EXERCISE 8(A)

PAGE: 103

1. Express each of the following in logarithmic form:

(i)

$5^3 = 125$

(ii)

$3^{-2} = \frac{1}{9}$

(iii)

$10^{-3} = 0.001$

(iv)

$(81)^{\frac{3}{4}} = 27$

Solution:

(i)

$5^3 = 125$

$\Rightarrow \log_5 125 = 3 \quad [a^b = c \Rightarrow \log_a c = b]$

(ii)

$3^{-2} = \frac{1}{9}$

$\Rightarrow \log_3 \frac{1}{9} = -2 \quad [a^b = c \Rightarrow \log_a c = b]$

(iii)

$10^{-3} = 0.001$

$\Rightarrow \log_{10} 0.001 = -3 \quad [a^b = c \Rightarrow \log_a c = b]$

(iv)

$(81)^{\frac{3}{4}} = 27$

$\Rightarrow \log_{81} 27 = \frac{3}{4}$ [By definition of logarithm, $a^b = c \Rightarrow \log_a c = b$]

2. Express each of the following in exponential form:

(i)

$\log_8 0.125 = -1$

(ii)

$\log_{10} 0.01 = -2$

(iii)

$\log_a A = x$

(iv)

$\log_{10} 1 = 0$

Solution:

(i)

$\log_8 0.125 = -1$

$\Rightarrow 8^{-1} = 0.125 \quad [\log_a c = b \Rightarrow a^b = c]$

(ii)

$\log_{10} 0.01 = -2$

$\Rightarrow 10^{-2} = 0.01 \quad [\log_a c = b \Rightarrow a^b = c]$

(iii)

$\log_a A = x$

$\Rightarrow a^x = A \quad [\log_a c = b \Rightarrow a^b = c]$

(iv)

$\log_{10} 1 = 0$

$\Rightarrow 10^0 = 1 \quad [\log_a c = b \Rightarrow a^b = c]$

3. Solve for x:

$\log_{10} x = -2$

Solution:

$\log_{10} x = -2$

$\Rightarrow 10^{-2} = x \quad [\log_a c = b \Rightarrow a^b = c]$

$\Rightarrow x = 10^{-2}$

$\Rightarrow x = \frac{1}{10^2}$

$\Rightarrow x = \frac{1}{100}$

$\Rightarrow x = 0.01$

4. Find the logarithm of:

i. 100 to the base 10
ii. 0.1 to the base 10
iii. 0.001 to the base 10
iv. 32 to the base 4
v. 0.125 to the base 2
vi. $\frac{1}{16}$ to the base 4
vii. 27 to the base 9
viii. $\frac{1}{81}$ to the base 27

Solution:

(i)

Let $\log_{10}100 = x$

$\therefore 10^x = 100$

$\Rightarrow 10^x = 10 \times 10$

$\Rightarrow 10^x = 10^2$

$\Rightarrow x = 2$ [if $a^m = a^n$; then m=n]

$\therefore \log_{10}100 = 2$

(ii)

Let $\log_{10}0.1 = x$

$\therefore 10^x = 0.1$

$\Rightarrow 10^x = \frac{1}{10}$

$\Rightarrow 10^x = 10^{-1}$

$\Rightarrow x = -1$ [if $a^m = a^n$; then m=n]

$\therefore \log_{10}0.1 = -1$

(iii)

Let $\log_{10}0.001 = x$

$\therefore 10^x = 0.001$

$\Rightarrow 10^x = \frac{1}{1000}$

$\Rightarrow 10^x = \frac{1}{10^3}$

$\Rightarrow 10^x = 10^{-3}$

$\Rightarrow x = -3$ [if $a^m = a^n$; then m=n]

$\therefore \log_{10}0.001 = -3$

(iv)

Let $\log_4 32 = x$

$\therefore\ 4^x = 32$

$\Rightarrow (2^2)^x = 2 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow 2^{2x} = 2^5$

$\Rightarrow 2x = 5$ [if $a^m = a^n$; then m=n]

$\Rightarrow x = \frac{5}{2}$

$\therefore \log_4 32 = \frac{5}{2}$

(v)

Let $\log_2 0.125 = x$

$\therefore\ 2^x = 0.125$

$\Rightarrow 2^x = \frac{125}{1000}$

$\Rightarrow 2^x = \frac{1}{8}$

$\Rightarrow 2^x = 8^{-1}$

$\Rightarrow 2^x = (2 \times 2 \times 2)^{-1}$

$\Rightarrow 2^x = (2^3)^{-1}$

$\Rightarrow 2^x = 2^{-3}$

$\Rightarrow x = -3$ [if $a^m = a^n$; then m=n]

$\therefore \log_2 0.125 = -3$

(vi)

$$\text{Let } \log_4 \frac{1}{16} = x$$
$$\therefore \quad 4^x = \frac{1}{16}$$
$$\Rightarrow 4^x = \frac{1}{4 \times 4}$$
$$\Rightarrow 4^x = (4 \times 4)^{-1}$$
$$\Rightarrow 4^x = (4^2)^{-1}$$
$$\Rightarrow 4^x = 4^{-2}$$
$$\Rightarrow x = -2 \quad [\text{if } a^m = a^n; \text{then } m=n]$$
$$\therefore \log_4 \frac{1}{16} = -2$$

(vii)

$$\text{Let } \log_9 27 = x$$
$$\therefore 9^x = 27$$
$$\Rightarrow (3 \times 3)^x = 3 \times 3 \times 3$$
$$\Rightarrow \quad (3^2)^x = (3^3)$$
$$\Rightarrow \quad 3^{2x} = (3^3)$$
$$\Rightarrow \quad 2x = 3 \quad [\text{if } a^m = a^n; \text{then } m=n]$$
$$\Rightarrow \quad x = \frac{3}{2}$$
$$\therefore \log_9 27 = \frac{3}{2}$$

(viii)

Let $\log_{27}\frac{1}{81} = x$

$\therefore 27^x = \frac{1}{81}$

$\Rightarrow (3\times3\times3)^x = \frac{1}{3\times3\times3\times3}$

$\Rightarrow \quad (3^3)^x = \frac{1}{3^4}$

$\Rightarrow \quad (3^3)^x = (3^4)^{-1}$

$\Rightarrow \quad 3^{3x} = (3^{-4})$

$\Rightarrow \quad 3x = -4 \quad [\text{if } a^m = a^n; \text{then } m=n]$

$\Rightarrow \quad x = \frac{-4}{3}$

$\therefore \log_{27}\frac{1}{81} = \frac{-4}{3}$

5. State, true or false:

(i) If $\log_{10}x=a$, then $10^x=a$.

(ii) If $x^y=z$, then $y=\log_z x$.

(iii) $\log_2 8=3$ and $\log_8 2=\frac{1}{3}$

Solution:

(i)

Consider the equation

$\log_{10} x = a$

$\Rightarrow 10^a = x$

Thus the statement, $10^x = a$ is false

(ii)

Consider the equation

$x^y = z$

$\Rightarrow \log_x z = y$

Thus the statement, $\log_z x = y$ is false

(iii)

Consider the equation

$\log_2 8 = 3$

$\Rightarrow 2^3 = 8....(1)$

Now consider the equation

$\log_8 2 = \frac{1}{3}$

$\Rightarrow 8^{\frac{1}{3}} = 2$

$\Rightarrow \left(2^3\right)^{\frac{1}{3}} = 2....(2)$

Both the equations (1) and (2) are correct

Thus the given statements, $\log_2 8 = 3$ and $\log_8 2 = \frac{1}{3}$ are true

6. Find x if:

(i) $\log_3 x=0$

(ii) $\log_x 2=-1$

(iii) $\log_9 243=x$

(iv) $\log_5 (x-7)=1$

(v) $\log_4 32=x-4$

(vi) $\log_7 (2x^2-1)=2$

Solution:

(i)

Consider the equation

$\log_3 x = 0$

$\Rightarrow 3^0 = x$

$\Rightarrow 1 = x$ or $x=1$

(ii)

Consider the equation

$\log_x 2 = -1$

$\Rightarrow x^{-1} = 2$

$\Rightarrow \frac{1}{x} = 2$

$\Rightarrow x=\frac{1}{2}$

(iii)

Consider the equation

$\log_9 243 = x$

$\Rightarrow 9^x = 243$

$\Rightarrow (3^2)^x = 3^5$

$\Rightarrow 3^{2x} = 3^5$

$\Rightarrow 2x = 5$

$\Rightarrow x = \frac{5}{2}$

$\Rightarrow x = 2\frac{1}{2}$

(iv)

Consider the equation

$\log_5 (x - 7) = 1$

$\Rightarrow 5^1 = x - 7$

$\Rightarrow 5 = x - 7$

$\Rightarrow x = 5 + 7$

$\Rightarrow x = 12$

(v)

Consider the equation

$\log_4 32 = x - 4$

$\Rightarrow 4^{x-4} = 32$

$\Rightarrow (2^2)^{x-4} = 2^5$

$\Rightarrow 2^{2(x-4)} = 2^5$

$\Rightarrow 2x - 8 = 5$

$\Rightarrow 2x = 5 + 8$

$\Rightarrow 2x = 13$

$\Rightarrow x = \frac{13}{2}$

$\Rightarrow x = 6\frac{1}{2}$

(vi)

