EXERCISE 6(A) PAGE: 79

Solve the following pairs of linear (simultaneous) equations using method of elimination by substitution:

1. 8x + 5y = 9
3x + 2y = 4

Solution:

8x + 5y = 9
3x + 2y = 4
8x + 5y = 9...(1)
3x + 2y = 4...(2)

(2) $\Rightarrow$ y = $\frac{9-8x}{5}$

Putting this value of y in (2)

3x + 2 $\left(\frac{9-8x}{5}\right) = 4$

15x + 18 - 16x = 20
x = -2

From (1) y = $\left(\frac{9-8x}{5}\right) = \frac{9-8(-2)}{5} = \frac{25}{5} = 5$

y = 5

2. 2x - 3y = 7
5x + y = 9

Solution:

2x - 3y = 7...(1)
5x + y = 9...(2)
(2) $\Rightarrow$ y = 9 - 5x
Putting this value of y in (1)
2x - 3 (9 - 5x) = 7
2x - 27 + 15x = 7
17x = 34
x = 2
From (2)
y = 9 - 5(2)
y = -1

3. 2x + 3y = 8
2x = 2 + 3y

Solution:

2x + 3y = 8...(1)
2x = 2 + 3y...(2)

(2) $\Rightarrow 2x = 2 + 3y$
Putting this value of 2x in (1)
$2 + 3y + 3y = 8$
$6y = 6$
$y = 1$
From (2) $2x = 2 + 3(1)$
$x = \frac{5}{2}$
$x = 2.5$

4.
$$0.2x + 0.1y = 25$$
$$2(x-2) - 1.6y = 116$$

Solution:

The given pair of linear equations are
$0.2x + 0.1y = 25 \ldots\ldots\ldots\ldots(i)$
$2(x-2) - 1.6y = 116 \ldots\ldots\ldots(ii)$

Consider equation (i)
$0.2x + 0.1y = 25$
$\Rightarrow 0.2x = 25 - 0.1y$
$\Rightarrow x = \frac{(25 - 0.1y)}{0.2} \ldots\ldots\ldots(iii)$

Substitute the value of x from equation (iii) in equation (ii).
$2(x-2) - 1.6y = 116$
$\Rightarrow 2\left(\frac{(25 - 0.1y)}{0.2} - 2\right) - 1.6y = 116$
$\Rightarrow 10(25 - 0.1y) - 4 - 1.6y = 116$
$\Rightarrow 250 - y - 4 - 1.6y = 116$
$\Rightarrow -2.6y = -130$
$\Rightarrow y = 50 \ldots\ldots\ldots(iv)$

Substitute the value of y from equation (iv) in equation (iii).
$x = \frac{(25 - 0.1y)}{0.2}$
$\Rightarrow x = \frac{(25 - 0.1(50))}{0.2}$
$\Rightarrow x = \frac{(25 - 5)}{0.2}$
$\Rightarrow x = 100$

$\therefore$ Solution is $x = 100$ and $y = 50$.

5. 6x = 7y + 7
7y - x = 8

Solution:

6x = 7y + 7...(1)
7y - x = 8...(2)
(2) $\Rightarrow$ x = 7y - 8
Putting this value of x in (1)
6(7y - 8) = 7y + 7
42y - 48 = 7y + 7
35y = 55
$y = \frac{11}{7}$

From (2) x = $7\left(\frac{11}{7}\right) - 8$
x = 3

6. y = 4x -7
16x- 5y = 25

Solution:

y = 4x -7...(1)
16x- 5y = 25...(2)
(1) $\Rightarrow$ y = 4x - 7
Putting this value of y in (2)
16x - 5 (4x - 7) = 25
16x - 20x + 35 = 25
-4x = -10
$x = \frac{5}{2}$

From (1)
$y = 4\left(\frac{5}{2}\right) - 7$
$\Rightarrow y = 10 - 7$
$\Rightarrow y = 7$
y = 10-7=3
Solution is $x = \frac{5}{2}$ and $y = 3$.

7. 2x + 7y = 39
3x + 5y = 31

Solution:

2x + 7y = 39...(1)
3x + 5y = 31...(2)

(1) $\Rightarrow x = \frac{39-7y}{2}$

Putting this value of x in (2)

$3\left(\frac{39-7y}{2}\right)+5y=31$

117 - 21y + 10y = 62

-11y = -55

y = 5

From (1) $x = \frac{39-7(5)}{2}$

$x=\frac{4}{2}$

$x=2$

8.

$1.5x + 0.1y = 6.2$

$3x - 0.4y = 11.2$

Solution:

The given pair of linear equations are

$1.5x + 0.1y = 6.2 \ldots\ldots\ldots\ldots\ldots(i)$

$3x - 0.4y = 11.2 \ldots\ldots\ldots\ldots\ldots(ii)$

Consider equation (i)

$1.5x + 0.1y = 6.2$

$\Rightarrow 1.5x = 6.2 - 0.1y$

$\Rightarrow x = \frac{(6.2 - 0.1y)}{1.5} \ldots\ldots\ldots(iii)$

Substitute the value of x from equation (iii) in equation (ii).

$3x - 0.4y = 11.2$

$\Rightarrow 3\left(\frac{(6.2-0.1y)}{1.5}\right) - 0.4y = 11.2$

$\Rightarrow 2(6.2 - 0.1y) - 0.4y = 11.2$

$\Rightarrow 12.4 - 0.2y - 0.4y = 11.2$

$\Rightarrow -0.6y = -1.2$

$\Rightarrow y = 2 \ldots\ldots\ldots(iv)$

Substitute the value of y from equation (iv) in equation (iii).

$x = \frac{(6.2 - 0.1y)}{1.5}$

$\Rightarrow x = \frac{(6.2 - 0.1(2))}{1.5}$

$\Rightarrow x = \frac{(6.2 - 0.2)}{1.5}$

$\Rightarrow x = 4$

$\therefore$ Solution is $x = 4$ and $y = 2$.

9.

$2(x - 3) + 3(y - 5) = 0$

$5(x - 1) + 4(y - 4) = 0$

Solution:

Given equations are

$2(x - 3) + 3(y - 5) = 0 \quad ....(1)$

$5(x - 1) + 4(y - 4) = 0 \quad ....(2)$

From (1), we get

$2x - 6 + 3y - 15 = 0$

$\Rightarrow 2x - 21 + 3y = 0$

$\Rightarrow 2x = 21 - 3y$

$\Rightarrow x = \frac{21 - 3y}{2}$

From (2), we get

$5(x - 1) + 4(y - 4) = 0$

$\Rightarrow 5x - 5 + 4y - 16 = 0$

$\Rightarrow 5x + 4y - 21 = 0 \quad ....(3)$

Substituting $x = \frac{21 - 3y}{2}$ in (3), we get

$5\left(\frac{21 - 3y}{2}\right) + 4y - 21 = 0$

$\Rightarrow \frac{105 - 15y}{2} + 4y - 21 = 0$

$\Rightarrow 105 - 15y + 8y - 42 = 0$

$\Rightarrow -7y + 63 = 0$

$\Rightarrow 7y = 63$

$\Rightarrow y = 9$

Substituting $y = 9$ in $x = \frac{21-3y}{2}$, we get

$x = \frac{21-3(9)}{2} = \frac{21-27}{2} = \frac{-6}{2} = -3$

$\therefore$ Solution is $x = -3$ and $y = 9$

10.

$$\frac{2x+1}{7} + \frac{5y-3}{3} = 12$$

$$\frac{3x+2}{2} - \frac{4y+3}{9} = 13$$

Solution:

$\frac{2x+1}{7} + \frac{5y-3}{3} = 12$ (given)

$\Rightarrow \frac{3(2x+1)+7(5y-3)}{21} = 12$

$\Rightarrow 6x + 3 + 35y - 21 = 252$

$\Rightarrow 6x + 35y - 18 = 252$

$\Rightarrow 6x + 35y = 270$

$\Rightarrow 6x = 270 - 35y$

$\Rightarrow x = \frac{270-35y}{6}$

$\frac{3x+2}{2} - \frac{4y+3}{9} = 13$ (given)

$\Rightarrow \frac{9(3x+2)-2(4y+3)}{18} = 13$

$\Rightarrow 27x + 18 - 8y - 6 = 234$

$\Rightarrow 27x - 8y + 12 = 234$

$\Rightarrow 27x - 8y = 222$ ....(1)

Substituting $x = \frac{270-35y}{6}$ in (1), we get

$27\left(\frac{270-35y}{6}\right) - 8y = 222$

$\Rightarrow 7290 - 945y - 48y = 1332$

$\Rightarrow -993y = -5958$

$\Rightarrow y = 6$

Substituting $y = 6$ in $x = \frac{270-35y}{6}$, we get

$x = \frac{270-35\times 6}{6} = \frac{270-210}{6} = \frac{60}{6} = 10$

$\therefore$ Solution is $x = 10$ and $y = 6$

11.

$3x + 2y = 11$

$2x - 3y + 10 = 0$

Solution:

$3x + 2y = 11$

$\Rightarrow 3x = 11 - 2y$

$\Rightarrow x = \dfrac{11 - 2y}{3} \quad ....(i)$

And,

$2x - 3y + 10 = 0$

$\Rightarrow 2 \times \left(\dfrac{11 - 2y}{3}\right) - 3y + 10 = 0 \quad ....[\text{From (i)}]$

$\Rightarrow \dfrac{22 - 4y}{3} - 3y = -10$

$\Rightarrow \dfrac{22 - 4y - 9y}{3} = -10$

$\Rightarrow 22 - 13y = -30$

$\Rightarrow 13y = 52$

$\Rightarrow y = 4$

Substituting the value of y in (i), we have

$x = \dfrac{11 - 2(4)}{3} = \dfrac{11 - 8}{3} = \dfrac{3}{3} = 1$

$\therefore$ Solution is $x = 1$ and $y = 4$.

12.

$2x - 3y + 6 = 0$

$\Rightarrow 2x = 3y - 6$

Solution:

$2x - 3y + 6 = 0$

$\Rightarrow 2x = 3y - 6$

$\Rightarrow x = \dfrac{3y - 6}{2} \quad ....(i)$

And,

$2x + 3y - 18 = 0$

$\Rightarrow 2 \times \left(\dfrac{3y - 6}{2}\right) + 3y = 18 \quad ....[\text{From (i)}]$

$\Rightarrow 3y - 6 + 3y = 18$

$\Rightarrow 6y = 24$

$\Rightarrow y = 4$

Substituting the value of y in (i), we have

$$x = \frac{3 \times 4 - 6}{2} = \frac{12 - 6}{2} = \frac{6}{2} = 3$$

$\therefore$ Solution is x = 3 and y = 4.

