Government of Karnataka

MATHEMATICS

8

EIGHTH STANDARD

Part-II

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TABLE OF CONTENTS

Part - II

UNIT 9
COMMERCIAL ARITHMETIC

After studying this unit, you learn to:

- identify the processes of mathematics in commercial transactions.
- define the term percentage.
- solve the numerical problems involving percentage.
- identify profit or loss in commercial transactions.
- workout problems on profit, loss, selling price and marked price.
- calculate discount, discount percentage and selling price after discount.
- solve problems on commission and commission percentage.
- define simple interest and other terminologies associated with simple interest.
- calculate simple interest principal time, rate of interest and amount in numerical problems.

Introduction

In this unit, we study several aspects of commercial mathematics which are useful in daily life. You go to a market and buy some essential item. You pay money for it. But behind this transaction a separate world of arithmetic exists. The vendor brings the goods from somewhere and he has to decide how much he should charge for the goods before selling. Depending on the market strategy, he has to fix his selling prices. Sometimes, he may have to sell it for a price lower than the price for which he has procured the goods. Some other times, he has to announce attractive incentives to get a good number of customers. When you do some transactions in buying and selling land, houses and cattle, you may have to give some money for the service you get from an agent. All these things are generally dealt with money. This is the reason why money is an important part of human life. Commercial mathematics is an insight into this kind of transactions in daily life.

Percentage

You are familiar with the meaning of percent. **Per cent** means **for every hundred**. Thus, 4 percent (or 4%) means **4 for every hundred** or $\frac{4}{100}$. Percentage is also a fraction whose denominator is 100. The numerator of the fraction is called **rate percent**. Thus 12% means 12 out of hundred or $\frac{12}{100}$. The concept of percentage is used in business transactions, calculating interest, comparison of quantities and the like.

Suppose a basket has, say, 6 pineapples and 14 oranges. Then the number of pineapples and oranges can be compared using the fraction $\frac{6}{14} = \frac{3}{7}$ The number of pineapples is $\frac{3}{7}$ times the number of oranges. In the same way, the number of oranges is $\frac{7}{3}$ times the number of pineapples. Comparison can also be done using percentages.

There are 6 pineapples out of 20=(6+14) fruits. Therefore percentage of pineapples

$$\frac{6}{20} = \frac{30}{100} = 30\%$$

(here denominator is made 100)

Out of 20 fruits, number of pineapples is 6. So out of 100 fruits, number of pineapples is

$$\frac{6}{20} \times 100 = 30\%.$$

This is called unitary method.

The basket contains only pineapples and oranges.

Therefore percentage of pineapples + percentage of oranges =100.

Or 30% + % of oranges =100 or % of oranges = 100 -30= 70.

Thus the basket has 30% pineapples and 70% oranges.

Percentage is a convenient way of comparing quantities.

Example 1. A man spends 78% of his monthly income and saves ₹ 1,100. What is his monthly income?

Solution: Let his monthly income be ₹ 100. Then his expenditure is ₹ 78. Therefore, his savings is 100-78 = ₹ 22.

Let us put this in reverse way. If the savings is ₹ 22, then the income is ₹ 100. Hence if the savings is ₹ 1, then the income is ₹ $\frac{100}{22}$.

If the savings is ₹ 1,100, then the income is $\frac{100}{22} \times 1100 = 5{,}000$ rupees. Hence his monthly income is ₹ 5,000.

Alternate Method : Given expenditure is 78%. Therefore savings is $(100 - 78) = 22\%$. Let the monthly income of the man be ₹ x. Then 22% of x is ₹ 1,100. This means

$$\frac{22}{100}x = 1100.$$

Solving for x, we obtain

$$x = 1100 \times \frac{100}{22} = 5000.$$

Hence his monthly income is ₹ 5,000.

Verification: 78% of ₹ 5,000 = $\frac{78}{100} \times 5000 =$ ₹ 3900. Hence, his savings is 5,000 - 3,900 = ₹ 1,100.

Example 2. An athlete won 8 events out of a number of events. If the win percentage was 40, how many events were there in total?

Solution: We are given that win percentage is 40. So,

40 events were won out of 100 events.

$\Rightarrow$ 8 events were won out of $\frac{100}{40} \times 8 = 20$ events.

Hence, there were 20 events in total.

Activity 1: Try this in another method.

Example 3. Ravi's income is 25% more than that of Raghu. What percent is Raghu's income less than that of Ravi?

Solution: Let Raghu's income be ₹ 100. Then. Ravi's income is ₹ 125. Put this in the reverse way. If Ravi's income is ₹ 125, Raghu's income is ₹ 100. Hence if Ravi's income is ₹ 1, Raghu's income is ₹ $\frac{100}{125}$ Changing the scale to 100, if Ravi's income is ₹ 100, then Raghu's income is ₹ $\frac{100}{125} \times 100$ = ₹ 80. Therefore Raghu's income is 100 - 80 = 20% less than that of Ravi.

Example 4. The salary of an employee is increased by 15%. If his new salary is ₹ 12,650, what was his salary before enhancement?

Solution: Let the salary before enhancement be ₹ 100. Since his increment is 15%, his salary after enhancement is ₹ 100 + ₹ 15 = ₹ 115.

Now we reverse the role. If the new salary is ₹ 115, salary before enhancement is ₹ 100.

If the new salary is ₹ 12,650, salary before enhancement is $\frac{100}{115} \times 12650$ = ₹ 11,000

Therefore, his salary before enhancement is ₹ 11,000.

Exercise 9.1

1. In a school, 30% of students play chess, 60% play carrom and the rest play other games. If the total number of students in the school is 900, find the exact number of students who play each game.
2. In a school function ₹ 360 remained after spending 82% of the money. How much money was there in the beginning? Verify your answer.
3. Akshay's income is 20% less than that of Ajay. What percent is Ajay's income more than that of Akshay?
4. A daily wage employee spends 84% of his weekly earning. If he saves ₹ 384, find his weekly earning.
5. A factory announces a bonus of 10% to its employees. If an employee gets ₹ 10,780, find his actual salary.

Profit and Loss

We purchase goods from shops. The shopkeeper purchases goods either directly from a manufacturer or through a wholesale dealer. The money paid to buy goods is called **Cost Price** and abbreviated as **C.P.** The price at which goods are sold in shops is called **Selling Price** abbreviated as **S.P.** When an article is bought, some additional expenses such as freight charges, labour charges, transportation charges, maintenance charges etc., are made before selling. These expenses are known as **Overhead Charges**. These expenses have to be included in the cost price. Hence, you may conclude that

the real cost price = total investment

= price for buying goods + overhead charges.

If S.P > C.P, there is a gain or **profit**. If S.P < C.P, there is a **loss**. Therefore,

$$\boxed{\textbf{profit = S.P - C.P.}} \quad \text{and} \quad \boxed{\textbf{loss = C.P - S.P.}}$$

Gain (or profit) or loss on ₹ 100 is called as **gain percent** or **loss percent**.

Note: Profit or loss is always calculated on cost price.

Remember the following formulae:

1. Gain or profit % $= \dfrac{\text{gain}}{\text{C.P.}} \times 100$

2. Loss % $= \dfrac{\text{loss}}{\text{C.P.}} \times 100$

3. Selling Price $= \dfrac{(100+\text{gain}\%)}{100} \times \text{C.P.}$

4. Selling Price $= \dfrac{(100-\text{loss}\%)}{100} \times \text{C.P.}$

5. Cost Price $= \dfrac{100}{(100+\text{gain}\%)} \times \text{S.P.}$

6. Cost Price $= \dfrac{100}{(100-\text{loss}\%)} \times \text{S.P.}$

Example 5. The cost price of a computer is ₹ 19,500. An additional ₹ 450 was spent on installing a software. If it is sold at 12% profit, find the selling price of the computer.

Solution : Cost price of the computer is ₹ 19,500 + ₹ 450 (overhead expenses) = ₹ 19,950. The computer is sold at a profit of 12%. Therefore,

$$\text{selling price} = \frac{(100+\text{gain\%})}{100} \times \text{C.P.}$$

$$= \frac{(100+12)}{100} \times 19950$$

$$= \frac{112}{10} \times 1995$$

$$= 22344.$$

Thus the selling price is ₹ 22,344.

Alternate method : (Unitary method)

Profit = 12%. Hence selling price is ₹100 + ₹12 = ₹112. If the cost price is ₹100, selling price is ₹112.

If the cost price is ₹ 19,950,

$$\text{selling price} = \frac{112}{100} \times 19950$$

$$= 22344.$$

Hence, the selling price of computer is ₹ 22,344.

Example 6. On selling a bicycle for ₹ 4,300, a dealer loses 14%. For how much should he sell it to gain 14%?

Solution : Selling price of the bicycle is ₹ 4,300, and the loss is 14%. Therefore,

$$\text{cost price} = \frac{100}{(100\text{-loss\%})} \times S.P.$$

$$= \frac{100}{(100-14)} \times 4300$$

$$= \frac{100}{86} \times 4300$$

$$= 5000.$$

Hence the cost price of the bicycle ₹ is 5,000.

Alternate method : (Unitary method)

We know that the loss is 14%. Therefore selling price = ₹100 - ₹ 14 = ₹ 86.

If the selling price is ₹ 86, cost price is ₹ 100. If the selling price is ₹ 4300,

$$\text{cost price} = \frac{100}{86} \times 4300 = ₹\ 57{,}000.$$

Now, the cost price of the bicycle is ₹ 5,000. Let us find out what should be the selling price to get 14% profit.

Expected gain = 14%.
Therefore,

$$\text{selling price} = \frac{(100+\text{gain\%})}{100} \times \text{C.P.}$$

$$= \frac{(100 + 14)}{100} \times 5000$$

$$= \frac{114}{100} \times 5000$$

$$= 5700.$$

Alternate method : (Unitary method)

Expected gain = 14%.
Therefore selling price
= ₹100 + ₹ 14 = ₹ 114.
If the selling price is ₹ 100, selling price is ₹ 114
If the cost price ₹ 5.000.

$$\text{selling price} = \frac{114}{100} \times 5000 = ₹5700.$$

Hence, the selling price of the bicycle to gain 14% is ₹ 5,700.

Example 7. Two cows were sold for ₹ 12,000 each, one at a gain of 20% and the other at a loss of 20%. Find the loss or gain in the entire transaction.

Solution: We first find out the cost price of each cow and then add them to find the total amount spent. We know the total money received. Comparing them, we will know whether there is a loss or gain in the whole transaction.

First cow:
S.P. = ₹ 12,000.
Gain = 20%. Therefore,

$$\text{cost price} = \frac{100}{(100+\text{gain\%})} \times \text{S.P.}$$

$$= \frac{100}{(100 + 20)} \times 12000$$

$$= \frac{100}{120} \times 12000$$

$$= 10000.$$

Second cow:
S.P. = ₹ 12,000.
Loss = 20%. Therefore,

$$\text{cost price} = \frac{100}{(100\text{-loss\%})} \times \text{S.P.}$$

$$= \frac{100}{(100 - 20)} \times 12000$$

$$= \frac{100}{80} \times 12000$$

$$= 15000.$$

Activity 2:

Find the cost price of the cow in both the cases using unitary method.

Let us compute the total cost price and selling price in the combined transaction.

Total C.P. of both cows = ₹ (10,000+ 15,000) = ₹ 25,000. Total S.P. of both cows = ₹ 12,000 × 2 = ₹ 24,000. Here, we observe that S.P. $<$ C.P. Therefore, loss = ₹ (25,000-24,000) = ₹ 1,000. Hence,

$$\text{loss percentage} = \frac{\text{loss}}{\text{C.P.}} \times 100 = \frac{1000}{25000} \times 100 = 4$$

Therefore, there is a loss of 4% in the whole transaction.

Example 8. If the cost price of 21 cell phones is equal to selling price of 18 cell phones, find the profit percent.

Solution: Let the C.P. of each cell phone be ₹ 1.Then the C.P. of 21 cell phones is ₹ 21.

By the given data, S.P. of 18 cell phones = C.P. of 21 cell phones = ₹ 21. Therefore, S.P. of 1 cell phone is ₹ $\frac{21}{18}$. This gives

$$\text{Profit} = \text{S.P.} - \text{C.P.} = \frac{21}{18} - 1 = \frac{3}{18} = \frac{1}{6}.$$

Thus, there is a profit of ₹ $\frac{1}{6}$ on each cell phone. Now we can calculate profit percentage:

$$\text{Profit percentage} = \frac{\text{profit}}{\text{C.P.}} \times 100 = \frac{\frac{1}{6}}{1} \times 100 = \frac{50}{3} = 16\frac{2}{3}.$$

Therefore, the profit percentage is $16\frac{2}{3}$%.

To find the overall profit or loss, we need to find the combined C.P. and combined S.P.

Example 9. A dealer sells a radio at a profit of 8%. Had he sold it for less, he would have lost 2%. Find the cost price of the radio.

Solution: Let the cost price of the radio be x. Let us calculate the selling price with 8% profit and 2% loss, separately.

With 8% profit :
We have here

$$\text{S.P.} = \frac{(100+\text{gain}\%)}{100} \times \text{C.P.}$$
$$= \frac{(100+8)}{100} \times \text{C.P.}$$
$$= \frac{54}{50}x.$$

With 2% loss:
We obtain here

$$\text{S.P.} = \frac{(100-\text{loss }\%)}{100} \times \text{C.P.}$$
$$= \frac{(100-2)}{100} \times \text{C.P.}$$
$$= \frac{49}{50}x.$$

Activity 3:

Calculate S.P. in both the cases using unitary method.

Thus the difference in the selling price with 8% profit and 2% loss is

$$\frac{54}{50}x - \frac{49}{50}x = \frac{5}{50}x = \frac{x}{10}.$$

But, this difference is given to be equal to ₹ 85, so that $\frac{x}{10} = 85$. This implies that, $x = 850$. Hence, the cost price of radio is ₹ 850.

Exercise 9.2

1. Sonu bought a bicycle for ₹ 3,750 and spent ₹ 250 on its repairs. He sold it for ₹ 4,400. Find his loss or profit percentage.
2. A shopkeeper purchases an article for ₹ 3,500 and pays transport charge of ₹ 100. He incurred a loss of 12% in selling this. Find the selling price of the article.
3. By selling a watch for ₹ 720, Ravi loses 10%. At what price should he sell it, in order to gain 15%?
4. Hari bought two fans for ₹ 2,400 each. He sold one at a loss of 10% and the other at a profit of 15%. Find the selling price of each fan and find also the total profit or loss.
5. A storekeeper sells a book at 15% gain. Had he sold it for 18 more, he would have gained 18%. Find the cost price of the book.
6. The cost price of 12 pens is equal to selling price of 10 pens. Find the profit percentage.

Discount

A reduction on the marked price of articles is called **discount**. Generally discount is given to attract customers to buy goods or to promote the sale of goods. The following completely describe the facts related to discount:

- discount is always given on the marked price of the article;
- discount = marked price - selling price;
- marked price is sometimes called list price;
- discount = rate of discount times the marked price;
- net price = marked price - discount.

Example 10. A computer marked at ₹ 18,000 was sold at ₹ 15,840. Find the percentage of discount.

Solution: Marked price is ₹ 18,000 and selling price is ₹ 15,480. Therefore, discount = ₹ 18,000 - ₹ 15,840 = ₹ 2,160.

Thus, for a marked price of ₹ 18,000, discount is ₹ 2,160.

For a marked price of ₹ 100, discount = $\frac{2160}{18000} \times 100 = 12\%$. Therefore the percentage of discount is 12.

Example 11. A tape recorder is sold at ₹ 5,225 after being given a discount of 5%. What is its marked price?

Solution: We are given that the discount is 5%. This means that for ₹ 100, the discount is ₹ 5.

Therefore, selling price = ₹ 100 - ₹ 5 = ₹ 95.

Thus on a selling price of ₹ 95, the marked price is ₹ 100.

On a selling price of ₹ 5,225 the marked price = $\frac{100}{95} \times 5225 = 5500$.

Therefore, marked price of the tape recorder is ₹ 5,500.

Example 12. A shopkeeper buys an article for ₹ 500. He marks it at 20% above the cost price. If he sells it at 12% discount, find the selling price.

Solution: Cost price = ₹ 500;

profit = 20% of ₹ 500 = $\frac{20}{100} \times 500$ = ₹ 100;

marked price = cost price + profit = ₹ 500 + ₹ 100 = ₹ 600.

Now the discount given is 12%.

For ₹ 600, the discount = $\frac{12}{100} \times 600$ = ₹ 72

Therefore, selling price = cost price - discount = ₹ 600 - ₹ 72 = ₹ 528. Hence the selling price of the article is 528.

Example 13. A cloth seller marks a dress at 45% above the cost price and allows a discount of 20%. What profit does he make in selling the dress?

Solution: Suppose the cost price of the dress material is ₹ 100. Since the seller marks it 45% above the C.P., the marked price would be

₹ 100 + ₹ 45 = ₹ 145.

Discount of 20% on this marked price is = 20% of ₹ 145 = $\frac{20}{100} \times 145$ = ₹ 29.

Therefore, selling price = ₹ 145 - ₹ 29 = ₹ 116. We get

profit = S.P. - C.P. = ₹ 116 - ₹ 100 = ₹ 16.

Hence, profit percent = $\frac{\text{profit}}{\text{C.P.}} \times 100 = \frac{16}{100} \times 100 = 16.$

Thus the merchant makes a profit of 16% on the marked price. (Observe it is not 45% - 20% = 25%)

Exercise 9.3

1. An article marked ₹ 800 is sold for ₹ 704. Find the discount and discount percent.
2. A dress is sold at ₹ 550 after allowing a discount of 12%. Find its marked price.
3. A shopkeeper buys a suit piece for ₹ 1,400 and marks it 60% above the cost price. He allows a discount of 15% on it. Find the marked price of the suit piece and also the discount given.
4. A dealer marks his goods 40% above the cost price and allows a discount of 10%. Find the profit percent.
5. A dealer is selling an article at a discount of 15%. Find:
   (i) the selling price if the marked price is ₹ 500;
   (ii) the cost price if he makes 25% profit.

Commission

You might have observed some advertisements in news papers regarding the availability of houses, sites, vehicles etc., for sale. Many a times, these transactions are mediated by a person other than the owner and the buyer. This mediator who helps in buying and selling is called **commission agent** or **broker**. The money that the broker or agent receives in the deal is called **brokerage** or **commission**. Commission is calculated on the transaction amount in percentage. Commission per hundred rupees is called **commission rate**.

Example 14. A real estate agent receives a commission of 1.5% in selling a land for ₹ 1,60,000. What is the commission amount?

Solution: Selling price of the land is ₹ 1,60,000 and the commission rate is 1.5%.

If the selling price is ₹ 100, commission = ₹ 1.5.

If the selling price is ₹ 1,60,000, commission = $\frac{1.5}{100} \times 160000$ = ₹ 2,400.

Therefore commission amount is ₹ 2,400.

The commission amount can also be calculated directly.

Commission = commission rate × selling price

In the example above,

commission = 1.5% of ₹ 1,60,000 = $\frac{1.5}{100} \times 160000$ = ₹ 2,400.

Example 15. The price of a long note book is ₹ 18. A shopkeeper sells 410 note books in a month and receives ₹ 1,033.20 as commission. Find the rate of commission.

Solution: Price of one note book is ₹ 18.

$\Rightarrow$ Price of 410 note books = 410×18 = ₹ 7,380.

Now the commission received for this amount is ₹ 1,033.20. Hence for ₹ 100, the commission is

$$\frac{1033.20}{7380} \times 100 = 14.$$

Therefore, the rate of commission is 14%.

Example 16. Abdul sold his house through a broker by paying ₹ 6,125 as brokerage. If the rate of brokerage is 2.5%, find the selling price of the house.

Solution: Brokerage given is ₹ 6,125. The brokerage rate is 2.5%.

If the brokerage is ₹ 2.5, selling price would be ₹ 100.

If the commission is ₹ 6,125, selling price is

$$\frac{100}{2.5} \times 6125 = 245000.$$

Thus the selling price of the house is ₹ 2,45,000.

The selling price can also be calculated directly;

$$\text{selling price} = \frac{100}{\text{commission rate}} \times \text{commission}.$$

In this example, S.P. $= \frac{100}{2.5} \times 6125 =$ ₹ 2,45,000.

Exercise 9.4

1. Sindhu sells her scooty for ₹ 28,000 through a broker. The rate of brokerage is $2\frac{1}{2}$ %. Find the commission that the agent gets and the net amount Sindhu gets.
2. A share agent sells 2000 shares at ₹ 45 each and gets the commission at the rate of 1.5%. Find the amount the agent gets.
3. A person insures ₹ 26,000 through an insurance agent. If the agent gets ₹ 650 as the commission, find the rate of commission.
4. A selling agent gets 10,200 in a month. This includes his monthly salary of ₹ 6000 and 6% commission for the sales. Find the value of goods he sold.

Simple interest

People borrow money from banks or financial institutions or money lenders for various purposes. While returning the borrowed money after a period of time, they need to pay some extra amount. This extra amount paid on the borrowed money after a period of time is called **interest**. In this context, we define the following terms.

1. **Principal:** the money borrowed is called principal or sum.
2. **Interest:** the extra money paid on the principal after a period of time is called interest.
3. **Amount:** the total money paid is called amount. Thus **Amount = Principal + Interest**.
4. **Rate:** interest for every ₹ 100 for one year is known as rate percent per annum.
5. **Time:** time is the duration for which the borrowed money is utilised. Time is expressed in years or months or days.
6. **Simple interest:** the interest calculated uniformly on the principal alone throughout the loan period is called simple interest. In other words, it is the interest paid on the principal alone.

In the world of finance (Bankers rule), time is often expressed in days also.

<u>Formula to find the simple interest:</u>

Let P = principal; R = rate of interest per annum; T = time in years; I = simple interest. These are related by the formula

$$I = \frac{P \times T \times R}{100}$$

From the above formula, we obtain different formulae:

$$P = \frac{100 \times I}{T \times R}, \quad T = \frac{100 \times I}{P \times R}, \quad R = \frac{100 \times I}{P \times T}.$$

Amount = principal + interest.

Example 17. Calculate the interest on ₹ 800 at $6\frac{1}{2}$ % per annum, for $3\frac{1}{2}$ years.

Solution: Given: P = ₹ 800; $T = 3\frac{1}{2}$ years $= \frac{7}{2}$ years; $R = 6\frac{1}{2}\% = \frac{13}{2}\%$. We use the formula for I;

$$I = \frac{PTR}{100} = \frac{800 \times \frac{7}{2} \times \frac{13}{2}}{100} = 2 \times 7 \times 13 = 182$$

Thus the interest is ₹ 182.

Example 18. Find the simple interest on ₹ 3,000 at 16% per annum for the period from 4th February 2010 to 16th June 2010.

Solution: Here the principal is P = ₹ 3,000, and the rate of interest $R = 16\%$ p.a. However, we are not given time in years; we have only the period, 4th February 2010 to 16th June 2010. We have to convert this to years. Observe that

February 5th to 28th	=	24 days (2010 is not a leap year)
March	=	31 days
April	=	30 days
May	=	31 days
June	=	16 days.

Adding, we get the total time is equal to 132 days. Converting this to years, $T = \frac{132}{365}$ years. Now we have all the required data to apply formula:

$$I = \frac{PTR}{100} = \frac{(3000 \times 132 \times 16)}{365 \times 100} = 173.58.$$

Thus the interest is approximately ₹ 174.

Note: 1. For calculating interest, the day on which money deposited is not counted, while the day on which money is withdrawn is counted.

2. When the time is given in days or months, it is to be expressed in years.

Example 19. A sum at a simple interest of $12\frac{1}{2}\%$ amounts to ₹ 2,502.50 after 3 years. Find the sum, and the interest.

Solution: Given: Amount = ₹ 2,502.50, and $R = 12\frac{1}{2}\% = \frac{25}{2}\%$. Let the sum be x. Using the formula, we have

$$I = \frac{PTR}{100} = \frac{x \times 3 \times 25}{2 \times 100} = \frac{75x}{200} = \frac{3x}{8}.$$

But we know that

Amount = Principal + Simple interest $= x + \dfrac{3x}{8} = \dfrac{(8x+3x)}{8} = \dfrac{11x}{8}$.

