0193 A

Total No. of Questions—24

Total No. of Printed Pages—4 Regd. No.

Part III

MATHEMATICS, Paper - I(B)

(English Version)

Time : 3 Hours] *[Max. Marks : 75*

Note :—This question paper consists of **THREE** sections A, B and C.

SECTION A 10×2=20

I. Very short answer type questions :

(*i*) Attempt **ALL** questions.

(*ii*) Each question carries **TWO** marks.

1. Compute :

$$\lim_{x\to 0} \frac{a^x - 1}{b^x - 1}.$$

2. Find the value of p, if the straight lines $3x + py - 1 = 0$, $7x - 3y + 3 = 0$ are mutually perpendicular.

3. If $f(x) = \log(\tan e^x)$, then find $f'(x)$.

4. Find the ratio in which the xz-plane divides the line joining A(–2, 3, 4) and B(1, 2, 3).

5. Reduce the equation of the plane $x + 2y - 3z - 6 = 0$ to the normal form.

6. Evaluate :

$$\lim_{x\to 0} \frac{\log_e(1 + 5x)}{x}.$$

7. If $f(x) = 1 + x + x^2 + \ldots\ldots + x^{100}$, then find $f'(1)$.

8. Find the angle which the straight line $y = \sqrt{3}x - 4$ makes with the y-axis.

9. Verify Rolle's theorem for the function $y = f(x) = x^2 + 4$ in $[-3, 3]$.

10. Find Δy and dy for the function $y = \cos x$ at $x = 60°$ with $\Delta x = i$.

(cos 60° = 0.4848, i = 0.0174 radians)

SECTION B

5×4=20

II. Short answer type questions :

(*i*) Attempt **ANY FIVE** questions.

(*ii*) Each question carries **FOUR** marks.

11. Check the continuity of the following function at '2' :

$$f(x) = \begin{cases} \frac{1}{2}(x^2 - 4) & \text{if} \quad 0 < x < 2 \\ 0 & \text{if} \quad x = 2 \\ 2 - 8x^{-3} & \text{if} \quad x > 2 \end{cases}$$

12. A(1, 2), B(2, –3) and C(–2, 3) are three points. If a point P moves such that $PA^2 + PB^2 = 2PC^2$, then show that the equation to the locus of P is $7x - 7y + 4 = 0$.

13. A straight line through $Q(\sqrt{3}, 2)$ makes an angle of $\frac{\pi}{6}$ with the positive direction of the X-axis. If the straight line intersects the line $\sqrt{3}x - 4y + 8 = 0$ at P, then find the distance of PQ.

14. When the axes are rotated through an angle α, find the transformed equation of $x \cos \alpha + y \sin \alpha = p$.

15. Show that the tangent at any point θ on the curve $x = c \sec \theta$, $y = c \tan \theta$ is $y \sin \theta = x - c \cos \theta$.

16. Find the derivative of $\cos^2 x$ from the first principle.

17. A container is in the shape of an inverted cone has height 8 m and radius 6 m at the top. If it is filled with water at the rate of 2 m^3/minute, how fast is the height of water changing when the level is 4 m.

SECTION C

5×7=35

III. Long answer type questions :

(*i*) Attempt **ANY FIVE** questions.

(*ii*) Each question carries **SEVEN** marks.

18. Find the orthocentre of the triangle whose vertices are (5, –2), (–1, 2) and (1, 4).

19. Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\left| \frac{n^2 \sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2} \right|$.

20. Find the angle between the lines whose direction cosines satisfy the equations :

$$l + m + n = 0, \quad l^2 + m^2 - n^2 = 0.$$

21. If $x^{\log y} = \log x$, then show that :

$$\frac{dy}{dx} = \frac{y}{x}\left(\frac{1 - \log x \log y}{(\log x)^2}\right).$$

22. If the tangent at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ intersects the co-ordinate axes in A and B, then show that the length AB is a constant.

23. Find the values of k, if the lines joining the origin to the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.

24. Find the maximum area of the rectangle that can be formed with fixed perimeter 20.