13.

$$\frac{3x}{2} - \frac{5y}{3} + 2 = 0$$

$$\frac{x}{3} + \frac{y}{2} = 2\frac{1}{6}$$

Solution:

$$\frac{x}{3} + \frac{y}{2} = 2\frac{1}{6}$$

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\Rightarrow \frac{2x + 3y}{6} = \frac{13}{6}$$

$\Rightarrow 2x + 3y = 13$

$\Rightarrow 2x = 13 - 3y$

$$\Rightarrow x = \frac{13 - 3y}{2} \quad \ldots.(i)$$

And,

$$\frac{3x}{2} - \frac{5y}{3} + 2 = 0$$

$$\Rightarrow \frac{3}{2}\left(\frac{13 - 3y}{2}\right) - \frac{5y}{3} = -2$$

$$\Rightarrow \frac{39 - 9y}{4} - \frac{5y}{3} = -2$$

$$\Rightarrow \frac{117 - 27y - 20y}{12} = -2$$

$$\Rightarrow \frac{117 - 47y}{12} = -2$$

$\Rightarrow 117 - 47y = -24$

$\Rightarrow 47y = 141$

$\Rightarrow y = 3$

Substituting the value of y in (i), we have

$$x = \frac{13 - 3 \times 3}{2} = \frac{13 - 9}{2} = \frac{4}{2} = 2$$

$\therefore$ Solution is x = 2 and y = 3.

14.

$$\frac{x}{6}+\frac{y}{15}=4$$

$$\frac{x}{3}-\frac{y}{12}=4\frac{3}{4}$$

Solution:

$$\frac{x}{6}+\frac{y}{15}=4$$

$$\Rightarrow \frac{5x+2y}{30}=4$$

$$\Rightarrow 5x+2y=120$$

$$\Rightarrow 5x=120-2y$$

$$\Rightarrow x=\frac{120-2y}{5} \qquad \ldots.(i)$$

And,

$$\frac{x}{3}-\frac{y}{12}=4\frac{3}{4}$$

$$\Rightarrow \frac{1}{3}\left(x-\frac{y}{4}\right)=\frac{19}{4}$$

$$\Rightarrow \frac{1}{3}\left(\frac{120-2y}{5}-\frac{y}{4}\right)=\frac{19}{4}$$

$$\Rightarrow \frac{480-8y-5y}{20}=\frac{57}{4}$$

$$\Rightarrow \frac{480-13y}{20}=\frac{57}{4}$$

$$\Rightarrow 480-13y=285$$

$$\Rightarrow 13y=195$$

$$\Rightarrow y=15$$

Substituting the value of y in (i), we have

$$x=\frac{120-2\times 15}{5}=\frac{120-30}{5}=\frac{90}{5}=18$$

$\therefore$ Solution is $x = 18$ and $y = 15$.

EXERCISE 6(B) PAGE: 81

For solving each pair of equations, in this exercise, use the method of elimination by equating coefficients:

1. $13 + 2y = 9x$
$3y = 7x$

Solution:

$13 + 2y = 9x$...(1)
$3y = 7x$...(2)
Multiplying equation no. (1) by 3 and (2) by 2, we get,

$$\begin{array}{r} 39 + 6y = 27x \quad \ldots(1) \\ 6y = 14x \quad \ldots(2) \\ - \quad - \quad - \\ \hline 39 = 13x \end{array}$$

$x = 3$
From (2)
$3y = 7(3)$
$y = 7$

2. $3x - y = 23$
$\frac{x}{3} + \frac{y}{4} = 4$

Solution:

$3x - y = 23$...(1)
$\frac{x}{3} + \frac{y}{4} = 4$
$4x + 3y = 48$...(2)
Multiplying equation no. (1) by 3

$$\begin{array}{r} 9x - 3y = 69 \quad \ldots(3) \\ 4x + 3y = 48 \\ \hline 13x = 117 \end{array}$$

$x = 9$
From (1)
$3(9) - y = 23$
$y = 27 - 23$
$y = 4$

3.
$\frac{5y}{2} - \frac{x}{3} = 8$
$\frac{y}{2} + \frac{5x}{3} = 12$

Solution:

The given pair of linear equations are

$$\frac{5y}{2} - \frac{x}{3} = 8$$

$$\Rightarrow -\frac{x}{3} + \frac{5y}{2} = 8 \ldots\ldots\ldots\ldots\ldots(i) \text{ [On simplifying]}$$

$$\frac{y}{2} + \frac{5x}{3} = 12$$

$$\Rightarrow \frac{5x}{3} + \frac{y}{2} = 12 \ldots\ldots\ldots\ldots\ldots(ii) \text{ [On simplifying]}$$

Multiply equation (i) by 5, we get:

$$-\frac{5x}{3} + \frac{25y}{2} = 40$$

$$\frac{5x}{3} + \frac{y}{2} = 12 \qquad \text{[Equation } (ii)\text{]}$$

$$+ \qquad + \qquad + \qquad \text{[Adding]}$$

$$\frac{26y}{2} = 52$$

$$\Rightarrow 13y = 52$$

$$\Rightarrow y = 4$$

Substituting $y = 4$ in equation (i), we get

$$-\frac{x}{3} + \frac{5(4)}{2} = 8$$

$$\Rightarrow -\frac{x}{3} = 8 - 10$$

$$\Rightarrow x = 6$$

$\therefore$ Solution is $x = 6$ and $y = 4$.

4.

$$\frac{1}{5}(x-2) = \frac{1}{4}(1-y)$$

$$26x + 3y = -4$$

Solution:

$$\frac{1}{5}(x-2) = \frac{1}{4}(1-y) \Rightarrow 4x + 5y = 13 \quad \ldots(1)$$

$$26x + 3y = -4 \quad \ldots(2)$$

Multiplying equation no. (1) by 3 and(2) by 5.

$$\begin{aligned} 12x + 15y &= 39 \quad \ldots(3) \\ 130x + 15y &= -20 \\ - \quad - \quad &+ \\ \hline -115x &= 59 \end{aligned}$$

$$x = -\frac{59}{118}$$

$$x = -\frac{1}{2}$$

From (1)

$$4\left(-\frac{1}{2}\right) + 5y = 13$$

$$5y = 13 + 2$$

$$y = 3$$

5. y = 2x - 6
y = 0

Solution:

y = 2x - 6
y = 0

$$\Rightarrow 2x - y = 6 \quad \ldots(1)$$

$$y = 0 \quad \ldots(2)$$

$$2x = 6$$

x = 3 ; y = 0

6.

$$\frac{x-y}{6} = 2(4-x)$$

$$2x + y = 3(x - 4)$$

Solution:

The given pair of linear equations are

$$\frac{x-y}{6} = 2(4-x)$$

$$\Rightarrow 13x - y = 48 \ldots\ldots\ldots\ldots\ldots(i) \text{ [On simplifying]}$$

$$2x + y = 3(x - 4)$$

$$\Rightarrow x - y = 12 \ldots\ldots\ldots\ldots\ldots(ii) \text{ [On simplifying]}$$

Multiply equation $(ii)$by 13, we get:

$$\begin{array}{l} 13x - 13y = 156 \\ 13x - y = 48 \qquad [\text{Equation } (i)] \\ \underline{- \quad + \quad -} \qquad [\text{Subtracting}] \\ \quad -12y = 108 \end{array}$$

$\Rightarrow y = -9$

Substituting $y = -9$ in equation (i), we get

$13x - (-9) = 48$

$\Rightarrow 13x = 39$

$\Rightarrow x = 3$

$\therefore$ Solution is $x = 3$ and $y = -9$.

7. 3 - (x - 5) = 4 + 2
2(x + y) = 4 - 3y

Solution:

3 - (x - 5) = 4 + 2
2(x + y) = 4 - 3y
$\Rightarrow$ -x - y = -6
$\Rightarrow$ x + y = 6...(1)
2x + 5y = 4...(2)
Multiplying equation no. (1) by 2.

$$\begin{array}{l} 2x + 2y = 12 \\ 2x + 5y = 4 \\ \underline{- \quad - \quad -} \\ \quad -3y = 8 \quad \Rightarrow y = \frac{-8}{3} \end{array}$$

From (1)

$x - \frac{8}{3} = 6 \qquad \Rightarrow x = \frac{26}{3}$

8. 2x -3y - 3 = 0
$\frac{2x}{3} + 4y + \frac{1}{2} = 0$

Solution:

2x -3y - 3 = 0
$\frac{2x}{3} + 4y + \frac{1}{2} = 0$
$\Rightarrow$ 2x - 3y = 3...(1)
$\Rightarrow$ 4x + 24y = -3...(2)
Multiplying equation no. (1) by 8.

$$\begin{array}{r} 16x - 24y = 24 \quad \ldots(3) \\ \underline{4x + 24y = -3} \\ 20x = 21 \quad \Rightarrow x = \frac{21}{20} \end{array}$$

From (1)

$$2\left(\frac{21}{20}\right) - 3y = 3$$

$$-3y = 3 - \frac{21}{10} \Rightarrow y = \frac{-3}{10}$$

9. 13x + 11y = 70
11x+ 13y = 74

Solution:

13x + 11y = 70...(1)
11x+ 13y = 74...(2)
Adding (1) and (2)
24x + 24y = 144
x + y = 6...(3)
Subtracting (2) from (1)
2x - 2y = -4
x - y = -2...(4)
x + y = 6...(3)

$$2x = 4 \qquad \Rightarrow x = 2$$

From (3)
2 + y = 6 ⇒ y = 4

10. 41x + 53y = 135
53x + 41y = 147

Solution:

41x + 53y = 135...(1)
53x + 41y = 147...(2)
Adding (1) and (2)
94x + 94y = 282
x + y = 3...(3)
Subtracting (2) from (1)

$$\begin{array}{r} -12x + 12y = -12 \\ -x + \quad y = -1 \quad \ldots(4) \\ \underline{x + \quad y = 3} \\ 2y = 2 \quad \Rightarrow y = 1 \end{array}$$

From (3)
x + 1 = 3 ⇒ x = 2

11. If 2x + y = 23 and 4x - y = 19; find the values of x-3y and 5y-2x.

Solution:

2x + y = 23...(1)

4x - y = 19...(2)

Adding equation (1) and (2) we get,

$$\begin{array}{r} 2x + y = 23 \\ 4x - y = 19 \\ \hline 6x = 42 \end{array} \quad \Rightarrow x = 7$$

From (1)

2(7) + y = 23

y = 23 - 14

$\Rightarrow$ y = 9

$\therefore$ x - 3y = 7 - 3(9) = -20

And 5y - 2x = 5(9) - 2(7) = 45- 14 = 31

12. If 10 y = 7x – 4 and 12x + 18y = 1; find the values of 4x+6y and 8y-x.

Solution:

10 y = 7x - 4

-7x + 10y = -4...(1)

12x + 18y = 1...(2)

Multiplying equation no. (1) by 12 and (2) by 7.

$$\begin{array}{r} -84x + 120y = -48 \quad \ldots(3) \\ 84x + 126y = 7 \\ \hline 246y = -41 \end{array} \quad \Rightarrow y = \frac{-1}{6}$$

From (1)

$-7x + 10\left(\frac{-1}{6}\right) = -4$

$-7x = -4 + \frac{5}{3} \Rightarrow x = \frac{1}{3}$

$\therefore 4\left(\frac{1}{3}\right) + 6\left(\frac{-1}{6}\right) = \frac{1}{3}$ and 8y - x = $8\left(\frac{-1}{6}\right) - \frac{1}{3} = \frac{-5}{3}$

13. Solve for x and y:

(i)

$$\frac{y+7}{5} = \frac{2y-x}{4} + 3x - 5$$

$$\frac{7-5x}{2} + \frac{3-4y}{6} = 5y - 18$$

(ii)

$$4x = 17 - \frac{x-y}{8}$$

$$2y + x = 2 + \frac{5y+2}{3}$$

Solution:

(i)

The given pair of linear equations are

$$\frac{y+7}{5} = \frac{2y-x}{4} + 3x - 5$$

$\Rightarrow 55x + 6y = 128 \ldots\ldots\ldots\ldots\ldots (i)$ [On simplifying]

$$\frac{7-5x}{2} + \frac{3-4y}{6} = 5y - 18$$

$\Rightarrow 15x + 34y = 132 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$ [On simplifying]

Multiply equation (i) by 3 and equation (ii) by 11, we get:

$$\begin{array}{rcl} 165x + 18y &=& 384 \\ 165x + 374y &=& 1452 \\ - \quad\quad - & & - \\ \hline -356y &=& -1068 \end{array} \quad \text{[Subtracting]}$$

$\Rightarrow y = 3$

Substituting $y = 3$ in equation (i), we get

$55x + 6(3) = 128$

$\Rightarrow 55x = 110$

$\Rightarrow x = 2$

$\therefore$ Solution is $x = 2$ and $y = 3$.