This amount is given to be ₹ 2,502.50. Therefore, we obtain $2502.50 = \dfrac{11x}{8}$

Solving for x, we get

$$x = \frac{2502.5 \times 8}{11} = 1820.$$

Hence, the sum is ₹ 1,820. The interest is

$$₹\ 2,502.50 - ₹\ 1,820 = ₹\ 682.50.$$

Example 20. ₹ 800 amounts to ₹ 920 in 3 years at a certain rate of interest. If the rate of interest is increased by 3%, what would the amount will become?

Solution: Recall, interest (I) = amount(A) - principal(P). Hence,

$$I = ₹\ 920 - ₹\ 800 = ₹\ 120.$$

This interest is accrued in T = 3 years, for the principal ₹ 800. Using, $I = \dfrac{PTR}{100}$ we get,

$$R = \frac{100 \times I}{P \times T} = \frac{100 \times 120}{800 \times 3} = 5.$$

Thus the original rate of interest is 5%. After the increase of interest by 3%, the new rate of interest, which we again denote by R = 5% +3% = 8%. The principal P = ₹ 800 and the period T = 3 years remain the same. Therefore,

$$I = \frac{PTR}{100} = \frac{800 \times 3 \times 8}{100} = 192.$$

Therefore, the new amount = ₹ 800 + ₹ 192 = ₹ 992.

Exercise 9.5

1. Find the simple interest on ₹ 2,500 for 4 years at $6\frac{1}{4}$% per annum.
2. Find the simple interest on ₹ 3,500 at the rate of $2\frac{1}{2}$% per annum for 165 days.
3. In what period will ₹ 5,200 amounts to ₹ 7,384 at 12% per annum simple interest?

4. Ramya borrowed a loan from a bank for buying a computer. After 4 years she paid ₹ 26,640 and settled the accounts. If the rate of interest is 12% per annum, what was the sum she borrowed?
5. A sum of money triples itself in 8 years. Find the rate of interest.

Tax

The Government requires money for its functioning. Money required for a Government is collected from the public in the form of taxes. One such method of collecting money is **Sales Tax**.

Sales Tax is the tax we pay when we buy goods/articles from a shop. Sales Tax is charged by the Government on the sale of every good/article. Sales tax is called indirect tax as it is collected from the manufacturer, wholesaler and retailer(shopkeeper) who in turn collects it from the customer.

Value added tax (VAT)

Value Added Tax (VAT) is a revised version of sales tax. Normally an article, before it reaches consumer, passes through various stages as given here:

Manufacturer → Wholesaler → Retailer → Consumer.

The person/company who/which manufactures an article is **Manufacturer**. Depending on the cost of producing an article, manufacturer marks the price higher than the cost price. On this marked price manufacturer has to charge sales tax, which he pays to the Government.

The person who purchases large quantities of articles from the manufacturer is the **Wholesaler**. He marks a price higher than the price he purchased from the manufacturer (as he has to get his profit). On this marked price, he also charges the sales tax.

The person who buys articles in smaller quantities from the whole saler is the **Retailer**. He marks the price higher than his cost price (he again takes his profit). On this marked price, he charges sales tax.

The common people who purchase the articles from the shop are **Consumers**. For the consumer, the cost price is the marked price of the retailer plus the sales tax on the marked price. Thus, **VAT** is a tax on the value added at each stage for a product that has to pass through various stages in the channel of distribution.

Remember:
No shopkeeper sells any article at loss. Even when a high discount is given, he makes a profit. The discount is given to attract customers and to expedite sales so that he can re invest his principal.

Example 21. Abdul purchases a pair of clothes with a marked price ₹ 1,350. If the rate of sales tax is 4%, calculate the amount to be paid by him.

Solution: Marked price of the item is ₹ 1,350, and sales tax is 4%, on the marked price. Hence the total sales tax on the item is $\frac{4}{100} \times 1350 =$ ₹54.
Amount to be paid =Marked price + tax = ₹ 1,350 + ₹ 54 = ₹ 1,404.
Hence, the amount to be paid by him is ₹ 1,404.

Example 22. A blazer marked at ₹ 1,600 is billed at ₹ 1,696. Find the rate of sales tax.

Solution: Selling price is ₹ 1,696 and marked price is ₹ 1,600. Hence sales tax paid is ₹ 96. On ₹ 1,600, sales tax is ₹ 96. Hence the percentage of sales tax is

$$\frac{96}{1600} \times 100 = 6.$$

Hence, the rate of sales tax is 6%.

Exercise 9.6

1. A person purchases the following items from a mall for which the sales tax is mentioned against:
   (a) Stationery materials for ₹ 250 and sales tax of 4% there on;
   (b) Electronic goods worth ₹ 2,580 and sales tax of 10% there on;
   (c) Groceries worth ₹ 1,200 on which sales tax of 3% is levied;
   (d) Medicines worth ₹ 200 with sales tax of 6%. Find the bill amount for each item.
2. A person buys electronic goods worth ₹ 10,000 for which the sales tax is 4% and other material worth ₹ 15,000 for which the sales tax is 6%. He manufactures a gadget using all these and sells it at 15% profit.What is his selling price?

3. A trader purchases 70 Kg of tea at the rate of ₹ 200/Kg and another 30 kg at the rate of ₹ 250/Kg. He pays a sales tax of 4% on the transaction. He mixes both of them and sells the product at the rate of ₹ 240/Kg. What is the percentage gain or loss? (find approximate value)

Glossary

Percentage: it means for every hundred; thus 6% means 6 for hundred.

Rate percent: when the percent is expressed as a fraction with denominator equal to 100, the numerator is called rate percent.

Cost price: the money paid to procure an item is called its cost price and abbreviated as C.P.

Overhead charges: it is the additional charges on the item before it is ready for sale, like, labour, transportation, etc.

Selling price: it is the price at which goods are sold; it is abbreviated as S.P.

Profit: it is the gain in a transaction; it is equal to S.P-C.P. whenever S.P. > C.P.

Loss: if S.P. is less than C.P., then C.P.-S.P. is the loss.

Discount: it is the reduction on the marked price of an item given by the seller to attract customers.

Brokerage or commission: it is the money charged by a mediator for the service provided for a smooth transaction of sales between a seller and a buyer.

Simple interest: it is the money charged by a lender on the receiver for the amount lent for a period of time.

Principal: it is the money lent for a period of time by a lender to a receiver.

Rate: it is the money charged as interest for every 100 for one year.

Time: it is the period for which money is lent.

Amount: principal plus interest is called amount.

Sales tax: on every sales done, the government levies a certain amount of money; it is called sales tax; this is a certain percent on the marked price of an item.

VAT: Value Added Tax; the sales tax goes on adding when there is multiple sales of goods.

Manufacturer: a company or a factory at which goods originate for sales.

Wholesaler: the purchaser who buys goods in bulk, and inturn sells it to small vendors.

Retailer: the vendor who buys goods in small quantities and sells it.

Consumer: the ultimate user of the goods.

Points to remember

- Percentage is a method of comparing quantities of the same kind.
- If SP > CP, then SP − CP is the profit; if SP < CP, then CP − SP is the loss.
- Profit or loss is always calculated on the cost price.
- Discount is the reduction given in the marked price.
- The money that an agent (or broker) receives in a deal is the commission (or brokerage).
- While calculating the simple interest, the day on which money is deposited is not counted where as the day on which it is withdrawn is taken in to account.
- VAT means value added tax.

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Answers

Exercise 9.1

1. chess: 270, carom: 540, other games: 90 **2.** ₹ 2,000. **3.** 25%.

4. ₹ 2,400. **5.** ₹ 9,800.

Exercise 9.2

1. 10% profit. **2.** ₹ 3,168. **3.** ₹ 920. **4.** Profit ₹ 120 **5.** 600 **6.** 20%.

Exercise 9.3

1. 12%. **2.** ₹ 625. **3.** ₹ 1904 (marked price) and ₹ 336 (discount).

4. 26%. **5.** (i) ₹ 425. (ii) ₹ 340.

Exercise 9.4

1. ₹ 700 (commission); Sindhu gets ₹ 27,300. **2.** ₹ 1,350. **3.** 2.5%. **4.** ₹ 70,000.

Exercise 9.5

1. ₹ 625. **2.** ₹ 39.55 **3.** $3\frac{1}{2}$ years **4.** ₹ 18,000. **5.** 25%.

Exercise 9.6

1. (i) ₹ 260; (ii) ₹ 2,838; (iii) ₹ 1,236. (iv) ₹ 212. **2.** ₹ 30,245 **3.** 7.33% gain.

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UNIT 10
EXPONENTS

After studying this unit you learn:

- the concept of an integral power to a non zero base.
- to write large numbers in exponential form.
- about the various laws of exponents and their use in simplifying complicated expressions.
- about the validity of these laws of exponents for algebraic variables.

Introduction

Suppose somebody asks you: how far is the Sun from the Earth? What is your answer? Perhaps some search in books will give you an idea how far is the Sun from us. A ray of light travels approximately at the speed of 2,99,792 Km per second. It takes roughly eight and a half minutes for a ray of light to reach the Earth starting from the Sun. Hence the distance from the Earth to the Sun is about15,29,00,000Km. Apart from the Sun, do you know how far is the nearest star to us? Proxima Centauri is the closest star to us and it is at a distance of 4.3 **light years** from us; that is, the distance a ray of light travels in 4.3 years at the speed of 2,99,792Km per second. This is equal to

$$4.3 \times 365 \times 24 \times 60 \times 60 \times 299792 \text{ Km},$$

which you can see is a very huge number.

At the other end what would be the size of an atom? As you can expect, it should be very small. The diameter of an atom is roughly $\frac{1}{100000000000}$ meters. The subatomic particles have still smaller sizes.

Hence there is a need to represent either big or small numbers in a compact form. The **exponential notation** is a very useful and handy notation used for this purpose. As we shall see there is a methodical way of handling exponentials.

Consider the number 128. An easy factorisation gives

$$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2.$$

Thus 128 is obtained by multiplying 2 with itself seven times. We write this as 2^7. Here 2 is called the **base** and 7 is called the **exponent**. We read this as *2 raised to the power 7*. The number 2^7 is called the exponential form of 128. Similarly, we observe that

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5.$$

Here 3 is the base and 5 is the exponent. We read this as *3 raised to the power 5*.

On the other hand, consider $72 = 2 \times 2 \times 2 \times 3 \times 3$. In the case of 128, the only prime factor is 2 and we are able to write $128 = 2^7$. Similarly, $343 = 3^5$, as 3 is the only prime factor of 343. However, 72 has two prime factors, namely, 2 and 3. We see that 2 occurs 3 times and 3 occurs 2 times. We write this in the form $72 = 2^3 \cdot 3^2$. We read this as *2 to the power of 3 times 3 to the power of 2*.

Let us consider $81 = 3 \times 3 \times 3 \times 3 = 3^4$. We also observe that $81 = 9 \times 9 = 9^2$. Thus the same number 81 can be written in two ways: $81 = 3^4 = 9^2$. In the first representation, 3 is the base and 4 is the exponent. In the second representation, 9 is the base and 2 is the exponent. Thus the same number may be represented using different bases and exponents.

Observe!

$9 = 3 \times 3 = 3^2$. Hence $9^2 = (3^2)^2$. Thus you obtain $(3^2)^2 = 9^2 = 81 = 3^4$. Can you recognise some thing?

It is not necessary that we use only numbers. For example, if we have an algebraic variable a, we write $a \times a \times a \times a = a^4$ and read this as a **raised to the power four**. Many times we suppress the word *raised*, and read a **to the power four** or simply a **power four**. An expression of the form ab^4 is read as a **times** b **power four**.

Observe:

$ab^4 = a \times b \times b \times b \times b = a(b^4)$, but not $ab \times ab \times ab \times ab$ which is actually $a^4 b^4$

To avoid such confusions, it is always better to use brackets: instead of writing ab^4, write this as $a(b^4)$. If you want $529 = 23 \times 23$, write this as $(23)^2$.

The notation 23^2 may some time be confused with 2×3^2 which is 18. Thus use proper brackets: $2(3^2)$ to represent 18 and $(23)^2$ to represent 529.

Another thing you may notice is that it is not necessary to use only integers as bases. In fact you may use any real number for base:

$$(0.1)^5 = 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 = 0.00001,$$
$$(-1.2)^3 = (-1.2) \times (-1.2) \times (-1.2) = -1.728,$$
$$(\sqrt{2})^2 = 2.$$

Think it over!

Is it possible to take for exponent a real number which is not an integer?

In general, given a number a or an algebraic variable which we again denote by a, and a natural number n, we define

$$a^n = \underbrace{a \times a \times a \times \cdots \times a}_{n \text{ times}}.$$

This is read as a **raised to the power** n or simply a **power** n. Here, we say a is the **base** and n is the **exponent**.

Observe:

$0^n = 0$ for any natural number n and $a^1 = a$ for any number a. Moreover, $a^m = a^n$ if and only if $m = n$ for any number $a \neq 0$, $a \neq 1$ or $a \neq -1$

Let us look at more examples:

Number	Expanded form	Expanded form	Base and index
1000	$10 \times 10 \times 10$	10^3	base 10 index 3
m^6	$m \times m \times m \times m \times m \times m$	m^6	base m index 6
$\frac{1}{1024}$	$\frac{1}{2} \times \frac{1}{2} \times \cdots$ (10 times)	$\left(\frac{1}{2}\right)^{10}$	base $\frac{1}{2}$, index 10
625	$5 \times 5 \times 5 \times 5$	5^4	base 5, index 4
625	$(-5) \times (-5) \times (-5) \times (-5)$	$(-5)^4$	base -5, index 4

Look at the last two examples. The number 625 is represented as 5^4 (with base 5 and exponent 4) and also as $(-5)^4$ (with base −5 and exponent 4). Thus the same number has representation in base 5 and base −5.

For a number $a \neq 0$, and a natural number n, we define

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}.$$

This extends the definition of power to negative integer exponents. Here are some more examples:

$$3^{-4} = \left(\frac{1}{3}\right) \times \left(\frac{1}{3}\right) \times \left(\frac{1}{3}\right) \times \left(\frac{1}{3}\right);\ \text{(read as 3 to the power −4.)}$$

$$(-0.1)^{-5} = \left(\frac{1}{(-0.1)}\right)^5 = (-10)^5 = -100000;\ \text{(read as (−0.1) to the power −5.)}$$

$$\left(\frac{4}{5}\right)^{-6} = \left(\frac{5}{4}\right)^6 = \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{5^6}{4^6}.$$

If $a = b^n$ for some integer $n \neq 0, 1$ and number $b \neq 0$, we say a is in **exponential form**. Here b is called the **base** and n is called the **exponent**. Earlier you have observed that $81 = 3^4 = 9^2$. Thus the same number may have different exponential forms.

Caution!
b^n **is not defined for** $b = 0$**, and** $n = 0$ **or** $n < 0$**.**

Example 1. Express 1024 and 0.1024 using base 2.

Solution: Consider the number $1024 = 2^{10}$. Here we have base 2 and exponent 10. However, 1024 is a number in decimal notation: here 4 is in unit's place; 2 in 10's place; 0 in 100's place; and 1 in 1000's place. Thus we have

$$\begin{aligned} 1024 &= (1 \times 1000) + (0 \times 100) + (2 \times 10) + (4 \times 1) \\ &= (1 \times 10^3) + (2 \times 10^1) + 4. \end{aligned}$$

Suppose we define $b^0 = 1$ for any number $b \neq 0$. Then we see that $1024 = 1 \cdot 10^3 + 2 \cdot 10^1 + 4 \cdot 10^0$. Do you see that we have done this in the unit **playing with numbers**? Similarly, consider 0.1024. You observe that

$$\begin{aligned} 0.1024 &= (0.1) + (0.002) + (0.0004) \\ &= \frac{1}{10} + \frac{2}{1000} + \frac{4}{10000} \\ &= \frac{1}{10} + \frac{2}{10^3} + \frac{4}{10^4} \\ &= 1 \cdot 10^{-1} + 2 \cdot 10^{-3} + 4 \cdot 10^{-4}. \end{aligned}$$

Example 2. Express 1000 using base 2 and exponents.

Solution: Consider 1000. How do you express this using base 2 and exponents? You observe that $512 = 2^9$, $256 = 2^8$, $128 = 2^7$ and $2^6 = 64$. Thus you get the sum

$$512 + 256 + 128 + 64 = 960.$$

You are still short by 40 to reach 1000. But $40 = 32 + 8 = 2^5 + 2^3$. Thus you obtain

$$\begin{aligned} 1000 &= 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^3 \\ &= 1 \cdot 2^9 + 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0. \end{aligned}$$

You observe that this is precisely the **binary** representation of 1000. We write this $1000 = (1111101000)_2$.

Some uses of exponents in daily life

Some examples of how exponents do connect with our everyday lives: when we speak about square feet, square meters or any such area units; or about cubic feet, cubic meters, cubic centimeters or any other such volume units.

The unit **square cm** is actually $1\text{cm} \times 1\text{cm} = 1\text{cm}^2$.

Similarly, a cubic cm is $1\text{cm} \times 1\text{cm} \times 1\text{cm} = 1\text{cm}^3$.

Another kind of indirect example is if you talk about extremely small or extremely big quantities. For example, the term 'nanometer' means 10^{-9} m. The prefix 'nano' means the number 10^{-9} an extremely small number. Or, within computer world you often see megabytes, gigabytes, terabytes. **Mega** means 10^6 or one million; **giga** means 10^9, and **tera** means 10^{12}.

Suppose you have a chemical concentration 0.000442 grams per litre, or a star having huge mass of 8,290,000,000,000,000,000,000 Kg. Those zeroes take up a lot of space. Scientific notation simplifies this by writing the last number, for example, as 829×10^{19} Kg, which means 829 followed by 19 zeroes. On the other-hand, the chemical concentration is written with a negative exponent: 4.42×10^{-4} grams per litre.

The mass of a proton is 0.000000000000000000000001673 g. You see how much space you need to write such a tiny number. If you use exponents, you may write this in the form 1.673×10^{-24} g.

Another place where exponential notation helps is in calculators. In a calculator, no display is possible once the number exceeds certain fixed number of digits. This fixed number varies from calculator to calculator depending on the capacity of the calculator. Scientific calculators, in which large or small numbers are calculated, use exponential notation. For example, a calculator may show only *eight* significant digits in its display. So to display a number of the form 234587643214878, which has 15 digits, calculators round it off to 234587640000000 and write the resulting number in the form 23458764×10^{7} (or some calculators in the form $.23458764\times 10^{15}$).

Activity 1:

Collect information about many more life situations where the exponential notation is useful.

Indian contribution to large numbers

The Indians had a passion for high numbers, which is intimately related to their religious thought. For example, in texts belonging to the Vedic literature dated from 1200 BC to 500 BC, we find individual Sanskrit names for each of the powers of 10 up to a trillion and even 10^{62}.

The Lalita vistara Sutra (a Mahayana Buddhist work) recounts a contest including writing, arithmetic, wrestling and archery, in which the Buddha was pitted against the great mathematician Arjuna and showed off his numerical skills by citing the names of the powers of ten up to 1 'tallakshana', which equals 10^{53}, but then going on to explain that this is just one of a series of counting systems that can be expanded geometrically. The last number at which he arrived after going through nine successive counting systems was 10^{421}, that is 1 followed by 421 zeros.

There is also an analogous system of Sanskrit terms for fractional numbers, capable of dealing with both very large and very small numbers.

Larger number in Buddhism seems

$$10^{7 \times 2^{122}} \text{ or } 10^{37218383881977644441306597687849648128}$$

which appeared as Bodhisattva's maths in the Avatamsaka Sutra.

Here are a few large numbers used in India by about 5th century BC (See Georges Ifrah: A Universal History of Numbers, pp 422-423):

koti -10^7, *ayuta* -10^9, *niyuta* -10^{11}, *kankara* -10^{13}, *pakoti* -10^{14}, *vivara* -10^{15}, *kshobhya* -10^{17}, *vivaha* -10^{19}, *kotippakoti* -10^{21}, *bahula* -10^{23}, *nagabala* -10^{25}, and so on. They also had name for 10^{421} as *dhvajagran ishamani*. (wikipedia.org/wiki/History_of_large_numbers)

An Indian Legend about Large numbers

King Shiraham of India was pleased with his grand *Vizier* (Chief minister) for inventing the game of Chess and wanted to reward him. Vizier's request was simple. He said: " Your Majesty, give me a grain of wheat to put on the first square of the chess board, two grains to put on the second square, four grains to put on the third, eight on the fourth, each timedoubling the number ofgrains,pleasegive me enoughgrains to cover all the 64 squares." King thought that his minister's request was very very modest. He ordered his men to bring the wheat grains and fulfill his Vizier's desire. But he soon realised his folly. Do you know how many grains are needed to comply with the minister's request? It is $2^{64} - 1$ grains. This is roughly equal to the world's wheat production for more than 2000 years.(At the current rate of production.)

Exercise 10.1

1. Express the following numbers in the exponential form:

   (i) 1728 (ii) $\frac{1}{512}$ (iii) 0.000169.

2. Write the following numbers using base 10 and exponents:

   (i) 12345 (ii) 1010.0101 (iii) 0.1020304

3. Find the value of $(-0.2)^{-4}$.

The first law of exponents

Consider $1024 = 2^{10}$. Observe

$$2^{10} = 1024 = 2 \times 512 = 2^1 \times 2^9 : \quad 1 + 9 = 10 \text{ and } 2^{10} = 2^{1+9};$$

$$2^{10} = 1024 = 4 \times 256 = 2^2 \times 2^8: \quad 2 + 8 = 10 \text{ and } 2^{10} = 2^{2+8};$$

$$2^{10} = 1024 = 8 \times 128 = 2^3 \times 2^7: \quad 3 + 7 = 10 \text{ and } 2^{10} = 2^{3+7};$$

$$2^{10} = 1024 = 16 \times 64 = 2^4 \times 2^6: \quad 4 + 6 = 10 \text{ and } 2^{10} = 2^{4+6};$$

$$2^{10} = 1024 = 32 \times 32 = 2^5 \times 2^5: \quad 5 + 5 = 10 \text{ and } 2^{10} = 2^{5+5};$$

What do you observe? Consider more examples:

$$5^4 = 625 = 25 \times 25 = 5^2 \times 5^2: \quad 2 + 2 = 4 \text{ and } 5^4 = 5^{2+2};$$

$$a^7 = (a \times a \times a \times a) \times (a \times a \times a) = a \times a: \quad 4 + 3 = 7 \text{ and } a^7 = a^{4+3}.$$

Looking at these examples, can you formulate a law?

For any number a and positive integers m, n, $a^m \times a^n = a^{m+n}$

This is also true for an algebraic variable x: $x^m x^n = x^{m+n}$. This is called the **first law of exponents**. This is useful in simplifying large expressions.

Examples:

3. $2^5 \times 2^6 = 2^{5+6} = 2^{11}$;

4. $3^3 \times 3^6 \times 3^7 = (3^3 \times 3^6) \times 3^7 = 3^9 \times = 3^7 = 3^{9+7} = 3^{16}$.

5. $2^5 \times 5^2 \times 2^3 \times 5 = (2^5 \times 2^3) \times (5^2 \times 5) = 2^8 \times 5^3$.

Exercise 10.2

1. Simplify:

   (i) $3^1 \times 3^2 \times 3^3 \times 3^4 \times 3^5 \times 3^6$. (ii) $2^2 \times 3^3 \times 2^4 \times 3^5 \times 3^6$.