Consider the equation

$\log_7(2x^2 - 1) = 2$

$\Rightarrow 7^2 = 2x^2 - 1$

$\Rightarrow 7 \times 7 = 2x^2 - 1$

$\Rightarrow 2x^2 - 1 - 49 = 0$

$\Rightarrow 2x^2 - 50 = 0$

$\Rightarrow 2x^2 = 50$

$\Rightarrow x^2 = \frac{50}{2}$

$\Rightarrow x^2 = 25$

$\Rightarrow x = \pm\sqrt{25}$

$\Rightarrow x = 5$ [neglecting the negative value]

7. Evaluate:

(i)

$\log_{10} 0.01$

(ii)

$\log_2 \frac{1}{8}$

(iii)

$\log_5 1$

(iv)

$\log_5 125$

(v)

$\log_{16} 8$

(vi)

$\log_{0.5} 16$

Solution:

(i)

Let $\log_{10} 0.01 = x$

$\Rightarrow 10^x = 0.01$

$\Rightarrow 10^x = \frac{1}{100}$

$\Rightarrow 10^x = \frac{1}{10 \times 10}$

$\Rightarrow 10^x = \frac{1}{10^2}$

$\Rightarrow 10^x = 10^{-2}$

$\Rightarrow x = -2$

Thus, $\log_{10} 0.01 = -2$

(ii)

Let $\log_2 \frac{1}{8} = x$

$\Rightarrow 2^x = \frac{1}{8}$

$\Rightarrow 2^x = \frac{1}{2 \times 2 \times 2}$

$\Rightarrow 2^x = \frac{1}{2^3}$

$\Rightarrow 2^x = 2^{-3}$

$\Rightarrow x = -3$

Thus, $\log_2 \frac{1}{8} = -3$

(iii)

Let $\log_5 1 = x$

$\Rightarrow 5^x = 1$

$\Rightarrow 5^x = 5^0$

$\Rightarrow x = 0$

Thus, $\log_5 1 = 0$

(iv)

Let $\log_5 125 = x$

$\Rightarrow 5^x = 125$

$\Rightarrow 5^x = 5 \times 5 \times 5$

$\Rightarrow 5^x = 5^3$

$\Rightarrow x = 3$

Thus, $\log_5 125 = 3$

(v)

Let $\log_{16} 8 = x$

$\Rightarrow 16^x = 8$

$\Rightarrow (2 \times 2 \times 2 \times 2)^x = 2 \times 2 \times 2$

$\Rightarrow (2^4)^x = 2^3$

$\Rightarrow 2^{4x} = 2^3$

$\Rightarrow 4x = 3$

$\Rightarrow x = \frac{3}{4}$

Thus, $\log_{16} 8 = \frac{3}{4}$

(vi)

Let $\log_{0.5} 16 = x$

$\Rightarrow 0.5^x = 16$

$\Rightarrow \left(\frac{5}{10}\right)^x = 2 \times 2 \times 2 \times 2$

$\Rightarrow \left(\frac{1}{2}\right)^x = 2^4$

$\Rightarrow \frac{1}{2^x} = 2^4$

$\Rightarrow 2^{-x} = 2^4$

$\Rightarrow -x = 4$

$\Rightarrow x = -4$

Thus, $\log_{0.5} 16 = -4$

8. If $\log_a m = n$, express a^{n-1} in terms of a and m.

Solution:

$\log_a m = n$

$\Rightarrow a^n = m$

$\Rightarrow \frac{a^n}{a} = \frac{m}{a}$

$\Rightarrow a^{n-1} = \frac{m}{a}$

9. Given $\log_2 x = m$ and $\log_5 y = n$.

(i) Express 2^{m-3} in terms of x.

(ii) Express 5^{3n+2} in terms of y.

Solution:

$\log_2 x = m$ and $\log_5 y = n$

$\Rightarrow 2^m = x$ and $5^n = y$

(i) Consider $2^m = x$

$\Rightarrow \frac{2^m}{2^3} = \frac{x}{2^3}$

$\Rightarrow 2^{m-3} = \frac{x}{8}$

(ii) Consider $5^n = y$

$\Rightarrow (5^n)^3 = y^3$

$\Rightarrow 5^{3n} = y^3$

$\Rightarrow 5^{3n} \times 5^2 = y^3 \times 5^2$

$\Rightarrow 5^{3n+2} = 25y^3$

10.If $\log_2 x=a$ and $\log_5 y=a$, write 72 in terms of x and y.

Solution:

Given that :

$\log_2^x = a$ and $\log_3^y = a$

$\Rightarrow 2^a = x$ and $3^a = y$ $\quad \left[\begin{array}{l} \because \log_a^m = n \\ \Rightarrow a^n = m \end{array}\right]$

Now prime factorization of 72 is

$72 = 2\times2\times2\times3\times3$

Hence,

$(72)^a = (2\times2\times2\times3\times3)^a$

$= (2^3 \times 3^2)^a$

$= 2^{3a} \times 3^{2a}$

$= (2^a)^3 \times (3^a)^2$ $\quad \left[\begin{array}{l} \text{as } 2^a = x \\ 3^a = y \end{array}\right]$

$= x^3y^2$

11.Solve for x: $\log(x-1)+\log(x+1)=\log_2 1$.

Solution:

$\log(x-1) + \log(x+1) = \log_2 1$

$\Rightarrow \log(x-1) + \log(x+1) = 0$

$\Rightarrow \log[(x-1)(x+1)] = 0$

$\Rightarrow (x-1)(x+1) = 1$....(Since $\log 1 = 0$)

$\Rightarrow x^2 - 1 = 1$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

$-\sqrt{2}$ cannot be possible, since log of a negative number is not defined.

So, $x = \sqrt{2}$.

12.If $\log(x^2-21)=2$, show that $x=\pm11$.

Solution:

$\log(x^2-21) = 2$

$\Rightarrow x^2 - 21 = 10^2$

$\Rightarrow x^2 - 21 = 100$

$\Rightarrow x^2 = 121$

$\Rightarrow x = \pm 11$

EXERCISE 8(B) PAGE: 106

1. Express in terms of log 2 and log 3.

(i)

$\log 36$

(ii)

$\log 144$

(iii)

$\log 4.5$

(iv)

$\log \frac{26}{51} - \log \frac{91}{119}$

(v)

$\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243}$

Solution:

(i)

$$\log 36 = \log(2 \times 2 \times 3 \times 3)$$
$$= \log(2^2 \times 3^2)$$
$$= \log(2^2) + \log(3^2) \quad [\log_a mn = \log_a m + \log_a n]$$
$$= 2\log 2 + 2\log 3 \quad [\log_a m^n = n\log_a m]$$

(ii)

$$\log 144 = \log(2 \times 2 \times 2 \times 2 \times 3 \times 3)$$
$$= \log(2^4 \times 3^2)$$
$$= \log(2^4) + \log(3^2) \quad [\log_a mn = \log_a m + \log_a n]$$
$$= 4\log 2 + 2\log 3 \quad [\log_a m^n = n\log_a m]$$

(iii)

$$\log 4.5 = \log \frac{45}{10}$$
$$= \log \frac{5 \times 3 \times 3}{5 \times 2}$$
$$= \log \frac{3^2}{2}$$
$$= \log 3^2 - \log 2 \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$$
$$= 2\log 3 - \log 2 \quad [\log_a m^n = n\log_a m]$$

(iv)

$$\log\frac{26}{51}-\log\frac{91}{119}=\log\frac{\frac{26}{51}}{\frac{91}{119}} \quad [\log_a m-\log_a n=\log_a\frac{m}{n}]$$

$$=\log\frac{26}{51}\times\frac{119}{91}$$

$$=\log\frac{2\times13}{3\times17}\times\frac{7\times17}{7\times13}$$

$$=\log\frac{2}{3}$$

$$=\log2-\log3 \quad [\log_a\frac{m}{n}=\log_a m-\log_a n]$$

(v)

$$\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}$$

$$=\log\frac{75}{16}-\log\left(\frac{5}{9}\right)^2+\log\frac{32}{243} \quad [n\log_a m=\log_a m^n]$$

$$=\log\frac{75}{16}-\log\frac{5}{9}\times\frac{5}{9}+\log\frac{32}{243}$$

$$=\log\frac{75}{16}-\log\frac{25}{81}+\log\frac{32}{243}$$

$$=\log\left(\frac{\frac{75}{16}}{\frac{25}{81}}\right) \quad [\log_a m-\log_a n=\log_a\frac{m}{n}]$$

$$=\log\frac{75}{16}\times\frac{81}{25}+\log\frac{32}{243}$$

$$=\log\frac{3\times25}{16}\times\frac{81}{25}+\log\frac{32}{243}$$

$$=\log\frac{3\times81}{16}+\log\frac{32}{243}$$

$$=\log\frac{243}{16}+\log\frac{32}{243}$$

$$=\log\frac{243}{16}\times\frac{32}{243} \quad [\log_a m+\log_a n=\log_a mn]$$

$$=\log\frac{32}{16}$$

$$=\log2$$

2. Express each of the following in a form free from logarithm:
   (i) 2logx−logy=1
   (ii) 2logx+3logy=loga
   (iii) alogx−blogy=2log3

Solution:

(i)

Consider the given equation

$2\log x - \log y = 1$

$\Rightarrow \log x^2 - \log y = 1$

$\Rightarrow \log \frac{x^2}{y} = \log 10$

$\Rightarrow \frac{x^2}{y} = 10$

$\Rightarrow x^2 = 10y$

(ii)

Consider the given equation

$2\log x + 3\log y = \log a$

$\Rightarrow \log x^2 + \log y^3 = \log a$

$\Rightarrow \log x^2y^3 = \log a$

$\Rightarrow x^2y^3 = a$

(iii)