(ii)

The given pair of linear equations are

$$4x = 17 - \frac{x-y}{8}$$

$\Rightarrow 33x - y = 136 \ldots\ldots\ldots\ldots\ldots (i)$ [On simplifying]

$$2y + x = 2 + \frac{5y+2}{3}$$

$\Rightarrow 3x + y = 8 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$ [On simplifying]

Multiply equation (ii) by 11, we get:

$$\begin{array}{rcl} 33x + 11y &=& 88 \\ 33x - y &=& 136 \quad \text{[Equation (i)]} \\ - \quad\quad + & & - \\ \hline 12y &=& -48 \end{array} \quad \text{[Subtracting]}$$

$\Rightarrow y = -4$

Substituting $y = -4$ in equation (i), we get:

$33x - (-4) = 136$

$\Rightarrow 33x = 132$

$\Rightarrow x = 4$

$\therefore$ Solution is $x = 4$ and $y = -4$.

14. Find the value of m, if x=2, y=1 is a solution of the equation 2x+3y=m.

Solution:

Let x = 2 and y = 1 be a solution of the equation

$2x + 3y = m.$

$\Rightarrow 2(2) + 3(1) = m$

$\Rightarrow 4 + 3 = m$

$\Rightarrow m = 7$

$\therefore$ If x = 2 and y = 1 is the solution of the equation $2x + 3y = m$ then the value of m is 7.

15.

10% of x + 20% of y = 24

3x - y = 20

Solution:

10% of x + 20% of y = 24

$\Rightarrow 0.1x + 0.2y = 24 \ldots\ldots\ldots\ldots\ldots(i)$ [On simplyfying]

$3x - y = 20 \ldots\ldots\ldots\ldots\ldots(ii)$

Multiply equation (*ii*) by 0.2, we get:

$$\begin{array}{l} 0.6x - 0.2y = 4 \\ 0.1x + 0.2y = 24 \quad \text{[Equation (i)]} \\ \underline{+ \quad\; + \qquad\; +} \quad \text{[Adding]} \\ \quad 0.7x = 28 \end{array}$$

$\Rightarrow x = 40$

Substituting $x = 40$ in equation (i), we get

$0.1(40) + 0.2y = 24$

$\Rightarrow 0.2y = 20$

$\Rightarrow y = 100$

$\therefore$ Solution is $x = 40$ and $y = 100$.

16. The value of expression mx-ny is 3 when x=5 and y=6. And its value is 8 when x=6 and y=5. Find the values of m and n.

Solution:

The value of expression mx - ny is 3 when x = 5 and y = 6.

$\Rightarrow 5m - 6n = 3..............(i)$

The value of expression mx - ny is 8 when x = 6 and y = 5.

$\Rightarrow 6m - 5n = 8..............(ii)$

Multiply equation (i) by 6 and equation (ii) by 5, we get:

$$\begin{array}{rl} 30m - 36n = 18 & \text{[Equation (i)]} \\ 30m - 25n = 40 & \text{[Equation (ii)]} \\ - \quad + \quad\quad - & \text{[Subtracting]} \\ \hline -11n = -22 & \end{array}$$

$\Rightarrow n = 2$

Substituting $n = 2$ in equation (i), we get

$5m - 6(2) = 3$

$\Rightarrow 5m = 15$

$\Rightarrow m = 3$

$\therefore$ Solution is $m = 3$ and $n = 2$.

17. Solve:

$11(x-5)+10(y-2)+54=0$

$7(2x-1)+9(3y-1)=25$

Solution:

$11(x-5)+10(y-2)+54=0$ (given)

$\Rightarrow 11x-55+10y-20+54=0$

$\Rightarrow 11x+10y-21=0$

$\Rightarrow 11x+10y=21 \quad ....(1)$

$7(2x-1)+9(3y-1)=25$ (given)

$\Rightarrow 14x-7+27y-9=25$

$\Rightarrow 14x+27y-16=25$

$\Rightarrow 14x+27y=41 \quad ....(2)$

Multiplying equation (1) by 27 and equation (2) by 10, we get

$297x+270y=567 \quad ....(3)$

$140x+270y=410 \quad ....(4)$

Subtracting equation (4) from equation (3), we get

$157x = 157$

$\Rightarrow x = 1$

Substituting x = 1 in equation (1), we get

$11 \times 1 + 10y = 21$

$\Rightarrow 10y = 10$

$\Rightarrow y = 1$

$\therefore$ Solution set is x = 1 and y = 1.

18.

$$\frac{7+x}{5} - \frac{2x-y}{4} = 3y - 5$$

$$\frac{5y-7}{2} + \frac{4x-3}{6} = 18 - 5x$$

Solution:

$\frac{7+x}{5} - \frac{2x-y}{4} = 3y - 5 \quad \text{(given)}$

$\Rightarrow 4(7+x) - 5(2x-y) = 20(3y-5)$

$\Rightarrow 28 + 4x - 10x + 5y = 60y - 100$

$\Rightarrow -6x - 55y = -128 \quad \ldots.(1)$

$\frac{5y-7}{2} + \frac{4x-3}{6} = 18 - 5x \quad \text{(given)}$

$\Rightarrow 3(5y-7) + 4x - 3 = 6(18-5x)$

$\Rightarrow 15y - 21 + 4x - 3 = 108 - 30x$

$\Rightarrow 34x + 15y = 132 \quad \ldots.(2)$

Multiplying equation (1) by 34 and equation (2) by 6, we get

$-204x - 1870y = -4352 \quad \ldots.(3)$

$204x + 90y = 792 \quad \ldots.(4)$

Adding equations (3) and (4), we get

$-1780y = -3560$

$\Rightarrow y = 2$

Substituting y = 2 in equation (1), we get

$-6x - 55 \times 2 = -128$

$\Rightarrow -6x - 110 = -128$

$\Rightarrow -6x = -18$

$\Rightarrow x = 3$

$\therefore$ Solution is x = 3 and y = 2

19.

$$4x + \frac{x - y}{8} = 17$$
$$2y + x - \frac{5y + 2}{3} = 2$$

Solution:

$$4x + \frac{x - y}{8} = 17 \quad \text{(given)}$$
$$\Rightarrow 32x + x - y = 136$$
$$\Rightarrow 33x - y = 136 \quad ....(1)$$
$$2y + x - \frac{5y + 2}{3} = 2 \quad \text{(given)}$$
$$\Rightarrow 6y + 3x - 5y - 2 = 6$$
$$\Rightarrow 3x + y = 8 \quad ....(2)$$

Adding equations (1) and (2), we get

$$36x = 144$$
$$\Rightarrow x = 4$$

Substituting $x = 4$ in equation (2), we get

$$3 \times 4 + y = 8$$
$$\Rightarrow 12 + y = 8$$
$$\Rightarrow y = -4$$

$\therefore$ Solution is $x = 4$ and $y = -4$

EXERCISE 6(C) PAGE: 85

Solve, using cross-multiplication:

1.

$4x + 3y = 17$
$3x - 4y + 6 = 0$

Solution:

Given equations are $4x + 3y = 17$ and $3x - 4y + 6 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = 3, c_1 = -17$ and $a_2 = 3, b_2 = -4, c_2 = 6$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{3 \times 6 - (-4) \times (-17)}{4 \times (-4) - 3 \times 3}$ and $y = \dfrac{-17 \times 3 - 6 \times 4}{4 \times (-4) - 3 \times 3}$

$\Rightarrow x = \dfrac{18 - 68}{-16 - 9}$ and $y = \dfrac{-51 - 24}{-16 - 9}$

$\Rightarrow x = \dfrac{-50}{-25}$ and $y = \dfrac{-75}{-25}$

$\Rightarrow x = 2$ and $y = 3$

2.

$3x + 4y = 11$
$2x + 3y = 8$

Solution:

Given equations are $3x + 4y = 11$ and $2x + 3y = 8$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3, b_1 = 4, c_1 = -11$ and $a_2 = 2, b_2 = 3, c_2 = -8$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{4 \times (-8) - 3 \times (-11)}{3 \times 3 - 2 \times 4}$ and $y = \dfrac{-11 \times 2 - (-8) \times 3}{3 \times 3 - 2 \times 4}$

$\Rightarrow x = \dfrac{-32 + 33}{9 - 8}$ and $y = \dfrac{-22 + 24}{9 - 8}$

$\Rightarrow x = 1$ and $y = 2$

3.

$6x + 7y - 11 = 0$
$5x + 2y = 13$

Solution:

Given equations are $6x + 7y - 11 = 0$ and $5x + 2y = 13$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 6$, $b_1 = 7$, $c_1 = -11$ and $a_2 = 5$, $b_2 = 2$, $c_2 = -13$

$$\text{Now, } x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{7 \times (-13) - 2 \times (-11)}{6 \times 2 - 5 \times 7} \text{ and } y = \frac{-11 \times 5 - (-13) \times 6}{6 \times 2 - 5 \times 7}$$

$$\Rightarrow x = \frac{-91 + 22}{12 - 35} \text{ and } y = \frac{-55 + 78}{12 - 35}$$

$$\Rightarrow x = \frac{-69}{-23} \text{ and } y = \frac{23}{-23}$$

$$\Rightarrow x = 3 \text{ and } y = -1$$

4.

$5x + 4y + 14 = 0$

$3x = -10 - 4y$

Solution:

Given equations are $5x + 4y + 14 = 0$ and $3x = -10 - 4y$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 5$, $b_1 = 4$, $c_1 = 14$ and $a_2 = 3$, $b_2 = 4$, $c_2 = 10$

$$\text{Now, } x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{4 \times 10 - 4 \times 14}{5 \times 4 - 3 \times 4} \text{ and } y = \frac{14 \times 3 - 10 \times 5}{5 \times 4 - 3 \times 4}$$

$$\Rightarrow x = \frac{40 - 56}{20 - 12} \text{ and } y = \frac{42 - 50}{20 - 12}$$

$$\Rightarrow x = \frac{-16}{8} \text{ and } y = \frac{-8}{8}$$

$$\Rightarrow x = -2 \text{ and } y = -1$$

5.