2. How many zeros are there in $10^4 \times 10^3 \times 10^2 \times 10$?

3. Which is larger: $(5^3 \times 5^4 \times 5^5 \times 5^6)$ or $(5^7 \times 5^8)$?

The second law of exponents

Look at the following example:

$$2^9 = 512 = \frac{1024}{2} = \frac{2^{10}}{2^1} \quad : \quad 9 = 10 - 1 \text{ and } 2^9 = 2^{10-1}$$

$$2^8 = 256 = \frac{1024}{4} = \frac{2^{10}}{2^2} \quad : \quad 8 = 10 - 2 \text{ and } 2^8 = 2^{10-2}$$

$$2^7 = 128 = \frac{1024}{8} = \frac{2^{10}}{2^3} \quad : \quad 7 = 10 - 3 \text{ and } 2^7 = 2^{10-3}$$

$$2^6 = 64 = \frac{1024}{16} = \frac{2^{10}}{2^4} \quad : \quad 6 = 10 - 4 \text{ and } 2^6 = 2^{10-4}$$

$$2^6 = 64 = \frac{256}{4} = \frac{2^8}{2^2} \quad : \quad 6 = 8 - 2 \text{ and } 2^6 = 2^{8-2}.$$

Study one more:

$$3^3 = 27 = \frac{243}{9} = \frac{3^5}{3^2} \quad : \quad 3 = 5 - 2 \text{ and } 3^3 = 5^{5-2}$$

Can you see some pattern? Is it apparent that some law is followed here again?

For any number a and positive integers m,n, with $m > n$,

$$\frac{a^m}{a^n} = a^{m-n}.$$

The natural question is: what happens if $m = n$ or $m<n$. A few examples will help to clarify the situation.

$$\frac{1}{2^5} = \frac{1}{32} = \frac{2}{64} = \frac{2^1}{2^6}.$$

Recall: we have defined $a^{-n} = \frac{1}{a^n}$ for any $a \neq 0$ and positive integer n. Thus

$$2^{-5} = \frac{2^1}{2^6} \qquad (\because -5 = 1-6)$$

Similarly,

$$3^{-4} = \frac{1}{3^4} = \frac{1}{81} = \frac{9}{729} = \frac{3^2}{3^6}; \qquad -4 = 2 - 6 \text{ and } 3^{-4} = 3^{2-6};$$

$$10^{-1} = \frac{1}{10} = \frac{1000}{10000} = \frac{10^3}{10^4}; \qquad -1 = 3 - 4 \text{ and } 10^{-1} = 10^{3-4}.$$

The above observations may be put in the following form:

For any number $a \neq 0$ and positive integers m,n, with $m \neq n$,

$$\frac{a^m}{a^n} = a^{m-n}.$$

Again recall, we have defined $a^0 = 1$ for all $a \neq 0$. Observe

$$\frac{2^5}{2^5} = 1 = 2^0 : 5 - 5 = 0 \text{ and } 2^{5-5} = 2^0;$$

$$\frac{3^4}{3^4} = 1 = 3^0 : 4 - 4 = 0 \text{ and } 3^{4-4} = 3^0;$$

This shows that $\frac{a^m}{a^m} = 1 = a^0$ for all $a \neq 0$. We can now reformulate our law:

For any number $a \neq 0$ and positive integers m,n, not necessarily distinct,

$$\frac{a^m}{a^n} = a^{m-n}.$$

The important thing here is that the only condition on m, n is that they are positive integers. It may happen that $m < n$, or $m = n$ or $m > n$. In all these cases, the above law holds.

There is another way of looking at the second law. Suppose $a \neq 0$, and m,n are positive integers. Then

$$a^{m-n} = \frac{a^m}{a^n} = a^m \times \frac{1}{a^n} = a^m \times a^{-n},$$

where we have used $a^{-n} = \frac{1}{a^n}$

Thus we obtain $a^{m-n} = a^m \times a^{-n}$.

Observe that this resembles the first law except that we have negative integer $-n$. In this sense, the second law simply extends the first law. Again observe that for natural numbers m,n,

$$a^{-(m+n)} = \frac{1}{a^{m+n}} = \frac{1}{a^m \times a^n} \quad \text{(using the first law)}$$

$$= \frac{1}{a^m} \times \frac{1}{a^n} \quad \text{(property of fractions)}$$

$$= a^{-m} \times a^{-n}.$$

Thus we see that the first law is also valid for negative integers m,n. We can combine both the first and the second law and state them together:

If $a \neq 0$ and m,n are integers, then $a^m \times a^n = a^{m+n}$

Example 6. Simplify

$$\frac{3^5 \times 2^2 \times 7^{-3}}{7^{-2} \times 3^{-3} \times 2^4}.$$

Solution: The expression is equal to

$$\left(\frac{3^5}{3^{-3}}\right) \times \left(\frac{2^2}{2^4}\right) \times \left(\frac{7^{-3}}{7^{-2}}\right) = 3^{5-(-3)} \times 2^{2-4} \times 7^{-3-(-2)}$$

$$= 3^{5+3} \times 2^{-2} \times 7^{-3+2}$$

$$= 3^8 \times 2^{-2} \times 7^{-1} = \frac{3^8}{2^2 \times 7} = \frac{6561}{28}.$$

Example 7. If $3^{l} \times 3^{2} = 3^{5}$, find the value of l.

Solution: Using the second law of exponents, we obtain $3^{l+2} = 3^{5}$. But we know that for any $a \neq 0, 1, -1$, if $a^{m} = a^{n}$, then $m = n$. Hence $l + 2 = 5$ or $l = 3$.

Exercise 10.3

1. Simplify: (i) $10^{-1} \times 10^{2} \times 10^{-3} \times 10^{4} \times 10^{-5} \times 10^{6}$; (ii) $\dfrac{2^3 \times 3^2 \times 5^4}{3^3 \times 5^2 \times 2^4}$.
2. Which is larger: $(3^{4} \times 2^{3})$ or $(2^{5} \times 3^{2})$?
3. Suppose m and n are distinct integers. Can $\dfrac{3^m \times 2^n}{2^m \times 3^n}$ be an integer? Give reasons.
4. Suppose b is a positive integer such that $\dfrac{2^4}{b^2}$ is also an integer. What are the possible values of b?

The third law of exponents

Consider the following examples:

$2^{10} = 2^{2+2+2+2+2} = 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 = (2^2)^5$:

$2 \times 5 = 10$ and $(2^2)^5 = 2^{10} = 2^{2 \times 5}$

$2^{10} = 2^{5+5} = 2^5 \times 2^5 = (2^5)^2$:

$5 \times 2 = 10$ and $(2^5)^2 = 2^{10} = 2^{5 \times 2}$;

$3^{12} = 3^{2+2+2+2+2+2} = 3^2 \times 3^2 \times 3^2 \times 3^2 \times 3^2 \times 3^2 = (3^2)^6$:

$2 \times 6 = 12$ and $(3^2)^6 = 3^{12} = 3^{2 \times 6}$;

$3^{12} = 3^{3+3+3+3} = 3^3 \times 3^3 \times 3^3 \times 3^3 = (3^3)^4$:

$3 \times 4 = 12$ and $(3^3)^4 = 3^{12} = 3^{3 \times 4}$;

$3^{12} = 3^{6+6} = 3^6 \times 3^6 = (3^6)^2$:

$6 \times 2 = 12$ and $(3^6)^2 = 3^{12} = 3^{6 \times 2}$;

What do you see? Can you put your observations in to a rule?

If $a \neq 0$ is a number and m, n are positive integers, then $(a^{m})^{n} = a^{mn}$.

Study more examples:

$$2^{2 \times (-4)} = 2^{-8} = \frac{1}{2^8} = \frac{1}{(2^2)^4} = (2^2)^{-4};$$

$$3^{(-5) \times 2} = 3^{-10} = \frac{1}{3^{10}} = \frac{1}{(3^5)^2} = \left(\frac{1}{3^5}\right)^2 = (3^{-5})^2.$$

Do you see that negative integer exponents follow similar rule?

If $a \neq 0$ **is a number and** m, n **are positive integers, then**

$(a^m)^{-n} = a^{-mn} = (a^{-m})^n$.

What do you expect in the case when exponents are negative integers? Consider the example:

$$5^{(-4)\times(-3)} = 5^{12} = \frac{1}{5^{-12}} = \frac{1}{(5^{-4})^3} = \left(5^{-4}\right)^{-3}.$$

Are you convinced that the negative exponents also follow the same rule: $(a^{-m})^{-n} = a^{mn}$ for all numbers $a \neq 0$ and positive integers m,n?

What if one of m,n is equal to 0? You observe that for $a \neq 0$, $a^0 = 1$ and hence $(a^0)^n = 1^n = 1 = a^0$ and $0 \times n = 0$.

Thus we can formulate the third law of exponents:

If $a \neq 0$ **is a number and** m,n **are integers , then** $(a^m)^n = a^{mn}$.

Example 8. Simplify . $\dfrac{(1024)^3 \times (81)^4}{(243)^2 \times (128)^4}$

Solution: Observe $1024 = 2^{10}$, $81 = 3^4$, $243 = 3^5$, and $128 = 2^7$. Hence the expression is

$$\frac{(2^{10})^3 \times (3^4)^4}{(3^5)^2 \times (2^7)^4} = \frac{(2^{10})^3 \times (3^4)^4}{(3^5)^2 \times (2^7)^4}$$

$$= 2^{30-28} \times 3^{16-10}$$

$$= 2^2 \times 3^6$$

Exercise 10.4

1. Simplify: (i) $\dfrac{(2^5)^6 \times (3^3)^2}{(2^6)^5 \times (3^2)^3}$; (ii) $\dfrac{(5^{-3})^2 \times 3^4}{(3^{-2})^{-3} \times (5^3)^{-2}}$
2. Can you find two integers m,n such that $2^{m+n} = 2^{mn}$?
3. If $(2^m)^4 = 4^6$, find the value of m.

The fourth law of exponents

Study the following examples.

(1) $10^5 = 10 \times 10 \times 10 \times 10 \times 10$

$= 2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5$

$= (2 \times 2 \times 2 \times 2 \times 2) \times (5 \times 5 \times 5 \times 5 \times 5)$

$= 2^5 \times 5^5$.

Here you may observe that $10 = 2 \times 5$.

$$(2) \qquad 3^4 \times 2^4 = (3 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2)$$
$$= (3 \times 2) \times (3 \times 2) \times (3 \times 2) \times (3 \times 2)$$
$$= 6 \times 6 \times 6 \times 6$$
$$= 6^4.$$

Here again $3 \times 2 = 6$. Consider one more example:

$$(3) \qquad (-2)^4 \times (4^4) = (-2) \times (-2) \times (-2) \times (-2) \times 4 \times 4 \times 4 \times 4$$
$$= (-2 \times 4) \times (-2 \times 4) \times (-2 \times 4) \times (-2 \times 4)$$
$$= (-8) \times (-8) \times (-8) \times (-8) = (-8)^4.$$

Once again you find that $(-2) \times 4 = (-8)$. Let us see what happens if both the numbers are negative.

$$(4) \qquad (-3)^3 \times (-5)^3 = (-3) \times (-3) \times (-3) \times (-5) \times (-5) \times (-5)$$
$$= \big((-3) \times (-5)\big) \times \big((-3) \times (-5)\big) \times \big((-3) \times (-5)\big)$$
$$= 15 \times 15 \times 15$$
$$= 15^3.$$

Again you see that expected rule is true: $(-3) \times (-5) = 15$. Now you can formulate new law:

If a and b are two non zero numbers and m is any positive integer, then $(a \times b)^m = a^m \times b^m$.

Observe that this is also true if $m = 0$. Since $a \neq 0$ and $b = 0$, you may conclude that $a \times b \neq 0$. Thus $(a \times b)^0 = 1$. But $a = 1$ and $b^0 = 1$ so that $a^0 \times b^0 = 1$. It follows that $(a \times b)^0 = a^0 \times b^0$. What happens if m is negative? If a, b are non zero numbers and n is a positive integer, you see that

$$(a \times b)^{-n} = \frac{1}{(a \times b)^n} = \frac{1}{a^n \times b^n} = \frac{1}{a^n} \times \frac{1}{b^n} = a^{-n} \times b^{-n}.$$

Thus the negative exponents also obey the same rule. We may now reformulate the law:

For any two numbers $a \neq 0$ and $b \neq 0$, and integer m, $(a \times b)^m = a^m \times b^m$.

Here are more examples .

Example 9. Simplify: $\dfrac{15^4 \times 14^2}{21^3 \times 10^3}$

Solution: The numerator is

$$15^4 \times 14^2 = (3 \times 5)^4 \times (2 \times 7)^2 = 3^4 \times 5^4 \times 2^2 \times 7^2.$$

Similarly the denominator is

$$21^3 \times 10^3 = (7 \times 3)^3 \times (2 \times 5)^3 = 7^3 \times 3^3 \times 2^3 \times 5^3.$$

The given expression is therefore

$$\frac{3^4 \times 5^4 \times 2^2 \times 7^2.}{7^3 \times 3^3 \times 2^3 \times 5^3} = 3^{4-3} \times 5^{4-3} \times 2^{2-3} \times 7^{2-3} = \frac{3 \times 5}{2 \times 7} = \frac{15}{14}.$$

Example 10. Which is larger: $(0.25)^4$ or $(0.35)^3$?

Solution: We use the simple observation: if a and b are non zero numbers, then $a<b$ is equivalent to $\dfrac{a}{b} < 1$.

Write $0.25 = \dfrac{1}{4}$ and $0.35 = \dfrac{35}{100} = \dfrac{7}{20}$. We have to compare $\dfrac{1}{4^4}$ and $\dfrac{7^3}{20^3}$ But $20^3 = (4 \times 5)^3 = 4^3 \times 5^3$. Thus we see that

$$\frac{(0.25)^4}{(0.35)^3} = \frac{4^3 \times 5^3}{4^4 \times 7^3} = \frac{5^3}{4 \times 7^3} < 1,$$

since $5 < 7$. Thus $(0.25)^4 < (0.35)^3$.

Exercise 10.5

1. Simplify: (i) $\dfrac{6^8 \times 5^3}{10^3 \times 3^4}$; (ii) $\dfrac{(15)^{-3} \times (-12)^4}{5^{-6} \times (36)^2}$; (iii) $\dfrac{(0.22)^4 \times (0.222)^3}{(0.2)^5 \times (0.2222)^2}$.
2. Is $\dfrac{(10^4)^3}{5^{13}}$ an integer? Justify your answer.
3. Which is larger : $(100)^4$ or $(125)^3$?

The fifth law of exponents

Consider

$$\left(\frac{3}{4}\right)^5 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{3 \times 3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4 \times 4} = \frac{3^5}{4^5}.$$

Similarly, you observe that

$$\left(\frac{-4}{5}\right)^3 = \left(\frac{-4}{5}\right) \times \left(\frac{-4}{5}\right) \times \left(\frac{-4}{5}\right) = \frac{(-4) \times (-4) \times (-4)}{5 \times 5 \times 5} = \frac{(-4)^3}{5^3}.$$

What do you infer from these two examples? Do you see that there is yet another law of exponent?

If a, b **are non zero numbers and** m **is a positive integer,** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$. You may also observe that this is valid if $m = 0$.

In this case

$$\left(\frac{a}{b}\right)^0 = 1 = \frac{1}{1} = \frac{a^0}{b^0}.$$

What do you think if m happens to be a negative integer? If $m < 0$, then $n = -m$ is apositive integer. Hence you can use the law for positive integral exponent:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

However, we know that $a^n = \frac{1}{a^{-n}} = \frac{1}{a^m}$.

Similarly $b^n = \frac{1}{b^{-n}} = \frac{1}{b^m}$ or $\frac{1}{b^n} = b^m$.

Thus you may obtain

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = a^n \times \frac{1}{b^n} = \frac{1}{a^m} \times b^m = \frac{b^m}{a^m}.$$

But we know

$$\frac{1}{\left(\frac{a}{b}\right)^m} = \left(\frac{a}{b}\right)^n = \frac{b^m}{a^m}.$$

This implies that

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m.$$

We have obtained law for negative exponent. Thus we may write the fifth law of exponents:

> **If** $a \neq 0$ **and** $b \neq 0$**, and** m **is an integer, then** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 11. Simplify $\left(\frac{0.2}{5}\right)^4 \times \left(\frac{2}{0.5}\right)^{-3}$.

Solution: You may simplify the two fractions separately. Thus

$$\left(\frac{0.2}{5}\right) = \frac{1}{5^2}, \quad \left(\frac{2}{0.5}\right) = 2 \times 2 = 2^2.$$

Thus the expression simplifies to

$$\left(\frac{1}{5^2}\right)^4 \times (2^2)^{-3} = \frac{1}{5^8 \times 2^6} = \frac{1}{5^2 \times 10^6} = 0.00000004$$

Example 12. Which is larger: $(2.5)^6$ or $(1.25)^{12}$?

Solution: We write $(2.5)^6 = \left(\frac{5}{2}\right)^6 = \frac{5^6}{2^6}$. Similarly, you may obtain

$$(1.25)^{12} = \left(\frac{5}{4}\right)^{12} = \frac{5^{12}}{4^{12}} = \frac{5^{12}}{2^{24}}.$$

Thus you have to check which of the numbers $\frac{5^6}{2^6}, \frac{5^{12}}{2^{24}}$ is larger. Equivalently, you must determine the larger number between 5^6 and 2^{18} (why?). However $5^2 = 25 < 32 = 2^5$. Hence

$$5^6 = (5^2)^3 < (2^5)^3 = 2^{15} < 2^{18}.$$

Thus

$$\frac{5^{12}}{2^{24}} = \frac{5^6}{2^{18}} \times \frac{5^6}{2^6} < \frac{5^6}{2^6}.$$

Hence $(1.25)^{12} < (2.5)^6$.

Exercise 10.6

1. Simplify: (i) $\left(\frac{2}{3}\right)^8 \times \left(\frac{6}{4}\right)^3$; (ii) $(1.8)^6 \times (4.2)^{-3}$; (iii) $\frac{(0.0006)^9}{(0.015)^{-4}}$.
2. Can it happen that for some integer $m \neq 0$, $\left(\frac{4}{25}\right)^m = \left(\frac{2}{5}\right)^{m^2}$?
3. Find all positive integers m,n such that $(3^m)^n = 3^m \times 3^n$.

Can you see that the fifth law of exponents can be deduced from the fourth law? You may write $\frac{a}{b} = a \times b^{-1}$. Thus you obtain

$$\left(\frac{a}{b}\right)^m = (a \times b^{-1})^m = a^m \times (b^{-1})^m = a^m \times b^{-m} = \frac{a^m}{b^m}.$$

You may notice that you needed $(b^{-1})^m = b^{-m}$ which is a consequence of the third law. Thus the fifth law is a consequence of the fourth and third laws.

Exercise 10.7

1. Use the laws of exponents and simplify.

$$\text{(i)}\ \frac{(12)^6}{162} \qquad \text{(ii)}\ \frac{3^{-4}\times 10^{-5}\times (625)}{5^{-3}\times 6^{-4}} \qquad \text{(iii)}\ \frac{2^{3^2}}{(2^3)^2}.$$

2. What is the value of $\frac{(10^3)^2\times 10^{-4}}{10^2}$?

3. Simplify :

$$\left(\frac{b^{-3}\cdot b^{7}\cdot (b^{-1})^{2}}{(-b)^{2}\cdot (b^{2})^{3}}\right)^{-2}.$$

4. Find the value of each of the following expressions:

(a) $(3^2)^2-((-2)^3)^2-(-(5^2))^2$;

(b) $((0.6)^2)^0-((4.5)^0)^{-2}$;

(c) $(4^{-1})^4\times 2^5\times\left(\frac{1}{16}\right)^3\times (8^{-2})^5\times (64^2)^3$;

Important points

All these laws are also valid for algebraic variables. So far you have seen the validity of these laws for numbers. If x is a variable, we have

$$x^m\cdot x^n = x^{m+n}, \text{ for all integers } m,n. \qquad (1)$$

Here again, we define $x^0 = 1$. Unlike for numbers, where we have defined $a^{-n} = \frac{1}{a^n}$ for $a \neq 0$, we cannot so easily introduce $\frac{1}{x}$. Hence we define x^{-n}, for a natural number n, as that expression for which $x^n\cdot x^{-n} = 1$ holds. We write x^{-n} in the form $\frac{1}{x^n}$. Thus we have the result: $x^n\cdot\frac{1}{x^n} = 1$, for all natural numbers n. Note that $x^n = \frac{1}{x^{-n}}$ also holds for all natural numbers n. With this understanding the above law (1) holds. Similarly, you may write laws:

$$(x^m)^n = x^{mn}, \text{ for all integers } m,n; \qquad (2)$$

$$(x\cdot y)^m = x^m\cdot y^m, \text{ for all integers } m \text{ and variables } x,y. \qquad (3)$$

Glossary

1) Light year: this is the distance travelled by a light ray in one year; it is equal to $4.3\times 365\times 24\times 60\times 60\times 299792$ km.

2) Exponential notation: writing $a \times a \times \cdots \times a$, where the product is taken n times in the form a^n **3) Base:** in the abbreviation a^n, a is the base.

4) Exponent: in the abbreviation a, n is the exponent.

5) Laws of exponents: $a^m = a^n = a^{m+n}$; $(a^m)^n = a^{mn}$; $(ab)^m = a^m \times b^m$ hold for non zero numbers and integers m,n. (These are also true for algebraic variables a,b and real numbers m,n.)

Points to remember

- $0^n = 0$ for $n > 0$; 0^n is not defined for $n \leq 0$.
- $a^m = a^n$ if and only if $m = n$ for any $a \neq 0, 1$ or -1.
- For any $a \neq 0$, and natural number n, $a^{-n} = \dfrac{1}{a^n}$.
- For any $a \neq 0$, and integers m,n, $a^m \times a^n = a^{m+n}$
- For any $a \neq 0$, and integers m,n, $a = a$.
- For any $a \neq 0$, $b \neq 0$ and integer m, $(ab)^m \times a^m = b^m$

Answers

Exercise 10.1

1. (i) 12^3 (ii) 2^{-9} (iii) $(0.013)^2$.

2. (i) $10^4 + 2 \cdot 10^3 + 3 \cdot 10^2 + 4 \cdot 10 + 5$

(ii) $10^3 + 10^1 + \dfrac{1}{10^2} + \dfrac{1}{10^4}$ (iii) $\dfrac{1}{10} + \dfrac{2}{10^2} + \dfrac{3}{10^5} + \dfrac{4}{10^7}$. **3.** $\dfrac{1}{0.0016}$

Exercise 10.2

1. (i) 3^{21} (ii) $2^6 \cdot 3^{14}$. **2.** 10 zeroes. **3.** $5^3 \times 5^4 \times 5^5 \times 5^6 > 5^7 \times 5^8$.

Exercise 10.3 : **1.** (i) 10^3 (ii) $\dfrac{25}{6}$. **2.** $3^4 \times 2^3 > 2^5 \times 3^2$. **4.** $b = 1, 2$ or 4.

Exercise 10.4 : **1.** (i) 1 (ii) $\dfrac{1}{9}$ **2.** $m = 2$, $n = 2$. **3.** $m = 3$.

Exercise 10.5 : **1.** (i) 2592 (ii) $\dfrac{2000}{27}$ **2.** No. **3.** $100^4 > 125^3$.

Exercise 10.6 : **1.** (i) $\dfrac{32}{243}$ (ii) $\dfrac{19683}{42875}$ (iii) $\dfrac{3^{13}}{2^{39} \times 5^{44}}$.

2. can happen for $m = 2$. **3.** $(m,n) = (2, 2)$.

Exercise 10.7 : **1.** (i) 1152 (ii) $\dfrac{25}{2}$ (iii) 8. **2.** 1. **3.** b^{12}. **4.** (a) 642 (b) 0 (c) 2^{-9}

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UNIT 11

CONGRUENCY OF TRIANGLES

After studying this unit, you learn to:

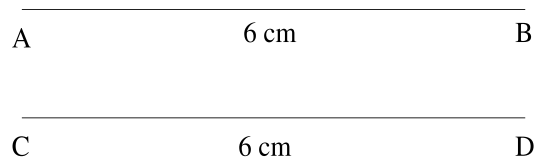
- identify the congruent figures.
- identify the congruent triangles.
- identify the corresponding sides and corresponding angles of congruent triangles.
- state the postulates for congruency of triangles.
- understand that particular triples of elements determine the congruency of triangles.
- deduce logical methods for proving theorems.
- solve problems based on different postulates of congruency.
- appreciate the use of congruency of triangles in solving practical day to day problems.

