a^2 + b^2 - c^2 - d^2 + 2ab - 2cd$$
$$= (a^2 + b^2 + 2ab) - (c^2 + d^2 + 2cd)$$
$$= (a + b)^2 - (c + d)^2$$
$$= \left[(a + b) - (c + d)\right]\left[(a + b) + (c + d)\right] \quad \left[\because a^2 - b^2 = (a + b)(a - b)\right]$$
$$= (a + b - c - d)(a + b + c + d)$$

19. $4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2$

Solution:

$$4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2$$
$$= 4x^2 + 9a^2 - 12ax - y^2 - z^2 - 2yz$$

$$= (2x)^2 + (3a)^2 - 12ax - (y^2 + z^2 + 2yz)$$
$$= (2x - 3a)^2 - (y + z)^2$$
$$= [(2x - 3a) - (y + z)][(2x - 3a) + (y + z)]$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= [2x - 3a - y - z][2x - 3a + y + z]$$

20. $(a^2 - 1)(b^2 - 1) + 4ab$

Solution:

$$(a^2 - 1)(b^2 - 1) + 4ab = a^2b^2 - a^2 - b^2 + 1 + 4ab$$
$$= a^2b^2 + 1 + 2ab - a^2 - b^2 + 2ab$$
$$= (a^2b^2 + 1 + 2ab) - (a^2 + b^2 - 2ab)$$
$$= (ab + 1)^2 - (a - b)^2$$
$$= [(ab + 1) - (a - b)][(ab + 1) + (a - b)]$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= [ab + 1 - a + b][ab + 1 + a - b]$$

21. $x^4 + x^2 + 1$

Solution:

$$x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$$
$$= (x^2)^2 + 2x^2 + (1)^2 - x^2$$
$$= (x^2 + 1)^2 - (x)^2$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= (x^2 + 1 - x)(x^2 + 1 + x)$$

22. $(a^2 + b^2 - 4c^2)^2 - 4a^2b^2$

Solution:

$$\left(a^2+b^2-4c^2\right)^2-4a^2b^2=\left(a^2+b^2-4c^2\right)^2-(2ab)^2$$
$$=\left(a^2+b^2-4c^2-2ab\right)\left(a^2+b^2-4c^2+2ab\right)$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=\left(a^2+b^2-2ab-4c^2\right)\left(a^2+b^2+2ab-4c^2\right)$$
$$=\left((a-b)^2-(2c)^2\right)\left((a+b)^2-(2c)^2\right)$$
$$=(a-b+2c)(a-b-2c)(a+b+2c)(a+b-2c)$$

23. $\left(x^2+4y^2-9z^2\right)^2-16x^2y^2$

Solution:

$$\left(x^2+4y^2-9z^2\right)^2-16x^2y^2$$
$$=\left(x^2+4y^2-9z^2\right)^2-(4xy)^2$$
$$=\left(x^2+4y^2-9z^2-4xy\right)\left(x^2+4y^2-9z^2+4xy\right)$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=\left(x^2+4y^2-4xy-9z^2\right)\left(x^2+4y^2+4xy-9z^2\right)$$
$$=\left[(x-2y)^2-(3z)^2\right]\left[(x+2y)^2-(3z)^2\right]$$
$$=\left[(x-2y)-3z\right]\left[(x-2y)+3z\right]\left[(x+2y)-3z\right]\left[(x+2y)+3z\right]$$
$$=\left[x-2y-3z\right]\left[x-2y+3z\right]\left[x+2y-3z\right]\left[x+2y+3z\right]$$

24. $(a+b)^2-a^2+b^2$

Solution:

$$(a+b)^2-a^2+b^2$$
$$=a^2+2ab+b^2-a^2+b^2$$
$$=2ab+2b^2$$
$$=2b(a+b)$$

25. $a^2-b^2-(a+b)^2$

Solution:

$$a^2-b^2-(a+b)^2$$
$$=a^2-b^2-(a^2+2ab+b^2)$$
$$=a^2-b^2-a^2-2ab-b^2$$
$$=-2ab-2b^2$$
$$=-2b(a+b)$$