$x - y + 2 = 0$

$7x + 9y = 130$

Solution:

Given equations are $x - y + 2 = 0$ and $7x + 9y = 130$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1, b_1 = -1, c_1 = 2$ and $a_2 = 7, b_2 = 9, c_2 = -130$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{-1 \times (-130) - 9 \times 2}{1 \times 9 - 7 \times (-1)}$ and $y = \dfrac{2 \times 7 - (-130) \times 1}{1 \times 9 - 7 \times (-1)}$

$\Rightarrow x = \dfrac{130 - 18}{9 + 7}$ and $y = \dfrac{14 + 130}{9 + 7}$

$\Rightarrow x = \dfrac{112}{16}$ and $y = \dfrac{144}{16}$

$\Rightarrow x = 7$ and $y = 9$

6.

$4x - y = 5$

$5y - 4x = 7$

Solution:

Given equations are $4x - y = 5$ and $5y - 4x = 7$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = -1, c_1 = -5$ and $a_2 = -4, b_2 = 5, c_2 = -7$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{-1 \times (-7) - 5 \times (-5)}{4 \times 5 - (-4) \times (-1)}$ and $y = \dfrac{(-5) \times (-4) - (-7) \times 4}{4 \times 5 - (-4) \times (-1)}$

$\Rightarrow x = \dfrac{7 + 25}{20 - 4}$ and $y = \dfrac{20 + 28}{20 - 4}$

$\Rightarrow x = \dfrac{32}{16}$ and $y = \dfrac{48}{16}$

$\Rightarrow x = 2$ and $y = 3$

7.

$4x - 3y = 0$

$2x + 3y = 18$

Solution:

Given equations are $4x - 3y = 0$ and $2x + 3y = 18$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = -3, c_1 = 0$ and $a_2 = 2, b_2 = 3, c_2 = -18$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{-3 \times (-18) - 3 \times 0}{4 \times 3 - 2 \times (-3)}$ and $y = \dfrac{0 \times 2 - (-18) \times 4}{4 \times 3 - 2 \times (-3)}$

$\Rightarrow x = \dfrac{54 - 0}{12 + 6}$ and $y = \dfrac{0 + 72}{12 + 6}$

$\Rightarrow x = \dfrac{54}{18}$ and $y = \dfrac{72}{18}$

$\Rightarrow x = 3$ and $y = 4$

8.

$8x + 5y = 9$

$3x + 2y = 4$

Solution:

Given equations are $8x + 5y = 9$ and $3x + 2y = 4$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 8, b_1 = 5, c_1 = -9$ and $a_2 = 3, b_2 = 2, c_2 = -4$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{5 \times (-4) - 2 \times (-9)}{8 \times 2 - 3 \times 5}$ and $y = \dfrac{-9 \times 3 - (-4) \times 8}{8 \times 2 - 3 \times 5}$

$\Rightarrow x = \dfrac{-20 + 18}{16 - 15}$ and $y = \dfrac{-27 + 32}{16 - 15}$

$\Rightarrow x = \dfrac{-2}{1}$ and $y = \dfrac{5}{1}$

$\Rightarrow x = -2$ and $y = 5$

9.

$4x - 3y - 11 = 0$

$6x + 7y - 5 = 0$

Solution:

Given equations are $4x - 3y - 11 = 0$ and $6x + 7y - 5 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = -3, c_1 = -11$ and $a_2 = 6, b_2 = 7, c_2 = -5$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{-3 \times (-5) - 7 \times (-11)}{4 \times 7 - 6 \times (-3)}$ and $y = \dfrac{-11 \times 6 - (-5) \times 4}{4 \times 7 - 6 \times (-3)}$

$\Rightarrow x = \dfrac{15 + 77}{28 + 18}$ and $y = \dfrac{-66 + 20}{28 + 18}$

$\Rightarrow x = \dfrac{92}{46}$ and $y = \dfrac{-46}{46}$

$\Rightarrow x = 2$ and $y = -1$

10.

$4x + 6y = 15$

$3x - 4y = 7$

Solution:

Given equations are $4x + 6y = 15$ and $3x - 4y = 7$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = 6, c_1 = -15$ and $a_2 = 3, b_2 = -4, c_2 = -7$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{6 \times (-7) - (-4) \times (-15)}{4 \times (-4) - 3 \times 6}$ and $y = \dfrac{-15 \times 3 - (-7) \times 4}{4 \times (-4) - 3 \times 6}$

$\Rightarrow x = \dfrac{-42 - 60}{-16 - 18}$ and $y = \dfrac{-45 + 28}{-16 - 18}$

$\Rightarrow x = \dfrac{-102}{-34}$ and $y = \dfrac{-17}{-34}$

$\Rightarrow x = 3$ and $y = \dfrac{1}{2}$

11.

$0.4x - 1.5y = 6.5$

$0.3x + 0.2y = 0.9$

Solution:

Given equations are $0.4x - 1.5y = 6.5$ and $0.3x + 0.2y = 0.9$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 0.4, b_1 = -1.5, c_1 = -6.5$ and $a_2 = 0.3, b_2 = 0.2, c_2 = -0.9$

$$\text{Now, } x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$
$$\Rightarrow x = \frac{(-1.5)\times(-0.9) - (0.2)\times(-6.5)}{0.4\times(0.2) - (0.3)\times(-1.5)} \text{ and } y = \frac{(-6.5)\times(0.3) - (-0.9)\times(0.4)}{0.4\times(0.2) - (0.3)\times(-1.5)}$$
$$\Rightarrow x = \frac{1.35 + 1.3}{0.08 + 0.45} \text{ and } y = \frac{-1.95 + 0.36}{0.08 + 0.45}$$
$$\Rightarrow x = \frac{2.65}{0.53} \text{ and } y = \frac{-1.59}{0.53}$$
$$\Rightarrow x = 5 \text{ and } y = -3$$

12.
$\sqrt{2}x - \sqrt{3}y = 0$
$\sqrt{5}x + \sqrt{2}y = 0$

Solution:

Given equations are $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{5}x + \sqrt{2}y = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = \sqrt{2}, b_1 = -\sqrt{3}, c_1 = 0$ and $a_2 = \sqrt{5}, b_2 = \sqrt{2}, c_2 = 0$

$$\text{Now, } x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$
$$\Rightarrow x = \frac{\left(-\sqrt{3}\right)\times 0 - \sqrt{2}\times 0}{\sqrt{2}\times\sqrt{2} - \sqrt{5}\times\left(-\sqrt{3}\right)} \text{ and } y = \frac{0\times\sqrt{5} - 0\times\sqrt{2}}{\sqrt{2}\times\sqrt{2} - \sqrt{5}\times\left(-\sqrt{3}\right)}$$
$$\Rightarrow x = \frac{0}{2 + \sqrt{15}} \text{ and } y = \frac{0}{2 + \sqrt{15}}$$
$$\Rightarrow x = 0 \text{ and } y = 0$$

EXERCISE 6(D)

Solve:

1.

$$\frac{9}{x}-\frac{4}{y}=8$$

$$\frac{13}{x}+\frac{7}{y}=101$$

Solution:

$$\frac{9}{x}-\frac{4}{y}=8 \quad \ldots(1)$$

$$\frac{13}{x}+\frac{7}{y}=101 \quad \ldots(2)$$

Multiplying equation no. (1) by 7 and (2) by 4.

$$\frac{63}{x}-\frac{28}{y}=56 \quad \ldots(3)$$

$$\frac{52}{x}+\frac{28}{y}=404 \quad \ldots(4)$$

$$\frac{115}{x}=460$$

$$x=\frac{115}{460} \Rightarrow x=\frac{1}{4}$$

From (1)

$$9\times\left(\frac{4}{1}\right)-\frac{4}{y}=8$$

$$-\frac{4}{y}=-28 \Rightarrow y=\frac{1}{7}$$

2.

$$\frac{3}{x}+\frac{2}{y}=10$$

$$\frac{9}{x}-\frac{7}{y}=10.5$$

Solution:

$$\frac{3}{x}+\frac{2}{y}=10 \quad \ldots.(i)$$

$$\frac{9}{x}-\frac{7}{y}=10.5 \quad \ldots.(ii)$$

Multiplying equation (i) by 3, we get

$$\frac{9}{x}+\frac{6}{y}=30 \quad \ldots.(iii)$$

Subtracting (ii) from (iii), we get

$$\frac{13}{y} = 19.5$$

$$\Rightarrow y = \frac{13}{19.5} = \frac{2}{3}$$

From (i),

$$\frac{3}{x} + \frac{2 \times 3}{2} = 10$$

$$\Rightarrow \frac{3}{x} + 3 = 10$$

$$\Rightarrow \frac{3}{x} = 7$$

$$\Rightarrow x = \frac{3}{7}$$

3.

$$5x + \frac{8}{y} = 19$$

$$3x - \frac{4}{y} = 7$$

Solution:

$$5x + \frac{8}{y} = 19 \qquad ....(i)$$

$$3x - \frac{4}{y} = 7 \qquad ....(ii)$$

Multiplying equation (ii) by 2, we get

$$6x - \frac{8}{y} = 14 \qquad ....(iii)$$

Adding (i) and (iii), we get

$$11x = 33$$

$$\Rightarrow x = 3$$

Substituting $x = 3$ in equation 91), we get

$$5(3) + \frac{8}{y} = 19$$

$$\Rightarrow \frac{8}{y} = 19 - 15$$

$$\Rightarrow y = \frac{8}{4} = 2$$

4. Solve:

$4x + \frac{6}{y} = 15$

$3x - \frac{4}{y} = 7$

Hence, find 'a' if y=ax-2.

Solution:

$4x + \frac{6}{y} = 15 \quad ....(i)$

$3x - \frac{4}{y} = 7 \quad ....(ii)$

Multiplying (i) by 4 and (ii) by 6

$16x + \frac{24}{y} = 60 \quad ....(iii)$

$18x - \frac{24}{y} = 42 \quad ....(iv)$

Adding (iii) and (iv), we get

$34x = 102$

$\Rightarrow x = 3$

Substituting $x = 3$ in (i), we get

$4(3) + \frac{6}{y} = 15$

$\Rightarrow \frac{6}{y} = 15 - 12$

$\Rightarrow y = \frac{6}{3} = 2$

Now, $y = ax - 2$

$\Rightarrow 2 = a(3) - 2$

$\Rightarrow 2 = 3a - 2$

$\Rightarrow 3a = 4$

$\Rightarrow a = \frac{4}{3} = 1\frac{1}{3}$

5. Solve:

$\frac{3}{x} - \frac{2}{y} = 0$

$\frac{2}{x} + \frac{5}{y} = 19$

Hence, find 'a' if y=ax+3

Solution:

$$\frac{3}{x}-\frac{2}{y}=0 \qquad \ldots(1)$$

$$\frac{2}{x}+\frac{5}{y}=19 \qquad \ldots(2)$$

Multiplying equation no. (1) by 5 and (2) by 2.

$$\frac{15}{x}-\frac{10}{y}=0 \qquad \ldots(3)$$

$$\frac{4}{x}+\frac{10}{y}=38 \qquad \ldots(4)$$

$$\frac{19}{x}=38 \qquad \Rightarrow x=\frac{1}{2}$$

From (1) $3\left(\frac{1}{2}\right)-\frac{2}{y}=0 \quad \Rightarrow y=\frac{1}{3}$

