EXERCISE 7(A) PAGE: 98

1. **Evaluate:**

(i)

$$3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}}$$

Solution:

$$3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}} = 3^3 \times (3\times3\times3\times3\times3)^{-\frac{2}{3}} \times (3\times3)^{-\frac{1}{3}}$$

$$= 3^3 \times (3^5)^{-\frac{2}{3}} \times (3^2)^{-\frac{1}{3}}$$

$$= 3^3 \times 3^{\left(-\frac{10}{3}\right)} \times 3^{-\frac{2}{3}} \qquad [(a^m)^n = a^{mn}]$$

$$= 3^{3-\frac{10}{3}-\frac{2}{3}} \qquad [a^m \times a^n \times a^p = a^{m+n+p}]$$

$$= 3^{\frac{9-10-2}{3}}$$

$$= 3^{\frac{9-12}{3}}$$

$$= 3^{-\frac{3}{3}}$$

$$= 3^{-1}$$

$$= \frac{1}{3}$$

(ii)

$$5^{-4} \times (125)^{\frac{5}{3}} \div (25)^{-\frac{1}{2}}$$

Solution:

$$5^{-4} \times (125)^{\frac{5}{3}} \div (25)^{-\frac{1}{2}} = 5^{-4} \times (5\times5\times5)^{\frac{5}{3}} \div (5\times5)^{-\frac{1}{2}}$$

$$= 5^{-4} \times (5^3)^{\frac{5}{3}} \div (5^2)^{-\frac{1}{2}}$$

$$= 5^{-4} \times \left(5^{3\times\frac{5}{3}}\right) \div \left(5^{2\times\left(-\frac{1}{2}\right)}\right)$$

$$= \frac{5^{-4} \times 5^5}{5^{-1}}$$

$$= \frac{5^{5-4}}{5^{-1}}$$

$$= \frac{5^1}{5^{-1}}$$

$$= 5^{1-(-1)}$$

$$= 5^2$$

$$= 5 \times 5$$

$$= 25$$

(iii)

$$\left(\frac{27}{125}\right)^{\frac{2}{3}} \times \left(\frac{9}{25}\right)^{-\frac{3}{2}}$$

Solution:

$$\left(\frac{27}{125}\right)^{\frac{2}{3}} \times \left(\frac{9}{25}\right)^{-\frac{3}{2}} = \left(\frac{3 \times 3 \times 3}{5 \times 5 \times 5}\right)^{\frac{2}{3}} \times \left(\frac{3 \times 3}{5 \times 5}\right)^{-\frac{3}{2}}$$

$$= \left[\left(\frac{3}{5}\right)^{3}\right]^{\frac{2}{3}} \times \left[\left(\frac{3}{5}\right)^{2}\right]^{-\frac{3}{2}}$$

$$= \left(\frac{3}{5}\right)^{3 \times \frac{2}{3}} \times \left(\frac{3}{5}\right)^{2 \times \left(-\frac{3}{2}\right)}$$

$$= \left(\frac{3}{5}\right)^{2} \times \left(\frac{3}{5}\right)^{-3}$$

$$= \left(\frac{3}{5}\right)^{2-3}$$

$$= \left(\frac{3}{5}\right)^{-1}$$

$$= \frac{1}{\frac{3}{5}}$$

$$= \frac{5}{3}$$

(iv)

$$7^{0} \times (25)^{-\frac{3}{2}} - 5^{-3}$$

Solution:

$$7^{0} \times (25)^{-\frac{3}{2}} - 5^{-3} = 7^{0} \times (5 \times 5)^{-\frac{3}{2}} - 5^{-3}$$

$$= 7^{0} \times (5^{2})^{-\frac{3}{2}} - \frac{1}{5^{3}}$$

$$= 7^{0} \times 5^{2\left(-\frac{3}{2}\right)} - \frac{1}{5^{3}}$$

$$= 7^{0} \times 5^{-3} - \frac{1}{5^{3}}$$

$$= 1 \times 5^{-3} - \frac{1}{5^{3}}$$

$$= \frac{1}{5^{3}} - \frac{1}{5^{3}}$$

$$= \frac{1-1}{5 \times 5 \times 5}$$

$$= \frac{0}{125}$$

$$= 0$$

(v)

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$$

Solution:

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2\times2\times2\times2}{3\times3\times3\times3}\right)^{-\frac{3}{4}} \times \left(\frac{7\times7}{3\times3}\right)^{\frac{3}{2}} \div \left(\frac{7\times7\times7}{6\times6\times6}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2}{3}\right)^4\right]^{-\frac{3}{4}} \times \left[\left(\frac{7}{3}\right)^2\right]^{\frac{3}{2}} \div \left[\left(\frac{7}{6}\right)^3\right]^{\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{4\times\left(-\frac{3}{4}\right)} \times \left(\frac{7}{3}\right)^{2\times\frac{3}{2}} \div \left(\frac{7}{6}\right)^{3\times\frac{2}{3}}$$

$$= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{7}{3}\right)^{3} \div \left(\frac{7}{6}\right)^{2}$$

$$= \frac{1}{\left(\frac{2}{3}\right)^3} \times \left(\frac{7}{3}\right)^{3} \times \frac{1}{\left(\frac{7}{6}\right)^2}$$

$$= \frac{1}{\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}} \times \frac{7}{3}\times\frac{7}{3}\times\frac{7}{3} \times \frac{1}{\frac{7}{6}\times\frac{7}{6}}$$

$$= \frac{1\times3\times3\times3}{2\times2\times2} \times \frac{7}{3}\times\frac{7}{3}\times\frac{7}{3} \times \frac{1\times6\times6}{7\times7}$$

$$= \frac{7\times3\times3}{2}$$

$$= \frac{63}{2}$$

$$= 31.5$$

2. Simplify:

(i)

$$\left(8x^3 \div 125y^3\right)^{\frac{2}{3}}$$

Solution:

$$\left(8x^3 \div 125y^3\right)^{\frac{2}{3}} = \left(\frac{8x^3}{125y^3}\right)^{\frac{2}{3}}$$

$$= \left(\frac{2x \times 2x \times 2x}{5y \times 5y \times 5y}\right)^{\frac{2}{3}}$$

$$= \left[\left(\frac{2x}{5y}\right)^3\right]^{\frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^{3 \times \frac{2}{3}}$$

$$= \left(\frac{2x}{5y}\right)^2$$

$$= \frac{2x}{5y} \times \frac{2x}{5y}$$

$$= \frac{4x^2}{25y^2}$$

(ii)

$$(a+b)^{-1}.\left(a^{-1}+b^{-1}\right)$$

Solution:

$$(a+b)^{-1}.\left(a^{-1}+b^{-1}\right) = \frac{1}{(a+b)} \times \left(\frac{1}{a}+\frac{1}{b}\right)$$

$$= \frac{1}{(a+b)} \times \left(\frac{b+a}{ab}\right)$$

$$= \frac{1}{(a+b)} \times \frac{(a+b)}{ab}$$

$$= \frac{1}{ab}$$

(iii)

$$\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2}$$

Solution:

$$\frac{5^{n+3}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}=\frac{5^{n+1}\times5^{2}-6\times5^{n+1}}{9\times5^{n}-5^{n}\times2^{2}}$$
$$=\frac{5^{n+1}\times\left(5^{2}-6\right)}{5^{n}\times(9-4)}$$
$$=\frac{5^{n}\times5^{1}\times(25-6)}{5^{n}\times(9-4)}$$
$$=\frac{5^{1}\times19}{5}$$
$$=19$$