Introduction

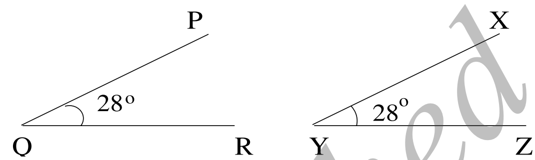
Suppose you have two equilateral triangles, each of side length, say 1cm. Can you place one on the other so that all the three sides coincide? That is all the three sides of one triangle sit exactly on three sides of the other triangle. Take two equilateral triangles of side lengths 1 cm and 2 cm respectively. Can you put one on the others so that one exactly fits the other? No matter how you adjust them, you see that it is impossible to superimpose one on the other. The exact geometrical idea which needed to understand these is **congruency**.

Congruency is one of the fundamental concepts in geometry. This concept is used to classify the geometrical figures on the basis of their shapes. Two geometrical figures are said to be **congruent**, if they have same shape and size. For example

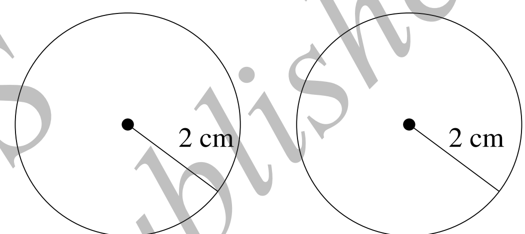
1. Two line segments are congruent if they have same length.



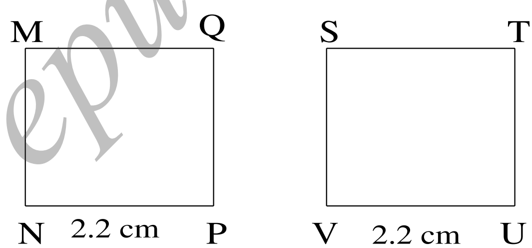
2. Two angles are congruent if they have same measure.



3. Two circles are congruent if they have same radii.



4. Two squares are congruent if they have sides of same length.

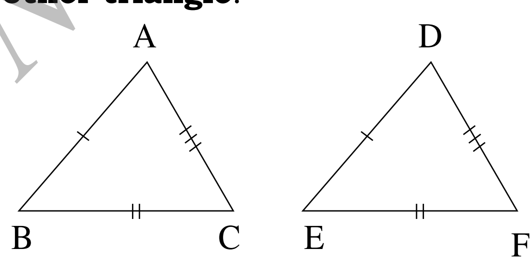


Note: 1. Two geometrical congruent figures can be made to superimpose one on the other so that one exactly covers the other.

2. Two congruent geometrical figures have same parameters.

<u>Congruency of triangles</u>

Two triangles are said to be congruent **if all the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle**.



In triangles ABC and DEF, you observe AB = DE, AC = DF and BC = EF; ∠A = ∠D, ∠B = ∠E, and ∠C = ∠F.

Therefore, triangles ABC and DEF are congruent.

We write this as $\Delta ABC \cong \Delta DEF$.

A few words about the use of this notation. When we write $\Delta ABC \cong \Delta DEF$, the important thing is to observe that the vertices A,B,C correspond to the vertices D,E,F in that order. If we write $\Delta ABC \cong \Delta EFD$, this gives a different meaning. This means $\angle A = \angle E$, $\angle B = \angle F$ and $\angle C = \angle D$; and $AB = EF$, $BC = FD$ and $CA = DE$. Can you see the difference? So while using the notation for congruency, keep the order of the vertices in proper way.

Recall that there are **six** elements associated with a triangle. Two triangles are congruent if and only if these six elements match in a suitable sense. This will be made clear by understanding corresponding sides and angles.

Corresponding sides and angles

Let us say that, on superposition, triangle ABC covers triangle DEF exactly in such way that

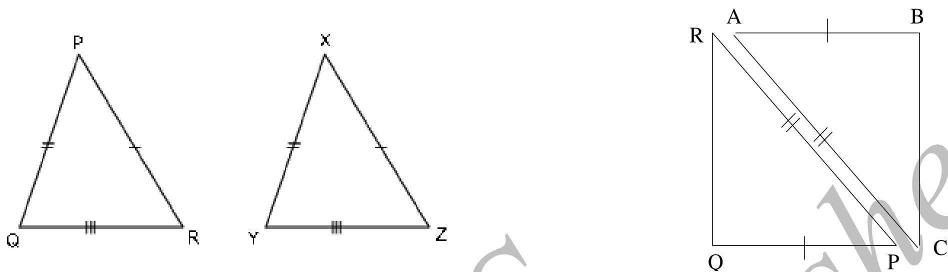
1. $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$;
2. $AB = DE$, $BC = EF$, $AC = DF$.

Then $\Delta ABC \cong \Delta DEF$ Angles which coincide on superposition are called **corresponding angles**. Sides which coincide on superposition are called **corresponding sides**. Generally it is not always possible to superimpose one triangle over other triangle to know which angles are the corresponding angles and which are the corresponding sides. Can you see that if two triangles are congruent, then there are **corresponding vertices** as well. However,vertices are points and do not have any numerical quantities associated with them. Hence we use only sides and angles for determining congruency.

Geometrically, in two congruent triangles ABC and DEF, angles opposite to equal sides are corresponding angles and so they are equal. Therefore, $BC = EF$ implies that angle opposite to BC = angle opposite to EF. Symbolically $BC = EF \Rightarrow \angle A = \angle D$. Similarly, $AC = DF \Rightarrow \angle B = \angle E$ and $AB = DE \Rightarrow \angle C = \angle F$.

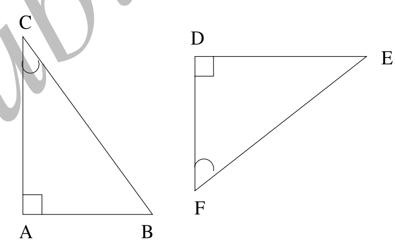
Exercise 11.1

1. Identify the corresponding sides and corresponding angles in the following congruent triangles:

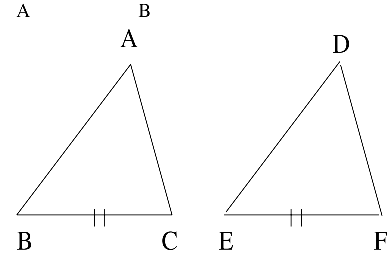


2. Pair of congruent triangles and incomplete statements related to them are given below. Observe the figures carefully and fill up the blanks:

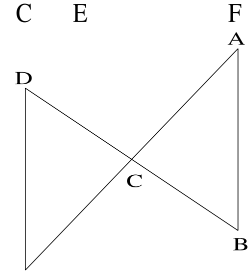
(a) In the adjoining figure if $\angle C = \angle F$, then AB = ––––– and BC = –––––.



(b) In the adjoining figure if BC = EF, then $\angle C$ = ––– and $\angle A$ = –––––.



(c) In the adjoining figure, if AC = CE and $\Delta ABC \cong \Delta DEC$, then $\angle D$ = ––––– and $\angle A$ = –––––.



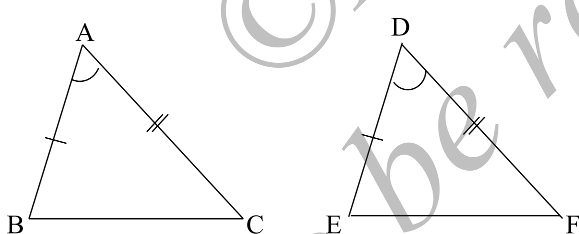
SAS postulate for the congruency of triangles

<u>Least number of conditions for congruency</u>

Now we know that two triangles are congruent if and only if the corresponding sides and corresponding angles of two triangles are equal (six elements), i.e., three for corresponding angles and three for corresponding sides. Now the natural question is; what is the least number of conditions required to ensure the congruency of two triangles? Do we need the corresponding equality of all the six elements or a lesser number of conditions will suffice to ensure the congruency of two triangles? In this and subsequent sections, we shall see that if three properly chosen elements out of six of a triangle are equal to the corresponding three elements of another triangle, then other three elements of two triangles coincide in order and we have the congruency of two triangles. Let us discuss those three conditions which will ensure the congruency of two triangles. One has to be careful in choosing three conditions. For example three angles will not do. (Draw two non-congruent triangles which have same set of angles.)

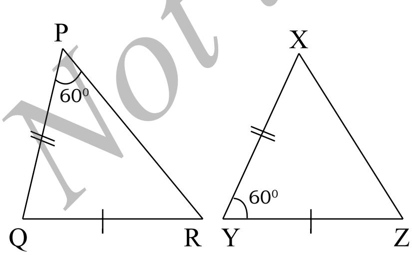
Side-angle-side postulate [SAS postulate]

If the two sides and included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle, then the two triangles are congruent.



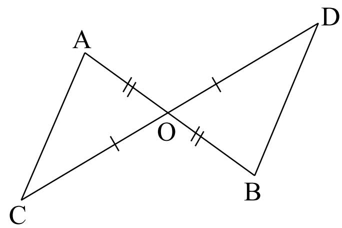
In triangles ABC and DEF, you observe that AB = DE, AC = DF and $\angle A = \angle D$. Hence SAS postulate tells

$$\Delta ABC \cong \Delta DEF.$$



Look at the following triangles PQR and XYZ. You observe that PQ = XY and QR = YZ. Moreover $\angle P = 60^\circ = \angle Y$. Still the triangles PQR and XYZ need not be congruent because the included angles $\angle Q$ and $\angle Y$, which are corresponding angles, need not be equal.

Example 1. In the figure O is the midpoint of AB and CD. Prove that (i) $\Delta AOC \cong \Delta BOD$; (ii) $AC = BD$.



Solution: In triangles AOC and BOD, we have

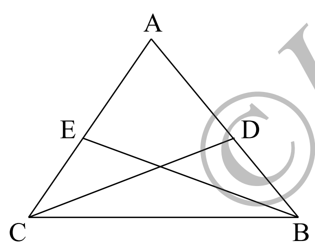
$AO = BO$, (O, the midpoint of AB);
$\angle AOC = \angle BOD$, (vertically opposite angles);
$CO = OD$, (O, the midpoint of CD).

So by SAS postulate we have

$$\Delta AOC \cong \Delta BOD$$

Hence $AC = BD$, as they are corresponding parts of congruent triangles.

Example 2. In the figure, it is given that $AE = AD$ and $BD = CE$. Prove that Δ AEB is congruent to Δ ADC.

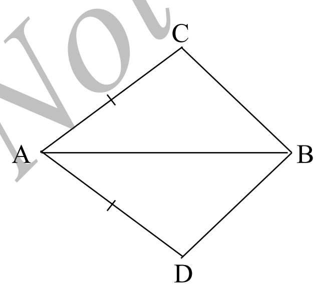


Solution: We have $AE = AD$ and $CE = BD$. Adding, we get $AE + CE = AD + BD \Rightarrow AC = AB$. In triangles AEB and ADC, we have

$AE = AD$, (given);
$AB = AC$, (proved);
$\angle EAB = \angle DAC$, (common angle).

By SAS postulate $\Delta AEB \cong \Delta ADC$.

Example 3. In a quadrilateral ACBD, $AC = AD$ and AB bisect $\angle A$. Show that ΔABC is congruent to ΔABD.



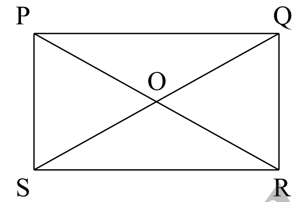
Solution: In triangles ABC and ABD, we have

$AC = AD$, (given);
$\angle CAB = \angle DAB$, (AB bisects $\angle A$);
$AB = AB$, (common side).

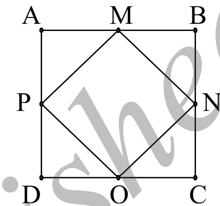
Hence $\Delta ABC \cong \Delta ABD$

(Can you see that *BA* bisects $\angle CBD$?)

Exercise 11.2

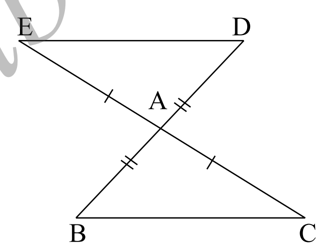


1. In the adjoining figure PQRS is a rectangle. Identify the congruent triangles formed by the diagonals.



2. In the figure ABCD is a square, M,N,O and P are the midpoints of sides AB, BC, CD and DA respectively. Identify the congruent triangles.

3. In a triangle ABC, AB = AC. Points E on AB and D on AC are such that AE = AD. Prove that triangles BCD and CBE are congruent.

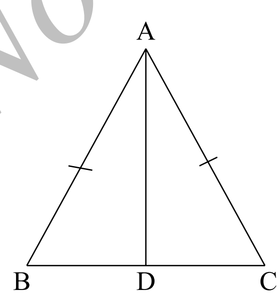


4. In the adjoining figure, the sides BA and CA have been produced such that BA = AD and CA = AE. Prove that DE ∥ BC. [hint:-use the concept of alternate angles.]

Consequences of SAS postulate

Now you have learnt how to compare two triangles using SAS condition. This comparison will lead to some very interesting consequences about the properties of triangles. We study a few of them here.

Theorem1. In a triangle, the angles opposite to equal sides are equal.



Given: A triangle ABC in which AB = AC.

To prove: $\angle C = \angle B$.

Construction: Draw the angle bisector of $\angle A$. Let it cut BC at D. Let us compare triangles ABD and ACD:

Proof:

Statement	**Reasons**
$AB = AC$;	given
$AD = AD$;	common side
$\angle BAD = \angle CAD$;	by construction.

We can use SAS postulate to conclude that $\Delta ADB \cong \Delta ADC$. Hence $\angle ABC = \angle ACB$, since these are corresponding angles of congruent triangles. Thus the theorem is proved.

Corollary 1: In an isosceles triangle, the angle bisector of the apex angle is the perpendicular bisector of the base.

Given: ABC is a triangle in which $AB = AC$ and apex angle $\angle A$.

Construction: Draw the angle bisector AD from A on BC.

To prove: AD is the perpendicular bisector of BC. Equivalently, we have to show that $BD = DC$ and $\angle ADB = \angle ADC = 90^\circ$.

Proof:

$\Delta ADB \cong \Delta ADC$, by (theorem 1)

$\Rightarrow DB = DC$ and. $\angle ADB = \angle ADC$.

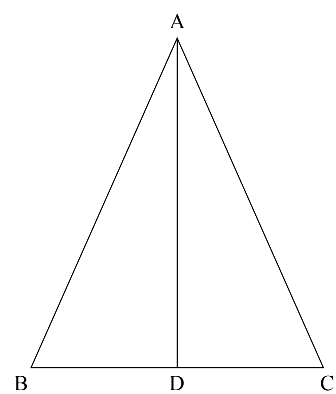
But $\angle ADB + \angle ADC = 180^\circ$ (linear pair)

$\Rightarrow \angle ADB + \angle ADC = 180^\circ$

$\Rightarrow 2\angle ADB = 180^\circ$

$\Rightarrow \angle ADB = \dfrac{180^\circ}{2} = 90^\circ$.

$\Rightarrow \angle ADC = 180^\circ - \angle ADB = 90^\circ$.

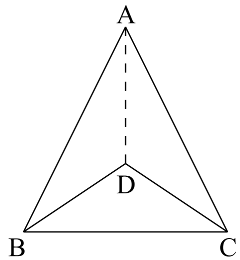


We also observe that $\Delta ABD \cong \Delta ACD$ by SAS postulate. Hence $BD = CD$.

This completes the proof.

Now you may wonder whether the converse of the theorem is true: in any triangle, the sides opposite to the equal angles are equal. It is also true, but its proof needs a different condition of congruency, which you will study later.

Example 4. In the figure $AB = AC$ and $DB = DC$. Prove that $\angle ABD = \angle ACD$.



Solution: In ΔABC, we have

$$AB = AC \Rightarrow \angle ABC = \angle ACB$$

(angles opposite to equal sides).

Again in ΔDBC, we have

$DB = DC$ (given) $\Rightarrow \angle DBC = \angle DCB$ (angles opposite to equal sides).

Hence we obtain

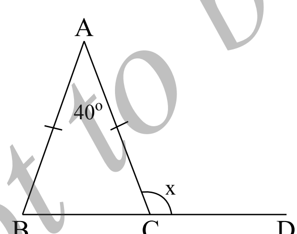
$$\angle ABC - \angle DBC = \angle ACB - \angle DCB.$$

This gives

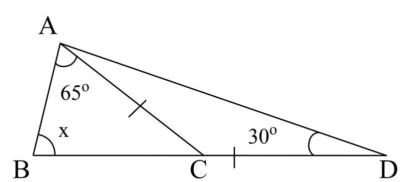
$$\angle ABD = \angle ACD.$$

Exercise 11.3

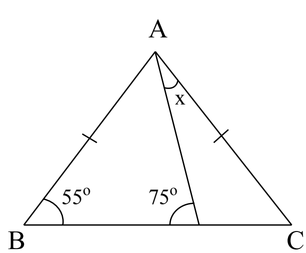
1. In a ΔABC, $AB = AC$ and $\angle A = 50^\circ$. Find $\angle B$ and $\angle C$.
2. In ΔABC, $AB = BC$ and $\angle B = 64^\circ$. Find $\angle C$.
3. In each of the following figure, find the value of x:



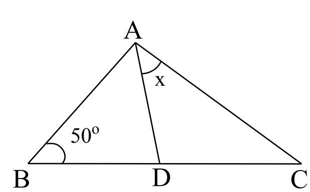
AB=AC



AC=CD



AB=AC

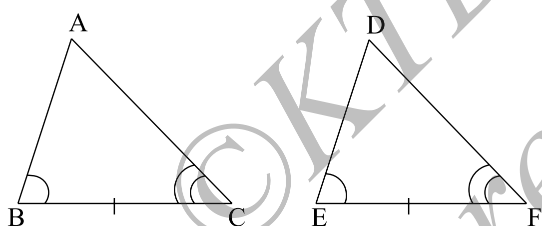


BD=DC=AD

4. Suppose ABC is an equilateral triangle. Its base BC is produced to D such that BC = CD. Calculate (i) $\angle ACD$ and (ii) $\angle ADC$.
5. Show that the perpendiculars drawn from the vertices of the base of an isosceles triangle to the opposite sides are equal.
6. Prove that a ΔABC is an isosceles triangle if the altitude AD from A on BC bisects BC.
7. Suppose a triangle is **equilateral**. Prove that it is **equiangular**.

ASA postulate for congruency

If two angles and the common side of one triangle are equal to the corresponding two angles and the common side of the other triangle, then the two triangles are congruent.



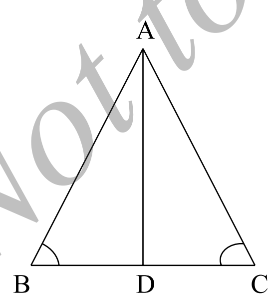
Given two triangles ABC and DEF such that

$\angle B = \angle E$, $\angle C = \angle F$ and BC = EF, the ASA postulates tells

$$\Delta ABC \cong \Delta DEF.$$

Earlier you have seen that the angles opposite to equal sides of a triangle are equal. Now we are ready to prove the converse of this result.

Theorem 2. If in a triangle two angles are equal, then the sides opposite to them are equal. (Converse of Theorem 1.)



Given: triangle ABC in which

$$\angle B = \angle C.$$

To prove: AC = AB.

Construction: Draw $AD \perp BC$.

Proof: Then $\angle ADB = \angle ADC = 90°$. We are given $\angle DBA = \angle DCA$. Consider triangles ADB and ADC. We have

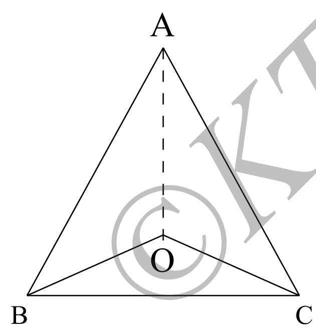
$$\angle ADB + \angle DBA + \angle BAD = 180° = \angle ADC + \angle DCA + \angle CAD.$$

It follows that $\angle BAD = \angle CAD$ (why?). Consider triangles ADB and ADC. We have

$\angle BAD = \angle CAD$	(proved);
$\angle ADB = \angle ADC$	(both are right angles);
$AD = AD$	(common side.)

$\Rightarrow \Delta ADB \cong \Delta ADC$, by ASA condition. We conclude that AB = AC, by property of congruency. This completes the proof of the converse of theorem 1.

Example 5. In a triangle ABC, AB = AC and the bisectors of angles B and C intersect at O. Prove that BO = CO and AO is the bisector of angle $\angle BAC$.



Solution: Since the angles opposite to equal sides are equal,

AB = AC

$$\Rightarrow \angle C = \angle B$$

$$\Rightarrow \frac{\angle B}{2} = \frac{\angle C}{2}$$

Since BO and CO are bisectors of $\angle B$ and $\angle C$, we also have

$$\angle ABO = \frac{\angle B}{2} \text{ and } \angle ACO = \frac{\angle C}{2}.$$

Hence

$$\angle ABO = \frac{\angle B}{2} = \frac{\angle C}{2} = \angle ACO$$

Consider ΔBCO:

$\angle OBC = \angle OCB$	(Why?)
$\Rightarrow BO = CO,$	(Theorem 2).

Finally, consider triangles ABO and ACO.

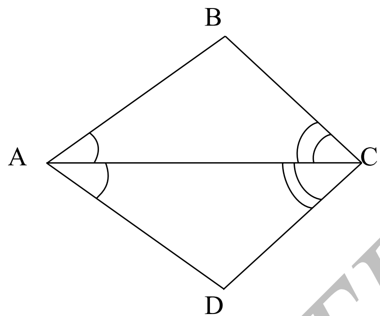
$BA = CA$	(given);
$BO = CO$	(proved);
$\angle ABO = \angle ACO$	(proved).

Hence by SAS postulate

$$\Delta ABO \cong \Delta ACO$$

$$\Rightarrow \angle BAO = \angle CAO \Rightarrow AO \text{ bisects } \angle A.$$

Example 6. Diagonal AC of a quadrilateral ABCD bisects the angles $\angle A$ and $\angle C$. Prove that AB = AD and CB = CD.



Solution: Since diagonal AC bisects the angles $\angle A$ and $\angle C$, we have $\angle BAC = \angle DAC$ and $\angle BCA = \angle DCA$. In triangles ABC and ADC, we have

$\angle BAC = \angle DAC$ (given);

$\angle BCA = \angle DCA$ (given);

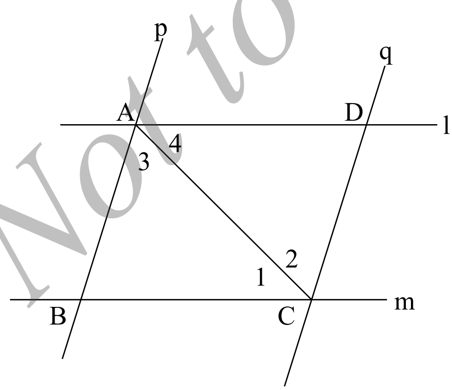
$AC = AC$ (common side).

So, by ASA postulate, we have

$$\Delta BAC \cong \Delta DAC$$

$\Rightarrow$ BA = AD and CB = CD (Corresponding parts of congruent triangle).

Example 7. Two parallel lines l and m are intersected by another pair of parallel lines p and q as in the figure. Show that triangles ABC and CDA are congruent.



Solution: Since l and m are parallel lines and AC is a transversal, $\angle 1 = \angle 4$. Similarly transversal AC cuts parallel lines p and q, so that $\angle 2 = \angle 3$. In triangles ABC and CDA, we have

$\angle 1 = \angle 4$ (proved);

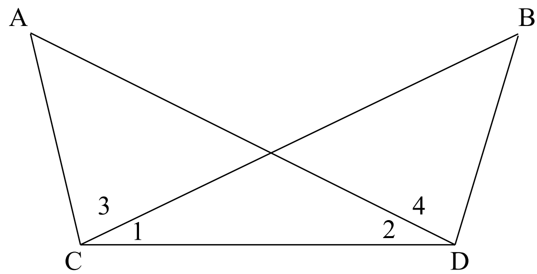
$\angle 2 = \angle 3$ (proved);

$AC = AC$ (common side).