$\therefore y = ax + 3$

$$\frac{1}{3}=a\left(\frac{1}{2}\right)+3$$

$$\frac{a}{2}=\frac{-8}{3}\Rightarrow a=\frac{-16}{3}$$

6. Solve:

(i)

$$\frac{20}{x+y}+\frac{3}{x-y}=7$$

$$\frac{8}{x+y}-\frac{15}{x+y}=5$$

(ii)

$$\frac{34}{3x+4y}+\frac{15}{3x-2y}=5$$

$$\frac{25}{3x-2y}-\frac{8.50}{3x+4y}=4.5$$

Solution:

(i)

$$\frac{20}{x+y}+\frac{3}{x-y}=7 \qquad \ldots(1)$$

$$\frac{8}{x+y}-\frac{15}{x+y}=5 \qquad \ldots(2)$$

Multiplying equation no. (1) by 8 and (2) by 3.

$$\frac{160}{x+y}+\frac{24}{x-y}=56 \quad ...(3)$$

$$\frac{-45}{x+y}+\frac{24}{x-y}=15 \quad ...(4)$$

$$- \quad - \quad\quad -$$

$$\frac{205}{x+y}=41$$

$x+y=5 \quad ...(5)$

From (1)

$$\frac{20}{5}+\frac{3}{x-y}=7$$

$$\frac{3}{x-y}=3$$

$x-y=1 \quad ...(6)$

$x+y=5 \quad ...(5)$

$\underline{x-y=1} \quad ...(6)$

$2x=6$

$x=3$

from (5)

$3+y=5 \Rightarrow y=2$

(ii)

Let $a=3x+4y$ and $b=3x-2y$

$$\therefore \frac{34}{3x+4y}+\frac{15}{3x-2y}=5$$

$$\Rightarrow \frac{34}{a}+\frac{15}{b}=5..........(i)$$

$$\frac{25}{3x-2y}-\frac{8.50}{3x+4y}=4.5$$

$$\Rightarrow -\frac{8.50}{a}+\frac{25}{b}=4.5..........(ii)$$

Multiply equation (ii)by 4, we get :

$$-\frac{34}{a}+\frac{100}{b}=18$$

$$\frac{34}{a}+\frac{15}{b}=5 \qquad \text{[Equation (i)]}$$

$$+ \quad + \qquad + \qquad \text{[Adding]}$$

$$\frac{115}{b}=23$$

$$\Rightarrow b=5$$

$$\Rightarrow 3x-2y=5 \text{..........(iii)}$$

Substituting $b = 5$ in equation (i), we get

$$\frac{34}{a}+\frac{15}{5}=5$$

$$\Rightarrow 2a=34$$

$$\Rightarrow a=17$$

$$\Rightarrow 3x+4y=17 \text{..........(iv)}$$

Subtracting equation (iv)from equation (iii), we get::

$$3x-2y=5$$

$$3x+4y=17$$

$$- \quad - \qquad -$$

$$-6y=-12$$

$$\Rightarrow y=2$$

SubStituting $y = 2$ in equation (iii), we get

$$3x-2(2)=5$$

$$\Rightarrow 3x=9$$

$$\Rightarrow x=3$$

$\therefore$ Solution is $x = 3$ and $y = 2$.

7.

(i)

$$x+y=2xy$$

$$x-y=6xy$$

(ii)

x + y = 7xy
2x - 3 = -xy

Solution:

(i)

$$x + y = 2xy \qquad \ldots(1)$$
$$\underline{x - y = 6xy} \qquad \ldots(2)$$
$$2x = 8xy$$
$$2 = 8y$$
$$y = \frac{1}{4}$$

From (1)

$$x + \frac{1}{4} = 2x\left(\frac{1}{4}\right)$$
$$\frac{1}{2}x = \frac{-1}{4}$$
$$x = \frac{-1}{2}$$

(ii)

x + y = 7xy...(1)

2x - 3 = -xy...(2)

Multiplying equation no. (1) by 3.

$$3x + 3y = 21xy \qquad \ldots(3)$$
$$\underline{2x - 3y = -xy} \qquad \ldots(4)$$
$$5x = 20xy$$
$$y = \frac{1}{4}$$

From (1)

$$x + \frac{1}{4} = 7x\left(\frac{1}{4}\right)$$
$$\frac{1}{4} = \frac{3}{4}x$$
$$x = \frac{1}{3}$$

8. Solve:

$$\frac{a}{x} - \frac{b}{y} = 0$$
$$\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$$

Solution:

Given equations are $\frac{a}{x} - \frac{b}{y} = 0$ and $\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the above system of equations become

$au - bv + 0 = 0$

$ab^2u + a^2bv - (a^2 + b^2) = 0$

By cross-multiplication, we have

$$\frac{u}{-b \times [-(a^2+b^2)] - a^2b \times 0} = \frac{-v}{a \times [-(a^2+b^2)] - ab^2 \times 0} = \frac{1}{a \times a^2b - ab^2 \times (-b)}$$

$$\Rightarrow \frac{u}{b(a^2+b^2)} = \frac{-v}{-a(a^2+b^2)} = \frac{1}{a^3b + ab^3}$$

$$\Rightarrow \frac{u}{b(a^2+b^2)} = \frac{v}{a(a^2+b^2)} = \frac{1}{ab(a^2+b^2)}$$

$$\Rightarrow u = \frac{b(a^2+b^2)}{ab(a^2+b^2)} \quad \text{and} \quad v = \frac{a(a^2+b^2)}{ab(a^2+b^2)}$$

$$\Rightarrow u = \frac{1}{a} \text{ and } v = \frac{1}{b}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{a} \text{ and } \frac{1}{y} = \frac{1}{b}$$

$$\Rightarrow x = a \text{ and } y = b$$

9.

$$\frac{2xy}{x+y} = \frac{3}{2}$$

$$\frac{xy}{2x-y} = -\frac{3}{10}$$

Solution:

$$\frac{2xy}{x+y} = \frac{3}{2}$$

$$\Rightarrow \frac{x+y}{xy} = \frac{4}{3}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{4}{3} \quad \ldots.(1)$$

$$\frac{xy}{2x-y} = -\frac{3}{10}$$

$$\Rightarrow \frac{2x-y}{xy} = -\frac{10}{3}$$

$$\Rightarrow -\frac{1}{x} + \frac{2}{y} = -\frac{10}{3} \quad \ldots.(2)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Then, equations(1) and (2) become

$u + v = \frac{4}{3}$ and $-u + 2v = -\frac{10}{3}$

$\Rightarrow 3u + 3v = 4$ and $-3u + 6v = -10$

Adding, we have

$9v = -6$

$\Rightarrow v = -\frac{6}{9} = -\frac{2}{3}$

$\Rightarrow \frac{1}{y} = -\frac{2}{3} \Rightarrow y = -\frac{3}{2}$

Substituting $y = -\frac{3}{2}$ in (1), we have

$\frac{1}{x} - \frac{2}{3} = \frac{4}{3}$

$\Rightarrow \frac{1}{x} = \frac{6}{3} = 2$

$\Rightarrow x = \frac{1}{2}$

Hence, $x = \frac{1}{2}$ and $y = -\frac{3}{2}$

10.

$\frac{3}{2x} + \frac{2}{3y} = -\frac{1}{3}$

$\frac{3}{4x} + \frac{1}{2y} = -\frac{1}{8}$

Solution:

Given equations are $\frac{3}{2x} + \frac{2}{3y} = -\frac{1}{3}$ and $\frac{3}{4x} + \frac{1}{2y} = -\frac{1}{8}$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Then, the system of equations become

$\frac{3}{2}u + \frac{2}{3}v = -\frac{1}{3}$ and $\frac{3}{4}u + \frac{1}{2}v = -\frac{1}{8}$

$\Rightarrow \frac{9u + 4v}{6} = -\frac{1}{3}$ and $\frac{3u + 2v}{4} = -\frac{1}{8}$

$\Rightarrow 27u + 12v = -6$ and $24u + 16v = -4$

$\Rightarrow 27u + 12v + 6 = 0$ and $24u + 16v + 4 = 0$

$$\Rightarrow \frac{u}{12 \times 4 - 16 \times 6} = \frac{-v}{27 \times 4 - 24 \times 6} = \frac{1}{27 \times 16 - 24 \times 12}$$

$$\Rightarrow \frac{u}{48 - 96} = \frac{-v}{108 - 144} = \frac{1}{432 - 288}$$

$$\Rightarrow \frac{u}{-48} = \frac{-v}{-36} = \frac{1}{144}$$

$$\Rightarrow \frac{u}{-48} = \frac{v}{36} = \frac{1}{144}$$

$$\Rightarrow u = \frac{-48}{144} = -\frac{1}{3} \text{ and } v = \frac{36}{144} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{x} = -\frac{1}{3} \text{ and } \frac{1}{y} = \frac{1}{4}$$

$$\Rightarrow x = -3 \text{ and } y = 4$$

EXERCISE 6(E)

1. **The ratio of two number is 2/3. If 2 is subtracted from the first and 8 from the second, the ratio becomes the reciprocal of the original ratio. Find the numbers.**

Solution:

Let the two numbers be x and y
According to the question,

$$\frac{x}{y} = \frac{2}{3}$$

3x - 2y = 0 ...(1)

Also, $\frac{x-2}{y-8} = \frac{3}{2}$

2x - 3y = -20 ...(2)

Multiplying equation no. (1) by 2 and (2) by 3and subtracting

$$\begin{array}{l} 6x - 4y = 0 \\ 6x - 9y = -60 \\ \underline{- \quad + \qquad +} \\ \quad 5y = 60 \end{array}$$

$$y = 12$$

From (1), we get
3x - 2(12) = 0

$$x = \frac{24}{3}$$

x = 8
Thus, the numbers are 8 and 12.

2. **Two numbers are in the ratio 4:7. If thrice the larger be added to twice the smaller, the sum is 59. Find the numbers.**

Solution:

Let the smaller number be x
and the larger number bey.
According to the question,

$$\frac{x}{y} = \frac{4}{7}$$

7x -; 4y = 0...(1)
and,3y + 2x = 59...(2)
Multiplying equation no. (1) by 3 and (2) by 4.and adding them

$$\begin{array}{ll} 21x - 12y = 0 & \ldots(3) \\ \underline{8x + 12y = 236} & \ldots(4) \\ \quad 29x = 236 & \end{array}$$

$x = \frac{236}{29}$

From (1)

$7\left(\frac{236}{29}\right) = 4y$

$y = 7\left(\frac{59}{29}\right)$

$y = \frac{413}{29}$

Hence, the number are $\frac{236}{29}$ and $\frac{413}{29}$.

3. **When the greater of the two numbers increased by 1 divides the sum of the numbers, the result is 3/2. When the difference of these numbers is divided by the smaller, the result is ½. Find the numbers.**

Solution:

Let the two numbers be a and b respectively such that b > a.

According to given condition,

$\frac{a+b}{b+1} = \frac{3}{2}$

$\Rightarrow 2a + 2b = 3b + 3$

$\Rightarrow 2a - b = 3 \quad ....(i)$

Also,

$\frac{b-a}{a} = \frac{1}{2}$

$\Rightarrow 2b - 2a = a$

$\Rightarrow 2b - 3a = 0 \quad ....(ii)$

Multiplying (i) by 2, we get

$4a - 2b = 6 \quad ....(iii)$

Adding (ii) and (ii), we get

$a = 6$

Substituting $a = 6$ in (i), we get

$2(6) - b = 3$

$\Rightarrow 12 - b = 3$

$\Rightarrow b = 9$

Thus, the two numbers are 6 and 9 respectively.