26. $9a^2-(a^2-4)^2$

Solution:

$$9a^2 - (a^2 - 4)^2$$
$$= (3a)^2 - (a^2 - 4)^2$$
$$= [3a - (a^2 - 4)][3a + (a^2 - 4)]$$
$$= [3a - a^2 - 4][3a + a^2 - 4]$$
$$= [-a^2 + 3a - 4][a^2 + 3a - 4]$$
$$= [-a^2 + 4a - a - 4][a^2 + 4a - a - 4]$$
$$= [a(-a + 4) + 1(-a + 4)][a(a + 4) - 1(a + 4)]$$
$$= [(a + 1)(4 - a)][(a + 4)(a - 1)]$$
$$= (a + 1)(4 - a)(a + 4)(a - 1)$$

27. $x^2 + \frac{1}{x^2} - 11$

Solution:

$$x^2 + \frac{1}{x^2} - 11$$
$$= x^2 + \frac{1}{x^2} - 2 - 9$$
$$= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} - 9$$
$$= \left(x - \frac{1}{x}\right)^2 - (3)^2$$
$$= \left(x - \frac{1}{x} + 3\right)\left(x - \frac{1}{x} - 3\right)$$

28. $4x^2 + \frac{1}{4x^2} + 1$

Solution:

$$4x^2 + \frac{1}{4x^2} + 1$$
$$= 4x^2 + \frac{1}{4x^2} + 2 - 1$$
$$= 4x^2 + \frac{1}{4x^2} + 2 \times 2x \times \frac{1}{2x} - 1$$
$$= \left(2x + \frac{1}{2x}\right)^2 - (1)^2$$
$$= \left(2x + \frac{1}{2x} + 1\right)\left(2x + \frac{1}{2x} - 1\right)$$

29. $4x^4 - x^2 - 12x - 36$

Solution:

$4x^4 - x^2 - 12x - 36$

$= 4x^4 - (x^2 + 12x + 36)$

$= (2x^2)^2 - (x^2 + 2 \times x \times 6 + 6^2)$

$= (2x^2)^2 - (x + 6)^2$

$= (2x^2 + x + 6)(2x^2 - x - 6)$

$= (2x^2 + x + 6)(2x^2 - 4x + 3x - 6)$

$= (2x^2 + x + 6)[2x(x - 2) + 3(x - 2)]$

$= (2x^2 + x + 6)[(x - 2)(2x + 3)]$

$= (2x^2 + x + 6)(x - 2)(2x + 3)$

30. $a^2(b + c) - (b + c)^3$

Solution:

$a^2(b + c) - (b + c)^3$

$= (b + c)[a^2 - (b + c)^2]$

$= (b + c)[(a + b + c)(a - b - c)]$

$= (b + c)(a + b + c)(a - b - c)$

EXERCISE 5(D) PAGE: 74

Factorise:

1. $a^3 - 27$

Solution:

$$a^3 - 27 = (a)^3 - (3)^3$$
$$= (a-3)\left[(a)^2 + a \times 3 + (3)^2\right] \quad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$
$$= (a-3)\left[a^2 + 3a + 9\right]$$

2. $1 - 8a^3$

Solution:

$$1 - 8a^3 = (1)^3 - (2a)^3$$
$$= (1-2a)\left[(1)^2 + 1 \times 2a + (2a)^2\right]$$
$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$
$$= (1-2a)\left[1 + 2a + 4a^2\right]$$

3. $64 - a^3b^3$

Solution:

$$64 - a^3b^3 = (4)^3 - ab^3$$
$$= (4-ab)\left[(4)^2 + 4 \times ab + (ab)^2\right]$$
$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$
$$= (4-ab)\left[16 + 4ab + a^2b^2\right]$$

4. $a^6 + 27b^3$

Solution:

$$a^6 + 27b^3 = (a^2)^3 + (3b)^3$$
$$= (a^2 + 3b)\left[(a^2)^2 - a^2 \times 3b + (3b)^2\right]$$
$$\left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$$
$$= (a^2 + 3b)\left[a^4 - 3a^2b + 9b^2\right]$$