By ASA postulate,

$$\Delta ABC \cong \Delta CDA.$$

Example 8. In the figure, $\angle BCD = \angle ADC$ and $\angle ACB = \angle BDA$. Prove that $AD = BC$ and $\angle A = \angle B$.



Solution: We have $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$\Rightarrow \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle ACD = \angle BDC$.

Thus in triangles ACD and BDC, we have,

$\angle ADC = \angle BCD$ (given);

$CD = CD$ (common);

$\angle ACD = \angle BDC$ (proved.)

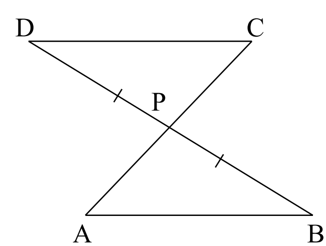
By ASA condition $\Delta ACD \cong \Delta BDC$. Therefore

$AD = BC$ and $\angle A = \angle B$.

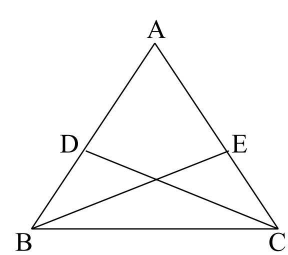
Think it over! We have taken ASA condition as a postulate. But actually, it can be proved as a theorem based on SAS postulate. Try to construct a proof. In deductive geometry, one takes only a minimum number of postulates and try to construct as many results as possible using these minimum number of postulates. One can also take ASA as a postulate and obtain SAS condition as a theorem.

Exercise 11.4

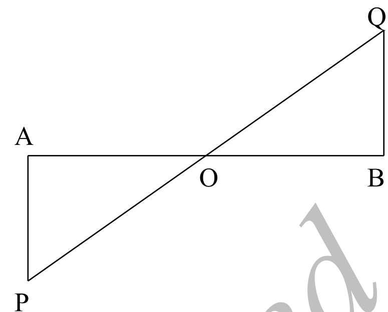
1. In the given figure, If $AB \parallel DC$ and P is the midpoint of BD, prove that P is also the midpoint of AC.



2. In the adjacent figure, CD and BE are altitudes of an isosceles triangle ABC with $AC = AB$. Prove that $AE = AD$.



3. In figure, AP and BQ are perpendiculars to the line segment AB and AP = BQ. Prove that O is the midpoint of line segment AB as well as PQ.



4. Suppose ABC is an isosceles triangle with AB = AC; BD and CE are bisectors of ∠B and ∠C. Prove that BD = CE.

5. Suppose ABC is an equiangular triangle. Prove that it is equilateral.(You have seen earlier that an equilateral triangleis equiangular. Thus for triangles equiangularity is equivalent to equilaterality.)

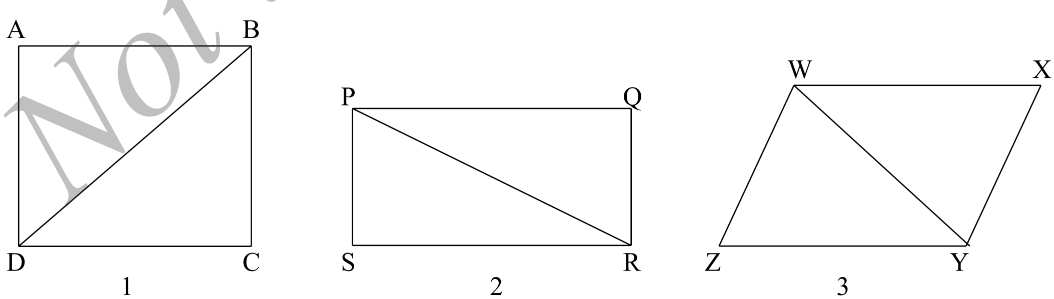
SSS postulate for congruency

We will study one more condition for congruency of two triangles. It depends only on the sides.

If three sides of one triangle are equal to the three corresponding sides of the other triangle, then the triangles are congruent.

Activity 1:

Take three sheets of papers; one in the shape of a square, another rectangular and a third one in the shape of a parallelogram (for this you may have to draw a parallelogram on a sheet of paper and cut it along the boundary). Draw diagonals as shown in the figure. Cut the sheets along the diagonals.

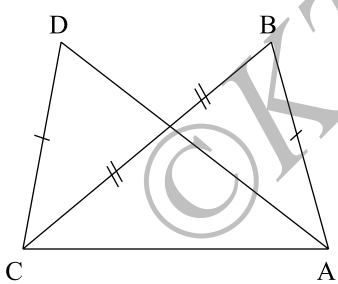


You will get two triangles from each sheet. Now you place one of the triangular sheet obtained from each figure on the other triangular sheet of the same figure such that it exactly covers the other triangular sheet. You will notice that each pair of triangles are congruent and in each case three sides of one triangle are equal to the corresponding sides of the other triangle.

Now we look for the converse of this. If three sides of one triangle are equal to the three corresponding sides of another triangle, can we put one on the other such that each covers the other exactly? SSS congruency condition says that it is indeed the case.

> **Think it over!**
> **Since there is SSS congruence postulate, can we have AAA postulate and SSA postulate?**

Example 9. In the figure, it is given that AB = CD and AD = BC. Prove that triangles ADC and CBA are congruent.



Solution: In triangles ADC and CBA, we have

AB = CD (given);

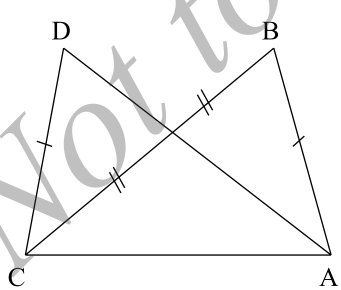
AD = BC (given);

AC = AC (common side.)

By SSS congruency condition, $\Delta ADC \cong \Delta CBA$.

Example 10. In the figure AD = BC and BD = CA. Prove that

$$\angle ADB = \angle BCA \text{ and } \angle DAB = \angle CBA.$$



Solution: In triangles ABD and BAC, we have

AD = BC given;

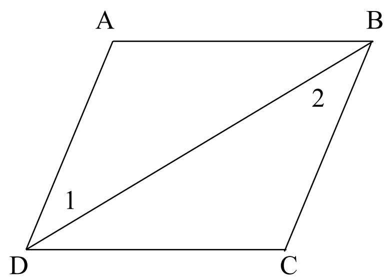
AB = AB (common);

AC = BD (given.)

We can use SSS condition to conclude that $\Delta ABD \cong \Delta BAC$. From this we conclude that

$$\angle ADB = \angle BCA \text{ and } \angle DAB = \angle CBA.$$

Example 11. In the adjoining figure, AB = CD and AD = BC. Show that $\angle 1 = \angle 2$.



Solution: In triangles ABD and CDB, we have

AB = CD (given);

AD = BC (given);

BD = DB (common side.)

Hence $\Delta ABD \cong \Delta CDB$, by SSS postulate. Comparing the angles, we get

$$\angle 1 = \angle 2.$$

(Later you will see that, under the given conditions, ABCD is a parallelogram so that AD ∥ BC, and $\angle 1$ and $\angle 2$ are alternate angles formed by the transversal BD.)

Think it over! We have taken SSS condition as a postulate. But this is also a consequence of SAS postulate. You can get SSS as a theorem from SAS postulate.

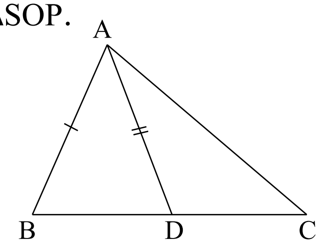
Exercise 11.5

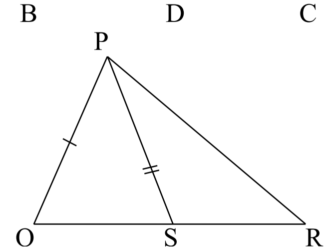
1. In a triangle ABC, AC = AB and the altitude AD bisects BC. Prove that $\Delta ADC \cong \Delta ADB$.
2. In a square PQRS, diagonals bisect each other at O. Prove that
$$\Delta POQ \cong \Delta QOR \cong \Delta ROS \cong \Delta SOP.$$
3. In the figure, two sides AB, BC and the median AD of ΔABC are respectively equal to two sides PQ, QR and median PS of ΔPQR. Prove that

   (i) $\Delta ADB \cong \Delta PSQ$;

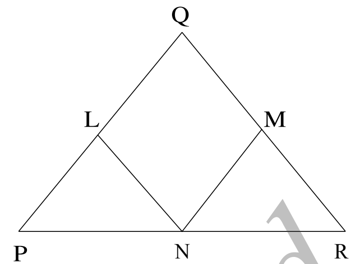
   (ii) $\Delta ADC \cong \Delta PSR$.

   Does it follow that triangles ABC and PQR are congruent?

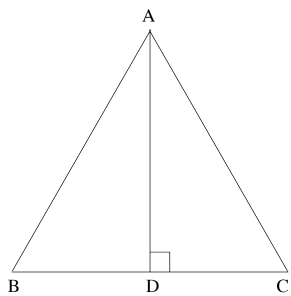




4. In ΔPQR, $PQ = QR$; L,M and N are the midpoints of the sides of PQ, QR and RP respectively. Prove that $LN = MN$.

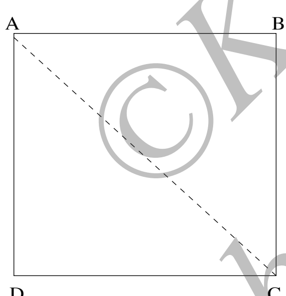


RHS theorem



Activity 2:

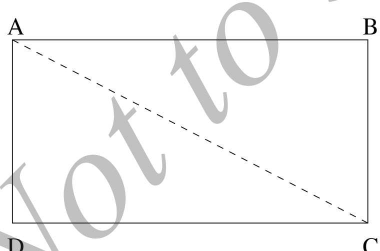
Draw an equilateral triangle ABC on a sheet of paper. From the vertex A, draw the perpendicular AD to the base the BC. Cut the sheet of paper along the triangle. Now fold it along the perpendicular line. You will notice that two right angled triangles superpose one on the other. So the two triangles are congruent.



Activity 3:

Take a square sheet of paper. Fold the sheet of the paper along one of its diagonal. You will notice that two triangles so formed by folding the sheet are right angled triangles and they superpose one on the other

Activity 4:

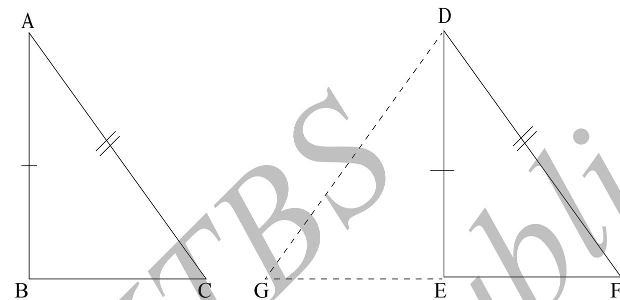


Take a rectangular sheet of paper, such that one of its side length is equal to the length of the square you had taken earlier and the other is different from the length of the square. Draw one of its diagonals.Cut the sheet along the diagonal, to obtain two right triangles. Can you see that here also you can superpose one right triangle on the other?

Now place one of the triangular sheets from the square and another from the rectangular sheet. You will notice that even though the two triangles are right angled and one set of corresponding sides are equal, they do not superpose one on the other.

In SAS postulate, You have noticed earlier that you need the equality of two corresponding sides and the included angle for congruency. However for a right angled triangle, you can relax this a bit to get what is known as RHS condition. We have the following theorem on right angled triangles.

Theorem 3. Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are equal to the hypotenuse and the corresponding side of the other triangle. (RHS theorem.)



Given: two right-angled triangles ABC and DEF such that

(i) $\angle B = \angle E = 90°$;

(ii) Hypotenuse AC = Hypotenuse DF; and

(iii) AB = DE.

To prove: $\Delta ABC \cong \Delta DEF$.

Construction: Produce FE to G so that EG = BC. Join DG.

Proof: In triangles ABC and DEG, observe that

AB = DE	(given);
BC = EG	(by construction);
$\angle ABC = \angle DEG$	(each equal to 90°.)

Hence by SAS, $\Delta ABC \cong \Delta DEG \Rightarrow \angle ACB = \angle DGE$ and AC = DG. But AC = DF, by the given hypothesis. We thus get

$$DG = AC = DF.$$

In triangle DGF, we have got DG = DF (just proved). This implies that

$\angle G = \angle F$ (angles opposite to equal sides are equal).

In triangles DEF and DEG,

$$\angle G = \angle F \qquad \text{(proved)}$$
$$\angle DEG = \angle DEF \qquad \text{(both equal to } 90^\circ\text{)}$$

Hence

$$GDE = 180^\circ - (\angle G + \angle DEG) = 180^\circ - (\angle F + \angle DEF) = \angle FDE.$$

Consider triangles DEG and DEF. We have

$$DG = DF \qquad \text{(proved);}$$
$$DE = DE \qquad \text{(common);}$$
$$\angle GDE = \angle FDE \qquad \text{(proved).}$$

Hence by SAS condition,

$$\Delta DEG \cong \Delta DEF.$$

But we have already proved that

$$\Delta ABC \cong \Delta DEG.$$

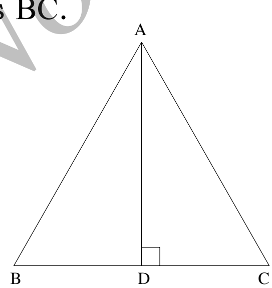
It follows that

$$\Delta ABC \cong \Delta DEF.$$

Note: We have used an important result: If there are three triangles ABC, DEF and JKL such that ABC is congruent to DEF and DEF is congruent to JKL, then ABC is congruent to JKL. This is precisely Axiom 3 in unit 11.

Think it over! If two sides of a right triangle are respectively equal to the corresponding sides of another right triangle, can you conclude that the two triangles are congruent? (We are not demanding that hypotenuse of two triangles be equal.)

Example 12. Suppose ABC is an isosceles triangle such that AB = AC and AD is the altitude from A on BC. Prove that(i) AD bisects $\angle A$,(ii) AD bisects BC.



Solution: We have to show that

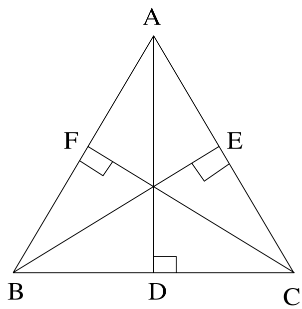
$\angle BAD = \angle CAD$ and BD = DC.

In right triangles ADB and ADC, we have

$$AB = AC \qquad \text{(given);}$$
$$AD = AD \qquad \text{(common side).}$$

So by RHS congruency of triangles, we have $\Delta ABD \cong \Delta ACD$. Hence $\angle BAD = \angle CAD$ and $BD = DC$.

Example 13. Suppose the altitudes AD, BE and CF of triangle ABC are equal. Prove that ABC is an equilateral triangle.



Solution: In right triangles BCE and CBF, we have,

$BC = BC$ (common hypotenuse);
$BE = CF$ (given).

Hence BCE and CBF are congruent, by RHS theorem. Comparing the triangles, we get $\angle B = \angle C$.

This implies that

$$AC = AB \text{ (sides opposite to equal angles).}$$

Similarly,

$$AD = BE \Rightarrow \angle B = \angle A$$
$$\Rightarrow AC = BC.$$

Together, we get $AB = BC = AC$ or ABC is equilateral.

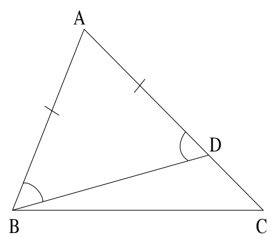
Exercise 11.6

1. Suppose ABCD is rectangle. Using RHS theorem,prove that triangles ABC and ADC are congruent.
2. Suppose ABC is a triangle and D is the midpoint of BC. Assume that the perpendiculars from D to AB and AC are of equal length. Prove that ABC is isosceles.
3. Suppose ABC is a triangle in which BE and CF are respectively the perpendiculars to the sides AC and AB. If $BE = CF$, prove that triangle ABC is isosceles.

Some consequences

You have seen earlier that in a triangle the angles opposite to equal sides are equal. And conversely, sides opposite to equal angles are equal. So the natural question is: if angles are of different measures, can we compare the sides opposite to them? Can we say something about angles if sides are of different lengths?

Proposition 1. Suppose two sides of a triangle are not equal. Then the angle opposite to a larger side is greater than the angle opposite to the smaller side.



Given: a triangle ABC in which

AC >AB.

To prove: ∠B> ∠C.

Construction: Take a point D on AC such that AB = AD. (This is possible since AC >AB.) Join BD.

Proof: In triangle ABD, we have

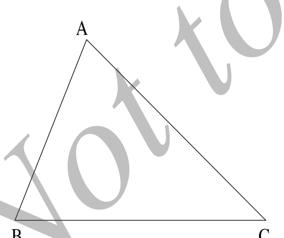
AB = AD (by construction) ⇒ ∠ABD = ∠ADB (angles opposite to equal sides).

Now ∠BDC is an exterior angle for triangle BCD. Hence it is larger than interior opposite angle ∠BCD. We thus get

$$\angle C < \angle BDA = \angle ABD < \angle ABC = \angle B.$$

This completes the proof.

Proposition 2. In a triangle, if two angles are unequal, then the side opposite to the larger angle is greater than the side opposite to the smaller angle.



Given: a triangle ABC in which
∠B >∠C.
To prove: AC >AB.
Proof: Observe that

$$\angle B > \angle C \Rightarrow AC \neq AB.$$

For, AC = AB implies that ∠B = ∠C (angles opposite to equal sides are equal).

Thus either

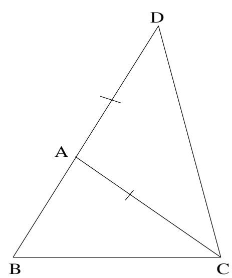
$$AC < AB \text{ or } AC > AB.$$

If AC <AB, then by previous proposition ∠B < ∠C; but this contradicts the given hypothesis. The only possibility left out is

$$AC > AB.$$

Note: Here we are using *law of trichotomy*. Given any two real numbers a and b, you have only one of the possibilities: $a < b$; $a = b$; or $a > b$.

Proposition 3. In a triangle, the sum of any two sides is greater than the third side.



Given: a triangle ABC.
To Proof : $AB + AC > BC$
Construction: Extend BA to D such that $AD = AC$ and join DC.
Proof: Then
$BD = BA + AD = BA + AC$.
Since $AD = AC$, we have
$\angle ADC = \angle ACD$ (angle opposite to equal sides).

Hence we obtain

$$\angle BCD > \angle ACD = \angle ADC = \angle BDC.$$

In triangle BCD, we have

$$\angle BCD > \angle BDC \Rightarrow BD > BC \text{ (by proposition 2).}$$

But $BD = BA + AC$, as we have observed earlier. We thus get

$$BA + AC > BC.$$

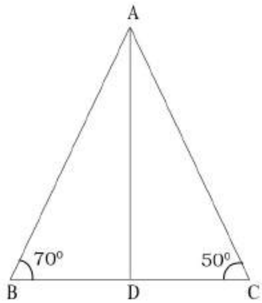
You can similarly prove $CA < AB + BC$ and $AB < BC + CA$.

Note: The inequalities $BC < CA + AB$, $CA < AB + BC$ and $AB < BC + CA$ are called *triangle inequalities*. They are necessary conditions for the existence of a triangle with sides AB, BC and CA. This tells that the straight line is the shortest path between any two point. Given three numbers a, b, c, necessary conditionsfor the existence of a triangle with sides a, b, c are that $a < b + c$, $b < c + a$ and $c < a + b$. These conditions are also sufficient which you will see while constructing a triangle with three given sides.

Example 14. Show that in a right angled triangle, hypotenuse is larger than any of the remaining sides.

Solution: Suppose ABC is a right angled triangle with $\angle B = 90°$. Then AC is the hypotenuse. Observe that $\angle BAC < \angle B$ and $\angle BCA < \angle B$. Now the side opposite to $\angle BAC$ is BC and side opposite $\angle BCA$ is AB. Hence by proposition 2, $BC < AC$ and $AB < AC$.

Example 15. In the adjoining figure, $\angle B = 70°$, $\angle C = 50°$ and AD is the bisector of $\angle A$. Prove that AB >AD >CD.



Solution: Observe that

$\angle A = 180° - (\angle B + \angle C)$

$= 180° - (70° + 50°) = 60°$.

Hence $\angle BAD = \angle DAC = 30°$. Consider triangle BAD. We can compute $\angle ADB$:

$\angle ADB = 180° - (70° + 30°) = 80° > \angle ABD$

Hence AB >AD, by proposition 2. In triangle ADC, we have

$$\angle DAC = 30° < 50° = \angle ACD.$$

Again proposition 2 gives CD <AD.

Exercise 11.7

1. In a triangle ABC, $\angle B = 28°$ and $\angle C = 56°$. Find the largest and the smallest sides.
2. In a triangle ABC, we have AB = 4cm, BC = 5.6cm and CA = 7.6cm. Write the angles of the triangle in ascending order of measures.
3. Let ABC be a triangle such that $\angle B = 70°$ and $\angle C = 40°$. Suppose D is a point on BC such that AB = AD. Prove that AB >CD.
4. Let ABCD be a quadrilateral in which AD is the largest side and BC is the smallest side. Prove that $\angle A < \angle C$.(Hint: Join AC)
5. Let ABC be a triangle and P be an interior point. Prove that $AB + BC + CA < 2\,(PA + PB + PC)$.

Glossary

Congruency: two geometrical figures are identical in both shape and size.

Superpose: one figure sitting exactly on the other.

Corresponding sides: while comparing two triangles, we index the sides of the triangles in an ordered way.

Corresponding angles: indexing of the angles in an ordered way.

SAS postulate: Side-Angle-Side postulate.

ASA postulate: Angle-Side-Angle postulate.

SSS postulate: Side-Side-Sidepostulate.

RHS theorem: Right angle-Hypotenuse-Side theorem.

Triangle inequality: any side of a triangle is smaller than the sum of the remaining two.

Points to remember

- Two triangles are congruent if we can superpose one on the other.
- Two triangles are congruent if two sides and the included angle of one triangle are respectively equal to the corresponding sides and the corresponding angle of the other.(SAS)
- Two triangles are congruent if two angles and the common side to these angles of one triangle are respectively equal to the corresponding angles and the corresponding side of the other. (ASA)
- Two triangles are congruent if three sides of one triangle are respectively equal tothree corresponding sides of the other.(SSS)
- Two right angled triangles are congruent if they have equal hypotenuse and, apart from hypotenuse, a side of one triangle is equal to a side of the other.(RHS)
- Any side of a triangle is smaller than the sum of the other two. (Triangle inequality)

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Answers

Exercise 11.3

1. $\angle B = \angle C = 65^0$ **2.** 58^0 **3.** (i) 110^0 (ii) 55^0 (iii) 20^0 (iv) 40^0.

4. $\angle ACD = 120^0$, $\angle ADC = 30^0$.

Exercise 11.7

1. BC is the largest and CA is the smallest. 2. $\angle C < \angle A < \angle B$.

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UNIT 12
CONSTRUCTION OF TRIANGLES

After studying this unit, you learn to construct a triangle:

- when three sides are given;
- when two sides and included angle are given;
- when two angles and included side are given;
- when two sides and altitude on third side are given; – one side and hypotenuse of a right angled triangle are given;
- an isosceles triangle, whose base and height are given;
- perimeter and ratio of the sides of a right triangle are given;
- whose perimeter and base angles are given;
- when the base, sum of the other two sides and one base angle of a right triangle are given;
- when the base, difference of the other two sides and one base angle of a right triangle are given.