4. The sum of two positive numbers x and y (x>y) is 50 and the difference of their squares is 720. Find the numbers.

Solution:

Two numbers are x and y such that x > y.
Now,
$x + y = 50$ ….(i)
And,
$y^2 - x^2 = 720$
$\Rightarrow (y - x)(y + x) = 720$
$\Rightarrow (y - x)(50) = 720$
$\Rightarrow y - x = 14.4$ ….(ii)
Adding (i) and (ii), we get
$2y = 64.4$
$\Rightarrow y = 32.2$
Substituting the value of y in (i), we have
$x + 32.2 = 50$
$\Rightarrow x = 17.8$
Thus, the two numbers are 17.8 and 32.2 respectively.

5. The sum of two numbers is 8 and the sum of their reciprocals is 8/15. Find the numbers.

Solution:

Let the two numbers be x and y respectively.
Then,
$x + y = 8$ ….(i)
$\Rightarrow x = 8 - y$
And,
$$\frac{1}{x} + \frac{1}{y} = \frac{8}{15}$$
$$\Rightarrow \frac{y + x}{xy} = \frac{8}{15}$$
$$\Rightarrow \frac{8}{xy} = \frac{8}{15} \quad \text{….[From (i)]}$$
$$\Rightarrow xy = 15$$
$$\Rightarrow (8 - y)y = 15$$
$$\Rightarrow 8y - y^2 = 15$$
$$\Rightarrow y^2 - 8y + 15 = 0$$
$$\Rightarrow y^2 - 3y - 5y + 15 = 0$$
$$\Rightarrow y(y - 3) - 5(y - 3) = 0$$
$$\Rightarrow (y - 3)(y - 5) = 0$$
$$\Rightarrow y = 3 \text{ or } y = 5$$
$$\Rightarrow x = 5 \text{ or } x = 3$$
Thus, the two numbers are 3 and 5 respectively.

6. The difference between two positive numbers x and y (x>y) is 4 and the difference between their reciprocals id 4/21. Find the numbers.

Solution:

Two numbers are x and y respectively such that x > y.
Then,
$x - y = 4$ ….(i)
$\Rightarrow x = 4 + y$
And,
$\frac{1}{y} - \frac{1}{x} = \frac{4}{21}$
$\Rightarrow \frac{x - y}{xy} = \frac{4}{21}$
$\Rightarrow \frac{4}{xy} = \frac{4}{21}$ ....[From (i)]
$\Rightarrow xy = 21$
$\Rightarrow (4 + y)y = 21$
$\Rightarrow 4y + y^2 = 21$
$\Rightarrow y^2 + 4y - 21 = 0$
$\Rightarrow y^2 + 7y - 3y - 21 = 0$
$\Rightarrow y(y + 7) - 3(y + 7) = 0$
$\Rightarrow (y - 3)(y + 7) = 0$
$\Rightarrow y = 3$ or $y = -7$
We reject $y = -7$ since y is positive.
$\Rightarrow y = 3$
$\Rightarrow x = 4 + y = 4 + 3 = 7$
Thus, the two numbers are 7 and 3 respectively.

7. Two numbers are in the ratio 4:5. If 30 is subtracted from each of the number, the ratio becomes 1:2. Find the numbers.

Solution:

Let the common multiple between the numbers be x.
So, the numbers are 4x and 5x.
According to the question,
$\frac{4x - 30}{5x - 30} = \frac{1}{2}$
$\Rightarrow 8x - 60 = 5x - 30$
$\Rightarrow 3x = 30$
$\Rightarrow x = 10$
So, $4x = 4(10) = 40$ and $5x = 5(10) = 50$
Thus, the numbers are 40 and 50.

8. If the numerator of a fraction is increased by 2 and denominator is decreased by 1, it becomes 2/3.

If the numerator is increased by 1 and denominator is increased by 2, it becomes 1/3. Find the fraction.

Solution:

Let the numerator and denominator a fraction be x and y respectively .

According to the question,

$$\frac{x+2}{y-1}=\frac{2}{3}$$

3x - 2y = -8...(1)

And,

$$\frac{x+1}{y+2}=\frac{1}{3}$$

$$3x-y=-1 \qquad \ldots(2)$$

Now subtracting,

$$\begin{array}{rl} 3x-y=-1 & \ldots(2) \\ 3x-2y=-8 & \ldots(1) \\ - \quad + \qquad + & \\ \hline y=7 & \end{array}$$

From (1) ,

3x - 2 (7) = -8

3x = -8 + 14

x = 2

Required fraction = $\frac{2}{7}$

9. The sum of the numerator and the denominator of a fraction is equal to 7. Four times the numerator is 8 less than 5 times the denominator. Find the fraction.

Solution:

Let the numerator and denominator of a fraction be x and y respectively .Then the fraction will be $\frac{x}{y}$

According to the question,

x + y = 7...(1)

5y - 4x = 8...(2)

Multiplying equation no. (1) by 4 and add with (2),

$$\begin{array}{rl} 4x+4y=28 & ..(3) \\ -4x+5y=8 & \\ \hline 9y=36 & \end{array}$$

$$y=4$$

From (1)

x + 4 = 7

x = 3

Required fraction = $\frac{3}{4}$

10. If the numerator of a fraction is multiplied by 2 and its denominator is increased by1, it becomes 1. However, if the numerator is increased by 4 and denominator is multiplied by 2, the fraction becomes ½ . Find the fraction.

Solution:

Let the numerator of the fraction be x and the denominator be y.

So, the fraction is $\frac{x}{y}$.

According to the question,

$\frac{2x}{y+1} = 1 \Rightarrow 2x = y + 1 \Rightarrow 2x - y = 1 \ldots(i)$

and $\frac{x+4}{2y} = \frac{1}{2} \Rightarrow 2x + 8 = 2y \Rightarrow 2x - 2y = -8 \ldots(ii)$

Solving equations (i) and (ii), we get

$y = 9$

Putting the value of y in (i), we get

$2x - (9) = 1 \Rightarrow 2x = 1 + 9 \Rightarrow x = 5$

So, the fraction is $\frac{5}{9}$.

11. A fraction becomes ½ if 5 is subtracted from its numerator and 3 is subtracted from its denominator. If the denominator of this fraction is 5 more than its numerator, find the fraction.

Solution:

Let the numerator of the fraction be x and denominator of the fraction be y.

Then, the fraction = $\frac{x}{y}$

According to given condition, we have

$\frac{x-5}{y-3} = \frac{1}{2}$

$\Rightarrow 2x - 10 = y - 3$

$\Rightarrow 2x - y = 7 \quad \ldots.(i)$

And,

$x + 5 = y$

$\Rightarrow x - y = -5 \quad \ldots.(ii)$

Subtracting (ii) from (i), we get

$x = 12$

$\Rightarrow y = x + 5 = 12 + 5 = 17$

hence, the fraction is $\frac{12}{17}$.

12. The sum of the digits of a two digit number is 5. If the digits are reversed, the number is reduced by 27. Find the number.

Solution:

Let the digit at unit's place be x and the digit at ten's place y.

Required no. = 10y + x

If the digit's are reversed,

Reversed no. = 10y + x

According to the question,

x + y = 5...(1)

and,

(10y + x) - (10x + y) = 27

$$9y - 9x = 27$$

$$y - x = 3 \quad \ldots(2)$$

Now adding the two equation,

$$\begin{array}{r} y - x = 3 \quad \ldots(2) \\ \underline{y + x = 5} \quad \ldots(1) \\ 2y = 8 \end{array}$$

$$y = 4$$

From (1)

x + 4 = 5

x = 1

Require no is

10 (4) + 1 = 41

13. The sum of the digits of a two digit number is 7. If the digits are reversed, the new number decreased by 2, equals twice the original number. Find the number.

Solution:

Let the digit at unit's place be x and the digit at ten's place be y.

Required no. = 10y + x

If the digits are reversed

Reversed no. = 10x + y

According to the question,

x + y = 7... (1)

And,

10x + y - 2 = 2(10y + x).

8x - 19y = 2... (2)

Multiplying equation no. (1) by 19.

$$19x + 19y = 133 \quad \ldots(3)$$

Now adding equation (2) and (3)

$$\begin{array}{r} 19x + 19y = 133 \quad \ldots(3) \\ \underline{8x - 19y = 2} \quad \ldots(2) \\ 27x = 135 \end{array}$$

x = 5

From (1)
5 +y = 7
y = 2
Required number is
10(2) + 5
= 25.

14. The ten's digit of a two digit number is three times the unit digit. The sum of the number and the unit digit is 32. Find the number.

Solution:

Let the digit at unit's place be x and the digit at ten's place be y.
Required no. = 10y + x
According to the question
y = 3x $\Rightarrow$ 3x - y = 0... (1)
And, 10y + x + x = 32
10y + 2x = 32... (2)
Multiplying equation no. (1) by 10

$$30x - 10y = 0 \quad \ldots(3)$$

Now adding (3) and (2)

$$\begin{array}{rl} 30x - 10y = 0 & \ldots(3) \\ \underline{2x + 10y = 32} & \ldots(2) \\ 32x = 32 & \end{array}$$

$x = 1$
From (1), we get
y = 3(1) = 3
Required no is
10(3) + 1 = 31

15. A two-digit number is such that the ten's digit exceeds twice the unit's digit exceeds twice the unit's digit by 2 and the number obtained by inter-changing the digits is 5 more than three times the sum of the digits. Find the two digit number.

Solution:

Let the digit a unit's place be x and the digit at ten's place be y.
Required no. = 10y + x.
According to the question,
y - 2x = 2
-2x + y = 2... (1)
And,
(10x + y) -3 (y + x) = 5
7x - 2y = 5... (2)
Multiplying equation no. (1) by 2.

$$-4x + 2y = 4 \quad \ldots(3)$$

Now adding (2) and (3)

$$\begin{array}{r} -4x + 2y = 4 \\ 7x - 2y = 5 \\ \hline 3x = 9 \end{array}$$

$x = 3$

From (1), we get

-2(3) + y = 2

⇒ y = 8

Required number is

10(8) + 3 = 83.

16. Four times a certain two digit number is seven times the number obtained on interchanging its digit. If the difference between the digits is 4; find the number.

Solution:

Let x be the number at the ten's place
and y be the number at the unit's place.
So, the number is 10x + y.

Four times a certain two-digit number is seven times the number obtained on interchanging its digits.

$\Rightarrow 4(10x + y) = 7(10y + x)$

$\Rightarrow 40x + 4y = 70y + 7x$

$\Rightarrow 33x - 66y = 0$

$\Rightarrow x - 2y = 0 \ldots\ldots\ldots\ldots\ldots(i)$

If the difference between the digits is 4, then

$\Rightarrow x - y = 4 \ldots\ldots\ldots\ldots\ldots(ii)$

Subtracting equation (i) from equation (ii), we get:

$$\begin{array}{rl} x - y = 4 & \\ x - 2y = 0 & \text{[Equation } (i)\text{]} \\ - \quad + & \text{[Subtracting]} \\ \hline y = 4 & \end{array}$$

Substituting $y = 4$ in equation (i), we get

$x - 2(4) = 0$

$\Rightarrow x = 8$

$\therefore$ The number is $10x + y = 10(8) + 4 = 84$.