(iv)

$$\left(3x^{2}\right)^{-3}\times\left(x^{9}\right)^{\frac{2}{3}}$$

Solution:

$$\left(3x^{2}\right)^{-3}\times\left(x^{9}\right)^{\frac{2}{3}}=\frac{1}{\left(3x^{2}\right)^{3}}\times x^{9\times\frac{2}{3}}$$
$$=\frac{1}{3^{3}x^{2\times3}}\times x^{6}$$
$$=\frac{1}{27x^{6}}\times x^{6}$$
$$=\frac{1}{27}$$

3. Evaluate:

(i)

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}$$

Solution:

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}=\sqrt{\frac{1}{2}\times\frac{1}{2}}+(0.1\times0.1)^{-\frac{1}{2}}-(3\times3\times3)^{\frac{2}{3}}$$

$$=\frac{1}{2}+\left[(0.1)^2\right]^{-\frac{1}{2}}-\left(3^2\right)^{\frac{2}{3}}$$

$$=\frac{1}{2}+(0.1)^{2\times\left(-\frac{1}{2}\right)}-3^{3\times\frac{2}{3}}$$

$$=\frac{1}{2}+(0.1)^{-1}-3^2$$

$$=\frac{1}{2}+\frac{1}{0.1}-9$$

$$=\frac{1}{2}+\frac{10}{1}-9$$

$$=\frac{1+20-18}{2}$$

$$=\frac{3}{2}$$

$$=1\frac{1}{2}$$

(ii)

$$\left(\frac{27}{8}\right)^{\frac{2}{3}}-\left(\frac{1}{4}\right)^{-2}+5^0$$

Solution:

$$\left(\frac{27}{8}\right)^{\frac{2}{3}}-\left(\frac{1}{4}\right)^{-2}+5^0=\left(\frac{3\times3\times3}{2\times2\times2}\right)^{\frac{2}{3}}-\left(\frac{1\times1}{2\times2}\right)^{-2}+5^0$$

$$=\left[\left(\frac{3}{2}\right)^3\right]^{\frac{2}{3}}-\left[\left(\frac{1}{2}\right)^2\right]^{-2}+1$$

$$=\left(\frac{3}{2}\right)^{3\times\frac{2}{3}}-\left(\frac{1}{2}\right)^{2\times(-2)}+1$$

$$=\left(\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^{-4}+1$$

$$=\frac{3}{2}\times\frac{3}{2}-\frac{1}{\left(\frac{1}{2}\right)^4}+1$$

$$=\frac{9}{4}-\frac{1}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}+1$$

$$=\frac{9}{4}-\frac{1}{\frac{1}{16}}+1$$

$$=\frac{9}{4}-16+1$$

$$=\frac{9-64+4}{4}$$

$$=\frac{-51}{4}$$

4. Simplify each of the following and express with positive index:

(i)

$$\left(\frac{3^{-4}}{2^{-8}}\right)^{\frac{1}{4}}$$

Solution:

$$\left(\frac{3^{-4}}{2^{-8}}\right)^{\frac{1}{4}} = \left(\frac{2^{8}}{3^{4}}\right)^{\frac{1}{4}}$$

$$= \frac{\left(2^{8}\right)^{\frac{1}{4}}}{\left(3^{4}\right)^{\frac{1}{4}}}$$

$$= \frac{2^{8\times\frac{1}{4}}}{3^{4\times\frac{1}{4}}}$$

$$= \frac{2^{2}}{3}$$

$$= \frac{4}{3}$$

(ii)

$$\left(\frac{27^{-3}}{9^{-3}}\right)^{\frac{1}{5}}$$

Solution:

$$\left(\frac{27^{-3}}{9^{-3}}\right)^{\frac{1}{5}} = \left(\frac{9^{3}}{27^{3}}\right)^{\frac{1}{5}}$$

$$= \left(\frac{\left(3^{2}\right)^{3}}{\left(3^{3}\right)^{3}}\right)^{\frac{1}{5}}$$

$$= \left[\left(\frac{3^{2}}{3^{3}}\right)^{3}\right]^{\frac{1}{5}}$$

$$= \left[\left(\frac{1}{3}\right)^{3}\right]^{\frac{1}{5}}$$

$$= \left(\frac{1}{3}\right)^{3\times\frac{1}{5}}$$

$$= \frac{1}{3^{\frac{3}{5}}}$$

(iii)

$$(32)^{-\frac{2}{5}} \div (125)^{-\frac{2}{3}}$$

Solution:

$$(32)^{-\frac{2}{5}} \div (125)^{-\frac{2}{3}} = \frac{(32)^{-\frac{2}{5}}}{(125)^{-\frac{2}{3}}}$$

$$= \frac{(125)^{\frac{2}{3}}}{(32)^{\frac{2}{5}}}$$

$$= \frac{(5 \times 5 \times 5)^{\frac{2}{3}}}{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}}$$

$$= \frac{(5^3)^{\frac{2}{3}}}{(2^5)^{\frac{2}{5}}}$$

$$= \frac{5^2}{2^2}$$

$$= \frac{25}{4}$$

$$= 6\frac{1}{4}$$

(iv)

$$\left[1 - \left\{1 - (1 - n)^{-1}\right\}^{-1}\right]^{-1}$$

Solution:

$$\left[1 - \left\{1 - (1 - n)^{-1}\right\}^{-1}\right]^{-1} = \frac{1}{\left[1 - \left\{1 - (1 - n)^{-1}\right\}^{-1}\right]^{+1}}$$

$$= \frac{1}{1 - \frac{1}{1 - (1 - n)^{-1}}}$$

$$= \frac{1}{1 - \frac{1}{1 - \frac{1}{(1 - n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{1(1-n)-1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{1-n-1}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{1}{\frac{-n}{(1-n)}}}$$

$$= \frac{1}{1 - \frac{(1-n)}{-n}}$$

$$= \frac{1}{1 + \frac{(1-n)}{n}}$$

$$= \frac{1}{\frac{n+(1-n)}{n}}$$

$$= \frac{1}{\frac{n+1-n}{n}}$$

$$= \frac{n}{1}$$

$$= n$$

5. If 2160= $2^a.3^b.5^c$, find a,b and c. Hence calculate the value of $3^a x 2^{-b} x 5^{-c}$.

Solution:

$2160 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^4 \times 3^3 \times 5^1 = 2^a \times 3^b \times 5^c$