Introduction

In earlier classes, you have learnt that given a triangle, there are **six** elements associated with it, namely, three sides and three angles. You may be wondering whether you need all these to be known for constructing a triangle. If all are known, it is well and good. But in a variety of practical situations, you may not know all these. If only two are known, you cannot hope to construct a triangle. Even if three of these are known,you may not be able to construct. For example, If two sides and an angle (not included angle) are given,then it is not possible to construct such triangle.

We will take up different situations when we will be able to construct a triangle. Of course along with these basic six elements, you can also associate many other elements like *medians, angle bisectors altitudes.* You get several other combinations. And constructing triangles with minimum number of data which includes these additional elements is interesting and challenging. We will not go beyond the traditional constructions.

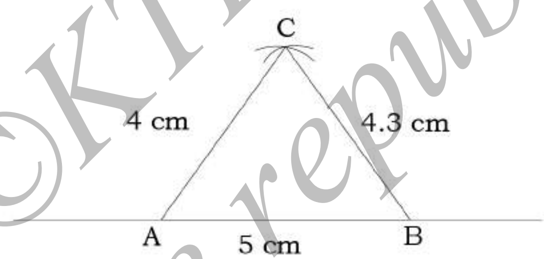
When three sides are given

Example 1 Construct a triangle ABC in which AB = 5 cm, BC = 4.3 cm and AC = 4 cm.

Solution: We follow several steps in the construction:

1. Draw a line segment which is sufficiently long using ruler.
2. Locate points A and B on it such that AB = 5 cm.
3. With A as centre and radius 4 cm, draw an arc(see figure).
4. With B as centre and radius 4.3 cm, draw another arc cutting the previous arc at C.
5. Join AC and BC.

Then ABC is the required triangle.



> **Think it over!**
>
> **The arc with B as centre and radius 4.3 cm always cuts the arc with A as center and radius 4 cm, whenever AB=5 cm. Can you relate this to triangle inequality?**

Exercise 12.1

1. Construct a triangle ABC in which AB = 5 cm and BC = 4.6 cm and AC = 3.7 cm.
2. Construct an equilateral triangle of side 4.8 cm.
3. Construct a triangle PQR, given that PQ = 5.6 cm, PR = 7 cm and QR = 4.5 cm.
4. Construct a triangle XYZ in which XY = 7.8 cm, YZ = 4.5 cm and XZ = 9.5 cm.

5. Construct a triangle whose perimeter is 12 cm and the ratio of their sides is 3:4:5.

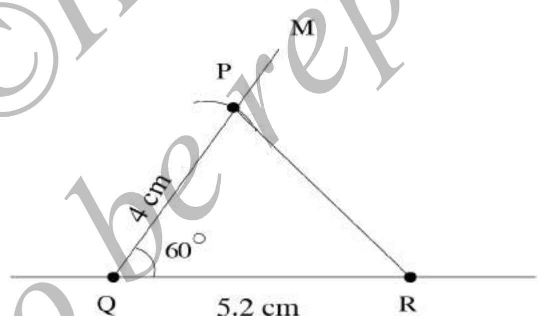
When two sides and their included angle are given

Example 2. Construct a triangle PQR, given that PQ = 4 cm, QR = 5.2 cm and $\angle Q = 60^\circ$.

Solution: Steps of construction:

1. Draw a line segment which is sufficiently long using ruler.
2. Locate points Q and R on it such that QR = 5.2 cm.
3. At Q, construct a line segment QM, sufficiently large, such that $\angle MQR = 60^\circ$; use protractor to measure 60°.
4. With Q as center and radius 4 cm., draw an arc cutting QM at P; join PR.

Then, PQR is required the triangle.



Think it over! Without using protractor, can you construct a line segment QM **at** Q **such that** $\angle MQR = 60^\circ$ **?**

Exercise 12.2

1. Construct a triangle ABC, in which AB = 4.5 cm, AC = 5.5 cm and $\angle BAC = 75^\circ$.
2. Construct a triangle PQR in which PQ = 5.4 cm, QR = 5.5 cm and $\angle PQR = 55^\circ$.

3. Construct a triangle XYZ in which XY = 5 cm, YZ = 5.5 cm and $\angle XYZ = 100°$.
4. Construct a triangle LMN in which LM = 7.8 cm, MN = 6.3 cm and $\angle LMN = 45°$.

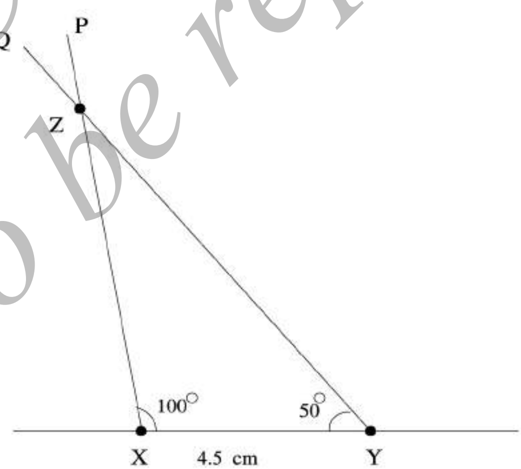
When two angles and included side are given

Example3. Construct a triangle XYZ in which XY = 4.5cm and $\angle X = 100°$ and $\angle Y = 50°$.

Solution: Steps of construction

1. Draw a line segment which is sufficiently long using ruler.
2. Locate points X and Y on it such that $XY = 4.5$ cm.
3. Construct a line segment XP such that $\angle PXY = 100°$; construct a line segment YQ such that $\angle XYQ = 50°$.
4. Extend XP and YQ to intersect at Z.

Then XYZ is the required triangle.



Think it over!

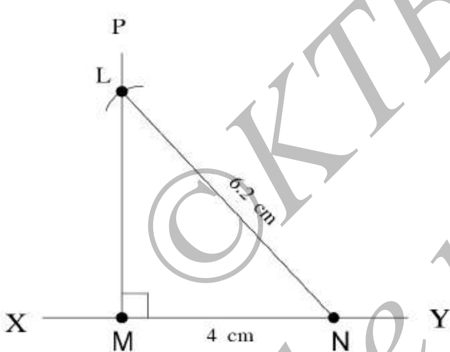
Suppose it is given that $XY = 4.5$ **cm,** $\angle X = 100°$ **and** $\angle Y = 80°$ **. Can you construct a triangle now? Where does the construction break down and why?**

Exercise 12.3

1. Construct a triangle ABC in which AB = 6.5 cm, $\angle A = 45^\circ$ and $\angle B = 60^\circ$.
2. Construct a triangle PQR in which QR = 4.8 cm, $\angle Q = 45^\circ$ and $\angle R = 55^\circ$.
3. Construct a triangle ABC in which BC = 5.2 cm, $\angle B = 35^\circ$ and $\angle C = 80^\circ$.
4. Construct a triangle ABC in which BC = 6 cm, $\angle B = 30^\circ$ and $\angle C = 125^\circ$.

To construct a right triangle whose one side and hypotenuse are given

Example 6. Construct a right triangle LMN in which $\angle M = 90^\circ$, MN = 4 cm and LN = 6.2 cm.



Solution:

Steps of construction

1. Draw a line segment XY.
2. Locate M,N on XY such that MN =4 cm.
3. Construct a line segment MP, sufficiently large, such that $\angle NMP = 90^\circ$.
4. 4. With N as centre and radius 6.2cm draw an arc, cutting MP at L; join NL.

Then, LMN is the required triangle.

> **Think it over!**
> **Why is that the arc with centre N and radius 6.2 cm cuts the line segment MP ? Which part of the data ensures it?**

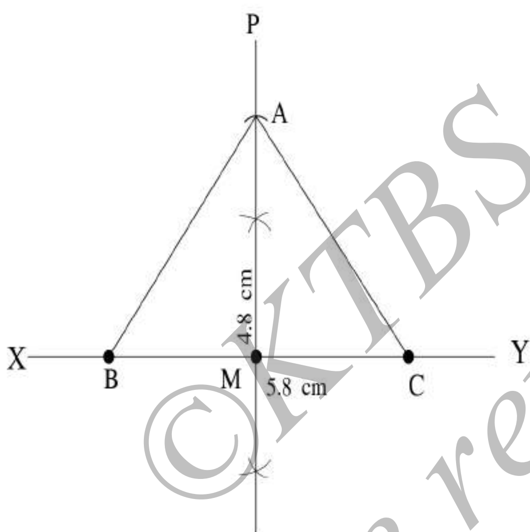
Exercise 12.4

1. Construct a right angle triangle ABC in which $\angle B = 90^\circ$, AB = 5 cm and AC = 7 cm.
2. Construct a right angle triangle PQR in which $\angle R = 90^\circ$, PQ = 4 cm and QR = 3 cm.

3. Construct a right angle triangle ABC in which $\angle B = 90°$, BC = 4 cm and AC = 5 cm.

To construct an isosceles triangle whose base and corresponding altitude are given

Example 7. Construct an isosceles triangle ABC in which base BC = 5.8 cm and altitude from A on BC is 4.8 cm.



Solution:

Steps of construction

1. Draw a line segment BC whose length is 5.8 cm.
2. Draw the perpendicular bisector of BC; call it MP, with M on BC.
3. With M as centre and radius 4.8 cm, draw an arc cutting MP at A; join AB and AC.

Then ABC is the required triangle.

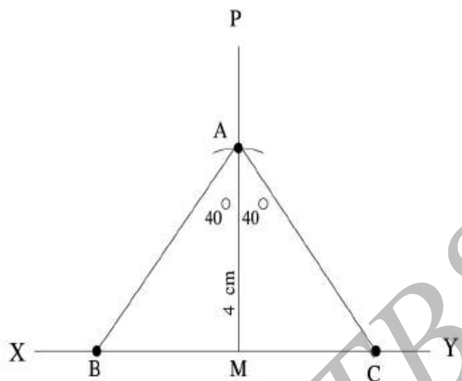
> **Think it over!**
>
> **Which results on triangle is used to conclude that ABC is the required triangle?**

Exercise 12.5

1. Construct an isosceles triangle ABC in which base BC = 6.5 cm and altitude from A on BC is 4 cm.
2. Construct an isosceles triangle XY Z in which base Y Z = 5.8 cm and altitude from X on Y Z is 3.8 cm.
3. Construct an isosceles triangle PQR in which base PQ = 7.2 cm and altitude from R on PQ is 5 cm.

To construct an isosceles triangle when its altitude and vertex angle are given

Example 8. Construct an isosceles triangle whose altitude is 4 cm and vertex angle is 80°.



Solution:

Steps of construction

1. Draw a line segment XY .
2. Take a point M on XY and draw a line MP $\perp$ XY .
3. With *M* as centre and radius 4 cm, draw an arc cutting MP at A.
4. Construct B and C on XY such that $\angle MAB = \frac{80^\circ}{2} = 40^\circ$ and $\angle MAC = \frac{80^\circ}{2} = 40^\circ$.

Then ABC is the required triangle.

> **Think it over!**
>
> **Why is triangle** ABC **isosceles? Which results on triangles are used to conclude that** ABC **is the required triangle?**

Exercise 12.6

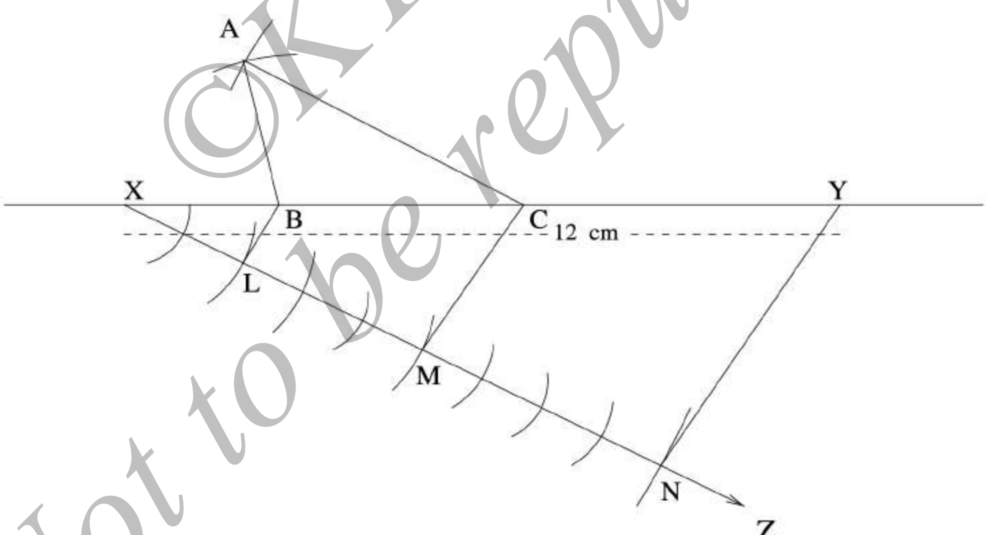
1. Construct an isosceles triangle whose altitude is 4.5 cm and vertex angle is 70°.
2. Construct an isosceles triangle whose altitude is 6.6 cm and vertex angle is 60°.
3. Construct an isosceles triangle whose altitude is 5 cm and vertex angle is 90°.

To construct a triangle whose perimeter and ratio of the sides are given

Example 9. Construct a triangle ABC, whose perimeter is 12 cm and whose sides are in the ratio 2:3:4.

Solution: Steps of construction

1. Draw a line segment and locate points X, Y such that XY = 12 cm.
2. Draw a ray XZ, making an acute angle with XY and drawn in the down-ward direction.
3. From X, locate (2+3+4) = 9 points at equal distances along XZ.
4. Mark point L,M,N on XZ such that XL = 2 parts, LM = 3 parts and MN = 4 parts.
5. Join NY . Through L and M, draw $LB \parallel NY$ and $MC \parallel NY$, intersecting XY in B and C respectively.
6. With B as center and BX as radius, draw an arc; with C as a centre and CY as radius, draw an arc cutting the previous arc at A.
7. Join AB and AC.



Then ABC is the required triangle.

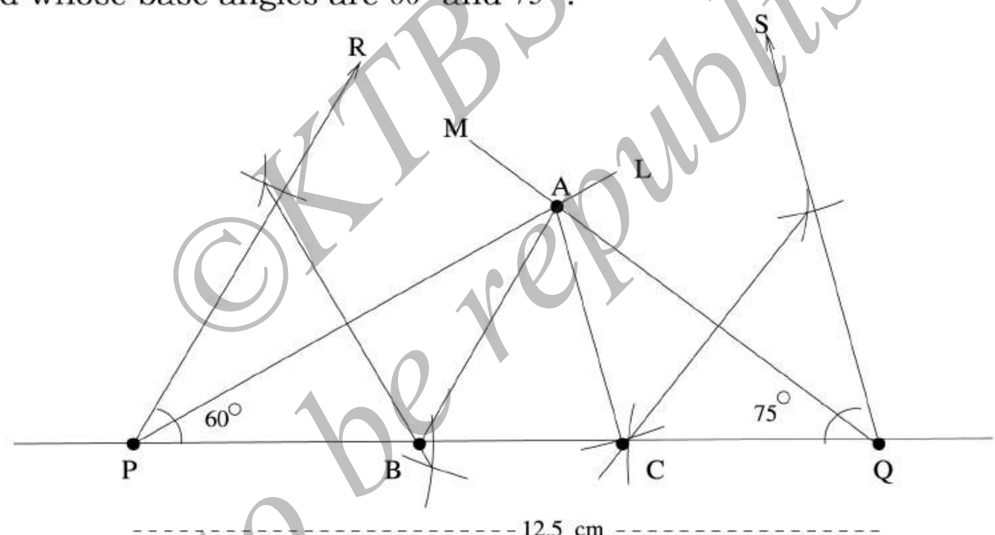
Think it over!
Why is that the sides of *ABC* are in the required ratio? Is it possible to construct a triangle if the sides are in the ratio 2:3:5?

Exercise 12.7

1. Construct a triangle ABC, whose perimeter is 13 cm and whose sides are in the ratio 3:4:5.
2. Construct a triangle PQR, whose perimeter is 14 cm and whose sides are in the ratio 2:4:5.
3. Construct a triangle MNP, whose perimeter is 15 cm and whose sides are in the ratio 2:3:4.

To construct a triangle whose perimeter and base angles are given

Example 10. Construct a triangle ABC whose perimeter is 12.5 cm and whose base angles are $60°$ and $75°$.



Solution: Steps of construction

1. Draw a line segment and locate points P,Q such that PQ =12.5 cm.
2. Construct rays PR such that $\angle QPR = 60°$ and QS such that $\angle PQS = 75°$.
3. Draw the bisectors PL and QM of $\angle QPR$ and $\angle PQS$ respectively. Let these intersect at A.
4. Draw the perpendicular bisector of AP and AQ and let these intersect PQ at B and C respectively.
5. Join AB and AC.

Then ABC is the required triangle.

Think it over!

What properties of triangles ensure that we get required base angles? And the perimeter is also as required?

Exercise 12.8

1. Construct a triangle ABC whose perimeter 12 cm and whose base angles are $50°$ and $80°$.
2. Construct a triangle XYZ whose perimeter 15 cm and whose base angles are $60°$ and $70°$.
3. Construct a triangle ABC whose perimeter 12 cm and whose base angles are $65°$ and $85°$.

These constructions are optional. They need not be considered for examination.

To construct an equilateral triangle of given height

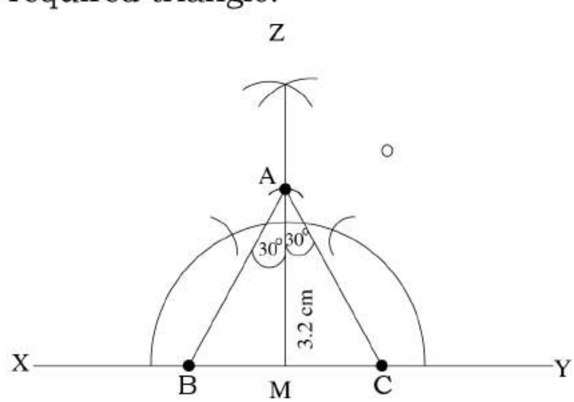
Example 4. Construct an equilateral triangle of height 3.2 cm.

Solution: First observe that the altitudes from any vertex to the opposite sides of an equilateral triangle are all of equal length (prove this statement). Hence we can define the height of an equilateral triangle as this common value of three altitudes.

<u>Steps of construction</u>

1. Draw any line segment XY.
2. Take any point M on XY. Draw $ZM \perp XY$.
3. With M as center and radius 3.2 cm, draw an arc, cutting MZ at A.
4. Construct $\angle MAB = 30°$ and $\angle MAC = 30°$, with B and C on XY.

Then ABC is the required triangle.



Think it over!

Why is that ABC **so constructed is equilateral? Can you think of a proof?**

Exercise 12.9

1. Construct an equilateral triangle of height 4.5 cm. Measure approximate length of the its side.
2. Construct an equilateral triangle of height 5.2 cm. Measure approximate length of the its side.
3. Construct an equilateral triangle of height 6. cm. Measure approximate length of the its side.

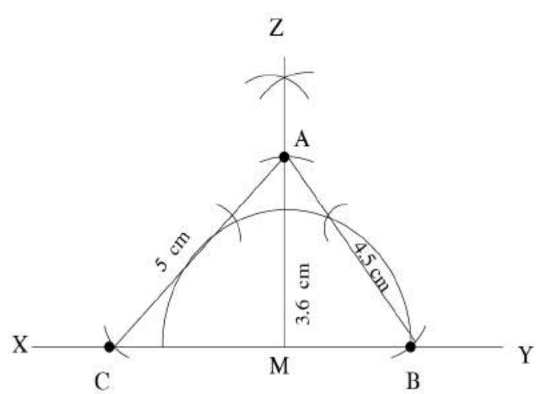
When two sides and altitude on the third side are given

Example 5. Construct a triangle ABC in which AB = 4.5 cm, AC = 5 cm and length of perpendicular from A on BC is 3.6 cm.

Solution: Steps of construction

1. Draw a line segment XY.
2. Take a point M on XY.
3. Draw ZM ⊥ XY, with MZ sufficiently large.
4. With M as center and radius 3.6 cm, draw an arc, cutting MZ at A.
5. With A as center and radii 4.5 cm and 5 cm, draw arcs cutting XY

   Then ABC is the required triangle.



Think it over!

(1) Why is that the arcs with centre A and radii 4.4 cm and 5 cm cut the line segment XY ? Which part of the data ensure it?

(2) There are two more triangles possible other than the one given. Construct them.

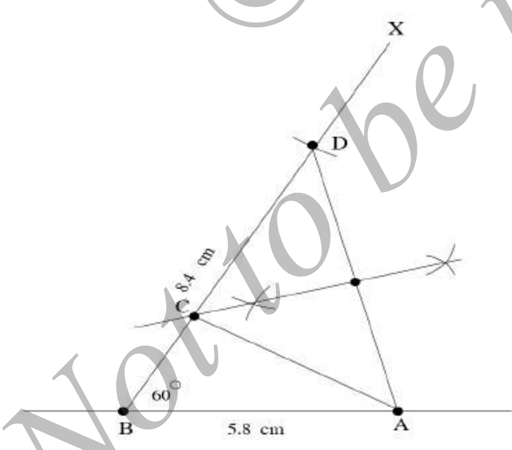
Exercise 12.10

1. Construct a triangle PQR in which PQ = 5.5 cm, PR = 6.2 cm and length of the perpendicular from P on QR is 4 cm.

4. Construct a triangle MNP in which MN = 4.5 cm, MP = 5.2 cm and length of perpendicular from M on NP is 3.8 cm.

To construct triangle when its base, sum of the other two sides and one base angle are given.

Example11. Construct a triangle ABC in which AB = 5.8 cm, $BC + CA = 8.4$ cm and $\angle B = 60°$.



Solution: Steps of construction

1. Draw a line segment AB of length 5.8 cm.
2. Draw a line segment BX, sufficiently large such that $\angle ABX = 60°$.
3. From the segment BX, cut off line segment BD of length 8.4 cm
4. Join AD.
5. Draw the perpendicular bisector of AD and let it meet BD at C.
6. Join AC.

Then ABC is the required triangle.

Think it over!

How is that the sum of *CA* and CB is equal to the given sum?

Exercise 12.11

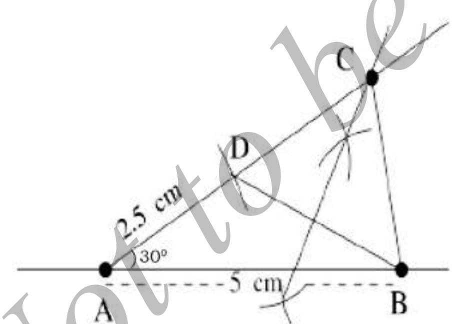
1. Construct a triangle ABC in which BC = 3.6 cm, AB + AC = 4.8 cm and $\angle B = 60°$.
2. Construct a triangle ABC in which AB + AC = 5.6 cm BC = 4.5 cm and $\angle B = 45°$.
3. Construct triangle PQR in which PQ + PR = 6.5 cm QR = 5.4 cm, $\angle Q = 40°$.

To construct a triangle when its base, difference of the other two sides and one base angle are given.

Example 12. Construct a triangle ABC in which AB = 5 cm, $\angle A = 30°$ and AC – BC = 2.5 cm.

Solution:

Steps of construction



1. Draw a line segment AB of length 5 cm.
2. Draw another line segment AX, sufficiently large such that $\angle BAX = 30°$.
3. From the segment AX, cut off line segment AD = 2.5 cm which is equal to (AC - BC)
4. Join BD.
5. Draw the perpendicular bisector of BD and let it meet AX at C.
6. Join BC.

Then ABC is the required triangle.

Think it over!

Can you see why AD **is equal to the difference** $AC - BC$ **? Can you take** $AC - BC > AB$ **and still construct a triangle?**

Exercise 12.12

1. Construct a triangle ABC in which BC = 3.4 cm AB - AC = 1.5 cm and $\angle B = 45^\circ$.
2. Construct a triangle ABC in which BC = 5 cm AB - AC = 2.8 cm and $\angle B = 40^\circ$
3. Construct a triangle ABC in which BC = 6 cm AB - AC = 3.1 cm and $\angle B = 30^\circ$.