17. The sum of a two digit number and the number obtained by interchanging the digits of the number is 121. If the digits of the number differ by 3, find the number.

Solution:

Let the tens digit of the number be x and the units digit be y.

So, the number is $10x + y$.

The number obtained by interchanging the digits will be $10y + x$.

According to question, we have

$10x + y + 10y + x = 121$

$\Rightarrow 11x + 11y = 121$

$\Rightarrow 11(x + y) = 121$

$\Rightarrow x + y = 11$ ....(i)

And,

$x - y = 3$ ....(ii)

Adding (i) and (ii), we get

$2x = 14$

$\Rightarrow x = 7$

$\Rightarrow y = 11 - x = 11 - 7 = 4$

Hence, the number is 74.

18. A two digit number is obtained by multiplying the sum of the digits by 8. Also, it is obtained by multiplying the difference of the digits by 14 and adding 2. Find the number.

Solution:

Let the tens digit of the number be x and the units digit be y.

So, the number is $10x + y$.

According to the question,

$10x + y = 8(x + y) \Rightarrow 2x = 7y$...(i)

and $10x + y = 14(x - y) + 2$ or $10x + y = 14(y - x) + 2$

$\Rightarrow 4x - 15y = -2$...(ii) or $24x - 13y = 2$...(iii)

Solving (i) and (ii), we get

$y = 2$ and $x = 7$

Solving (i) and (iii), we get

$y = \frac{2}{71}$

This is not possible, since y is a digit and cannot be in fraction form.

So the number is 72.

EXERCISE 6(A) PAGE: 79

1. Five years ago, A's age was four times the age of B. Five years hence, A's age will be twice the age of B. Find their present ages.

Solution:

Let present age of A = x years

And present age of B = y years

According to the question,

Five years ago,

$x - 5 = 4(y - 5)$

$x - 4y = -15...(1)$

Five years later,

$x + 5 = 2(y + 5)$

$x - 2y = 5 \quad ...(2)$

Now subtracting (1)from(2)

$$\begin{array}{l} x - 2y = 5 \quad ...(2) \\ x - 4y = -15 \quad ...(1) \\ \underline{- \quad + \qquad +} \\ \quad 2y = 20 \end{array}$$

$y = 10$

From (1)

$x - 4(10) = -15$

$x = 25$

Present ages of A and B are 25 years and 10 years respectively.

2. A is 20 years older than B. 5 years ago, A was 3 time as old as B. Find their present age.

Solution:

Let A's present age be x years

And B's present age be y years

According to the question

$x = y + 20$

$x - y = 20...(1)$

Five years ago,

$x - 5 = 3(y - 5)$

$x - 3y = -10 \quad ...(2)$

Subtracting (1) from (2),

$$\begin{array}{l} x - 3y = -10 \quad ...(2) \\ x - \ y = \ 20 \quad ...(1) \\ \underline{- \quad + \qquad -} \\ \quad -2y = -30 \end{array}$$

$y = 15$

From (1)

$x = 15 + 20$
$x = 35$
Thus, present ages of A and B are 35 years and 15 years.

3. **Four years ago, a mother was four times as old as her daughter. Six years later, the mother will be two and a half times as old as her daughter at that time. Find the present age of mother and her daughter.**

Solution:

Let the present age of the mother be x years
and the present age of the daughter be y year.
According to the question,
$x - 4 = 4(y - 4) \Rightarrow x - 4 = 4y - 16 \Rightarrow x - 4y = -12 \ldots(i)$
and $x + 6 = 2\frac{1}{2}(y + 6) \Rightarrow x + 6 = \frac{5}{2}y + 15 \Rightarrow x - \frac{5}{2}y = 9 \ldots(ii)$
Solving (i) and (ii), we get
$y = 14$ and $x = 44$
Hence, the present age of the mother is 44 years
and the present age of the daughter is 14 years.

4. **The age of a man is twice the sum of the ages of his children. After 20 years, his age will be equal to the sum of the ages of his children at that time. Find the present age of the man.**

Solution:

Let the present age of the man be x years
and let the sum of the ages of his two children be y years.
According to the question,
$x = 2y \ldots(i)$
and $x + 20 = y + 40 \ldots(ii) \ldots$(Since he has two children)
Solving (i) and (ii), we get
$2y + 20 = y + 40 \Rightarrow y = 20$
So, $x = 2y \Rightarrow x = 40$
Hence, the present age of the man is 40 years.

5. **The annual incomes of A and B are in the ratio 3:4 and their annual expenditures are in the ratio 5:7. If each saves Rs. 5,000; find their annual incomes.**

Solution:

Let A's annual income = Rs.x
And B's annual income = Rs. y
According to the question,
$$\frac{x}{y} = \frac{3}{4}$$
$4x - 3y = 0 \ldots (1)$

And, $\frac{x-5000}{y-5000}=\frac{5}{7}$

7x - 5y = 10000... (2)

Multiplying equation no. (1) by 7 and (2) by 4.and subtracting (4) from (3)

$$\begin{array}{l} 28x-21y=0 \quad \ldots(3) \\ 28x-20y=40000 \quad \ldots(4) \\ \underline{- \quad + \quad\quad -} \\ \quad -y=-40000 \end{array}$$

$y = 40,000$

From (1)

4x - 3 (40000) = 0

x = 30000

Thus, A’s income in Rs. 30,000 and B’s income is Rs. 40,000.

6. **In an examination, the ratio of passes to failures was 4:1. Had 30 less appeared and 20 less passed, the ratio of passes to failures would have been 5:1. Find the number of students who appeared for the examination.**

Solution:

Let the no. of pass candidates be x

And the no. of fail candidates be y.

According to the question,

$\frac{x}{y}=\frac{y}{1}$

x - 4y = 0...(1)

And $\frac{x-20}{y-10}=\frac{5}{1}$

x - 5y = -30...(2)

$$\begin{array}{l} x-4y=0 \quad \ldots(1) \\ x-5y=-30 \quad \ldots(2) \\ \underline{- \quad + \quad\quad +} \\ \quad y=30 \end{array}$$

From (1)

- 4(30) = 0

x = 120

Total students appeared = x + y

= 120 + 30

= 150

7. **A and B both have some pencils. If A gives 10 pencils to B, then B will have twice as many as A. And if B gives 10 pencils to A, then they will have the same number of pencils. How many pencils does each have?**

Solution:

Let the number of pencils with A = x
And the number of pencils with B = y.
If A gives 10 pencils to B,
y + 10 = 2(x - 10)
2x - y = 30... (1)
If B gives to pencils to A
y - 10 = x + 10

$$\begin{array}{rl} x - y = -20 & \dots(2) \\ 2x - y = 30 & \dots(1) \\ \underline{-\quad+\quad-} & \\ -x = -50 & \end{array}$$

$x = 50$
From (1)
2(50) - y = 30
y = 70
Thus, A has 50 pencils and B has 70 pencils.

8. 1250 persons went to see a circus-show. Each adult paid Rs.75 and each child paid Rs.25 for the admission ticket. Find the number of adults and number of children, if the total collection from them amounts to Rs.61250.

Solution:

Let the number of adults = x
And the number of children = y
According to the question,
x + y = 1250... (1)
And 75x + 25y = 61250

$$\begin{array}{rl} 3x + y = 2450 & \dots(2) \\ x + y = 1250 & \dots(1) \\ \underline{-\quad-\quad-} & \\ 2x = 1200 & \end{array}$$

x = 6000
From (1)
600 + y = 1250
y = 650
Thus, number of adults = 600
And the number of children = 650.

9. Two articles A and b are sold for Rs.1167making 5% profit on A and 7% profit on B. If the two articles are sold for Rs.1165, a profit of 7% is made on A and a profit of 5% is made on B. Find the cost price of each article.

Solution:

Let the cost price of article A = Rs. x
And the cost price of articles B = Rs. y
According to the question,

(x + 5% of x) +(y + 7% of y) = 1167

$$\left(x + \frac{5}{100}x\right) + \left(y\frac{7}{100}y\right) = 1167$$

$$\frac{21x}{20} + \frac{107y}{100} = 1167$$

105x + 107y = 1167... (1)

And $\frac{107x}{100} + \frac{105y}{100} = 1165$

107x + 105y = 116500... (2)

Adding (1) and (2)

212x + 212y = 233200

x + y = 1100... (3)

Subtracting (2) from (1)

-2x + 2y = 200

$$-x + y = 100 \qquad \ldots(4)$$
$$x + y = 1100 \qquad \ldots(3)$$
$$\overline{\quad 2y = 1200 \quad}$$

y = 600

From (3)

x +600 = 1100

x = 500

Thus, cost price of article A is Rs. 500.

And that of article B is Rs. 600.

10. Pooja and Ritu can do a piece of work in $17\frac{1}{7}$ days. If one day work of Pooja be three fourth of one day work of Ritu; find in how many days each will do the work alone.

Solution:

Let Pooja's 1 day work = $\frac{1}{x}$

and Ritu's 1 day work = $\frac{1}{y}$

According the question,

$$\frac{1}{x} + \frac{1}{y} = \frac{7}{120} \qquad \ldots(1)$$

and, $\frac{1}{x} = \frac{3}{4}\cdot\frac{1}{y}$

$$y = \frac{3}{4}x \qquad \ldots(2)$$

Using the value of y from (2) in (1)

$$\frac{1}{x}+\frac{4}{3x}=\frac{7}{120}$$

$$\frac{1}{x}\left(\frac{7}{3}\right)=\frac{7}{120}$$

$$x=40$$

From (2) $y=\frac{3}{4}(40)=30$

y = 30

EXERCISE 6(G) PAGE: 94

1. **Rohit says to Ajay, "Give me a hundred, I shall then become twice as rich as you." Ajay replies, "if you give me 10, I shall be six times as rich as you." How much does each have originally?**

Solution:

Let Rohit has Rs. x
and Ajay has Rs. y
When Ajay gives Rs. 100 to Rohit
x + 100 = 2(y - 100)
x - 2y = -300...(1)
When Rohit gives Rs. 10 to Ajay
6(x-10) = y + 10
6x - y = 70...(2)
Multiplying equation no. (2) By 2.

$$\begin{array}{rl} 12x - 2y = 140 & \quad ...(3) \\ x - 2y = -300 & \\ - \quad + \quad\quad + & \\ \hline 11x = 440 & \end{array}$$

x = 40
From (1)
40 - 2y = -300
⇒ -2y = -340
⇒ y = 170
Thus, Rohit has Rs. 40
and Ajay has Rs. 170

2. **The sum of a two digit number and the number obtained by reversing the order of the digits is 99. Find the number, if the digits differ by 3.**

Solution:

Let the digits in the tens place be x and the digit in the units place be y.
$\therefore$ Number = $10x + y$
Number on reversing the digits = $10y + x$
The difference between the digits = $x - y$ or $y - x$
Given :$(10x + y)+(10y + x) = 99$
$\Rightarrow 11x + 11y = 99$
$\Rightarrow x + y = 9$.....(i)
$x - y = 3$.....(ii)
or $y - x = 3$....(iii)
On solving equations (i) and (ii), we get
$2x = 12 \Rightarrow x = 6$
So, $y = 3$

On solving equations (i) and (iii), we get

$2y = 12 \Rightarrow y = 6$

So, $x = 3$

Number $= 10x + y = 10(6) + 3 = 63$

or Number $= 10x + y = 10(3) + 6 = 36$

$\therefore$ Required number $= 63$ or 36.