Consider the given equation

$a\log x - b\log y = 2\log 3$

$\Rightarrow \log x^a - \log y^b = \log 3^2$

$\Rightarrow \log \frac{x^a}{y^b} = \log 9$

$\Rightarrow \frac{x^a}{y^b} = 9$

$\Rightarrow x^a = 9y^b$

3. Evaluate each of the following without using tables:
   (i) $\log 5 + \log 8 - 2\log 2$
   (ii) $\log_{10}8 + \log_{10}25 + 2\log_{10}3 - \log_{10}18$
   (iii) $\log 4 + \frac{1}{3}\log 125 - \frac{1}{5}\log 32$

Solution:

(i) Consider the given expression

$\log 5+\log 8-2\log 2=\log 5+\log 8\times 8-\log 2^2 \quad [n\log_a m=\log_a m^n]$

$=\log 5\times 8-\log 2^2 \quad [\log_a m+\log_a n=\log_a mn]$

$=\log 40-\log 4$

$=\log\frac{40}{4} \quad [\log_a m-\log_a n=\log_a \frac{m}{n}]$

$=\log 10$

$=1$

(ii) Consider the given expression

$\log_{10}8+\log_{10}25+2\log_{10}3-\log_{10}18$

$=\log_{10}8+\log_{10}25+\log_{10}3^2-\log_{10}18$

$[n\log_a m=\log_a m^n]$

$=\log_{10}8+\log_{10}25+\log_{10}9-\log_{10}18$

$=\log_{10}8\times 25\times 9-\log_{10}18$

$[\log_a l+\log_a m+\log_a n=\log_a lmn]$

$=\log_{10}1800-\log_{10}18$

$=\log_{10}\frac{1800}{18} \quad [\log_a m-\log_a n=\log_a \frac{m}{n}]$

$=\log_{10}100$

$=2 \quad [\because \log_{10}100=2]$

(iii) Consider the given expression

$\log 4+\frac{1}{3}\log 125-\frac{1}{5}\log 32$

$=\log 4+\log(125)^{\frac{1}{3}}-\log(32)^{\frac{1}{5}} \quad [n\log_a m=\log_a m^n]$

$=\log 4+\log(5^3)^{\frac{1}{3}}-\log(2^5)^{\frac{1}{5}}$

$=\log 4+\log 5-\log 2$

$=\log 4\times 5-\log 2 \quad [\log_a m+\log_a n=\log_a mn]$

$=\log\frac{20}{2} \quad [\log_a m-\log_a n=\log_a \frac{m}{n}]$

$=\log 10$

$=1$

4. Prove that:

$$2\log\frac{15}{18} - \log\frac{25}{162} + \log\frac{4}{9} = \log 2$$

Solution:

We need to prove that

$$2\log\frac{15}{18} - \log\frac{25}{162} + \log\frac{4}{9} = \log 2$$

$$L.H.S = 2\log\frac{15}{18} - \log\frac{25}{162} + \log\frac{4}{9}$$

$$= \log\left(\frac{15}{18}\right)^2 - \log\frac{25}{162} + \log\frac{4}{9} \qquad [n\log_a m = \log_a m^n]$$

$$= \log\left[\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\right] - \log\frac{25}{162} + \log\frac{4}{9}$$

$$= \log\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9} - \log\frac{25}{162} \qquad [\log_a m + \log_a n = \log_a mn]$$

$$= \log\frac{\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}}{\frac{25}{162}} \qquad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$

$$= \log\left(\frac{15}{18}\right)\times\left(\frac{15}{18}\right)\times\frac{4}{9}\times\frac{162}{25}$$

$$= \log\frac{72}{36}$$

$$= \log 2$$

$$= R.H.S$$

5. Find x, if:

$$x - \log 48 + 3\log 2 = \frac{1}{3}\log 125 - \log 3$$

Solution:

Consider the given equation

$x - \log 48 + 3\log 2 = \frac{1}{3}\log 125 - \log 3$

$\Rightarrow x = \frac{1}{3}\log 125 - \log 3 + \log 48 - 3\log 2$

$\Rightarrow x = \log(125)^{\frac{1}{3}} - \log 3 + \log 48 - \log 2^3 \quad [n\log_a m = \log_a m^n]$

$\Rightarrow x = \log(5\times5\times5)^{\frac{1}{3}} - \log 3 + \log 48 - \log 8$

$\Rightarrow x = \log(5^3)^{\frac{1}{3}} - \log 3 + \log 48 - \log 8$

$\Rightarrow x = \log 5 - \log 3 + \log 48 - \log 8$

$\Rightarrow x = \log 5 + \log 48 - \log 3 - \log 8$

$\Rightarrow x = (\log 5 + \log 48) - (\log 3 + \log 8)$

$\Rightarrow x = (\log 5\times 48) - (\log 3\times 8) \quad [\log_a m + \log_a n = \log_a mn]$

$\Rightarrow x = \log\frac{5\times 48}{3\times 8} \quad [\log_a m - \log_a n = \log_a \frac{m}{n}]$

$\Rightarrow x = \log\frac{5\times 6\times 8}{3\times 8}$

$\Rightarrow x = \log 10$

$\Rightarrow x = 1$

6. Express $\log_{10}2+1$ in the form of $\log_{10}x$.

Solution:

$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10 \quad [\because \log_{10} 10 = 1]$

$= \log_{10} 2\times 10 \quad [\log_a m + \log_a n = \log_a mn]$

$= \log_{10} 20$

7. Solve for x:

(i) $\log_{10}(x-10)=1$

(ii) $\log(x^2-21)=2$

(iii) $\log(x-2)+\log(x+2)=\log 5$

(iv) $\log(x+5)+\log(x-5)=4\log 2+2\log 3$

Solution:

(i)

$\log_{10}(x-10) = 1$

$\Rightarrow \log_{10}(x-10) = \log_{10}10$

$\Rightarrow x - 10 = 10$

$\Rightarrow x = 10 + 10$

$\Rightarrow x = 20$

(ii)

$$\log\left(x^2-21\right)=2$$
$$\Rightarrow \log\left(x^2-21\right)=\log 100$$
$$\Rightarrow x^2-21=100$$
$$\Rightarrow x^2-21-100=0$$
$$\Rightarrow x^2-121=0$$
$$\Rightarrow x^2=121$$
$$\Rightarrow x=\pm\sqrt{121}$$
$$\Rightarrow x=\pm 11$$

(iii)

$$\log(x-2)+\log(x+2)=\log 5$$
$$\Rightarrow \log(x-2)(x+2)=\log 5 \quad [\log_a m+\log_a n=\log_a mn]$$
$$\Rightarrow \log\left(x^2-4\right)=\log 5$$
$$\Rightarrow x^2-4=5$$
$$\Rightarrow x^2=9$$
$$\Rightarrow x=\pm\sqrt{9}$$
$$\Rightarrow x=\pm\sqrt{3^2}$$
$$\Rightarrow x=\pm 3$$

(iv)

$$\log(x+5)+\log(x-5)=4\log 2+2\log 3$$
$$\Rightarrow \log(x+5)(x-5)=4\log 2+2\log 3 \quad [\log_a m+\log_a n=\log_a mn]$$
$$\Rightarrow \log\left(x^2-25\right)=\log 2^4+\log 3^2 \quad [n\log_a m=\log_a m^n]$$
$$\Rightarrow \log\left(x^2-25\right)=\log 16+\log 9$$
$$\Rightarrow \log\left(x^2-25\right)=\log 16\times 9 \quad [\log_a m+\log_a n=\log_a mn]$$
$$\Rightarrow \log\left(x^2-25\right)=\log 144$$
$$\Rightarrow x^2-25=144$$
$$\Rightarrow x^2=144+25$$
$$\Rightarrow x^2=169$$
$$\Rightarrow x=\pm\sqrt{169}$$
$$\Rightarrow x=\pm\sqrt{13^2}$$
$$\Rightarrow x=\pm 13$$

8. Solve for x:

(i)

$$\frac{\log 81}{\log 27}=x$$

(ii)

$$\frac{\log 128}{\log 32} = x$$

(iii)

$$\frac{\log 64}{\log 8} = \log x$$

(iv)

$$\frac{\log 225}{\log 15} = \log x$$

Solution:

(i)

$$\frac{\log 81}{\log 27} = x$$

$$\Rightarrow x = \frac{\log 81}{\log 27}$$

$$\Rightarrow x = \frac{\log 3 \times 3 \times 3 \times 3}{\log 3 \times 3 \times 3}$$

$$\Rightarrow x = \frac{\log 3^4}{\log 3^3}$$

$$\Rightarrow x = \frac{4\log 3}{3\log 3} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{4}{3}$$

$$\Rightarrow x = 1\frac{1}{3}$$

(ii)

$$\frac{\log 128}{\log 32} = x$$

$$\Rightarrow x = \frac{\log 128}{\log 32}$$

$$\Rightarrow x = \frac{\log 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{\log 2 \times 2 \times 2 \times 2 \times 2}$$

$$\Rightarrow x = \frac{\log 2^7}{\log 2^5}$$

$$\Rightarrow x = \frac{7\log 2}{5\log 2} \quad [n\log_a m = \log_a m^n]$$

$$\Rightarrow x = \frac{7}{5}$$

$$\Rightarrow x = 1.4$$

(iii)

$$\frac{\log 64}{\log 8} = \log x$$

$$\Rightarrow \log x = \frac{\log 64}{\log 8}$$

$$\Rightarrow \log x = \frac{\log 2 \times 2 \times 2 \times 2 \times 2 \times 2}{\log 2 \times 2 \times 2}$$

$$\Rightarrow \log x = \frac{\log 2^6}{\log 2^3}$$

$$\Rightarrow \log x = \frac{6 \log 2}{3 \log 2} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow \log x = \frac{6}{3}$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10 \times 10$$

$$\Rightarrow x = 100$$

(iv)

$$\frac{\log 225}{\log 15} = \log x$$

$$\Rightarrow \log x = \frac{\log 225}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15 \times 15}{\log 15}$$

$$\Rightarrow \log x = \frac{\log 15^2}{\log 15}$$

$$\Rightarrow \log x = \frac{2 \log 15}{\log 15} \quad [n \log_a m = \log_a m^n]$$

$$\Rightarrow \log x = 2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow 10^2 = x$$

$$\Rightarrow x = 10 \times 10$$

$$\Rightarrow x = 100$$

9. Given log x=m+n and log y=m−n, express the value of log $\frac{10x}{y^2}$ in terms of m and n.

Solution:

Given that

$\log x = m + n;$

$\log y = m - n;$

Consider the expression $\log \frac{10x}{y^2}$:

$$\log \frac{10x}{y^2} = \log 10x - \log y^2$$
$$= \log 10x - 2\log y \qquad [n\log_a m = \log_a m^n]$$
$$= \log 10 + \log x - 2\log y \quad [\log_a m + \log_a n = \log_a mn]$$
$$= 1 + \log x - 2\log y$$
$$= 1 + m + n - 2(m - n)$$
$$= 1 + m + n - 2m + 2n$$
$$\Rightarrow \log \frac{10x}{y^2} = 1 - m + 3n$$

10. State, true or false:

(i)

$\log 1 \times \log 1000 = 0$

(ii)

$\frac{\log x}{\log y} = \log x - \log y$

(iii) **If,**

$\frac{\log 25}{\log 5} = \log x$ **, then x=2.**

(iv)

$\log x + \log y = \log x \times \log y$

Solution:

(i)

We have,

$\log 1 = 0$ and $\log 1000 = 3$

$\therefore \log 1 \times \log 1000 = 0 \times 3 = 0$

Thus the statement, $\log 1 \times \log 1000 = 0$ is true

(ii)

We know that

$\log\left(\frac{m}{n}\right) = \log m - \log n$

$\therefore \frac{\log x}{\log y} \neq \log x - \log y$

Thus the statement, $\frac{\log x}{\log y} = \log x - \log y$ is false

(iii)

Given that

$\frac{\log 25}{\log 5} = \log x$

$\Rightarrow \frac{\log 5 \times 5}{\log 5} = \log x$

$\Rightarrow \frac{\log 5^2}{\log 5} = \log x$

$\Rightarrow \frac{2\log 5}{\log 5} = \log x \quad [\log_a m^n = n\log_a m]$

$\Rightarrow 2 = \log_{10} x$

$\Rightarrow 10^2 = x$

$\Rightarrow x = 100$

Thus the statement, $x = 2$ is false

(iv)

We know that

$\log x + \log y = \log xy$

$\therefore \log x + \log y \neq \log x \times \log y$

Thus the statement $\log x + \log y = \log x \times \log y$ is false

11. If $\log_{10}2=a$ and $\log_{10}3=b$; express each of the following in terms of 'a' and 'b':

(i) log 12

(ii) log 2.25

(iii) log $2\frac{1}{4}$

(iv) log 5.4

(v) log 60

(vi) log $3\frac{1}{8}$

Solution:

Given that $\log_{10} 2 = a$ and $\log_{10} 3 = b$

(i)

$\log 12 = \log 2 \times 2 \times 3$

$= \log 2 \times 2 + \log 3 \quad [\log_a mn = \log_a m + \log_a n]$

$= \log 2^2 + \log 3$

$= 2\log 2 + \log 3 \quad [n\log_a m = \log_a m^n]$

$= 2a + b \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$

(ii)

$$\log 2.25 = \log\frac{225}{100}$$
$$= \log\frac{25\times 9}{25\times 4}$$
$$= \log\frac{9}{4}$$
$$= \log\left(\frac{3}{2}\right)^2$$
$$= 2\log\left(\frac{3}{2}\right) \qquad [n\log_a m = \log_a m^n]$$
$$= 2(\log 3 - \log 2) \qquad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= 2(b - a) \qquad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$
$$= 2b - 2a$$

(iii)

$$\log 2\frac{1}{4} = \log\frac{9}{4}$$
$$= \log\left(\frac{3}{2}\right)^2$$
$$= 2\log\left(\frac{3}{2}\right) \qquad [n\log_a m = \log_a m^n]$$
$$= 2(\log 3 - \log 2) \qquad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= 2(b - a) \qquad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$
$$= 2b - 2a$$

(iv)

$$\log 5.4 = \log\frac{54}{10}$$
$$= \log\left(\frac{2\times 3\times 3\times 3}{10}\right)$$
$$= \log(2\times 3\times 3\times 3) - \log_{10} 10 \qquad [\log_a m - \log_a n = \log_a \frac{m}{n}]$$
$$= \log_{10} 2 + \log_{10} 3^3 - \log_{10} 10 \qquad [\log_a mn = \log_a m + \log_a n]$$
$$= \log_{10} 2 + 3\log_{10} 3 - \log_{10} 10 \qquad [n\log_a m = \log_a m^n]$$
$$= \log_{10} 2 + 3\log_{10} 3 - 1 \qquad [\because \log_{10} 10 = 1]$$
$$= a + 3b - 1 \qquad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]$$

(v)

$$\begin{aligned}\log 60 &= \log_{10} 10\times 2\times 3\\ &= \log_{10} 10 + \log_{10} 2 + \log_{10} 3 \quad [\log_a mn = \log_a m + \log_a n]\\ &= 1 + \log_{10} 2 + \log_{10} 3 \quad [\because \log_{10} 10 = 1]\\ &= 1 + a + b \quad [\because \log_{10} 2 = a \text{ and } \log_{10} 3 = b]\end{aligned}$$

(vi)

$$\begin{aligned}\log 3\frac{1}{8} &= \log_{10}\left(\frac{25}{8}\times\frac{4}{4}\right)\\ &= \log_{10}\left(\frac{100}{32}\right)\\ &= \log_{10} 100 - \log_{10} 32 \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]\\ &= \log_{10} 100 - \log_{10} 2^5\\ &= 2 - \log_{10} 2^5 \quad [\because \log_{10} 100 = 2]\\ &= 2 - 5\log_{10} 2 \quad [\log_a m^n = n\log_a m]\\ &= 2 - 5a \quad [\because \log_{10} 2 = a]\end{aligned}$$

12.If log 2= 0.3010 and 0.4771; find the value of:

(i) log 12

(ii) log 1.2

(iii) log 3.6

(iv) log 15

(v) log 25

(vi) $\frac{2}{3}$ log 8

Solution:

We know that log2 = 0.3010 and log3 = 0.4771

(i)

$$\begin{aligned}\log 12 &= \log 2\times 2\times 3\\ &= \log 2\times 2 + \log 3 \quad [\log_a mn = \log_a m + \log_a n]\\ &= \log 2^2 + \log 3\\ &= 2\log 2 + \log 3 \quad [n\log_a m = \log_a m^n]\\ &= 2(0.3010) + 0.4771 \quad \left[\begin{matrix}\because \log 2 = 0.3010 \text{ and}\\ \log 3 = 0.4771\end{matrix}\right]\\ &= 1.0791\end{aligned}$$

(ii)

$$\log 1.2 = \log\frac{12}{10}$$

$$= \log 12 - \log 10 \qquad \left[\log_a \frac{m}{n} = \log_a m - \log_a n\right]$$

$$= \log 2 \times 2 \times 3 - 1 \qquad [\because \log 10 = 1]$$

$$= \log 2 \times 2 + \log 3 - 1 \qquad [\log_a mn = \log_a m + \log_a n]$$

$$= \log 2^2 + \log 3 - 1$$

$$= 2\log 2 + \log 3 - 1 \qquad [n\log_a m = \log_a m^n]$$

$$= 2(0.3010) + 0.4771 - 1 \qquad \begin{bmatrix}\because \log 2 = 0.3010 \\ \text{and } \log 3 = 0.4771\end{bmatrix}$$

$$= 1.0791 - 1$$

$$= 0.0791$$

(iii)

$$\log 3.6 = \log\frac{36}{10}$$

$$= \log 36 - \log 10 \qquad \left[\log_a \frac{m}{n} = \log_a m - \log_a n\right]$$

$$= \log 2 \times 2 \times 3 \times 3 - 1 \qquad [\because \log 10 = 1]$$

$$= \log 2 \times 2 + \log 3 \times 3 - 1 \qquad [\log_a mn = \log_a m + \log_a n]$$

$$= \log 2^2 + \log 3^2 - 1$$

$$= 2\log 2 + 2\log 3 - 1 \qquad [n\log_a m = \log_a m^n]$$

$$= 2(0.3010) + 2(0.4771) - 1 \qquad \begin{bmatrix}\because \log 2 = 0.3010 \\ \text{and } \log 3 = 0.4771\end{bmatrix}$$

$$= 1.5562 - 1$$

$$= 0.5562$$

(iv)

$$\log 15 = \log\left(\frac{15}{10} \times 10\right)$$

$$= \log\left(\frac{15}{10}\right) + \log 10$$

$$= \log\left(\frac{3}{2}\right) + 1 \qquad [\because \log 10 = 1]$$

$$= \log 3 - \log 2 + 1 \qquad \left[\because \log m - \log n = \log\left(\frac{m}{n}\right)\right]$$

$$= 0.4771 - 0.3010 + 1$$

$$= 1.1761$$

(v)

$$\log 25 = \log\left(\frac{25}{4} \times 4\right)$$
$$= \log\left(\frac{100}{4}\right) \quad [\log_a mn = \log_a m + \log_a n]$$
$$= \log 100 - \log(2 \times 2) \quad [\log_a \frac{m}{n} = \log_a m - \log_a n]$$
$$= 2 - \log(2^2) \quad [\log 100 = 2]$$
$$= 2 - 2\log 2 \quad [\log_a m^n = n\log_a m]$$
$$= 2 - 2(0.3010) \quad [\because \log 2 = 0.3010]$$
$$= 1.398$$