Glossary

Perpendicular bisector: the line which is perpendicular to the given line segment and also bisects the line segment.

Angle bisector: the line which bisects the given angle.

Perimeter: the length of the boundary of any plane figure.

Altitude: the perpendicular from a point to a line; this is also used for the length of such a perpendicular.

Arc: part of the circumference of a circle.

Base angle: any of the angles formed by the base of a triangle, with the other sides.

Vertex angle: the angle at the top of an isosceles triangle.

Points to remember

- At least three parameters are needed to construct a triangle.
- Not all combination of three parameters will enable one to construct a triangle.

✡✡✡✡✡

UNIT 13
STATISTICS

After studying this chapter, you learn to:

- explain the terms data, observation, range, frequency, class interval, exclusive and inclusive class intervals, size of class interval, mid-point of class interval.
- construct frequency distribution table for exclusive and inclusive class intervals.
- draw histogram for the given frequency distribution.
- define mean, median and mode.
- calculate mean for grouped and un-grouped data.
- calculate median for grouped and un-grouped data.
- identify mode for un-grouped and grouped data.

Introduction

Statistics is considered to be a mathematical science pertaining to the collection, analysis, interpretation and presentation of data. Statistics is useful in drawing conclusions from numerical data. It is also useful to predict weather, to obtain information concerning business, import, export, education etc.. Many research and investigation require statistical interpretation.

A collection of numerical facts with particular information is called **data**.

Consider, the marks obtained by 20 students in mathematics in 8th standard mid-term examination:

56, 31, 44, 78, 67, 74, 38, 60, 56, 59, 87, 73, 38, 77, 84, 80, 49, 60, 60, 71. The above data is a collection numerical entries. It is called **observation**. Such a collection of data is called **raw data**. The data can be arranged in ascending or descending order. Arranged in descending order, we get, 87, 84, 80, 78, 77, 74, 73, 71, 67, 60, 60, 60, 59, 56, 56, 49, 44, 38, 38, 31. From this, we can infer that the highest score is 87 and lowest score is 31. The difference between the highest and the lowest score is called **range**. The range of above data is (87-31) = 56.

We can observe that the scores 38 and 60 are repeated. The number 38 is repeated twice and 60 is repeated thrice. We say that the **frequency** of 38 is 2 and the frequency of 60 is 3. The frequency of rest of the scores is 1.The number of times a particular observation (score) occurs in a data is called its **frequency**. The above data may be represented in a tabular form, showing the frequency of each distribution. This representation in tabular form is called **Frequency Distribution Table**. Tallies are used to mark the counts; III represents three (3) counts where as ~~IIII~~ represents 5 counts,

Example 1. The marks scored by 20 students in a unit test out of 25 marks are given below.

12, 10, 08, 12, 04, 15, 18, 23, 18, 16, 16, 12, 23, 18, 12, 05, 16, 16, 12, 20. Prepare a frequency distribution table.

Solution: The table looks like:

Marks	Tally marks	No. of students (frequency)
23	II	2
20	I	1
18	III	3
16	IIII	4
15	I	1
12	~~IIII~~	5
10	I	1
08	I	1
05	I	1
04	I	1
Total	20	20

Grouping Data

Organising the data in the form of frequency distribution table is called grouped frequency distribution of raw data. Sometimes, we have to deal with a large data.

Example 2. Consider the following marks (out of 50) scored in mathematics by 50 students of 8th class:

41, 31, 33, 32, 28, 31, 21, 10, 30, 22, 33, 37, 12, 05, 08, 15, 39, 26, 41, 46, 34, 22, 09, 11, 16, 22, 25, 29, 31, 39, 23, 31, 21, 45, 47, 30, 22, 17, 36, 18, 20, 22, 44, 16, 24, 10, 27, 39, 28, 17.

Prepare a frequency distribution table.

Solution: If we prepare a frequency distribution table for each observation, then the table would be too long. So for convenience, we make groups of observations like 0 - 9, 10 - 19 and soon.We obtain a frequency of distribution of the number of observations coming under each group. In this way, we prepare a frequency distribution table for the above data as below:

Groups	Tally marks	Frequency
0 - 9	III	03
10 - 19	~~IIII~~ ~~IIII~~	10
20 - 29	~~IIII~~ ~~IIII~~ ~~IIII~~ I	16
30 - 39	~~IIII~~ ~~IIII~~ ~~IIII~~ I	15
40 - 49	~~IIII~~ I	06
50 - 59		0
Total	50	50

The data presented in this manner is said to be grouped and the distribution obtained is called **grouped frequency distribution.** Grouped frequency distribution table helps us to draw meaningful inferences like:

1. most of the students have scored between 20 and 29;
2. only 3 students have scored less than 10;
3. no student has scored 50 or more than 50.

In the above table, marks are grouped into 0 - 9, 10 - 19 and the like. No score overlaps in any group. Each of these groups is called a **class interval** or a **class**. This method of grouping data is called **inclusive method.**

Class Limit: In the class interval, say (10 - 19), 9.5 is called the lower class limit and 19.5 is called the upper class limit.

> **Note: To find the class limit, in inclusive method, subtract 0.5 from lower score to get lower class limit and add 0.5 to the upper score to get upper class limit.**

Class Size: The number of scores in the class interval say, (10-19), including 10 and 19, is called the class size or width of the class. In this example, the class size is 10.

Class Mark: The midpoint of a class is called its class mark (or midpoint of class interval). It is obtained by adding the two limits and dividing by 2. For example, the class mark of (10-19) is $\frac{(10+19)}{2} = 14.5$. The class mark of (10-20) is $\frac{(10+20)}{2} = 15$.

The data in Example 2 can also be grouped in class intervals like 0 - 10, 10 - 20, 20 - 30 and so on. The frequency distribution table will then be as follows:

Groups	Tally marks	Frequency
0 - 10	III	03
10 - 20	~~IIII~~ ~~IIII~~	10
20 - 30	~~IIII~~ ~~IIII~~ ~~IIII~~ III	18
30 - 40	~~IIII~~ ~~IIII~~ III	13
40 - 50	~~IIII~~ I	06
Total	50	50

Here observe that 10 occurs in both the classes (0 - 10) as well as (10 -20). But it is not possible that an observation (say 10) can belong to two classes (0 - 10) and (10 - 20) simultaneously. In order to avoid this, we follow a convention that the common observation (here 10) will belong to the higher class, that is 10 belongs to (10 - 20) and not to (0 - 10). Similarly 30 belongs to (30 - 40) and not (20 -30). This method of grouping the data is called **exclusive method.**

Class Limit: In the class interval (10 - 20), 10 is called the lower limit and 20 is called the upper limit.

Class Size: The difference between the upper limit and the lower limit is called the class size or width. The width of class in (10 - 20) is 20 -10 = 10.

Example 3. Forty candidates from 10th class of a school appear for a test. The number of questions (out of 60) attempted by them in forty five minutes is given here.

52, 42, 40, 36, 12, 28, 15, 37, 35, 22, 39, 50, 54, 39, 21, 34, 46, 31, 10, 09, 13, 24, 29, 31, 49, 58, 40, 44, 37, 28, 13, 16, 29, 36, 39, 41, 47, 55, 52, 09.

Prepare a frequency distribution table with the class size 10 and answer the following:

(i) Which class has the highest frequency ? (ii) Which class has the lowest frequency? (iii) Write the upper and lower limits of the class (20-29). (iv) Which two classes have the same frequency?

Solution: Let us first prepare the frequency distribution table pertaining to this data.

Class-Interval	Tally marks	Frequency
0 - 9	II	2
10 - 19	~~IIII~~ I	6
20 - 29	~~IIII~~ II	7
30 - 39	~~IIII~~ ~~IIII~~ I	11
40 - 49	~~IIII~~ III	8
50 - 59	~~IIII~~ I	6
Total	40	40

Using this table, we can observe:

(i) (30-39) has the highest frequency;

(ii) (0-9) has the lowest frequency;

(iii) upper limit is 29.5 and lower limit is 19.5;

(iv) (10-19) and (50-59) have the same frequency.

Example 4. The heights of 25 children in centimetre are given below:
174, 168, 110, 142, 156, 119, 110, 101, 190, 102, 190, 111, 172, 140, 136, 174, 128, 124, 136, 147, 168, 192, 101, 129, 114.

Prepare a frequency distribution table, taking the size of the class interval as 20, and answer the following:

(i) Mention the class intervals of highest and lowest frequency.

(ii) What does the frequency 6 corresponding to class interval (160-180) indicate?

(iii) Find out the class mark (or midpoint) of (140 -160).

(iv) What is the range of heights?

Solution: The frequency distribution for the given data is as follows:

Class-Interval	Tally marks	Frequency
100 - 120	~~IIII~~ III	8
120 - 140	~~IIII~~	5
140 - 160	III	3
160 - 180	~~IIII~~ I	6
180 - 200	III	3
	Total 25	25

Answers:

(i) Highest frequency: (100-120); lowest frequency: (140-160). and (180-200).

(ii) There are 6 children whose heights are in the range 160 cm to 180 cm.

(iii) Mid-point = $\frac{(140+160)}{2}$ =150

(iv) Range = highest score-lowest score= 192 -101 = 91.

Exercise 13.1

1. The marks scored by 40 candidates in an examination (out of 100) is given below:

75, 65, 57, 50, 32, 54, 75, 67, 75, 88, 80, 42, 40, 41, 34, 78, 43, 61, 42, 46, 68, 52, 43, 49, 59, 49, 67, 34, 33, 87, 97, 47, 46, 54, 48, 45, 51, 47, 41, 43.

Prepare a frequency distribution table with the class size 10. Take the class intervals as (30-39), (40-49), ... and answer the following questions:

(i) Which class intervals have highest and lowest frequency?

(ii) Write the upper and lower limits of the class interval 30-39

(iii) What is the range of the given distribution?

2. Prepare the frequency distribution table for the given set of scores:

   39, 16, 30, 37, 53, 15, 16, 60, 58, 26, 28, 19, 20, 12, 14, 24, 59, 21, 57, 38, 25, 36, 34, 15, 25, 41, 52, 45, 60, 63, 18, 26, 43, 36, 18, 27, 59, 63, 46, 48, 25, 33, 46, 27, 46, 42, 48, 35, 64, 24.

   Take class intervals as (10-20), (20-30), ... and answer the following:

   (i) What does the frequency corresponding to the third class interval mean?

   ii) What is the size of each class interval? Find the midpoint of the class interval 30-40.

   iii) What is the range of the given set of scores?

Histogram

A histogram is a representation of a frequency distribution by means of rectangles whose widths represent class intervals and whose areas are proportional to the corresponding frequencies. In a histogram, frequency is plotted against class interval. Thus, a histogram is a two-dimensional graphical representation of data. However, if the length of all the class intervals are the same, then the frequency is proportional to the height of the rectangle.

Construction of a histogram

We will show how to construct histograms taking some examples.

Example 5. Draw the histogram of the following frequency distribution:

Class - Interval	Frequency
0 - 9	5
10 - 19	8
20 - 29	12
30 - 39	18
40 - 49	22
50 - 59	10

Solution: The given distribution is in inclusive form. It should be converted into exclusive form. This can be done by applying a correction factor $\frac{d}{2}$. where

d = (lower limit of a class) - (upper limit of a class before it)

Here, we have

$$\text{actual upper limit} = \text{stated limit} + \frac{d}{2};$$

$$\text{actual lower limit} = \text{stated limit} - \frac{d}{2}.$$

For example, consider the class limit 10 - 19. You get

d = lower limit of the class interval -upper limit of class before it =10-9 =1.

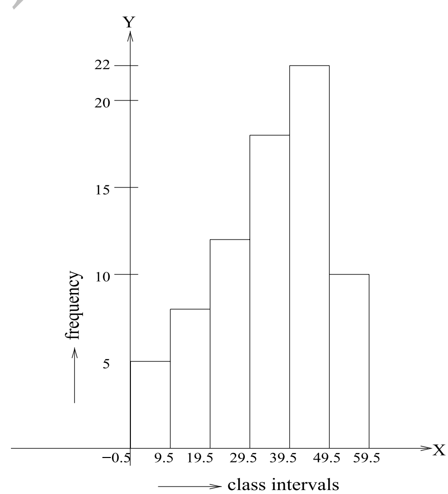
Hence, d = 1 or $\frac{d}{2}$ = 0.5. Now,

$$\text{actual upper limit} = (\text{stated upper limit}) + \frac{d}{2} = 19 + 0.5 = 19.5.$$

$$\text{actual lower limit} = (\text{stated lower limit}) - \frac{d}{2} = 10 - 0.5 = 9.5$$

Converting into exclusive form, we get the table as below:

Stated Class interval	Actual Class interval	Frequency
0 - 9	-0.5 - 9.5	5
10 - 19	9.5 - 19.5	8
20 - 29	19.5 - 29.5	12
30 - 39	29.5 - 39.5	18
40 - 49	39.5 - 49.5	22
50 - 59	49.5 - 59.5	10



Construction of a histogram:

1. Draw X-axis and Y-axis. Choose a proper scale for X and Y axes, say on X-axis: 1cm = 10 and on Y-axis: 1cm = 5.

2. Mark the class intervals on X-axis;(0.5 -9.5), (9.5 -19.5) and the like.
3. Draw a rectangle of height 5 cm on the first class interval (0.5 -9.5).
4. Draw a second rectangle of height 8 cm on the second class interval and follow the same procedure for the rest of the class intervals and the corresponding frequencies.

Then, the histogram takes the form shown above.

In the above histogram, we observe that,

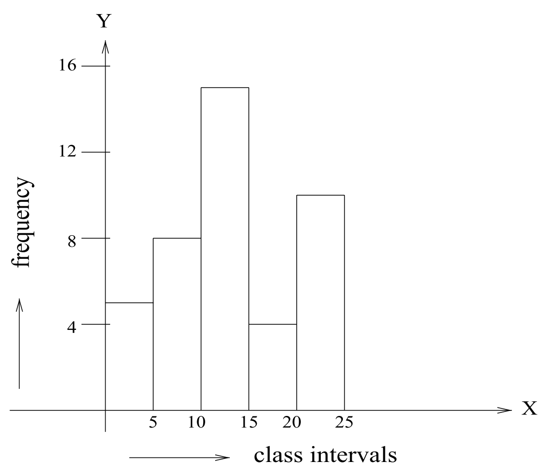
- there are no gaps between rectangles, showing that the distribution is continuous;
- the heights of rectangles represent the frequencies and the base represents class intervals.

Remember:

- **In a bar graph, the height of the bar represents the data. The bars may be separated or closed.**
- **In a histogram, the area of each rectangle represents corresponding data (frequency). There should be no gaps between rectangles.**

Example 6. Draw the histogram for the following frequency distribution.

Class - Interval	Frequency
0 - 5	5
5 - 10	8
10 - 15	15
15 - 20	4
20 - 25	10



Solution: The given distribution is in exclusive form. So, we can take the class-intervals as (0 -5) , (5 -10) etc., along the X-axis and frequency along Y-axis. Choosing a proper scale, we can construct a histogram as explained in the previous example.

Note: Histogram is drawn for class intervals which are exclusive (continuous). If the class intervals are given in inclusive (discrete), it should be converted into exclusive form by applying the correction factor.

Exercise 13.2

1. Draw a histogram to represent the following frequency distribution.

Class - Interval	Frequency
20 - 25	5
25 - 30	10
30 - 35	18
35 - 40	14
40 - 45	12

2. Draw a histogram to represent the following frequency distribution.

Class - Interval	Frequency
10 - 19	7
20 - 29	10
30 - 39	20
40 - 49	5
50 - 59	15

Mean, Median and Mode

Now we study three important quantities associated with a statistical data. They give a clear picture of the behaviour of an experiment. They are generally called **measures of central tendencies**.

Mean

Mean is commonly used as a measure, in a given statistical experiment, to get an idea how the experiment is behaving. This is simply the average of the numerical data collected during an experiment.

Mean for an un-grouped data:

It is the sum of the numerical values of all the observations divided by the total number of observations. If $x_1, x_2, x_3, \ldots, x_N$ are the values of N observations, then

$$\text{Mean} = \frac{\text{(sum of all values of observations)}}{\text{(the number of observations)}} = \frac{x_1 + x_2 + x_3 + \cdots + x_N}{N}$$

The sum of N values of x is represented by $\sum x$. Here $\sum$ stands for summation notation. Therefore,

$$\overline{x} = \frac{\sum x}{N}$$

> **Note:** Sum is denoted by $\sum$ and read as sigma.
>
> Mean is denoted by $\overline{x}$ This is read as x -bar.

Example 7. Find the mean of first six even natural numbers.

Solution: The first six even natural numbers are 2, 4, 6, 8, 10, 12. There are six scores. Therefore, $N = 6$. The observations are $x_1 = 2$, $x_2 = 4$, $x_3 = 6$, $x_4 = 8$, $x_5 = 10$, $x_6 = 12$. Hence

$$\sum x = 2 + 4 + 6 + 8 + 10 + 12 = 42$$

Hence the mean is given by

$$\overline{x} = \frac{\sum x}{N} = \frac{42}{6} = 7.$$

Example 8. Marks scored by Hari in 5 tests (out of 25 marks) are given: 24, 22, 23, 23, 25. Find his average score.

Solution: We observe that there are 5 scores, so that $\sum x = 24+22+23+ 23+25 =117$. Hence the mean is given by

$$\overline{x} = \frac{\sum x}{N} = \frac{117}{5} = 23.4.$$

In the above examples, the number of values is very less. So we could find the mean easily. If large number of values are given and we need to find the mean, it is difficult. In these cases, where the given data is more, we group the data and prepare a frequency distribution table. From frequency distribution table, we can find the mean.

Mean of a grouped data:

Example 9: The number of goals scored by a hockey team in 20 matches is given here:

4, 6, 3, 2, 2, 4, 1, 5, 3, 0, 4, 5, 4, 5, 4, 0, 4, 3, 6, 4.

Find the mean.

Solution: To find the mean, let us prepare a frequency distribution table first. We observe that some scores are repeated. So to find the sum of all scores, we have to multiply each score with its frequency and then find the sum.

Scores	Tally marks	Frequency
0	II	Z
1	I	1
2	II	2
3	III	3
4	~~IIII~~ II	7
5	III	3
6	II	2
		N = 20

Scores (x)	Frequency (f)	fx
0	2	0
1	1	1
2	2	4
3	3	9
4	7	28
5	3	15
6	2	12
	N = 20	$\sum fx = 69$

To do this, let us denote the scores by x and frequency by f, then multiply f and x and add the product fx. Here $\sum fx$ denotes the sum of all the products $f \times x$. Now the mean is

$$\overline{x} = \frac{\text{sum of the scores}}{\text{number of scores}} = \frac{\sum fx}{N} = \frac{69}{20}.$$

Thus we get $\overline{x} = 3.45$.

Example 10. Find the mean for the given frequency distribution table.

Class - Interval	Frequency
0 - 4	3
5 - 9	5
10 - 14	7
15 - 19	4
20 - 24	6
	$N = 25$

Solution: To find the mean, first we have to find the mid-point of each class interval.

Mid-point of 0-4 = $\frac{(0+4)}{2} = 2$;

mid-point of 5-9 = $\frac{(5+9)}{2} = 7$ and the like. Denote the mid-point of the class interval by x. Write down the frequencies f corresponding to each class interval.

Class interval	Mid-point of CI (x)	Frequency (f)	fx
0 - 4	2	3	6
5 - 9	7	5	35
10 - 14	12	7	84
15 - 19	17	4	68
20 - 24	22	6	132
		$N = 25$	$\sum fx = 325$

Multiply f and x to get fx. Add all fx and find out $\sum fx$. Now the mean is calculated using the formula,

$$\overline{x} = \frac{\text{sum of all scores}}{\text{total number of scores}} = \frac{\sum fx}{N}.$$

Thus we get

$$\overline{x} = \frac{\sum fx}{N} = \frac{325}{25} = 13.$$

Therefore mean is 13.

Activity 1:

Mark the height in centimetres on a wall in your school. (Take the help your teacher). Measure and record the height of 10 of your friends. Find the mean height.

Median

Median is the mid-point of the data (raw scores), after being arranged in ascending or descending order. Median divides the given set of scores into two equal halves, that is there are as many scores below the median as above the median.

The mean depends on the nature of scores. If some scores are very high (or low), that will influence the mean. For example; consider the data 5,8, 6, 9, 12, 110,130. If you compute the mean, it is 40 (the sum is 280 and there are 7 scores). But there are 5 scores below 40 and only two scores above 40. Hence it is not central. On the other hand the median is 9 and you see that it is in the centre. Thus mean is influenced by unusually high scores in the given data and may not be a true representative of the data. In such cases median is preferred.

Median for an un-grouped data:

Arrange the given set of scores in ascending or descending order of magnitudes (values). If the total number of scores is odd, then the middle most score is the median. If the total number of scores is even, then the average of the two middle most scores is the median.

Example 11. Find the median of the data: 26, 31, 33, 37, 43, 8, 26, 33.

Solution: Arranging the scores in ascending order, we have 26, 31, 33, 37, 38, 42, 43.

Here the number of terms is 7. The middle term is the 4^{th} one and it is 37. Therefore, median is 37.

> **Think it over:**
>
> **Do you get the same median if the scores are arranged in descending order?**

Example 12. Find the median of the data: 32, 30, 28, 31, 22, 26, 27, 21.

Solution: Arranging in descending order, we obtain 32, 31, 30, 28, 27, 26, 22, 21.

There are 8 terms. Therefore, median is the average of the two middle terms, which are 27 and 28. Thus the median is $\frac{(27 + 28)}{2} = 27.5$.

> **Think it over:**
>
> **Do you get the same median if the scores are arranged in ascending order?**

Note: When N scores are given, we can use the following method to find the median:

First arrange the scores in ascending or descending order; (i) If N is odd, then median is the score at $\frac{(N+1)}{2}^{th}$ place; (ii) If N is even, median is $\frac{1}{2}\left(\text{the score at } \frac{N}{2}^{th} \text{ place + the score at } \left(\frac{N}{2}+1\right)^{th} \text{ place}\right)$.

Median for a grouped data

In the case of an un-grouped data, you could compute its median as the middle score if the number of scores is odd (or the mean of its two middle scores in the case of even number of scores). We have to adopt a different way of computing the median of a grouped data. We describe it through examples.

Example 13. Find the median for the following grouped data.

Solution: Median is the middle score. Here actual scores are not given. So we have to use a different procedure to find the median. We have $N = 20$, an even number. Therefore, there are two middle scores: one corresponding to $\frac{N}{2} = \frac{20}{2} = 10^{th}$ score and the other is 11^{th} score.

Class - Interval	Frequency
1 - 5	4
6 - 10	3
11 - 15	6
16 - 20	5
21 - 25	2
	$N = 25$

We have to locate the median between 10^{th} and 11^{th} scores. Since we are not given individual scores, we have to get some idea about 10^{th} and 11^{th} scores. For this it is required to find the cumulative frequency. Observe the following table, to know how to find cumulative frequency.

Class Interval	Frequency (f)	Cumulative frequency (f_c)	
1 - 5	4	4	4
6 - 10	3	7	4+3=7
11 - 15	6	13	7+6=13
16 - 20	5	18	13+5=18
21 - 25	2	20	18+2=20
	$N = 20$		

Observe that the cumulative frequency corresponding to the last class interval is equal to N. Counting frequencies from first class interval downwards, we find that the 10^{th} score lies in the class interval (11-15). This class interval (11-15) is called the **median class**. The frequency corresponding to this is 6. Its **lower real limit** (LRL) is 10.5. The cumulative frequency above this class is 7. Now, knowing:

(a) lower real limit (LRL)=10.5;

(b) frequency of the median class (f_m)=6;

(c) cumulative frequency above the median class (f_c)=7; and

(d) size of the class interval (i)=5;

we can find the median, using the formula

$$\text{median} = \text{LRL} + \left(\frac{\frac{N}{2} - f_c}{f_m}\right) \times i.$$

Thus we get

$$\text{median} = 10.5 + \left(\frac{\frac{20}{2} - 7}{6}\right) \times 5 = 10.5 + \frac{(10-7)}{6} \times 5$$

$$= 10.5 + \frac{3}{6} \times 5 = 10.5 + 2.5 = 13.$$

Note: $\text{median} = \text{LRL} + \left(\frac{\frac{N}{2} - f_c}{f_m}\right) \times i.$ This formula can be derived from the basic principles.