5. $3x^7y - 81x^4y^4$

Solution:

$$3x^7y - 81x^4y^4 = 3xy\left(x^6 - 27x^3y^3\right)$$
$$= 3xy\left(\left(x^2\right)^3 - \left(3xy\right)^3\right)$$
$$= 3xy\left(x^2 - 3xy\right)\left[\left(x^2\right)^2 + x^2 \times 3xy + \left(3xy\right)^2\right]$$
$$\left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$$
$$= 3xy\left(x^2 - 3xy\right)\left[x^4 + 3x^3y + 9x^2y^2\right]$$
$$= 3xy\left\{x\left(x - 3y\right)x^2\left[x^2 + 3xy + 9y^2\right]\right\}$$
$$= 3x^4y\left(x - 3y\right)\left[x^2 + 3xy + 9y^2\right]$$

6. $a^3 - \frac{27}{a^3}$

Solution:

$$a^3 - \frac{27}{a^3} = (a)^3 - \left(\frac{3}{a}\right)^3$$
$$= \left(a - \frac{3}{a}\right)\left(a^2 + a \times \frac{3}{a} + \left(\frac{3}{a}\right)^2\right)$$
$$\left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$$
$$= \left(a - \frac{3}{a}\right)\left(a^2 + 3 + \frac{9}{a^2}\right)$$

7. $a^3 + 0.064$

Solution:

$$a^3 + 0.064 = (a)^3 + (0.4)^3$$
$$= (a + 0.4)\left[(a)^2 - a \times 0.4 + (0.4)^2\right]$$
$$\left[\because a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right]$$
$$= (a + 0.4)\left[a^2 - 0.4a + 0.16\right]$$

8. $a^4 - 343a$

Solution:

$$a^4 - 343a = a\left(a^3 - 7^3\right)$$
$$= a(a-7)\left[(a)^2 + a \times 7 + (7)^2\right]$$
$$\left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$$
$$= a(a-7)\left[a^2 + 7a + 49\right]$$

9. $(x - y)^3 - 8x^3$

Solution:

$(x - y)^3 - 8x^3 = (x - y)^3 - (2x)^3$

$= (x - y - 2x)[(x - y)^2 + 2x(x - y) + (2x)^2]$

[Using identity $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$]

$= (-x - y)[x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2]$

$=-(x + y)\,[7x^2 - 4xy + y^2]$

10. $\frac{8a^3}{27} - \frac{b^3}{8}$

Solution:

$$\frac{8a^3}{27} - \frac{b^3}{8} = \left(\frac{2a}{3}\right)^3 - \left(\frac{b}{2}\right)^3$$

$$= \left(\frac{2a}{3} - \frac{b}{2}\right)\left[\left(\frac{2a}{3}\right)^2 + \frac{2a}{3} \times \frac{b}{2} + \left(\frac{b}{2}\right)^2\right]$$

$[\because a^3 - b^3 = (a - b)(a^2 = ab + b^2)]$

$$= \left(\frac{2a}{3} - \frac{b}{2}\right)\left[\frac{4a^2}{9} + \frac{ab}{3} + \frac{b^2}{4}\right]$$

11. $a^6 - b^6$

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ ....(1)

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ ....(2)

$a^6 - b^6 = (a^3)^2 - (b^3)^2$

$= (a^3 + b^3)(a^3 - b^3)$

$= (a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2)$ [from (1) and (2)]

$= (a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

12. $a^6 - 7a^3 - 8$

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ ....(1)

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ ....(2)

$a^6 - 7a^3 - 8 = a^6 - 8a^3 + a^3 - 8$

$= a^3(a^3 - 8) + 1(a^3 - 8)$

$= (a^3 + 1)(a^3 - 8)$

$= (a^3 + 1^3)(a^3 - 2^3)$

$= (a + 1)(a^2 - a + 1)(a - 2)(a^2 + 2a + 4)$ [from (1) and (2)]

$= (a + 1)(a - 2)(a^2 - a + 1)(a^2 + 2a + 4)$

13. $a^3 - 27b^3 + 2a^2b - 6ab^2$

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ ....(1)