(vi)

$$\frac{2}{3}\log 8 = \frac{2}{3}\log 2 \times 2 \times 2$$
$$= \frac{2}{3}\log 2^3$$
$$= 3 \times \frac{2}{3}\log 2 \quad [\log_a m^n = n\log_a m]$$
$$= 2\log 2$$
$$= 2 \times 0.3010 \quad [\because \log 2 = 0.3010]$$
$$= 0.602$$

13. Given: $2\log_{10} x + 1 = \log_{10} 250$, find:

(i) x

(ii) $\log_{10} 2x$

Solution:

(i)

Consider the given equation:

$$2\log_{10} x + 1 = \log_{10} 250$$
$$\Rightarrow \log_{10} x^2 + 1 = \log_{10} 250 \quad [\log_a m^n = n\log_a m]$$
$$\Rightarrow \log_{10} x^2 + \log_{10} 10 = \log_{10} 250 \quad [\because \log_{10} 10 = 1]$$
$$\Rightarrow \log_{10}(x^2 \times 10) = \log_{10} 250 \quad [\log_a m + \log_a n = \log_a mn]$$
$$\Rightarrow x^2 \times 10 = 250$$
$$\Rightarrow x^2 = 25$$
$$\Rightarrow x = \sqrt{25}$$
$$\Rightarrow x = 5$$

(ii)

$$x = 5 \quad \text{(proved above in (i))}$$
$$\log_{10} 2x = \log_{10} 2(5)$$
$$= \log_{10} 10$$
$$= 1 \qquad [\because \log_{10} 10 = 1]$$

14. Given $3\log x + \frac{1}{2}\log y = 2$, express y in terms of x.

Solution:

$$3\log x + \frac{1}{2}\log y = 2$$
$$\Rightarrow \log x^3 + \log\sqrt{y} = 2$$
$$\Rightarrow \log x^3\sqrt{y} = 2$$
$$\Rightarrow x^3\sqrt{y} = 10^2$$
$$\Rightarrow \sqrt{y} = \frac{10^2}{x^3}$$

Squaring both sides, we get

$$y = \frac{10000}{x^6}$$
$$\Rightarrow y = 10000x^{-6}$$

15. If $x=(100)^a$, $y=(10000)^b$ and $z=(10)^c$, find $\log\frac{10\sqrt{y}}{x^2z^3}$ in terms of a ,b and c.

Solution:

$$x = (100)^a,\ y = (10000)^b \text{ and } z = (10)^c$$
$$\Rightarrow \log x = a\log 100,\ \log y = b\log 10000 \text{ and } \log z = c\log 10$$
$$\log\frac{10\sqrt{y}}{x^2z^3} = \log 10\sqrt{y} - \log(x^2z^3)$$
$$= \log(10y^{1/2}) - \log x^2 - \log z^3$$
$$= \log 10 + \log y^{1/2} - \log x^2 - \log z^3$$
$$= \log 10 + \frac{1}{2}\log y - 2\log x - 3\log z$$
$$= 1 + \frac{1}{2}\log(10000)^b - 2\log(100)^a - 3\log(10)^c \ldots\ldots(\text{Since } \log 10 = 1)$$
$$= 1 + \frac{b}{2}\log(10)^4 - a\log(10)^2 - 3c\log 10$$
$$= 1 + \frac{b}{2}\times 4\log 10 - 2\times 2a\log 10 - 3c\log 10$$
$$= 1 + 2b - 4a - 3c$$

16.If 3(log5−log3)−(log5−2log6)=2−logx, find x.

Solution:

$$3(\log 5 - \log 3) - (\log 5 - 2\log 6) = 2 - \log x$$

$$\Rightarrow 3\log 5 - 3\log 3 - \log 5 + 2\log(2 \times 3) = 2 - \log x$$
$$\Rightarrow 3\log 5 - 3\log 3 - \log 5 + 2\log 2 + 2\log 3 = 2 - \log x$$
$$\Rightarrow 2\log 5 - \log 3 + 2\log 2 = 2 - \log x$$
$$\Rightarrow 2\log 5 - \log 3 + 2\log 2 + \log x = 2$$
$$\Rightarrow \log 5^2 - \log 3 + \log 2^2 + \log x = 2$$
$$\Rightarrow \log\left(\frac{25 \times 4 \times x}{3}\right) = 2$$
$$\Rightarrow \log\left(\frac{100x}{3}\right) = 2$$
$$\Rightarrow \frac{100x}{3} = 10^2$$
$$\Rightarrow \frac{x}{3} = 1$$
$$\Rightarrow x = 3$$

EXERCISE 8(C)

PAGE: 108

1. **If $\log_{10}8=0.90$; find the value of:**
   (i) $\log_{10}4$
   (ii) $\log\sqrt{32}$
   (iii) $\log 0.125$

Solution:

Given that $\log_{10}8 = 0.90$

$\Rightarrow \log_{10} 2\times 2\times 2 = 0.90$

$\Rightarrow \log_{10} 2^3 = 0.90$

$\Rightarrow 3\log_{10} 2 = 0.90$

$\Rightarrow \log_{10} 2 = \frac{0.90}{3}$

$\Rightarrow \log_{10} 2 = 0.30....(1)$

(i)

$\log 4 = \log_{10}(2\times 2)$

$\Rightarrow \quad = \log_{10}(2^2)$

$\Rightarrow \quad = 2\log_{10} 2$

$\Rightarrow \quad = 2(0.30) \quad$ [from (1)]

$\Rightarrow \quad =0.60$

(ii)

$\log\sqrt{32} = \log_{10}(32)^{\frac{1}{2}}$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(32)$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(2\times 2\times 2\times 2\times 2)$

$\Rightarrow \quad = \frac{1}{2}\log_{10}(2^5)$

$\Rightarrow \quad = \frac{1}{2}\times 5\log_{10} 2$

$\Rightarrow \quad = \frac{1}{2}\times 5(0.30) \quad$ [from (1)]

$\Rightarrow \quad =5\times 0.15$

$\Rightarrow \quad = 0.75$

(iii)

$$\log 0.125 = \log_{10}\frac{125}{1000}$$
$$= \log_{10}\frac{1}{8}$$
$$= \log_{10}\frac{1}{2\times 2\times 2}$$
$$= \log_{10}\left(\frac{1}{2^3}\right)$$
$$= \log_{10} 2^{-3}$$
$$= -3\times(0.30) \quad [\text{from } (1)]$$
$$= -0.9$$

2. If log 27=1.431, find the value of:

(i) log 9

(ii) log 300

Solution:

$$\log 27 = 1.431$$
$$\Rightarrow \log 3\times 3\times 3 = 1.431$$
$$\Rightarrow \log 3^3 = 1.431$$
$$\Rightarrow 3\log 3 = 1.431$$
$$\Rightarrow \log 3 = \frac{1.431}{3}$$
$$\Rightarrow \log 3 = 0.477.....(1)$$

(i)

$$\log 9 = \log(3\times 3)$$
$$= \log 3^2$$
$$= 2\log 3$$
$$= 2\times 0.477 \quad [\text{from } (1)]$$
$$= 0.954$$

(iii)

$$\log 300 = \log(3\times 100)$$
$$= \log 3 + \log 100$$
$$= \log 3 + 2 \quad [\because \log_{10} 100 = 2]$$
$$= 0.477 + 2 \quad [\text{from } (1)]$$
$$= 2.477$$

3. If $\log_{10}a=b$, find 10^{3b-2} in terms of a.

Solution:

$\log_{10} a = b$

$\Rightarrow 10^b = a$

$\Rightarrow (10^b)^3 = (a)^3$ [cubing both sides]

$\Rightarrow \frac{10^{3b}}{10^2} = \frac{a^3}{10^2}$ [dividing both sides by 10^2]

$\Rightarrow 10^{3b-2} = \frac{a^3}{100}$

4. If $\log_5 x=y$, find 5^{2y+3} in terms of x.

Solution:

$\log_5 x = y$ [given]

$\Rightarrow 5^y = x$

$\Rightarrow (5^y)^2 = x^2$

$\Rightarrow 5^{2y} = x^2$

$\Rightarrow 5^{2y} \times 5^3 = x^2 \times 5^3$

$\Rightarrow 5^{2y+3} = 125x^2$

5. Given: $\log_3 m=x$ and $\log_3 n=y$.

(i) Express 3^{2x-3} in terms of m.

(ii) Write down $3^{1-2y+3x}$ in terms of m and n.

(iii) If $2\log_3 A=5x-3y$; find A in terms of m and n.