$\Rightarrow 2^a \times 3^b \times 5^c = 2^4 \times 3^3 \times 5^1$

Comparing powers of 2,3 and 5 on the both sides of equation, we have

a=4;b=3 and c=1

Hence value of $3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$

$$= 3 \times 3 \times 3 \times 3 \times \frac{1}{2^3} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{2 \times 2 \times 2} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{8} \times \frac{1}{5}$$

$$= \frac{81}{40}$$

$$= 2\frac{1}{40}$$

6. If $1960=2^a.3^b.5^c$, find a,b and c. Calculate the value of $2^{-a}x7^bx5^{-c}$.

Solution:

$1960 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2 \times 2 \times 2 \times 5 \times 7 \times 7 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$

$\Rightarrow 2^a \times 5^b \times 7^c = 2^3 \times 5^1 \times 7^2$

Comparing powers of 2,5 and 7 on the both sides of equation, we have

a=3;b=1 and c=2

Hence value of $2^{-a} \times 7^b \times 5^{-c} = 2^{-3} \times 7^1 \times 5^{-2}$

$$= \frac{1}{2^3} \times 7 \times \frac{1}{5^2}$$

$$= \frac{1}{8} \times 7 \times \frac{1}{5 \times 5}$$

$$= \frac{7}{200}$$

7. Simplify:

(i)

$$\frac{8^{3a} \times 2^5 \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}}$$

(ii)

$$\frac{3 \times 27^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 27^n}$$

Solution:

(i)

$$\frac{8^{3a} \times 2^5 \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}} = \frac{(2^3)^{3a} \times 2^5 \times 2^{2a}}{2^2 \times 2^{11a} \times 2^{-2a}}$$

$$= \frac{2^{3\times 3a} \times 2^5 \times 2^{2a}}{2^2 \times 2^{11a} \times 2^{-2a}}$$

$$= \frac{2^{9a} \times 2^5 \times 2^{2a}}{2^2 \times 2^{11a} \times 2^{-2a}}$$

$$= 2^{9a+5+2a-2-11a+2a}$$

$$= 2^{2a+3}$$

(ii)

$$\frac{3\times 27^{n+1}+9\times 3^{3n-1}}{8\times 3^{3n}-5\times 27^{n}}=\frac{3\times(3\times 3\times 3)^{n+1}+3\times 3\times 3^{3n-1}}{2\times 2\times 2\times 3^{3n}-5\times(3\times 3\times 3)^{n}}$$

$$=\frac{3\times\left(3^{3}\right)^{n+1}+3^{2}\times 3^{3n-1}}{2^{3}\times 3^{3n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3\times 3^{3n+3}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+3+1}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+4}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\times 3^{4}+3^{3n}\times 3^{1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{\left(3^{3}\right)^{n}(8-5)}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{3^{3n}\times 3}$$

$$=\frac{3\times 3\times 3\times 3+3}{3}$$

$$=\frac{81+3}{3}$$

$$=\frac{84}{3}$$

$$=28$$

8. Show that:

$$\left(\frac{a^{m}}{a^{-n}}\right)^{m-n}\times\left(\frac{a^{n}}{a^{-\ell}}\right)^{n-\ell}\times\left(\frac{a^{\ell}}{a^{-m}}\right)^{\ell-m}$$

Solution:

$$\left(\frac{a^m}{a^{-n}}\right)^{m-n} \times \left(\frac{a^n}{a^{-\ell}}\right)^{n-\ell} \times \left(\frac{a^\ell}{a^{-m}}\right)^{\ell-m}$$
$$= \left(a^m \times a^n\right)^{m-n} \times \left(a^n \times a^\ell\right)^{n-\ell} \times \left(a^\ell \times a^m\right)^{\ell-m}$$
$$= \left(a^{m+n}\right)^{m-n} \times \left(a^{n+\ell}\right)^{n-\ell} \times \left(a^{\ell+m}\right)^{\ell-m}$$
$$= a^{m^2-n^2} \times a^{n^2-\ell^2} \times a^{\ell^2-m^2}$$
$$= a^{m^2-n^2+n^2-\ell^2+\ell^2-m^2}$$
$$= a^0$$
$$= 1$$

9. If $a=x^{m+n}.y^l$; $b=x^{n+1}.y^m$ and $c=x^{l+m}.y^n$, prove that: $a^{m-n}.b^{n-l}.c^{l-m}=1$

Solution:

$$a = x^{m+n}.x^l$$
$$b = x^{n+l}.x^m$$
$$c = x^{l+m}.x^n$$

LHS

$$a^{m-n}.b^{n-l}.c^{l-m}$$
$$= (x^{m+n}.x^l)^{m-n}.(x^{n+l}.x^m)^{n-l}.(x^{l+m}.x^n)^{l-m} \text{ [Substituting a,b,c in LHS]}$$
$$= x^{(m+n)(m-n)}.x^{l(m-n)}.x^{(n+l)(n-l)}.x^{m(n-l)}.x^{(l+m)(l-m)}.x^{n(l-m)}$$
$$= x^{m^2-n^2+ml-nl+n^2-l^2+mn-nl+l^2-m^2+nl-mn}$$
$$= x^0$$
$$= 1 = RHS$$

10. Simplify:

(i)

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$$

(ii)

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$$

Solution:

(i)

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$$

$$= \left(x^{a-b}\right)^{a^2+ab+b^2} \times \left(x^{b-c}\right)^{b^2+bc+c^2} \times \left(x^{c-a}\right)^{c^2+ca+a^2}$$

$$= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$$

$$= x^0$$

$$= 1$$

(ii)

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$$

$$= \left(x^{a+b}\right)^{a^2-ab+b^2} \times \left(x^{b+c}\right)^{b^2-bc+c^2} \times \left(x^{c+a}\right)^{c^2-ca+a^2}$$

$$= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$= x^{\left(a^3+b^3+b^3+c^3+c^3+a^3\right)}$$

$$= x^{2\left(a^3+b^3+c^3\right)}$$

EXERCISE 7(A) PAGE: 100

1. Solve for x:

(i) $2^{2x+1} = 8$

(ii) $2^{5x-1} = 4 \times 2^{3x+1}$

(iii) $3^{4x+1} = 27^{(x+1)}$

(iv) $49^{x+4} = 7^2 (343)^{(x+1)}$

Solution:

(i)

$2^{2x+1} = 8$

$\Rightarrow 2^{2x+1} = 2^3$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x+1=3$

$\Rightarrow 2x=3-1$

$\Rightarrow 2x = 2$

$\Rightarrow x = \frac{2}{2}$

$\Rightarrow x = 1$

(ii)

$2^{5x-1} = 4 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^2 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^{3x+1+2}$

$\Rightarrow 2^{5x-1} = 2^{3x+3}$

We know that if bases are equal, the powers are equal

$\Rightarrow 5x-1=3x+3$

$\Rightarrow 5x-3x=3+1$

$\Rightarrow 2x = 4$

$\Rightarrow x = \frac{4}{2}$

$\Rightarrow x = 2$

(iii)

$3^{4x+1} = 27^{(x+1)}$

$\Rightarrow 3^{4x+1} = (3^3)^{x+1}$

$\Rightarrow 3^{4x+1} = 3^{3x+3}$

We know that if bases are equal, the powers are equal

$\Rightarrow 4x+1=3x+3$

$\Rightarrow 4x-3x=3-1$

$\Rightarrow x = 2$

(iv)