$a^3 - 27b^3 + 2a^2b - 6ab^2$

$= (a)^3 - (3b)^3 + 2ab(a - 3b)$

$= (a - 3b)\left[a^2 + a \times 3b + (3b)^2\right] + 2ab(a - 3b)$ [from (1)]

$= (a - 3b)\left[a^2 + 3ab + 9b^2\right] + 2ab(a - 3b)$

$= (a - 3b)\left[a^2 + 3ab + 9b^2 + 2ab\right]$

$= (a - 3b)\left[a^2 + 5ab + 9b^2\right]$

14. $8a^3 - b^3 - 4ax + 2bx$

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ ....(1)

$8a^3 - b^3 - 4ax + 2bx = \left[(2a)^3 - (b)^3\right] - 2x(2a - b)$

$= (2a - b)\left[(2a)^2 + 2a \times b + (b)^2\right] - 2x(2a - b)$

[from (1)]

$= (2a - b)\left[4a^2 + 2ab + b^2\right] - 2x(2a - b)$

$= (2a - b)\left[4a^2 + 2ab + b^2 - 2x\right]$

15. $a - b - a^3 + b^3$

Solution:

We know that,

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) \quad ....(1)$$

$$a - b - a^3 + b^3 = a - b - (a^3 - b^3)$$

$$= (a-b) - (a-b)[a^2 + ab + b^2] \quad [\text{from } (1)]$$

$$= (a-b)[1 - a^2 - ab - b^2]$$

16. $2x^3 + 54y^3 - 4x - 12y$

Solution:

$2x^3 + 54y^3 - 4x - 12y = 2(x^3 + 27y^3 - 2x - 6y)$

$= 2\{[(x)^3+(3y)^3] - 2(x + 3y)\}$

[Using identity $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$]

$=2\{[(x + 3y)(x^2 - 3xy + 9y^2)] - 2(x + 3y)\}$

$=2(x + 3y)(x^2 - 3xy + 9y^2 - 2)$

17. $1029 - 3x^3$

Solution:

$1029 - 3x^3$

$= 3(343 - x^3)$

$= 3(7^3 - x^3)$

$= 3(7 - x)(7^2 + 7x + x^2)$

$= 3(7 - x)(49 + 7x + x^2)$

18. Show that:

(i) $(13^3 - 5^3)$ is divisible by 8.

Solution:

$(13^3 - 5^3)$

[Using identity $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$]

$=(13 - 5)(13^2 + 13 \times 5 + 5^2)$

$=8(169 + 65 + 25)$

Therefore, the number is divisible by 8.

(ii) $(35^3 + 27^3)$

Solution:

[Using identity $(a^3 + b^3)=(a + b)(a^2 - ab + b^2)$]

$=(35 + 27)(35^2 + 35\times 27 + 27^2)$

$=62 \times (35^2 + 35 \times 27 + 27^2)$

Therefore, the number is divisible by 62.

19. Evaluate:

$$\frac{5.67 \times 5.67 \times 5.67 + 4.33 \times 4.33 \times 4.33}{5.67 \times 5.67 - 5.67 \times 4.33 + 4.33 \times 4.33}$$

Solution:

Let $a = 5.67$ and $b = 4.33$

Then,

$$\frac{5.67 \times 5.67 \times 5.67 + 4.33 \times 4.33 \times 4.33}{5.67 \times 5.67 - 5.67 \times 4.33 + 4.33 \times 4.33}$$

$$= \frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a + b)(a^2 - ab + b^2)}{a^2 - ab + b^2}$$

$$= a + b$$

$$= 5.67 + 4.33$$

$$= 10$$

EXERCISE 5(E)

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Factorise:

1. $x^2+\frac{1}{4x^2}+1-7x-\frac{7}{2x}$

Solution:

$$x^2+\frac{1}{4x^2}+1-7x-\frac{7}{2x}=(x)^2+\frac{1}{(2x)^2}+2\times x\times\frac{1}{2x}-7\left(x+\frac{1}{2x}\right)$$

$$=\left(x+\frac{1}{2x}\right)^2-7\left(x+\frac{1}{2x}\right)$$

$$=\left(x+\frac{1}{2x}\right)\left(x+\frac{1}{2x}-7\right)$$

$$=\left(x+\frac{1}{2x}\right)\left(x-7+\frac{1}{2x}\right)$$

2. $9a^2+\frac{1}{9a^2}-2-12a+\frac{4}{3a}$

Solution:

$$9a^2+\frac{1}{9a^2}-2-12a+\frac{4}{3a}=(3a)^2+\frac{1}{(3a)^2}-2\times 3a\times\frac{1}{3a}-4\left(3a-\frac{1}{3a}\right)$$

$$=\left(3a-\frac{1}{3a}\right)^2-4\left(3a-\frac{1}{3a}\right)$$

$$=\left(3a-\frac{1}{3a}\right)\left(\left(3a-\frac{1}{3a}\right)-4\right)$$

$$=\left(3a-\frac{1}{3a}\right)\left(3a-4-\frac{1}{3a}\right)$$

3. $x^2+\frac{a^2+1}{a}x+1$

Solution:

$$x^2+\frac{a^2+1}{a}x+1=x^2+ax+\frac{1}{a}x+1$$

$$=x(x+a)+\frac{1}{a}(x+a)$$

$$=(x+a)\left(x+\frac{1}{a}\right)$$

4. $x^4+y^4-27x^2y^2$

Solution:

$$x^4 + y^4 - 27x^2y^2 = \left(x^2\right)^2 + \left(y^2\right)^2 - 2x^2y^2 - 25x^2y^2$$
$$= \left(x^2 - y^2\right)^2 - 25x^2y^2$$
$$= \left(x^2 - y^2\right)^2 - (5xy)^2 \quad [\because a^2 - b^2 = (a+b)(a-b)]$$
$$= \left[\left(x^2 - y^2\right) + 5xy\right]\left[\left(x^2 - y^2\right) - 5xy\right]$$
$$= \left[x^2 + 5xy - y^2\right]\left[x^2 - 5xy - y^2\right]$$

5. $4x^4 + 9y^4 + 11x^2y^2$

Solution:

$$4x^4 + 9y^4 + 11x^2y^2 = \left(2x^2\right)^2 + \left(3y^2\right)^2 + 12x^2y^2 - x^2y^2$$
$$= \left(2x^2 + 3y^2\right)^2 - x^2y^2$$
$$= \left(2x^2 + 3y^2\right)^2 - (xy)^2$$
$$= \left(2x^2 + 3y^2 - xy\right)\left(2x^2 + 3y^2 + xy\right)$$
$$[\because a^2 - b^2 = (a+b)(a-b)]$$

6. $x^2 + \frac{1}{x^2} - 3$

Solution:

$$x^2 + \frac{1}{x^2} - 3 = x^2 + \frac{1}{x^2} - 2 - 1$$
$$= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} - 1$$
$$= \left(x - \frac{1}{x}\right)^2 - 1$$
$$= \left(x - \frac{1}{x}\right)^2 - (1)^2$$
$$= \left(x - \frac{1}{x} - 1\right)\left(x - \frac{1}{x} + 1\right) \quad [\because a^2 - b^2 = (a+b)(a-b)]$$

7. $a - b - 4a^2 + 4b^2$

Solution:

$$a-b-4a^2+4b^2=(a-b)-4(a^2-b^2)$$
$$=(a-b)-4(a-b)(a+b)\ [\because a^2-b^2=(a+b)(a-b)]$$
$$=(a-b)[1-4(a+b)]$$
$$=(a-b)[1-4a-4b]$$

8. $(2a-3)^2-2(2a-3)(a-1)+(a-1)^2$

Solution:

$$(2a-3)^2-2(2a-3)(a-1)+(a-1)^2$$
$$=[(2a-3)-(a-1)]^2$$
$$=[2a-3-a+1]^2$$
$$=(a-2)^2$$

9. $(a^2-3a)(a^2-3a+7)+10$

Solution:

Let us assume, $a^2-3a=x$

Then the given expression is,

$$(a^2-3a)(a^2-3a+7)+10=x(x+7)+10$$
$$=x^2+7x+10$$
$$=x^2+5x+2x+10$$
$$=x(x+5)+2(x+5)$$
$$=(x+5)(x+2)$$
$$=(a^2-3a+5)(a^2-3a+2)$$

[resubstitute the value of x]

$$=(a^2-3a+5)(a^2-2a-a+2)$$
$$=(a^2-3a+5)(a(a-2)-1(a-2))$$
$$=(a^2-3a+5)[(a-1)(a-2)]$$

10. $(a^2-a)(4a^2-4a-5)-6$

Solution:

Let us assume $a^2 - a = x$

Then the given expression is

$$(a^2 - a)(4a^2 - 4a - 5) - 6 = x(4x - 5) - 6$$

$$= 4x^2 - 5x - 6$$

$$= 4x^2 - 8x + 3x - 6$$

$$= 4x(x - 2) + 3(x - 2)$$

$$= (4x + 3)(x - 2)$$

$$= (4(a^2 - a) + 3)(a^2 - a - 2)$$

[resubstitute the value of x]

$$= (4a^2 - 4a + 3)(a^2 - a - 2)$$

$$= (4a^2 - 4a + 3)(a^2 - 2a + a - 2)$$

$$= (4a^2 - 4a + 3)(a(a - 2) + 1(a - 2))$$

$$= (4a^2 - 4a + 3)(a - 2)(a + 1)$$

11. $x^4 + y^4 - 3x^2y^2$

Solution:

$$x^4 + y^4 - 3x^2y^2 = x^4 + y^4 - 2x^2y^2 - x^2y^2$$

$$= (x^2)^2 + (y^2)^2 - 2x^2y^2 - x^2y^2$$

$$= (x^2 - y^2)^2 - (xy)^2$$

$$= (x^2 - y^2 - xy)(x^2 - y^2 + xy)$$

$[\because a^2 - b^2 = (a + b)(a - b)]$

12. $5a^2 - b^2 - 4ab + 7a - 7b$

Solution:

$$5a^2 - b^2 - 4ab + 7a - 7b$$

$$= 4a^2 + a^2 - b^2 - 4ab + 7a - 7b$$

$$= a^2 - b^2 + 4a^2 - 4ab + 7a - 7b$$

$$= (a^2 - b^2) + 4a(a - b) + 7(a - b)$$

$$= (a - b)(a + b) + 4a(a - b) + 7(a - b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

$$= (a - b)[(a + b) + 4a + 7]$$

$$= (a - b)[a + b + 4a + 7]$$

$$= (a - b)[5a + b + 7]$$

13. $12(3x-2y)^2-3x+2y-1$

Solution:

$12(3x-2y)^2-3x+2y-1=12(3x-2y)^2-(3x-2y)-1$

Let us assume that $3x-2y=a$

Then the given expression is

$12(3x-2y)^2-3x+2y-1=12a^2-3a-1$

$=12a^2-4a+3a-1$

$=4a(3a-1)+1(3a-1)$

$=(4a+1)(3a-1)$

$=(4(3x-2y)+1)(3(3x-2y)-1)$

[resubstitute the value of a]

$=(12x-8y+1)(9x-6y-1)$

14. $4(2x-3y)^2-8x+12y-3$

Solution:

$4(2x-3y)^2-8x+12y-3=4(2x-3y)^2-4(2x-3y)-3$

Let us assume that $2x-3y=a$

Then the given expression is

$4(2x-3y)^2-8x+12y-3=4a^2-4a-3$

$=4a^2-6a+2a-3$

$=2a(2a-3)+1(2a-3)$

$=(2a-3)(2a+1)$

$=(2(2x-3y)-3)(2(2x-3y)+1)$

$=(4x-6y-3)(4x-6y+1)$

15. $3-5x+5y-12(x-y)^2$

Solution:

$3-5x+5y-12(x-y)^2 = 3-5(x-y)-12(x-y)^2$

Let us assume that $x-y=a$

Then the given expression is

$3-5x+5y-12(x-y)^2 = 3-5a-12a^2$

$= 3-9a+4a-12a^2$

$= 3(1-3a)+4a(1-3a)$

$= (3+4a)(1-3a)$

[resubstitute the value of a]

$= (3+4(x-y))(1-3(x-y))$

$= (3+4x-4y)(1-3x+3y)$

16. $9x^2+3x-8y-64y^2$

Solution:

$9x^2+3x-8y-64y^2$

$= 9x^2-64y^2+3x-8y$

$= [(3x)^2-(8y)^2]+(3x-8y)$

$= [(3x+8y)(3x-8y)]+(3x-8y)$

$= (3x-8y)(3x+8y+1)$

17. $2\sqrt{3}x^2+x-5\sqrt{3}$

Solution:

$2\sqrt{3}x^2+x-5\sqrt{3}$

$= 2\sqrt{3}x^2+6x-5x-5\sqrt{3}$

$= 2\sqrt{3}x(x+\sqrt{3})-5(x+\sqrt{3})$

$= (2\sqrt{3}x-5)(x+\sqrt{3})$

18. $\frac{1}{4}(a+b)^2-\frac{9}{16}(2a-b)^2$

Solution:

$$\frac{1}{4}(a+b)^2-\frac{9}{16}(2a-b)^2$$
$$=\frac{1}{4}\left[(a+b)^2-\frac{9}{4}(2a-b)^2\right]$$
$$=\frac{1}{4}\left[(a+b)^2-\left(\frac{3}{2}(2a-b)\right)^2\right]$$
$$=\frac{1}{4}\left[\left(a+b+\frac{3}{2}(2a-b)\right)\left(a+b-\frac{3}{2}(2a-b)\right)\right]$$
$$=\frac{1}{4}\left[\left(a+b+3a-\frac{3b}{2}\right)\left(a+b-3a+\frac{3b}{2}\right)\right]$$
$$=\frac{1}{4}\left[\left(4a-\frac{b}{2}\right)\left(\frac{5b}{2}-2a\right)\right]$$
$$=\frac{1}{4}\left[\left(\frac{8a-b}{2}\right)\left(\frac{5b-4a}{2}\right)\right]$$
$$=\frac{1}{4}\left[\frac{1}{4}(8a-b)(5b-4a)\right]$$
$$=\frac{1}{16}(8a-b)(5b-4a)$$

19. $2(ab+cd)-a^2-b^2+c^2+d^2$

Solution:

$$2(ab+cd)-a^2-b^2+c^2+d^2$$
$$=2ab+2cd-a^2-b^2+c^2+d^2$$
$$=c^2+d^2+2cd-a^2-b^2+2ab$$
$$=(c^2+d^2+2cd)-(a^2+b^2-2ab)$$
$$=(c+d)^2-(a-b)^2$$
$$=(c+d+a-b)(c+d-a+b)$$

20. Find the value of:

Solution:

(i) $(987)^2-(13)^2$

(ii) $(67.8)^2-(32.2)^2$

(iii) $\dfrac{(6.7)^2-(3.3)^2}{6.7-3.3}$

(iv) $\dfrac{(18.5)^2-(6.5)^2}{18.5+6.5}$

(i) $(987)^2 - (13)^2$

$= (987 + 13)(987 - 13)$

$= 1000 \times 974$

$= 974000$

(ii) $(67.8)^2 - (32.2)^2$

$= (67.8 + 32.2)(67.8 - 32.2)$

$= 100 \times 35.6$

$= 3560$

(iii) $\dfrac{(6.7)^2 - (3.3)^2}{6.7 - 3.3}$

$= \dfrac{(6.7 + 3.3)(6.7 - 3.3)}{(6.7 - 3.3)}$

$= 10$

(iv) $\dfrac{(18.5)^2 - (6.5)^2}{18.5 + 6.5}$

$= \dfrac{(18.5 + 6.5)(18.5 - 6.5)}{(18.5 + 6.5)}$

$= 12$