Solution:

Given that $\log_3 m = x$ and $\log_3 n = y$

$\Rightarrow 3^x = m$ and $3^y = n$

(i)

Consider the given expression:

$3^{2x-3} = 3^{2x} \cdot 3^{-3}$

$= 3^{2x} \cdot \frac{1}{3^3}$

$= \frac{3^{2x}}{3^3}$

$= \frac{(3^x)^2}{3^3}$

$= \frac{m^2}{27}$

Therefore, $3^{2x-3} = \frac{m^2}{27}$

(ii)

Consider the given expression:

$3^{1-2y+3x} = 3^1 \cdot 3^{-2y} \cdot 3^{3x}$

$= 3 \cdot \frac{1}{3^{2y}} \cdot 3^{3x}$

$= \frac{3}{(3^y)^2} \cdot (3^x)^3$

$= \frac{3}{(n)^2} \cdot (m)^3$

$= \frac{3m^3}{n^2}$

Therefore, $3^{1-2y+3x} = \frac{3m^3}{n^2}$

(iii)

Consider the given expression:

$2 \log_3 A = 5x - 3y$

$\Rightarrow 2 \log_3 A = 5 \log_3 m - 3 \log_3 n$

$\Rightarrow \log_3 A^2 = \log_3 m^5 - \log_3 n^3$

$\Rightarrow \log_3 A^2 = \log_3 \left(\frac{m^5}{n^3}\right)$

$\Rightarrow A^2 = \left(\frac{m^5}{n^3}\right)$

$\Rightarrow A = \sqrt{\left(\frac{m^5}{n^3}\right)}$

6. Simplify:

(i)

$\log(a)^3 - \log a$

(ii)

$\log(a)^3 \div \log a$

Solution:

(i)

$\log(a)^3 - \log a = 3\log a - \log a$

$= 2\log a$

(ii)

$$\log(a)^3 \div \log a = 3\log a \div \log a$$
$$= \frac{3\log a}{\log a}$$
$$= 3$$

7. If log (a+b)=log a + log b, find a in terms of b.

Solution:

$$\log(a+b) = \log a + \log b$$
$$\Rightarrow \log(a+b) = \log ab$$
$$\Rightarrow a + b = ab$$
$$\Rightarrow a - ab = -b$$
$$\Rightarrow -ab + a = -b$$
$$\Rightarrow -a(b-1) = -b$$
$$\Rightarrow a(b-1) = b$$
$$\Rightarrow a = \frac{b}{b-1}$$

8. Prove that:

(i) $(\log a)^2 - (\log b)^2 = \log(a/b).\log(ab)$

(ii) If a log b+b log a−1=0, then $b^a.a^b=10$

Solution:

(i)

$$L.H.S = (\log a)^2 - (\log b)^2$$
$$\Rightarrow L.H.S = (\log a + \log b)(\log a - \log b)$$
$$\Rightarrow L.H.S = \log(ab)\log\left(\frac{a}{b}\right)$$
$$\Rightarrow L.H.S = \log\left(\frac{a}{b}\right) \times \log(ab)$$
$$\Rightarrow L.H.S = R.H.S$$

Hence proved.

(ii)

Given that

$a\log b + b\log a - 1 = 0$

$\Rightarrow a\log b + b\log a = 1$

$\Rightarrow \log b^a + \log a^b = 1$

$\Rightarrow \log b^a + \log a^b = \log 10$

$\Rightarrow \log(b^a \cdot a^b) = \log 10$

$\Rightarrow b^a \cdot a^b = 10$

9.

(i) If log (a+1)=log (4a−3)−log3; find a.

(ii) If 2logy−logx−3=0, express x in terms of y.

Solution:

(i)

Given that

$\log(a+1) = \log(4a-3) - \log 3$

$\Rightarrow \log(a+1) = \log\left(\frac{4a-3}{3}\right)$

$\Rightarrow a+1 = \frac{4a-3}{3}$

$\Rightarrow 3a+3 = 4a-3$

$\Rightarrow 4a-3a = 3+3$

$\Rightarrow a = 6$

(ii)

$2\log y - \log x - 3 = 0$

$\Rightarrow 2\log y - \log x = 3$

$\Rightarrow \log y^2 - \log x = 3$

$\Rightarrow \log y^2 - \log x = \log 1000$

$\Rightarrow \log\frac{y^2}{x} = \log 1000$

$\Rightarrow \frac{y^2}{x} = 1000$

$\Rightarrow x = \frac{y^2}{1000}$

(iii)

$\log_{10} 125 = 3(1 - \log_{10} 2)$

$L.H.S. = \log_{10} 125$

$= \log_{10} 5 \times 5 \times 5$

$= \log_{10} 5^3$

$= 3\log_{10} 5 .....(1)$

$R.H.S = 3(1 - \log_{10} 2)$

$= 3(\log_{10} 10 - \log_{10} 2)$

$= 3\log_{10}\left(\frac{10}{2}\right)$

$= 3\log_{10} 5 .....(2)$

From (1) and (2), we have

L.H.S.=R.H.S.

Hence proved.

10. Given log x=2m−n, log y=n−2m and log z=3m−2n, find in terms of m and n, the value of:

$\log \frac{x^2y^3}{z^4}$

Solution:

Given $\log x = 2m - n$, $\log y = n - 2m$ and $\log z = 3m - 2n$

$\log \frac{x^2y^3}{z^4} = \log x^2y^3 - \log z^4$

$= \log x^2 + \log y^3 - \log z^4$

$= 2\log x + 3\log y - 4\log z$

$= 2(2m - n) + 3(n - 2m) - 4(3m - 2n)$

$= 4m - 2n + 3n - 6m - 12m - 8n$

$= -14m - 7n$

11. Given $\log_x 25 - \log_x 5 = 2 - \log_x(1/125)$; find x.

Solution:

$$\log_x 25 - \log_x 5 = 2 - \log_x \frac{1}{125}$$

$$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x \left(\frac{1}{5}\right)^3$$

$$\Rightarrow \log_x 5^2 - \log_x 5 = 2 - \log_x 5^{-3}$$

$$\Rightarrow 2\log_x 5 - \log_x 5 = 2 + 3\log_x 5$$

$$\Rightarrow 2\log_x 5 - \log_x 5 - 3\log_x 5 = 2$$

$$\Rightarrow -2\log_x 5 = 2$$

$$\Rightarrow \log_x 5 = -1$$

$$\Rightarrow x^{-1} = 5$$

$$\Rightarrow \frac{1}{x} = 5$$

$$\Rightarrow x = \frac{1}{5}$$

EXERCISE 8(D) PAGE: 110

1. If $\frac{3}{2}\log a + \frac{2}{3}\log b - 1 = 0$ find the value of $a^9.b^4$

Solution:

$$\frac{3}{2}\log a + \frac{2}{3}\log b - 1 = 0$$
$$\Rightarrow \log a^{\frac{3}{2}} + \log b^{\frac{2}{3}} = 1$$
$$\Rightarrow \log\left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right) = 1$$
$$\Rightarrow \log\left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right) = \log 10$$
$$\Rightarrow a^{\frac{3}{2}} \times b^{\frac{2}{3}} = 10$$
$$\Rightarrow \left(a^{\frac{3}{2}} \times b^{\frac{2}{3}}\right)^6 = 10^6$$
$$\Rightarrow a^9 \cdot b^4 = 10^6$$

2. If x=1+log2−log5, y=2log3 and z=loga−log5; find the value of a, if x=y=2z.

Solution:

Given that

$x = 1 + \log 2 - \log 5, y = 2\log 3$ and $z = \log a - \log 5$

Consider

$x = 1 + \log 2 - \log 5$

$= \log 10 + \log 2 - \log 5$

$= \log(10 \times 2) - \log 5$

$= \log 20 - \log 5$

$= \log\frac{20}{5}$

$= \log 4 \ldots.(1)$

We have

$y = 2\log 3$

$= \log 3^2$

$= \log 9 \ldots.(2)$

Also we have

$z = \log a - \log 5$

$= \log\frac{a}{5} \ldots.(3)$

Given that x+y=2z

∴ Substitute the values of x,y and z from (1),(2) and (3), we have

$\Rightarrow \log 4 + \log 9 = 2\log\frac{a}{5}$

$\Rightarrow \log 4 + \log 9 = \log\left(\frac{a}{5}\right)^2$

$\Rightarrow \log 4 + \log 9 = \log\frac{a^2}{25}$

$\Rightarrow \log(4 \times 9) = \log\frac{a^2}{25}$

$\Rightarrow \log 36 = \log\frac{a^2}{25}$

$\Rightarrow \frac{a^2}{25} = 36$

$\Rightarrow a^2 = 36 \times 25$

$\Rightarrow a^2 = 900$

$\Rightarrow a = 30$

3. If x=log 0.6; y=log 1.25 and z=log 3−2 log 2, find the values of:

(i) x+y−z

(ii) 5^{x+y-z}

Solution:

Given that

x=log0.6, y=log1.25,z=log3 - 2log2

Consider

$z = \log 3 - 2\log 2$

$= \log 3 - \log 2^2$

$= \log 3 - \log 4$

$= \log\frac{3}{4}$

$= \log 0.75 \ldots (1)$

(i)

$x + y - z = \log 0.6 + \log 1.25 - \log 0.75$

$= \log\frac{0.6 \times 1.25}{0.75}$

$= \log\frac{0.75}{0.75}$

$= \log 1$

$= 0 \ldots (2)$

(ii)