3. **Seven times a two digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3, find the number.**

Solution:

Let the digit at ten's place be x
And the digit at unit's place be y
Required number = 10x + y
When the digits are interchanged,
Reversed number = 10y + x
According to the question,
$7(10x + y) = 4(10y + x)$
$66x = 33y$
$2x - y = 0$...(1)
Also,

$$\begin{array}{rl} y - x = 3 & \ldots(2) \\ -y + 2x = 0 & \ldots(1) \\ \hline x = 3 & \end{array}$$

From (1) $2(3) - y = 0$
$y = 6$
Thus, Required number = 10(3) + 6 = 36

4. **From Delhi station, if we buy 2 tickets for station A and 3 tickets for station B, the total cost is Rs. 77. But if we buy 3 tickets for station A and 5 tickets for station B, the total cost is rs. 124. What are the fares from Delhi to Station A and to Station B.**

Solution:

Let, the fare of ticket for station A be Rs. x
and the fare of ticket for station B be Rs. y
According, to the question
$2x + 3y = 77$....(1)
and$3x+5y = 124$...(2)
Multiplying equation no. (1) by 3 and (2) by 2.

$$\begin{array}{rl} 6x + 9y = 231 & \ldots(3) \\ 6x + 10y = 248 & \ldots(4) \\ - \quad - \quad\quad - & \\ \hline -y = -17 & \end{array}$$

$y = 17$
From (1) $2x + 3(17) = 77$
$2x = 77 - 51$

2x = 26
x = 13
Thus, fare for station A = Rs. 13
and, fare for station B = Rs. 17.

5. The sum of digits of a two digit number is 11. If the digit at ten's place is increased by 5 and the digit at unit' place is decreased by 5, the digits of the number are found to be reversed. Find the original number.

Solution:

Let x be the number at the ten's place
and y be the number at the unit's place.
So the number is 10x + y.

The sum of digit of a two digit number is 11.
$\Rightarrow x + y = 11 \ldots\ldots\ldots\ldots\ldots (i)$

If the digit at ten's place is increased by 5
and the digit at unit place is decreased by 5,
the digits of the number are found to be reversed.
$\Rightarrow 10(x+5) + (y-5) = 10y + x$
$\Rightarrow 9x - 9y = -45$
$\Rightarrow x - y = -5 \ldots\ldots\ldots\ldots\ldots (ii)$

Subtracting equation (i) from equation (ii), we get:

$$\begin{array}{rll} x - y &= -5 & \\ x + y &= 11 & \text{[Equation } (i)\text{]} \\ - \quad - & \quad - & \text{[Subtracting]} \\ \hline -2y &= -16 & \end{array}$$

$\Rightarrow y = 8$

Substituting $y = 8$ in equation (i), we get
$x + 8 = 11$
$\Rightarrow x = 3$

$\therefore$ The number is $10x + y = 10(3) + 8 = 38$.

6. 90% acid solution (90% pure acid and 10% water) and 97% acid solution are mixed to obtain 21 litres of 95% acid solution. How many litres of each solution are mixed?

Solution:

Let the quantity of 90% acid solution be x litres and
The quantity of 97% acid solution be y litres
According to the question,
x + y = 21...(1)
and 90% of x + 97% of y = 95% of 21

$90x + 97y = 1995$...(2)
Multiplying equation no. (1) by90, we get,

$$\begin{array}{l} 90x + 90y = 1890 \quad \ldots(3) \\ 90x + 97y = 1995 \quad \ldots(2) \\ \underline{- \quad - \quad\quad +} \\ \quad -7y = -105 \end{array}$$

$y = 15$
From (1)$x + 15 = 21$
$x = 6$
Hence, 90% acid solution is 6 litres and 97% acid solution is 15 litres.

7. Class XI students of a school wanted to give a farewell party to the outgoing students of Class XII. They decided to purchase two kinds of sweets, one costing Rs.250 per kg and the other costing Rs. 350 per kg. They estimated that 40 kg of sweets were needed. If the total budget for the sweets was Rs.11800; find how much sweets of each kind were bought.

Solution:

Assume x kg of the first kind costing Rs. 250 per kg
and y kg of the second kind costing Rs. 350 per kg
sweets were bought.

It is estimated that 40 kg of sweets were needed.
$\Rightarrow x + y = 40 \ldots\ldots\ldots\ldots\ldots (i)$

The total budget for the sweets was Rs. 11,800.
$\Rightarrow 250x + 350y = 11,800 \ldots\ldots\ldots\ldots\ldots (ii)$

Multiply equation (i)by 250, we get:

$$\begin{array}{ll} 250x + 250y = 10000 & \\ 250x + 350y = 11,800 & \text{[Equation } (ii)] \\ \underline{- \quad\quad - \quad\quad\quad -} & \text{[Subtracting]} \\ \quad -100y = -1800 & \end{array}$$

$\Rightarrow y = 18$

Substituting $y = 18$ in equation (i), we get
$x + 18 = 40$
$\Rightarrow x = 22$

$\therefore$ 22 kgs of the first kind costing Rs. 250 per kg
and 18 kgs of the second kind costing Rs. 350 per kg
sweets were bought.

8. **Mr. and Mrs Ahuja weigh x kg and y kg respectively. They both take a dieting course, at the end of which Mr. Ahuja loses 5 kg and weighs as much as his wife weighed before the course. Mrs. Ahuja loses 4 kg and weighs 7/8th of what her husband weighed before the course. Form two equation in x and y to find their weights before taking the dieting course,**

Solution:

Weight of Mr. Ahuja = x kg
And weight of Mrs. Ahuja = y kg.
After the dieting,
x - 5 = y
x - y = 5...(1)

And, $y - 4 = \frac{7}{8}x$

7x - 8y = -32... (2)
Multiplying equation no. (1) by 7, we get

$7x - 7y = 35 \qquad \ldots(3)$

Now subtracting (2) from (3)

$$\begin{array}{l} 7x - 7y = 35 \qquad \ldots(3) \\ 7x - 8y = -32 \qquad \ldots(2) \\ \underline{- \quad + \qquad +} \\ \qquad y = 67 \end{array}$$

From (1)
x - 67 = 5 ⇒ x = 72
Thus, weight of Mr. Ahuja = 72 kg.
And that of Mr. Ahuja = 67 kg.

9. **A part of monthly expenses of a family is constant and the remaining vary with the number of members in the family. For a family of 4 persons, the total monthly expenses are Rs. 10,400; whereas for a family of 7 persons, the total monthly expenses are Rs.15800. Find the constant expenses per month and the monthly expenses on each member of a family.**

Solution:

Let x be the constant expense per month of the family.
and y be the expense per month for a single member of the family.

For a family of 4 people,
the total monthly expense is Rs. 10,400.
$\Rightarrow x + 4y = 10,400 \ldots\ldots\ldots\ldots\ldots(i)$

For a family of 7 people,
the total monthly expense is Rs. 15,800.
$\Rightarrow x + 7y = 15,800 \ldots\ldots\ldots\ldots\ldots(ii)$

Subtracting equation (i) from equation (ii), we get:

$$\begin{array}{ll} x + 7y = 15800 & \\ x + 4y = 10400 & \text{[Equation } (i)\text{]} \\ - \quad - \qquad - & \text{[Subtracting]} \\ \hline 3y = 5400 & \end{array}$$

$\Rightarrow y = 1800$

Substituting $y = 1800$ in equation (i), we get

$x + 4(1800) = 10,400$

$\Rightarrow x = 3200$

$\therefore$ The constant expense is Rs. 3,200 per month and
the monthly expense of each member of a family is Rs. 1,800.

10. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10km, the charge paid is Rs.315 and for a distance of 15km, the charge paid is Rs, 465. What are the fixed charges and the charge per kilometre? How much does a person have to pay for travelling a distance of 32 km?

Solution:

Let the fixed charge be Rs. x and the charge per kilometer be Rs. y.

The charges for 10 km = Rs. 10y

The charges for 15 km = Rs. 15y

According to the question,

$x + 10y = 315.....(i)$

$x + 15y = 465.....(ii)$

Solving the equations, we get

$-5y = -150 \Rightarrow y = 30$

and $x = 315 - 10y = 315 - 10(30) = 15$

So, the fixed charges is Rs. 15 and the charges per kilometer is Rs. 30.

To travel 32 km, a person has to pay

Rs. 15 + Rs. 30(32) = Rs. 15 + Rs. 960 = Rs. 975

11. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Geeta paid Rs.27 for a book kept for seven days, while Mohit paid Rs.21 for the book he kept for five days. Find the fixed charges and the charges for each extra day.

Solution:

Let the fixed charges be Rs. x and the charge for each extra day be Rs. y.

According to the question,

$x + 4y = 27......(i)$

and $x + 2y = 21......(ii)$

Solving the equations, we get

$2y = 6 \Rightarrow y = 3$

and $x = 21 - 2y = 21 - 2(3) = 15$

Hence, the fixed charges is Rs. 15 and the charge for each extra day is Rs. 3.

12. The areas of rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. However, if the length of the rectangle increases by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solution:

Let the length of the rectangle be x units and the breadth of the rectangle be y units.

We know that, area of a rectangle = length x breadth = xy

According to the question,

$xy - 9 = (x - 5)(y + 3)$

$\Rightarrow xy - 9 = xy + 3x - 5y - 15$

$\Rightarrow 3x - 5y = 6 \ldots\ldots(i)$

$xy + 67 = (x + 3)(y + 2)$

$\Rightarrow xy + 67 = xy + 2x + 3y + 6$

$\Rightarrow 2x + 3y = 61 \ldots\ldots(ii)$

Multiply (i) by 2 and (ii) by 3, we get

$6x - 10y = 12 \ldots(iii)$

and $6x + 9y = 183 \ldots(iv)$

Solving (iii) and (iv), we get

$-19y = -171 \Rightarrow y = 9$

and $x = 17$

Hence, the length of the rectangle is 17 units and the breadth of the rectangle is 9 units.

13. It takes 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter is used for 9 hours, only half of the pool is filled. How long would each pipe take to fill the swimming pool?

Solution:

Let the pipe with larger diameter and smaller diameter be pipes A and B respectively.
Also, let pipe A work at a rate of x hours / unit and pipe B work at a rate of y hours / unit.
According to the question,

$x + y = \frac{1}{12} \Rightarrow 12x + 12y = 1 \ldots..(i)$

and $4x + 9y = \frac{1}{2} \Rightarrow 8x + 18y = 1 \ldots..(ii)$

Multiply (i) by 2 and (ii) by 3, we get

$24x + 24y = 2$ and $24x + 54y = 3$

On solving we get, $30y = 1 \Rightarrow y = \frac{1}{30}$

and $x = \frac{1}{20}$

Hence, the pipe with larger diameter will take 20 hours to fill the swimming pool and the pipe with smaller diameter will take 30 hours to fill the swimming pool.