$49^{x+4} = 7^2(343)^{(x+1)}$

$\Rightarrow (7 \times 7)^{x+4} = 7^2(7 \times 7 \times 7)^{(x+1)}$

$\Rightarrow (7^2)^{x+4} = 7^2(7^3)^{(x+1)}$

$\Rightarrow 7^{2x+8} = 7^2 \times 7^{3x+3}$

$\Rightarrow 7^{2x+8} = 7^{3x+3+2}$

$\Rightarrow 7^{2x+8} = 7^{3x+5}$

We know that if bases are equal, the powers are equal

$\Rightarrow 2x+8=3x+5$

$\Rightarrow 3x-2x=8-5$

$\Rightarrow x = 3$

2. Find x, if:

(i)

$4^{2x} = \frac{1}{32}$

(ii)

$\sqrt{2^{x+3}} = 16$

(iii)

$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$

(iv)

$\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$

Solution:

(i)

$4^{2x} = \frac{1}{32}$

$\Rightarrow (2 \times 2)^{2x} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$

$\Rightarrow (2^2)^{2x} = \frac{1}{2^5}$

$\Rightarrow 2^{2x2x} = 2^{-5}$

$\Rightarrow 2^{4x} = 2^{-5}$

We know that if bases are equal, the powers are equal

$\Rightarrow 4x = -5$

$\Rightarrow x = \frac{-5}{4}$

(ii)

$\sqrt{2^{x+3}} = 16$

$(2^{x+3})^{\frac{1}{2}} = 2 \times 2 \times 2 \times 2$

$\Rightarrow 2^{\frac{x+3}{2}} = 2^4$

We know that if bases are equal, the powers are equal

$\Rightarrow \frac{x+3}{2} = 4$

$\Rightarrow x + 3 = 8$

$\Rightarrow x = 8 - 3$

$\Rightarrow x = 5$

(iii)

$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$

$\Rightarrow \left[\left(\frac{3}{5}\right)^{\frac{1}{2}}\right]^{x+1} = \frac{5 \times 5 \times 5}{3 \times 3 \times 3}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{5}{3}\right)^3$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{3}{5}\right)^{-3}$

We know that if bases are equal, the powers are equal

$\Rightarrow \frac{x+1}{2} = -3$

$\Rightarrow x + 1 = -6$

$\Rightarrow x = -6 - 1$

$\Rightarrow x = -7$

(iv)

$$\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$$

$$\left[\left(\frac{2}{3}\right)^{\frac{1}{3}}\right]^{x-1} = \frac{3^3}{2^3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{3}{2}\right)^3$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{2}{3}\right)^{-3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x-1}{3} = -3$$

$$\Rightarrow x - 1 = -9$$

$$\Rightarrow x = -9 + 1$$

$$\Rightarrow x = -8$$

3. Solve:

(i)

$$4^{x-2} - 2^{x+1} = 0$$

(ii)

$$3^{x^2} : 3^x = 9 : 1$$

Solution:

(i)

$$4^{x-2} - 2^{x+1} = 0$$

$$\Rightarrow 4^{x-2} = 2^{x+1}$$

$$\Rightarrow (2^2)^{x-2} = 2^{x+1}$$

$$\Rightarrow 2^{2x-4} = 2^{x+1}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 2x - 4 = x + 1$$

$$\Rightarrow 2x - x = 4 + 1$$

$$\Rightarrow x = 5$$

(ii)

$3^{x^2} : 3^x = 9 : 1$

$\frac{3^{x^2}}{3^x} = \frac{9}{1}$

$\Rightarrow 3^{x^2} = 9 \times 3^x$

$\Rightarrow 3^{x^2} = 3^2 \times 3^x$

$\Rightarrow 3^{x^2} = 3^{x+2}$

We know that if bases are equal, the powers are equal

$\Rightarrow x^2 = x+2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x-2) + 1(x-2) = 0$

$\Rightarrow (x+1)(x-2) = 0$

$\Rightarrow x + 1 = 0$ or $x - 2 = 0$

$\Rightarrow x = -1$ or $x = 2$

4. Solve:

(i) $8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$

(ii) $2^{2x} + 2^{x+2} - 4 \times 2^3 = 0$

(iii) $(\sqrt{3})^{x-3} = (\sqrt[4]{3})^{x+1}$

Solution:

(i)

$8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$

$\Rightarrow 8 \times (2^x)^2 + 4 \times 2^x \times 2^1 = 1 + 2^x$

$\Rightarrow 8 \times (2^x)^2 + 4 \times (2^x) \times 2^1 - 1 - 2^x = 0$

$\Rightarrow 8 \times (2^x)^2 + (2^x) \times (8 - 1) - 1 = 0$

$\Rightarrow 8 \times (2^x)^2 + 7(2^x) - 1 = 0$

$\Rightarrow 8y^2 + 7y - 1 = 0 \quad [y = 2^x]$

$\Rightarrow 8y^2 + 8y - y - 1 = 0$

$\Rightarrow 8y(y+1) - 1(y+1) = 0$

$\Rightarrow (8y-1)(y+1) = 0$

$\Rightarrow 8y = 1$ or $y = -1$

$\Rightarrow y = \frac{1}{8}$ or $y = -1$

$\Rightarrow 2^x = \frac{1}{8}$ or $2^x = -1$

$\Rightarrow 2^x = \frac{1}{2^3}$ or $2^x = -1$

$\Rightarrow 2^x = 2^{-3}$ or $2^x = -1$

$\Rightarrow x = -3$

$[\because 2^x = -1$ is not possible$]$

(ii)

$$2^{2x}+2^{x+2}-4\times2^3=0$$
$$\Rightarrow\left(2^x\right)^2+2^x\cdot2^2-4\times2\times2\times2=0$$
$$\Rightarrow\left(2^x\right)^2+2^x\cdot2^2-32=0$$
$$\Rightarrow y^2+4y-32=0\quad[y=2^x]$$
$$\Rightarrow y^2+8y-4y-32=0$$
$$\Rightarrow y(y+8)-4(y+8)=0$$
$$\Rightarrow(y+8)(y-4)=0$$
$$\Rightarrow y+8=0 \text{ or } y-4=0$$
$$\Rightarrow y=-8 \text{ or } y=4$$
$$\Rightarrow 2^x=-8 \text{ or } 2^x=4$$
$$\Rightarrow 2^x=2^2\quad[\because 2^x=-8 \text{ is not possible}]$$
$$\Rightarrow x=2$$

(iii)

$$\left(\sqrt{3}\right)^{x-3}=\left(\sqrt[4]{3}\right)^{x+1}$$
$$\Rightarrow\left(3^{\frac{1}{2}}\right)^{x-3}=\left(3^{\frac{1}{4}}\right)^{x+1}$$
$$\Rightarrow 3^{\frac{x-3}{2}}=3^{\frac{x+1}{4}}$$
$$\Rightarrow\frac{x-3}{2}=\frac{x+1}{4}$$
$$\Rightarrow 4(x-3)=2(x+1)$$
$$\Rightarrow 4x-12=2x+2$$
$$\Rightarrow 4x-2x=12+2$$
$$\Rightarrow 2x=14$$
$$\Rightarrow x=\frac{14}{2}$$
$$\Rightarrow x=7$$