$5^{x+y-z} = 5^0 \ldots [\because x + y - z = 0 \text{ from } (2)]$

$= 1$

4. If $a^2=\log x$, $b^3=\log y$ and $3a^2-2b^3=6\log z$, express y in terms of x and z.

Solution:

Given that

$a^2 = \log x, b^3 = \log y$ and $3a^2 - 2b^3 = 6\log z$

Consider the equation,

$3a^2 - 2b^3 = 6\log z$

$\Rightarrow 3\log x - 2\log y = 6\log z$

$\Rightarrow \log x^3 - \log y^2 = \log z^6$

$\Rightarrow \log\left(\frac{x^3}{y^2}\right) = \log z^6$

$\Rightarrow \frac{x^3}{y^2} = z^6$

$\Rightarrow \frac{x^3}{z^6} = y^2$

$\Rightarrow y^2 = \frac{x^3}{z^6}$

$\Rightarrow y = \left(\frac{x^3}{z^6}\right)^{\frac{1}{2}}$

$\Rightarrow y = \left(\frac{x^{\frac{3}{2}}}{z^{\frac{6}{2}}}\right)$

$\Rightarrow y = \frac{x^{\frac{3}{2}}}{z^3}$

5. If $\log \frac{a-b}{2} = \frac{1}{2}(\log a+\log b)$, show that $a^2+b^2=6ab$.

Solution:

$$\log\left(\frac{a-b}{2}\right)=\frac{1}{2}(\log a+\log b)$$
$$\Rightarrow \log\left(\frac{a-b}{2}\right)=\frac{1}{2}(\log ab)$$
$$\Rightarrow \log\left(\frac{a-b}{2}\right)=\log(ab)^{\frac{1}{2}}$$
$$\Rightarrow \left(\frac{a-b}{2}\right)=(ab)^{\frac{1}{2}}$$

Squaring both sides we have,

$$\left(\frac{a-b}{2}\right)^2=ab$$
$$\Rightarrow \frac{(a-b)^2}{4}=ab$$
$$\Rightarrow (a-b)^2=4ab$$
$$\Rightarrow a^2+b^2-2ab=4ab$$
$$\Rightarrow a^2+b^2=4ab+2ab$$
$$\Rightarrow a^2+b^2=6ab$$

6. If $a^2+b^2=23ab$, show that:

$$\log\left(\frac{a+b}{5}\right)=\frac{1}{2}(\log a+\log b)$$

Solution:

Given that

$$a^2+b^2=23ab$$
$$\Rightarrow a^2+b^2+2ab=23ab+2ab$$
$$\Rightarrow a^2+b^2+2ab=25ab$$
$$\Rightarrow (a+b)^2=25ab$$
$$\Rightarrow \frac{(a+b)^2}{25}=ab$$
$$\Rightarrow \left(\frac{a+b}{5}\right)^2=ab$$
$$\Rightarrow \log\left(\frac{a+b}{5}\right)^2=\log ab$$
$$\Rightarrow 2\log\left(\frac{a+b}{5}\right)=\log ab$$
$$\Rightarrow \log\left(\frac{a+b}{5}\right)=\frac{1}{2}(\log a+\log b)$$

7. If m=log20 and n=log 25, find the value of x, so that: 2log(x−4)=2m−n.

Solution:

Given that

$m = \log 20$ and $n = \log 25$

We also have

$2\log(x-4) = 2m - n$

$\Rightarrow 2\log(x-4) = 2\log 20 - \log 25$

$\Rightarrow \log(x-4)^2 = \log 20^2 - \log 25$

$\Rightarrow \log(x-4)^2 = \log 400 - \log 25$

$\Rightarrow \log(x-4)^2 = \log\frac{400}{25}$

$\Rightarrow (x-4)^2 = \frac{400}{25}$

$\Rightarrow (x-4)^2 = 16$

$\Rightarrow x - 4 = 4$

$\Rightarrow x = 4 + 4$

$\Rightarrow x = 8$

8. Solve for x and y; if x>0 and y>0:

$\log xy = \log\left(\frac{x}{y}\right) + 2\log 2 = 2$

Solution:

$\log xy = \log\left(\frac{x}{y}\right) + 2\log 2 = 2$

$\log xy = 2$

$\Rightarrow \log xy = 2\log 10$

$\Rightarrow \log xy = \log 10^2$

$\Rightarrow \log xy = \log 100$

$\therefore xy = 100 \ldots.(1)$

Now consider the equation

$\log\left(\frac{x}{y}\right) + 2\log 2 = 2$

$\Rightarrow \log\left(\frac{x}{y}\right) + \log 2^2 = 2\log 10$

$\Rightarrow \log\left(\frac{x}{y}\right) + \log 4 = \log 10^2$

$\Rightarrow \log\left(\frac{x}{y}\right) + \log 4 = \log 100$

$\Rightarrow \left(\frac{x}{y}\right) \times 4 = 100$

$\Rightarrow 4x = 100y$

$\Rightarrow x = 25y$

$\Rightarrow xy = 25y \times y$

$\Rightarrow xy = 25y^2$

$\Rightarrow 100 = 25y^2 \ldots.[\text{from}(1)]$

$\Rightarrow y^2 = \frac{100}{25}$

$\Rightarrow y^2 = 4$

$\Rightarrow y = 2 \quad [\because y > 0]$

From (1),

$xy = 100$

$\Rightarrow x \times 2 = 100$

$\Rightarrow x = \frac{100}{2}$

$\Rightarrow x = 50$

Thus the values of x and y are x=50 and y=2

9. Find x, if:

(i) $\log_x 625 = -4$

(ii) $\log_x (5x - 6) = 2$

Solution:

(i)

$\log_x 625 = -4$

$\Rightarrow 625 = x^{-4}$ [Removing Logarithm]

$\Rightarrow 5^4 = \left(\frac{1}{x}\right)^4$

$\Rightarrow 5 = \frac{1}{x}$ [Powers are same, bases are equal]

$\Rightarrow x = \frac{1}{5}$

(ii)

$\log_x(5x-6)=2$
$\Rightarrow 5x-6=x^2$ [Removing Logarithm]
$\Rightarrow x^2-5x+6=0$
$\Rightarrow x^2-3x-2x+6=0$
$\Rightarrow x(x-3)-2(x-3)=0$
$\Rightarrow (x-2)(x-3)=0$
$\therefore\ x=2,3$

10.If p=log 20 and q=log 25, find the value of x, if 2 log (x+1) = 2p−q.

Solution:

Given that
$p=\log 20$ and $q=\log 25$
we also have
$2\log(x+1)=2p-q$
$\Rightarrow 2\log(x+1)=2\log 20-\log 25$
$\Rightarrow \log(x+1)^2=\log 20^2-\log 25$
$\Rightarrow \log(x+1)^2=\log 400-\log 25$
$\Rightarrow \log(x+1)^2=\log\frac{400}{25}$
$\Rightarrow \log(x+1)^2=\log 16$
$\Rightarrow \log(x+1)^2=\log 4^2$
$\Rightarrow x+1=4$
$\Rightarrow x=4-1$
$\Rightarrow x=3$

11.If $\log_2(x+y)=\log_3(x-y)=\frac{\log 25}{\log 0.2}$, find the values of x and y.

Solution:

$$\log_2(x+y) = \frac{\log 25}{\log 0.2}$$
$$\Rightarrow \log_2(x+y) = \log_{0.2} 25$$
$$\Rightarrow \log_2(x+y) = \log_{\frac{2}{10}} 25$$
$$\Rightarrow \log_2(x+y) = \log_{5^{-1}} 5^2$$
$$\Rightarrow \log_2(x+y) = -2\log_5 5$$
$$\Rightarrow \log_2(x+y) = -2$$
$$\Rightarrow x+y = 2^{-2} \text{[Removing logarithm]}$$
$$\Rightarrow x+y = \frac{1}{4} \ldots\ldots(i)$$
$$\log_3(x-y) = \frac{\log 25}{\log 0.2}$$
$$\Rightarrow \log_3(x-y) = \log_{0.2} 25$$
$$\Rightarrow \log_3(x-y) = \log_{\frac{2}{10}} 25$$
$$\Rightarrow \log_3(x-y) = \log_{5^{-1}} 5^2$$
$$\Rightarrow \log_3(x-y) = -2\log_5 5$$
$$\Rightarrow \log_3(x-y) = -2$$
$$\Rightarrow x-y = 3^{-2} \text{[Removing logarithm]}$$
$$\Rightarrow x-y = \frac{1}{9} \ldots\ldots(ii)$$

Solving (i) & (ii), we get

$$x = \frac{13}{72}, y = \frac{5}{72}$$

12. Given: $\frac{\log x}{\log y} = \frac{3}{2}$ and log (xy) = 5; find the values of x and y.

Solution:

$$\frac{\log x}{\log y} = \frac{3}{2}$$
$$\Rightarrow 2\log x = 3\log y$$
$$\Rightarrow \log y = \frac{2\log x}{3} \ldots\ldots.(i)$$
$$\log(xy) = 5$$
$$\Rightarrow \log x + \log y = 5$$
$$\Rightarrow \log x + \frac{2\log x}{3} = 5 \ \text{[Substituting } (i)\text{]}$$
$$\Rightarrow \frac{3\log x + 2\log x}{3} = 5$$
$$\Rightarrow \frac{5\log x}{3} = 5$$
$$\Rightarrow \log x = 3$$
$$\Rightarrow x = 10^3$$
$$\therefore x = 1000$$
Substituting $x = 1000$
$$\log y = \frac{2 \times 3}{3}$$
$$\Rightarrow \log y = 2$$
$$\Rightarrow y = 10^2$$
$$\therefore y = 100$$

13. Given $\log_{10}x=2a$ and $\log_{10}y=\frac{b}{2}$

(i) Write 10^a in terms of x

(ii) Write 10^{2b+1} in terms of y

(iii) If $\log_{10}P=3a-2b$, express P in terms of x and y.