5. Find the values of m and n if:

$$4^{2m}=\left(\sqrt[3]{16}\right)^{-\frac{6}{n}}=\left(\sqrt{8}\right)^2$$

Solution:

$$4^{2m} = \left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow 4^{2m} = \left(\sqrt{8}\right)^2 \ldots.(1)$$

and

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2 \ldots.(2)$$

From (1)

$$4^{2m} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(2^2\right)^{2m} = \left(\sqrt{2^3}\right)^2$$

$$\Rightarrow 2^{4m} = \left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m} = \left[2^{3\times\frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m} = 2^{3\times\frac{1}{2}\times 2}$$

$$\Rightarrow 2^{4m} = 2^3$$

$$\Rightarrow 4m = 3$$

$$\Rightarrow m = \frac{3}{4}$$

From (2), we have

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2\times2\times2\times2}\right)^{-\frac{6}{n}} = \left(\sqrt{2\times2\times2}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2^4}\right)^{-\frac{6}{n}} = \left(\sqrt{2^3}\right)^2$$

$$\Rightarrow \left[\left(2^4\right)^{\frac{1}{3}}\right]^{-\frac{6}{n}} = \left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow \left[2^{\frac{4}{3}}\right]^{-\frac{6}{n}} = \left[2^{\frac{3}{2}}\right]^2$$

$$\Rightarrow 2^{\frac{4}{3}\times\left(-\frac{6}{n}\right)} = 2^{\frac{3}{2}\times 2}$$

$$\Rightarrow 2^{\left(-\frac{8}{n}\right)} = 2^3$$

$$\Rightarrow -\frac{8}{n} = 3$$

$$\Rightarrow n = \frac{-8}{3} \qquad \text{Thus } m = \frac{3}{4}\ n = \frac{-8}{3}$$

6. Solve for x and y, if:

$\left(\sqrt{32}\right)^x \div 2^{y+1} = 1$ and $8^y - 16^{4-\frac{x}{2}} = 0$

Solution:

Consider the equation

$$\left(\sqrt{32}\right)^x \div 2^{y+1} = 1$$

$$\Rightarrow \left(\sqrt{2\times2\times2\times2\times2}\right)^x \div 2^{y+1} = 1$$

$$\Rightarrow \left(\sqrt{2^5}\right)^x \div 2^{y+1} = 1$$

$$\Rightarrow \left[\left(2^5\right)^{\frac{1}{2}}\right]^x \div 2^{y+1} = x^0$$

$$\Rightarrow 2^{\frac{5x}{2}} \div 2^{y+1} = x^0$$

$$\Rightarrow \frac{5x}{2} - (y+1) = 0$$

$$\Rightarrow 5x - 2(y+1) = 0$$

$$\Rightarrow 5x - 2y - 2 = 0.....(1)$$

Now consider the other equation

$$8^y - 16^{4-\frac{x}{2}} = 0$$

$$\Rightarrow \left(2^3\right)^y - \left(2^4\right)^{4-\frac{x}{2}} = 0$$

$$\Rightarrow 2^{3y} - 2^{4\left(4-\frac{x}{2}\right)} = 0$$

$$\Rightarrow 2^{3y} = 2^{4\left(4-\frac{x}{2}\right)}$$

$$\Rightarrow 3y = 4\left(4 - \frac{x}{2}\right)$$

$$\Rightarrow 3y = 16 - 2x$$

$$\Rightarrow 2x + 3y = 16.....(2)$$

Thus we have two equations,

$5x - 2y = 2$ .....(1)

$2x + 3y = 16$.....(2)

Multiplying (1) by 3 and (2) by 2, we have

$15x - 6y = 6$....(3)

$4x + 6y = 32$....(4)

Adding (3) and (4), we have

$19x = 38$

$\Rightarrow x = 2$

Substituting the value of x in equation (1), we have,

$5(2) - 2y = 2$

$\Rightarrow 10 - 2y = 2$

$\Rightarrow 2y = 10 - 2$

$\Rightarrow 2y = 8$

$\Rightarrow y = \frac{8}{2}$

$\Rightarrow y = 4$

Thus the values of x and y are:

$x=2$ and $y=4$

7. Prove that:

(i)

$$\left(\frac{x^a}{x^b}\right)^{a+b-c} \times \left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b} = 1$$

(ii)

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$$

Solution:

(i)

$$\text{L.H.S.} = \left(\frac{x^a}{x^b}\right)^{a+b-c} \times \left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b}$$

$$= \left(x^{a-b}\right)^{(a+b-c)} \times \left(x^{b-c}\right)^{(b+c-a)} \times \left(x^{c-a}\right)^{(c+a-b)}$$

$$= x^{(a-b)(a+b-c)} \times x^{(b-c)(b+c-a)} \times x^{(c-a)(c+a-b)}$$

$$= x^{a^2+ab-ac-ab-b^2+bc} \times x^{b^2+bc-ab-cb-c^2+ac} \times x^{c^2+ac-bc-ac-a^2+ab}$$

$$= x^{a^2-ac-b^2+bc+b^2-ab-c^2+ac+c^2-bc-a^2+ab}$$

$$= x^0$$

$$= 1$$

$$= \text{R.H.S}$$

(ii)

We need to prove that

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$$

$$\text{L.H.S.} = x^{a(b-c)-b(a-c)} \div \frac{x^{bc}}{x^{ac}}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc} \div x^{bc-ac}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc-(bc-ac)}$$

$$\Rightarrow \quad = x^{ab-ac-ab+bc-bc+ac}$$

$$\Rightarrow \quad = x^0$$

$$\Rightarrow \quad = 1$$

$$\Rightarrow \quad = \text{R.H.S}$$

8. If $a^x=b$, $b^y=c$ and $c^z=a$, prove that $xyz=1$.

Solution:

We are given that

$a^x = b, b^y = c$ and $c^z = a$

Consider the equation

$a^x = b$

$\Rightarrow a^{xyz} = b^{yz}$ [raising to the power yz on both sides]

$\Rightarrow a^{xyz} = (b^y)^z$

$\Rightarrow a^{xyz} = (c)^z$ [$\because b^y = c$]

$\Rightarrow a^{xyz} = c^z$

$\Rightarrow a^{xyz} = a$ [$\because c^z = a$]

$\Rightarrow a^{xyz} = a^1$

$\Rightarrow xyz = 1$

9. If $a^x=b^y=c^z$ and $b^2=ac$, prove that:

$$y=\frac{2xz}{z+x}$$

Solution:

Let $a^x=b^y=c^z=k$

$\therefore\ a=k^{\frac{1}{x}}; b=k^{\frac{1}{y}}; c=k^{\frac{1}{z}}$

Also, we have $b^2=ac$

$$\therefore \left(k^{\frac{1}{y}}\right)^2=\left(k^{\frac{1}{x}}\right)\times\left(k^{\frac{1}{z}}\right)$$

$$\Rightarrow k^{\frac{2}{y}}=k^{\frac{1}{x}+\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}}=k^{\frac{z+x}{xz}}$$

Comparing the powers we have

$$\frac{2}{y}=\frac{z+x}{xz}$$

$$\Rightarrow y=\frac{2xz}{z+x}$$

10. If $5^{-p}=4^{-q}=20^r$; show that:

$$\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=0$$

Solution:

Let $5^{-p}=4^{-q}=20^r=k$

$$5^{-p}=k\Rightarrow 5=k^{-\frac{1}{p}}\left[\because a^p=b^q\Rightarrow a=b^{\frac{q}{p}}\right]$$

$$4^{-q}=k\Rightarrow 4=k^{-\frac{1}{q}}\left[\because a^p=b^q\Rightarrow a=b^{\frac{q}{p}}\right]$$

$$20^r=k\Rightarrow 20=k^{\frac{1}{r}}\left[\because a^p=b^q\Rightarrow a=b^{\frac{q}{p}}\right]$$

$5\times 4=20$

$$\Rightarrow k^{-\frac{1}{p}}\times k^{-\frac{1}{q}}=k^{\frac{1}{r}}$$

$$\Rightarrow k^{-\frac{1}{p}-\frac{1}{q}}=k^{\frac{1}{r}}$$

$$\Rightarrow k^0=k^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$$

$$\Rightarrow \frac{1}{p}+\frac{1}{q}+\frac{1}{r}=0 \text{ [If bases are equal, powers are also equal]}$$

11. If $m \neq n$ and $(m+n)^{-1}(m^{-1}+n^{-1})=m^x n^y$;
Shot that: $x+y+2=0$

Solution:

$$(m+n)^{-1}(m^{-1}+n^{-1}) = m^x n^y$$

$$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{1}{m}+\frac{1}{n}\right) = m^x n^y$$

$$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{m+n}{mn}\right) = m^x n^y$$

$$\Rightarrow \frac{1}{mn} = m^x n^y$$

$$\Rightarrow m^{-1}n^{-1} = m^x n^y$$

Comparing the coefficients of x and y, we get

$x = -1$ and $y = -1$

LHS,

$x + y + 2 = (-1) + (-1) + 2 = 0 = RHS$

12. If $5^{x+1}=25^{x-2}$; find the value of:
$3^{x-3} \times 2^{3-x}$.

Solution:

$$5^{x+1} = 25^{x-2}$$

$$\Rightarrow 5^{x+1} = \left(5^2\right)^{x-2}$$

$\Rightarrow 5^{x+1} = 5^{2x-4}$ [If bases are equal, powers are also equal]

$$\Rightarrow x + 1 = 2x - 4$$

$$\Rightarrow 2x - x = 4 + 1$$

$$\Rightarrow x = 5$$

$$\therefore 3^{x-3} \times 2^{3-x} = 3^{5-3} \times 2^{3-5} = 3^2 \times 2^{-2} = 9 \times \frac{1}{4} = \frac{9}{4}$$

13. If $4^{x+3}=112+8 \times 4^x$; find $(18x)^{3x}$

Solution:

$4^{x+3} = 112 + 8 \times 4^x$
$\Rightarrow 4^x \times 4^3 = 112 + 8 \times 4^x$
$\Rightarrow 64 \times 4^x = 112 + 8 \times 4^x$
Let $4^x = y$
$64y = 112 + 8y$
$\Rightarrow 56y = 112$
$\Rightarrow y = 2$
Substituting we get,
$4^x = 2$
$\Rightarrow 2^{2x} = 2$
$\Rightarrow 2x = 1$
$\Rightarrow x = \frac{1}{2}$

$$(18x)^{3x} = \left(\frac{18}{2}\right)^{3\times\frac{1}{2}} = 9^{3\times\frac{1}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$$

14. Solve for x:

(i)

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^{-x}$$

(ii)

$$a^{2(3x+5)} \times a^{4x} = a^{8x+12}$$

(iii)

$$(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + x\left(\frac{1}{2}\right)^{-1} . 2^0 = 27$$

(iv)

$$2^{3x} \times 2^3 = 2^{3x} \times 2 + 48$$

(v)

$$3 \times 2^x + 3 - 2^x \times 2^2 + 5 = 0$$

(vi)

$$9^{x+2} = 720 + 9^x$$

Solution:

(i)

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^{-x}$$
$$\Rightarrow \left(2^2\right)^{x-1} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^{-x}$$
$$\Rightarrow 2^{2x-2} \times 2^{-(3-2x)} = \left(2^{-3}\right)^{-x}$$
$$\Rightarrow 2^{2x-2-3+2x} = 2^{3x}$$
$$\Rightarrow 2^{4x-5} = 2^{3x}$$
$$\Rightarrow 4x - 5 = 3x$$
$$\Rightarrow 4x - 3x = 5$$
$$\Rightarrow x = 5$$

(ii)

$$a^{2(3x+5)} \times a^{4x} = a^{8x+12}$$
$$\Rightarrow a^{6x+10+4x} = a^{8x+12}$$
$\Rightarrow 10x + 10 = 8x + 12$ [If bases are the same, powers are also same]
$$\Rightarrow 2x = 2$$
$$\Rightarrow x = 1$$

(iii)

$$(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + x\left(\frac{1}{2}\right)^{-1}.2^{0} = 27$$
$$\Rightarrow 3^{4\times\frac{3}{4}} - \left(2^{-5}\right)^{-\frac{2}{5}} + x(2) = 27$$
$$\Rightarrow 3^{3} - 2^{2} + 2x = 27$$
$$\Rightarrow 2x + 27 - 4 = 27$$
$$\Rightarrow 2x = 4$$
$$\Rightarrow x = 2$$

(iv)

$$2^{3x} \times 2^{3} = 2^{3x} \times 2 + 48$$
$$\Rightarrow 8 \times 2^{3x} = 2^{3x} \times 2 + 48$$
$$\Rightarrow 2^{3x}\ (8 - 2) = 48$$
$$\Rightarrow 2^{3x} \times 6 = 48$$
$$\Rightarrow 2^{3x} = 8$$
$$\Rightarrow 2^{3x} = 2^{3}$$
$$\Rightarrow 3x = 3$$
$$\Rightarrow x = 1$$

(v)

$3 \times 2^x + 3 - 2^x \times 2^2 + 5 = 0$

$\Rightarrow 2^x (3 - 4) + 8 = 0$

$\Rightarrow -2^x = -8$

$\Rightarrow 2^x = 8$

$x=3$

(vi)

$9^{x+2} = 720 + 9^x$

$\Rightarrow 9^{x+2} - 9^x = 720$

$\Rightarrow 9^x (9^2 - 1) = 720$

$\Rightarrow 9^x (81 - 1) = 720$

$\Rightarrow 9^x (80) = 720$

$\Rightarrow 9^x = 9$

$\Rightarrow 9^x = 9^1$

$\Rightarrow x = 1$

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1. Evaluate:

Solution:

(i) $9^{\frac{5}{2}} - 3 \times 8^{0} - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$

$= \left(3^2\right)^{\frac{5}{2}} - 3 \times 1 - \left(\frac{1}{3^4}\right)^{-\frac{1}{2}}$

$= 3^{2\times\frac{5}{2}} - 3 - 3^{-4\times\left(-\frac{1}{2}\right)}$

$= 3^5 - 3 - 3^2$

$= 243 - 3 - 9$

$= 231$

(ii) $(64)^{\frac{2}{3}} - \sqrt[3]{125} - \frac{1}{2^{-5}} + (27)^{-\frac{2}{3}} \times \left(\frac{25}{9}\right)^{-\frac{1}{2}}$

$= \left(4^3\right)^{\frac{2}{3}} - \sqrt[3]{5^3} - 2^5 + \left(3^3\right)^{-\frac{2}{3}} \times \left(\frac{5^2}{3^2}\right)^{-\frac{1}{2}}$

$= 4^2 - 5 - 2^5 + 3^{-2} \times \left(\frac{5}{3}\right)^{2\times\left(-\frac{1}{2}\right)}$

$= 16 - 5 - 32 + \frac{1}{3^2} \times \left(\frac{5}{3}\right)^{-1}$

$= -21 + \frac{1}{9} \times \frac{3}{5}$

$= -21 + \frac{1}{15}$

$= \frac{-315 + 1}{15}$

$= \frac{-314}{15}$

$= -20\frac{14}{15}$

(iii) $\left[\left(-\frac{2}{3}\right)^{-2}\right]^3 \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

$= \left[\left(-\frac{3}{2}\right)^{2}\right]^3 \times (3)^4 \times \frac{1}{3} \times \frac{1}{3\times 2}$

$= \left(-\frac{3}{2}\right)^{6} \times (3)^2 \times \frac{1}{2}$

$= \frac{3^{6+2}}{2^{6+1}}$

$= \frac{3^8}{2^7}$

2. Simplify:

$$\frac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$$

Solution:

$$\frac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$$

$$= \frac{3 \times \left(3^2\right)^{n+1} - 3^2 \times 3^{2n}}{3 \times 3^{2n+3} - \left(3^2\right)^{n+1}}$$

$$= \frac{3^{1+2n+2} - 3^{2+2n}}{3^{1+2n+3} - 3^{2n+2}}$$

$$= \frac{3^{3+2n} - 3^{2+2n}}{3^{4+2n} - 3^{2n+2}}$$

$$= \frac{3^{2n}\left(3^3 - 3^2\right)}{3^{2n}\left(3^4 - 3^2\right)}$$

$$= \frac{27 - 9}{81 - 9}$$

$$= \frac{18}{72}$$

$$= \frac{1}{4}$$

3. Solve:

$$3^{x-1} \times 5^{2y-3} = 225$$

Solution:

$$3^{x-1} \times 5^{2y-3} = 225$$

$$\Rightarrow 3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$$

$$\Rightarrow x - 1 = 2 \text{ and } 2y - 3 = 2$$

$$\Rightarrow x = 3 \text{ and } 2y = 5$$

$$\Rightarrow x = 3 \text{ and } y = \frac{5}{2}$$

$$\Rightarrow x = 3 \text{ and } y = 2\frac{1}{2}$$

4. If

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right)^{-5} = a^x \cdot b^y$$, find x+y.

Solution:

$$\left(\frac{a^{-1}b^{2}}{a^{2}b^{-4}}\right)^{7} \div \left(\frac{a^{3}b^{-5}}{a^{-2}b^{3}}\right)^{-5} = a^{x} \cdot b^{y}$$

$$\Rightarrow \left(\frac{b^{6}}{a^{3}}\right)^{7} \div \left(\frac{a^{5}}{b^{8}}\right)^{-5} = a^{x} \cdot b^{y}$$

$$\Rightarrow \left(\frac{b^{6}}{a^{3}}\right)^{7} \div \left(\frac{b^{8}}{a^{5}}\right)^{5} = a^{x} \cdot b^{y}$$

$$\Rightarrow \frac{b^{42}}{a^{21}} \div \frac{b^{40}}{a^{25}} = a^{x} \cdot b^{y}$$

$$\Rightarrow \frac{b^{42}}{a^{21}} \times \frac{a^{25}}{b^{40}} = a^{x} \cdot b^{y}$$

$$\Rightarrow b^{2} \times a^{4} = a^{x} \times b^{y}$$

$$\Rightarrow x = 4 \text{ and } y = 2$$

$$\Rightarrow x + y = 4 + 2 = 6$$

5. If $3^{x+1}=9^{x-3}$, find the value of 2^{1+x}.

Solution:

$$3^{x+1} = 9^{x-3}$$

$$\Rightarrow 3^{x} \times 3 = \left(3^{2}\right)^{x-3}$$

$$\Rightarrow 3^{x} \times 3 = 3^{2x-6}$$

$$\Rightarrow 3^{x} \times 3 = \frac{3^{2x}}{3^{6}}$$

$$\Rightarrow 3^{6} \times 3 = \frac{3^{2x}}{3^{x}}$$

$$\Rightarrow 3^{7} = 3^{x}$$

$$\Rightarrow x = 7$$

$$\Rightarrow 2^{1+x} = 2^{1+7} = 2^{8} = 256$$

6. If $2^{x}=4^{y}=8^{z}$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = 4$, find the value of x.

Solution:

$2^x = 4^y = 8^z$
$\Rightarrow 2^x = 2^{2y} = 2^{3z}$
$\Rightarrow x = 2y = 3z$
$\Rightarrow y = \frac{x}{2}$ and $z = \frac{x}{3}$

Now, $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = 4$

$\Rightarrow \frac{1}{2x} + \frac{1}{\frac{4x}{2}} + \frac{1}{\frac{8x}{3}} = 4$

$\Rightarrow \frac{1}{2x} + \frac{2}{4x} + \frac{3}{8x} = 4$

$\Rightarrow \frac{1}{2x} + \frac{1}{2x} + \frac{3}{8x} = 4$

$\Rightarrow \frac{4+4+3}{8x} = 4$

$\Rightarrow \frac{11}{8x} = 4$

$\Rightarrow x = \frac{11}{32}$

7. If

$$\frac{9^n \cdot 3^2 \cdot 3^n - (27)^n}{(3^m \cdot 2)^3} = 3^{-3}$$

Show that: m−n=1

Solution:

$$\frac{9^n \cdot 3^2 \cdot 3^n - (27)^n}{(3^m \cdot 2)^3} = 3^{-3}$$

$$\Rightarrow \frac{3^{2n} \cdot 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \cdot 3^2 - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{1}{3^{3(m-n)}} = \frac{1}{3^{3\times 1}}$$

$$\Rightarrow m - n = 1 \quad \text{(proved)}$$

8. Solve for x:

$$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

Solution:

$$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$
$$\Rightarrow (13)^{\sqrt{x}} = 256 - 81 - 6$$
$$\Rightarrow (13)^{\sqrt{x}} = 169$$
$$\Rightarrow (13)^{\sqrt{x}} = 13^2$$
$$\Rightarrow \sqrt{x} = 2$$
$$\Rightarrow x = 4$$

9. If $3^{4x} = (81)^{-1}$ and $(10)^{\frac{1}{y}} = 0.0001$ find the value of: $2^{-x} \times 16^{y}$.

Solution:

$$3^{4x} = (81)^{-1} \text{ and } (10)^{\frac{1}{y}} = 0.0001$$
$$\Rightarrow 3^{4x} = (3^4)^{-1} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10000}$$
$$\Rightarrow 3^{4x} = 3^{-4} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10^4}$$
$$\Rightarrow 4x = -4 \text{ and } (10)^{\frac{1}{y}} = 10^{-4}$$
$$\Rightarrow x = -1 \text{ and } \frac{1}{y} = -4$$
$$\Rightarrow x = -1 \text{ and } y = -\frac{1}{4}$$
$$\therefore 2^{-x} \times 16^{y} = 2^{-(-1)} \times 16^{-\frac{1}{4}}$$
$$= 2 \times 2^{4\times\left(-\frac{1}{4}\right)}$$
$$= 2 \times 2^{-1}$$
$$= 2^{1-1}$$
$$= 2^{0}$$
$$= 1$$

10. Solve:

$$3(2^x + 1) - 2^{x+2} + 5 = 0$$

Solution:

$$3(2^x+1)-2^{x+2}+5=0$$
$$\Rightarrow 3\times 2^x+3-2^x\times 2^2+5=0$$
$$\Rightarrow 2^x(3-2^2)+8=0$$
$$\Rightarrow 2^x(3-4)=-8$$
$$\Rightarrow 2^x\times(-1)=-8$$
$$\Rightarrow 2^x=8$$
$$\Rightarrow 2^x=2^3$$
$$\Rightarrow x=3$$

11.If $(a^m)^n=a^m.a^n$, find the value of:
$m(n-1)-(n-1)$

Solution:

$$(a^m)^n=a^m\cdot a^n$$
$$\Rightarrow a^{mn}=a^{m+n}$$
$$\Rightarrow mn=m+n \quad ....(1)$$
Now,
$$m(n-1)-(n-1)$$
$$=mn-m-n+1$$
$$=m+n-m-n+1 \quad ....[\text{From }(1)]$$
$$=1$$

12. If $m=\sqrt[3]{15}$ and $n=\sqrt[3]{14}$, find the value of $m-n-\dfrac{1}{m^2+mn+n^2}$

Solution:

$$m=\sqrt[3]{15} \text{ and } n=\sqrt[3]{14}$$
$$\Rightarrow m^3=15 \text{ and } n^3=14$$
$$\therefore m-n-\frac{1}{m^2+mn+n^2}=\frac{(m^3+m^2n+mn^2)-(m^2n+mn^2+n^3)-1}{m^2+mn+n^2}$$
$$=\frac{m^3+m^2n+mn^2-m^2n-mn^2-n^3-1}{m^2+mn+n^2}$$
$$=\frac{m^3-n^3-1}{m^2+mn+n^2}$$
$$=\frac{15-14-1}{m^2+mn+n^2}$$
$$=\frac{1-1}{m^2+mn+n^2}$$
$$=0$$

13. Evaluate:

$$\frac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$$

Solution:

$$\frac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$$

$$= \frac{2^n \times 6^m \times 6 \times 10^m \times 10^{-n} \times 15^m \times 15^n \times 15^{-2}}{4^m \times (3^2)^m \times 3^n \times 25^m \times 25^{-1}}$$

$$= \frac{\left(2 \times \frac{1}{10} \times 15\right)^n \times (6 \times 10 \times 15)^m \times 6 \times \frac{1}{15^2}}{3^n \times \left(4 \times 3^2 \times 25\right)^m \times \frac{1}{25}}$$

$$= \frac{3^n \times 900^m \times \frac{6}{225}}{3^n \times 900^m \times \frac{1}{25}}$$

$$= \frac{6}{225} \times \frac{25}{1}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

14. Evaluate:

$$\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$$

Solution:

$$\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$$

$$= \left(\frac{x^{q \times \frac{1}{qr}}}{x^{r \times \frac{1}{qr}}}\right) \times \left(\frac{x^{r \times \frac{1}{rp}}}{x^{p \times \frac{1}{rp}}}\right) \times \left(\frac{x^{p \times \frac{1}{pq}}}{x^{q \times \frac{1}{pq}}}\right)$$

$$= \frac{x^{\frac{1}{r}}}{x^{\frac{1}{q}}} \times \frac{x^{\frac{1}{p}}}{x^{\frac{1}{r}}} \times \frac{x^{\frac{1}{q}}}{x^{\frac{1}{p}}}$$

$$= 1$$

15. Prove that:

(i)

$$\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}=\frac{2b^2}{b^2-a^2}$$

(ii)

$$\frac{a+b+c}{a^{-1}b^{-1}+b^{-1}c^{-1}+c^{-1}a^{-1}}=abc$$

Solution:

(i)

$$\text{L.H.S.}=\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}$$

$$=\frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}}+\frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}}$$

$$=\frac{\frac{1}{a}}{\frac{b+a}{ab}}+\frac{\frac{1}{a}}{\frac{b-a}{ab}}$$

$$=\frac{1}{a}\times\frac{ab}{b+a}+\frac{1}{a}\times\frac{ab}{b-a}$$

$$=\frac{b}{b+a}+\frac{b}{b-a}$$

$$=\frac{b^2-ab+b^2+ab}{b^2-a^2}$$

$$=\frac{2b^2}{b^2-a^2}$$

$$=\text{R.H.S.}$$

(ii)

$$\text{L.H.S.}=\frac{a+b+c}{a^{-1}b^{-1}+b^{-1}c^{-1}+c^{-1}a^{-1}}$$

$$=\frac{a+b+c}{\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}}$$

$$=\frac{a+b+b}{\frac{c+a+b}{abc}}$$

$$=\frac{(a+b+c)(abc)}{(a+b+c)}$$

$$=abc$$

$$=\text{R.H.S.}$$

16. Evaluate:

$$\frac{4}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{2}{(243)^{-\frac{1}{5}}}$$

Solution:

$$\frac{4}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{2}{(243)^{-\frac{1}{5}}}$$

$$=\frac{4}{(6^3)^{-\frac{2}{3}}}+\frac{1}{(4^4)^{-\frac{3}{4}}}+\frac{2}{(3^5)^{-\frac{1}{5}}}$$

$$=\frac{4}{6^{-2}}+\frac{1}{4^{-3}}+\frac{2}{3^{-1}}$$

$$= 4\times 6^2 + 1\times 4^3 + 2\times 3$$

$$= 4\times 36 + 1\times 64 + 6$$

$$= 144 + 64 + 6$$

$$= 214$$