Solution:

(i) $\log_{10}x = 2a$

$\Rightarrow x = 10^{2a}$ [Removing logarithm from both sides]

$\Rightarrow x^{1/2} = 10^a$

$\Rightarrow 10^a = x^{1/2}$

(ii) $\log_{10} y = \frac{b}{2}$

$\Rightarrow y = 10^{b/2}$

$\Rightarrow y^4 = 10^{2b}$

$\Rightarrow 10y^4 = 10^{2b} \times 10$

$\Rightarrow 10^{2b+1} = 10y^4$

(iii)

We know $10^a = x^{1/2}$

$10^{b/2} = y$

$\Rightarrow 10^b = y^2$

$\log_{10}^{p} = 3a - 2b$

$\Rightarrow p = 10^{3a-2b}$

$\Rightarrow p = (10^3)^a \div (10^2)^b$

$\Rightarrow p = (10^a)^3 \div (10^b)^2$

Substituting 10^a & 10^b, we get

$\Rightarrow p = (x^{1/2})^3 \div (y^2)^2$

$\Rightarrow p = x^{3/2} \div y^4$

$\Rightarrow p = \frac{x^{3/2}}{y^4}$

14. Solve:

$\log_5(x + 1) - 1 = 1 + \log_5(x - 1)$

Solution:

$\log_5(x + 1) - 1 = 1 + \log_5(x - 1)$

$\Rightarrow \log_5(x + 1) - \log_5(x - 1) = 2$

$\Rightarrow \log_5 \frac{(x+1)}{(x-1)} = 2$

$\Rightarrow \frac{(x + 1)}{(x - 1)} = 5^2$

$\Rightarrow \frac{(x + 1)}{(x - 1)} = 25$

$\Rightarrow x + 1 = 25(x - 1)$

$\Rightarrow x + 1 = 25x - 25$

$\Rightarrow 25x - x = 25 + 1$

$\Rightarrow 24x = 26$

$\Rightarrow x = \frac{26}{24} = \frac{13}{12}$

15. Solve for x, if:

$$\log_x 49 - \log_x 7 + \log_x \frac{1}{343} = -2$$

Solution:

$$\log_x 49 - \log_x 7 + \log_x \frac{1}{343} = -2$$

$$\Rightarrow \log_x \frac{49}{7 \times 343} = -2$$

$$\Rightarrow \log_x \frac{1}{49} = -2$$

$$\Rightarrow -\log_x 49 = -2$$

$$\Rightarrow \log_x 49 = 2$$

$$\Rightarrow 49 = x^2 \text{ [Removing logarithm]}$$

$$\therefore x = 7$$

16. If $a^2 = \log x$, $b^3 = \log y$ and

$\frac{a^2}{2} - \frac{b^3}{3} = \log c$, find c in terms of x and y.

Solution:

$$\text{Given } a^2 = \log x, b^3 = \log y$$

$$\text{Now } \frac{a^2}{2} - \frac{b^3}{3} = \log c$$

$$\Rightarrow \frac{\log x}{2} - \frac{\log y}{3} = \log c$$

$$\Rightarrow \frac{3\log x - 2\log y}{6} = \log c$$

$$\Rightarrow 3\log x - 2\log y = 6\log c$$

$$\Rightarrow \log x^3 - \log y^2 = 6\log c$$

$$\Rightarrow \log\left(\frac{x^3}{y^2}\right) = \log c^6$$

$$\Rightarrow \frac{x^3}{y^2} = c^6$$

$$\Rightarrow c = \sqrt[6]{\frac{x^3}{y^2}}$$

17. Given $x = \log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ and $z = \log_{10} 0.4$, find:

(i) $x - y - z$

(ii) 13^{x-y-z}

Solution:

(i)

$$x - y - z$$
$$= \log_{10}12 - \log_4 2 \times \log_{10}9 - \log_{10}0.4$$
$$= \log_{10}(4 \times 3) - \log_4 2 \times \log_{10}9 - \log_{10}0.4$$
$$= \log_{10}4 + \log_{10}3 - \log_4 2 \times 2\log_{10}3 - \log_{10}\left(\frac{4}{10}\right)$$
$$= \log_{10}4 + \log_{10}3 - \frac{\log_{10}2}{2\log_{10}2} \times 2\log_{10}3 - \log_{10}4 + \log_{10}10$$
$$= \log_{10}4 + \log_{10}3 - \frac{2\log_{10}3}{2} - \log_{10}4 + 1$$
$$= 1$$

(ii) $13^{x-y-z} = 13^1 = 13$

18. Solve for x,

$\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$

Solution:

$$\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$$
$$\Rightarrow \log_x 15\sqrt{5} + \log_x 3\sqrt{5} = 2$$
$$\Rightarrow \log_x\left(15\sqrt{5} \times 3\sqrt{5}\right) = 2$$
$$\Rightarrow \log_x 225 = 2$$
$$\Rightarrow \log_x 15^2 = 2$$
$$\Rightarrow 2\log_x 15 = 2$$
$$\Rightarrow \log_x 15 = 1$$
$$\Rightarrow x = 15$$

19. Evaluate:

(i) $\log_b a \times \log_c b \times \log_a c$

(ii) $\log_3 8 \div \log_9 16$

(iii) $\dfrac{\log_5 8}{\log_{25}16 \times \log_{100}10}$

Solution:

$$(i)\log_b a \times \log_c b \times \log_a c$$
$$= \frac{\log_{10}a}{\log_{10}b} \times \frac{\log_{10}b}{\log_{10}c} \times \frac{\log_{10}c}{\log_{10}a}$$
$$= 1$$
$$(ii)\log_3 8 \div \log_9 16$$
$$= \frac{\log_3 8}{\log_9 16}$$
$$= \frac{\log_{10}8}{\log_{10}3} \times \frac{\log_{10}9}{\log_{10}16}$$
$$= \frac{3\log_{10}2}{\log_{10}3} \times \frac{2\log_{10}3}{4\log_{10}2}$$
$$= \frac{3}{2}$$
$$(iii)\frac{\log_5 8}{\log_{25}16 \times \log_{100}10}$$
$$= \frac{\frac{\log_{10}8}{\log_{10}5}}{\frac{\log_{10}16}{\log_{10}25} \times \frac{\log_{10}10}{\log_{10}100}}$$
$$= \frac{\frac{\log_{10}2^3}{\log_{10}5}}{\frac{\log_{10}2^4}{\log_{10}5^2} \times \frac{\log_{10}10}{\log_{10}10^2}}$$
$$= \frac{\log_{10}2^3}{\log_{10}5} \times \frac{\log_{10}5^2}{\log_{10}2^4} \times \frac{\log_{10}10^2}{\log_{10}10}$$
$$= \frac{3\log_{10}2}{\log_{10}5} \times \frac{2\log_{10}5}{4\log_{10}2} \times \frac{2\log_{10}10}{\log_{10}10}$$
$$= 3$$

20. Show that:

Solution:

$$\log_a m \div \log_{ab} m = \frac{\log_a m}{\log_{ab} m}$$

$$= \frac{\log_m ab}{\log_m a} \quad \left[\because \log_b a = \frac{1}{\log_a b} \right]$$

$$= \log_a ab \left[\because \frac{\log_x a}{\log_x b} = \log_b a \right]$$

$$= \log_a a + \log_a b$$

$$= 1 + \log_a b$$

21. If $\log_{\sqrt{27}} x = 2\frac{2}{3}$, find x.

Solution:

$$\log_{\sqrt{27}} x = 2\frac{2}{3}$$

$$\therefore \log_{\sqrt{27}} x = \frac{8}{3}$$

$$\therefore x = \left(\sqrt{27}\right)^{\frac{8}{3}} \qquad \because \log_a x = b \Rightarrow x = a^b$$

$$\therefore x = \left(27^{\frac{1}{2}}\right)^{\frac{8}{3}}$$

$$\therefore x = \left(3^{\frac{3}{2}}\right)^{\frac{8}{3}}$$

$$\therefore x = 3^{\frac{3}{2} \times \frac{8}{3}}$$

$$\therefore x = 3^4$$

$$\therefore x = 81$$

22. Evaluate:

$$\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1}$$

Solution:

$$\frac{1}{\log_a bc+1}+\frac{1}{\log_b ca+1}+\frac{1}{\log_c ab+1}$$

$$=\frac{1}{\log_a bc+\log_a a}+\frac{1}{\log_b ca+\log_b b}+\frac{1}{\log_c ab+\log_c c}$$

$$=\frac{1}{\log_a abc}+\frac{1}{\log_b aba}+\frac{1}{\log_c abc} \qquad \because \log_a b+\log_b c=\log_a bc$$

$$=\frac{1}{\frac{\log abc}{\log a}}+\frac{1}{\frac{\log abc}{\log b}}+\frac{1}{\frac{\log abc}{\log c}}$$

$$=\frac{\log a+\log b+\log c}{\log abc}$$

$$=\frac{\log abc}{\log abc} \qquad \because \log_a b+\log_b c=\log_a bc$$

$$=1$$