Example 14. Calculate the median for an exclusive (continuous) distribution given below.

Class Interval	Frequency
20 - 30	13
30 - 40	13
40 - 50	9
50 - 60	4

Class Interval	Frequency (f)	Cumulative frequency (fc)
10 - 20	11	11
20 - 30	13	24
30 - 40	13	37
40 - 50	9	46
50 - 60	4	50

Solution: We first prepare the cumulative frequency table (see above). Here you see that the total number of observation is $N = 50$. Therefore (30-40) is the *median class*. We also observe that LRL = 30, $f_c = 24$, $f_m = 13$ and $i = 20 - 10 = 10$. Now we are in a position to use the formula for median:

$$\text{median} = \text{LRL} + \left(\frac{\frac{N}{2} - f_c}{fm}\right) \times i. = 30 + \frac{(25-24)}{13} \times 10 = 30 + \frac{10}{13} = 30.77,$$

approximately.

Mode

There is another measure of central tendency which is used occasionally, called mode. **Mode** is the score that occurs frequently in a given set of scores. Mode is the value around which the other scores cluster around densely.

Mode for an un-grouped data:

Example 15. Find the mode for the data: 15, 20, 22, 25, 30, 20, 15, 20, 12, 20.

Solution: Here 20 appear maximum times (4 times). Therefore, mode is 20.

Example 16. Find the mode of the data: 5, 3, 3, 5, 7, 6, 3, 4, 3, 5, 8, 5.

Solution: Here 3 and 5 appear 4 times. Therefore modes are both 3 and 5.

Note: A collection of data can have more than one mode. If the data has only one mode, we say it has uni-mode, if it has 2 modes, we say it has bi-mode and if it has more than 2 modes, we say it has multi-mode.

Mode for a grouped data

For a grouped data, the same score having maximum frequency is the mode.

Example 17. Find the mode for the following data.

Number	12	13	14	15	16	17
Frequency	7	9	6	22	20	19

Solution: Here the maximum frequency is 22. Therefore, the number 15 corresponding to maximum frequency is the mode. Hence, the mode is 15.

Exercise 13.3

1. Runs scored by 10 batsmen in a one day cricket match are given. Find the average runs scored.

23, 54, 08, 94, 60, 18, 29, 44, 05, 86

2. Find the mean weight from the following table.

Weight (Kg)	29	30	31	32	33
No. of children	02	01	04	03	05

3. Calculate the mean for the following frequency distribution:

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	3	7	10	6	8	2	4

4. Calculate the mean for the following frequency distribution:

Marks	15-19	20-24	25-29	30-34	35-39	40-44
Frequency	6	5	9	12	6	2

5. Find the median of the data: 15, 22, 9, 20, 6, 18, 11, 25, 14.
6. Find the median of the data: 22, 28, 34, 49, 44, 57, 18, 10, 33, 41, 66, 59.
7. Find the median for the following frequency distribution table:

Class Intervals	110-119	120-129	130-139	140-149	150-159	160-169
Frequency	6	8	15	10	6	5

8. Find the median for the following frequency distribution table:

Class-interval	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30
Frequency	5	3	9	10	8	5

9. Find the mode for the following data:

   (i) 4, 3, 1, 5, 3, 7, 9, 6

   (ii) 22, 36, 18, 22, 20, 34, 22, 42, 46, 42

10. Find the mode for the following data:

x	5	10	12	15	20	30	40
f	4	8	11	13	16	12	9

Glossary

Data: collection of numerical facts with a particular information.
Observation: numerical display of data.
Score: the numerical entries in an observation.
Range: the difference between the highest and the lowest scores in an observation.

Frequency: the number of times a particular score appears in an observation.

GFD: Grouped frequency distribution; the data is collected into several groups and the frequency of scores in each group is recorded.

Class interval: in a grouped frequency distribution, each group is called a class interval.

Cumulative frequency: the sum of the frequencies up to the current class interval.

FDT: Frequency Distribution Table; it displays the frequencies of scores corresponding to various class intervals.

Inclusive method: while grouping, the end points of the groups do not overlap.

Exclusive method: the end points of consecutive groups overlap.

Class limit: the end points of a class in exclusive method; the end points of a class with correction factors in inclusive method.

Class size (or width) : the difference between the upper classlimit and the lower class limit.

Class mark: the mid point of the class interval; it is equal to the average between the upper class limit and the lower class limit.

Histogram: a graphical way of representing grouped data using rectangular bars in which the frequency is proportional to the area of the rectangle.

Mean: average of the scores; It is equal to the sum of the scores divided by the number of scores.

Median: the middle score.

Mode: the score which appears maximum number of times in an observation.

Median class: in a grouped data, it is difficult to say which is the median as scores are not given explicitly; but one can say in which class interval, the middle score lies. This is called the median class.

Points to remember

- Statistics is a branch of study which helps us to analyse the collected data in a systematic way.
- Mean, median and mode are three different measures which, in some sense, represent the given data. They are called measures of central tendency.

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Answers

Exercise 13.1

1.

Class Interval	Tally marks	Frequency
30-39	IIII	4
40-49	~~IIII~~ ~~IIII~~ ~~IIII~~ I	16
50-59	~~IIII~~ II	7
60-69	~~IIII~~	5
70-79	IIII	4
80-89	III	3
90-99	I	1

(i) highest: 40-49, lowest: 90 -99 (ii) 30.5 and 39.5 (iii) 65.

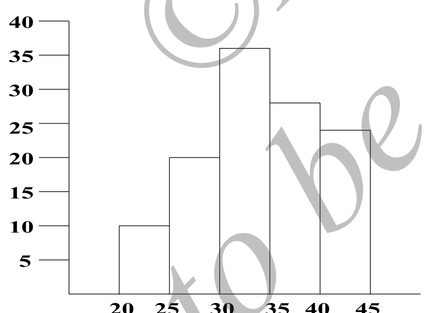
2.

Class Interval	Tally marks	Frequency
10-20	~~IIII~~ IIII	9
20-30	~~IIII~~ ~~IIII~~ II	12
30-40	~~IIII~~ ~~IIII~~	10
40-50	~~IIII~~ IIII	9
50-60	~~IIII~~ I	6
60-70	IIII	4

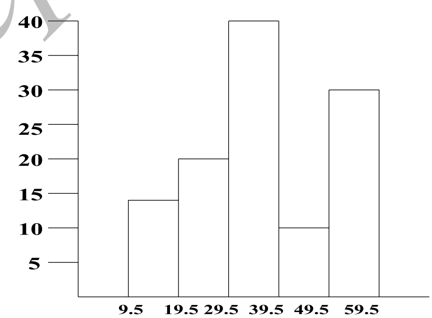
(i) 10 scores between 30 and 40. ii) 10, 35. (iii)52.

Exercise 13.2

1.



2.



Exercise 13.3

1. 42.1. **2.** 29.73. **3.** 42.75. **4.** 22.63. **5.** 15. **6.** 37.5. **7.** 136.8. **8.** 16.7. **9.** (i) 3;

(ii) 22 and 42 (bi-modal distribution). **10.** 20.

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UNIT 14
INTRODUCTION TO GRAPHS

After learning this unit, you learn to:

- identify a point in the plane using a rectangular coordinate system.
- set up a coordinate system and draw graphs of linear curves in such a system.
- get to know the way the coordinates of a point are related in two different rectangular coordinate systems with axes parallel to each other.
- construct the equation of a straight line by looking at the graph of the straight line on a graph paper.

Introduction

In May, when the summer is at its peak, you may find some days are hot and some days are not so hot. You may measure the maximum temperature on each day (you may use either Fahrenheit or Celsius to measure temperature). You get a data for the whole month. Perhaps you will tabulate it in a straight forward way:

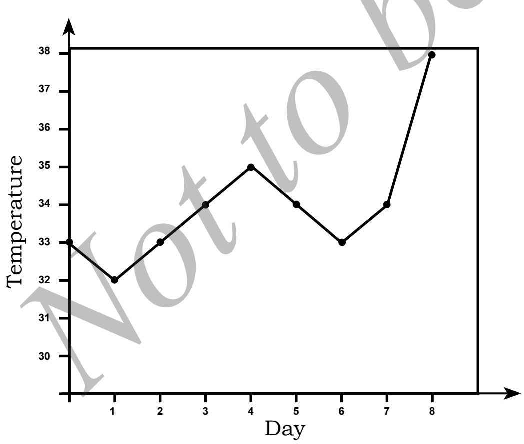
Day	1	2	3	4	5	6	7	8	......	30	31
Maximum Temperature	33°	32°	34°	35°	37°	32°	35°	38°	......	36°	36°.

Similarly, the rainfall for each month of a year can be measured in inches or centimeters and we obtain data. Recording these data is very useful for future plans. For example, you can see the rain pattern over several years and plan the agricultural crops accordingly. You may come across several such important data in daily life which need to be recorded for a better future. One easy way is to tabulate as earlier. However, this is a very cumbersome procedure. People look for efficient way of recording such data, which also help them to analyse it in a better way. One of the effective method is the use of **graphs**.

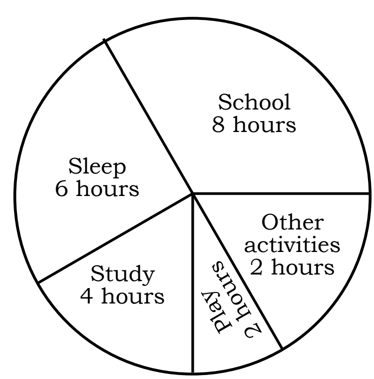
What is a graph? We can define a graph as a **visual representation of numerical data** collected during an experiment. By looking at the graph, one can easily understand the data. Moreover, the graphs also help us to analyse the data quickly.

There are different types of graphs: Bar graphs; Pie charts; Histograms; Cartesian graphs. Each of them is useful for describing a particular type of data. But the principle behind all these graphical representation is the same; an easy visual comprehension of the collected data.

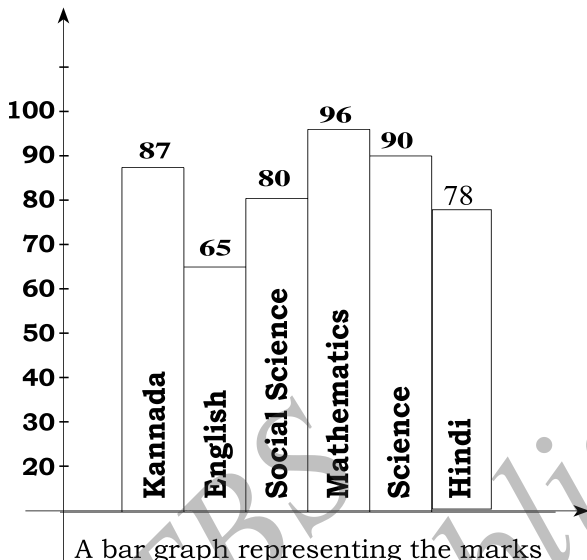
See the following graphical representations and understand the data represented in them.



Line graph representing temperature for 8 days in Bengaluru.



A pie-chart representing day-to-day activities of a student.



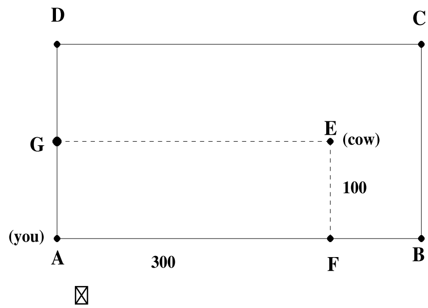
A bar graph representing the marks obtained by a student in six subjects.

Coordinate system

An extremely useful and important method of representing geometric shapes is the use of **Cartesian graphs**. These are also called **coordinate graphs** as the basic principle depends on the use of a **coordinate system**.A coordinate system is a useful device for representing points on the plane. If you take a line, then the points on the line correspond to real numbers. In a coordinate system in the plane, two lines, one perpendicular to the other are used. This will help us to identify the points on the plane in a definite manner.

To get an idea how a coordinate system works, suppose there is a rectangular grazing yard and you are standing at one corner of the yard. A cow is grazing at some point in the yard. You have to identify its location by some numerical data. What do you do?

An easy way is, walk in the direction of the cow from where you stand and measure the distance you move. You may measure it using some convenient measurement. But when you describe the situation to some one else, you have to tell the direction in which you had moved and the distance you had moved. Describing the direction is not so easy.You have to use a different mechanism.



The best way is to use the boundaries of the grazing yard. Suppose you are at the point A and the cow is grazing at E. (Look at the adjacent figure.) You may walk along the boundary AB to a point F where EF is parallel to AD. Then make a 90° turn anticlockwise and move in the direction of the cow; remember EF is parallel to AD. If AF = 300 m and FE = 100 m, you may say the cow is located at 300 m in the direction of AB and 100 m in the direction of AD.

Briefly we say E has coordinates (300,100) with respect to the point A and the system (AB, AD). Note that 300 is the distance in the direction of AB and 100 is the distance in the direction of AD.

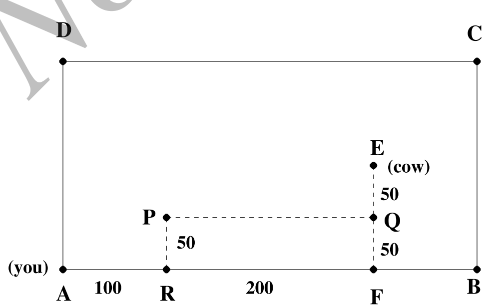
You may also walk along AD first to a point G where AG = 100 m, and then walk along GE (which is parallel to AB) to the point E where GE =300 m. Thus if you use the point A and the system (AD,AB), then E has coordinates (100, 300).

Observe!

The point E is described by the ordered pair (300,100) with respect to the point A and the system (AB,AD). The same point E is described by the ordered pair (100,300) now with respect to the point A and the system (AD,AC). Thus the same point E may have different coordinates with respect to different systems.

Caution!!

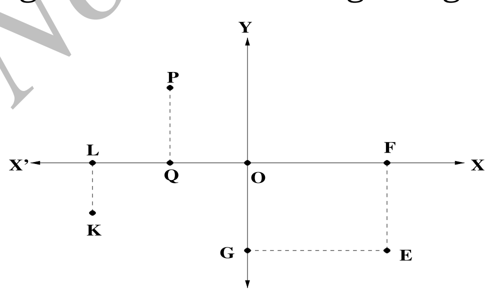
With respect to the point A and the system (AB,AD), the points (300,100) and (100,300) are totally different.



Instead being at the point A, suppose you are at the point P (See the figure). How do you describe the location of the cow with respect to new position? Again, you may walk along PQ to the point Q, where PQ is parallel to AB and QE is parallel to AD. Then you may move along QE to reach E. Now you can describe E with respect to the point P and the system (AB,AD). Note that PQ is parallel to AB and QE is parallel to AD; we are using the same system (AB,AD) but a different starting point P. If you know PQ and QE, you can easily describe E. Draw PR parallel to AD with R on AB. Suppose AR = 100 m. Then PQ = RF = 200 m. Similarly, if RP = 50 m, then QE = FE− FQ = FE− RP =100− 50 =50 m. Thus with respect to the point P and the system (AB,AD), the point E has coordinates (200,50)

> **Observe!**
>
> The point P can be described with respect to A and the system (AB,AD) by (100,50), since AR = 100 and RP = 50 units. Using the point P and the system (AB,AD), the point E is described by (200,50). But E is described by (300,100) using the point A and the system (AB,AD). Can you see that 300 =100+200 and 100 =50+50? What do you conclude?

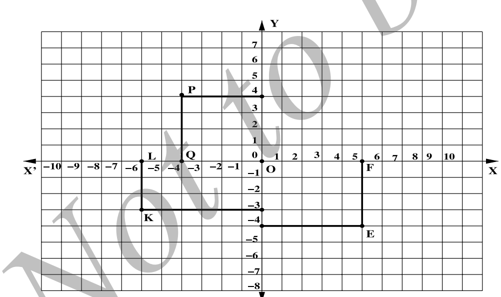
Now you imagine a situation of a huge grazing yard which is endless and having no boundaries. Again a cow is grazing at some point in the yard and you are standing at some other point. How do you describe the position of the cow? Earlier, you had a rectangular yard and you had a **rectangular frame** given by the boundaries. Now you are on your own and you do not have any such frame. You may consider two lines perpendicular to each other and passing through the point where you are standing. Let O be the point where you are standing and let $X'OX$, $Y'OY$ be two lines perpendicular to each other and passing through O. Let the cow be grazing at E (See the figure).



Again you may move along $\overrightarrow{OX}$ to F, where EF is parallel to $Y'OY$. Then you may move from F to E along FE. With respect to the point O and the system ($X'OX$,$Y'OY$), you can describe E using the lengths OF and FE = OG (here G is on the line Y 'OY such that GE is parallel to X 'OX).

But there is a difficulty. If the point E is below the line X 'OX, as shown in the figure, then you have to move along the ray $\overrightarrow{OX}$ to the point F, and then you have to move from F to E in the direction of the ray $\overrightarrow{OY'}$. If some other cow is located at the point K, then you have to move from O to L in the direction of the ray $\overrightarrow{OX'}$ and then you have to move from L to K in the direction of the ray $\overrightarrow{OY'}$. Similarly, you can reach the point P from O, by moving first along $\overrightarrow{OX'}$ and then from Q to P in the direction of $\overrightarrow{OY}$.

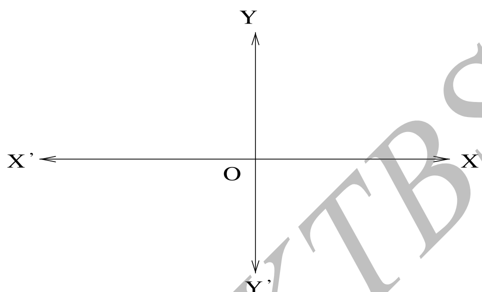
Now you may observe that there are four different directions to move starting from O; $\overrightarrow{OX}$; $\overrightarrow{OX'}$; $\overrightarrow{OY}$; and $\overrightarrow{OY'}$. You may think the line X 'OX as the real number line and O corresponding to the number *zero*. Thus all points to the left of O (along $\overrightarrow{OX'}$) correspond to *negative* numbers and all points to the right of O (along $\overrightarrow{OX}$) correspond to the positive numbers. Similarly, we can view the line Y 'OY as another copy of number line and O corresponding to *zero*; all points above O corresponding to positive numbers and all points below O corresponding to negative numbers. Note that O has the position 0 with respect to both the lines. We say O has coordinates (0,0).



For reaching E, you have to move 5 units to F in the direction $\overrightarrow{OX}$ and 4 units in the direction $\overrightarrow{OY'}$, from F to E. Since $\overrightarrow{OY'}$ corresponds to the negative part of the number line, we describe E by (5,−4). We say E has **coordinates** (5,−4). with respect to the **rectangular coordinate system** X 'OX ↔ Y 'OY. (Here X 'OX ↔ Y 'OY is a symbol used to represent the coordinate system.)

Here the line X 'OX is called the X-**axis** and Y 'OY is called the Y-**axis**. The point O is called the **origin** of the coordinate system. The ray $\overrightarrow{OX}$

is called as the **positive** X-**axis** and $\overrightarrow{OX'}$ is called as the **negative** X-**axis**. Similarly, $\overrightarrow{OY}$ is the **positive** Y-**axis** and $\overrightarrow{OY'}$ is the **negative** Y-**axis**. The point E has coordinates (5,–4) in this system. We say 5 is the *x*-**coordinate** of E and −4 is the y-**coordinate** of E. Note that the first number is always identifier along the **horizontal axis** and the second number is always the identifier along the **vertical axis**. Similarly, the point K has −6 as its x-coordinate and −3 as its y-coordinate. We say K has coordinates (−6,–4)in this system. Similarly, P has coordinates (−4,4).



The lines X 'OX and Y'OY divide the **plane** into 4 regions. The region bounded by the rays $\overrightarrow{OX}$ and $\overrightarrow{OY}$ is called the **first quadrant**. Similarly, you have the **second quadrant**, the **third quadrant** and the **fourth quadrant.** If you take any point in the first quadrant, you have to move in the directions of $\overrightarrow{OX}$ and $\overrightarrow{OY}$ to reach that point. Thus you use the positive X and Y-axes.

You will now recognise that all points in the first quadrant has non-negative x- and y- coordinates; each point on the ray $\overrightarrow{OX}$ has its y-coordinate equal to 0; and each point on the ray $\overrightarrow{OY}$ has its x-coordinate 0. In the second quadrant, you have to use the rays $\overrightarrow{OX'}$ and $\overrightarrow{OY}$; hence each point in the second quadrant has non-positive x-coordinate and non-negative y-coordinate. Similarly, the points in the third quadrant are described by the coordinates (x,y), where $x \leq 0$ and $y \leq 0$. Each point in the fourth quadrant is represented by the ordered pair (x,y) where $x \geq 0$ and $y \leq 0$.

The x-coordinate of a point is also called the **abscissa** and the y-coordinate is called the **ordinate** of that point.

Observe that the lines X'OX and Y 'OY are completely left to your choice.The only condition is that they must be perpendicular to each other for obtaining a *rectangular* coordinate system.

We use only rectangular coordinate system throughout this chapter. However, it is possible to have a coordinate system in which the two axes need not be perpendicular to each other. Such a system called *oblique coordinate system* **is also useful in solving many practical problems.**

The idea of introducing coordinates to identify the points on the plane is due to René Descartes, a french mathematician and philosopher. This has revolutionised the thinking as it helps to convert geometrical problems to equivalent algebraic problems. To commemorate his contribution, the system we study is also known as **Cartesian coordinate system**. This has developed in to a new branch of mathematics known as **Analytic geometry**.

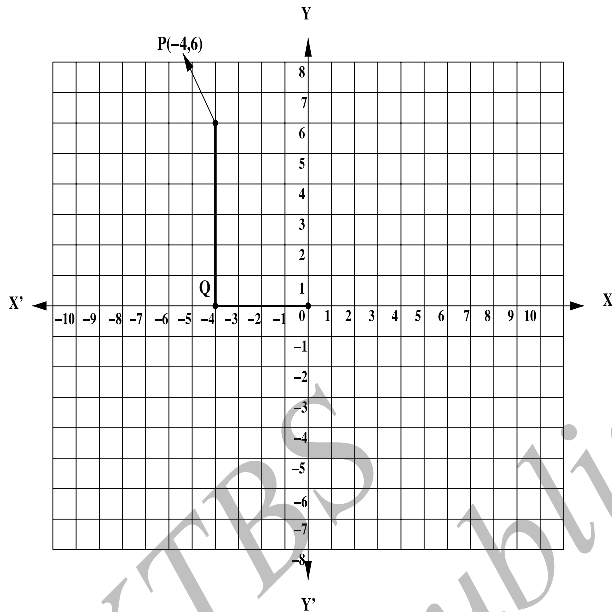
René Descartes was born on March 31st, 1596 in the town of La Haye in the south of France. In 1606, at the age of 8, René Descartes started studying literature, grammar, science, and mathematics. In 1616, he received his baccalaureate and licentiate degrees in Law. Aside from his Law degrees, Descartes also spent time studying philosophy, theology, and medicine.

After a short stay in the military, Descartes went on to lead a quiet life, continuing his intellectual pursuits, writing philosophical essays, and exploring the world of science and mathematics. In 1637, he published "Geometry", in which his combination of algebra and geometry gave birth to analytical geometry, better known as Cartesian geometry.

Activity 1:

<u>Locating a given point on a graph paper</u>

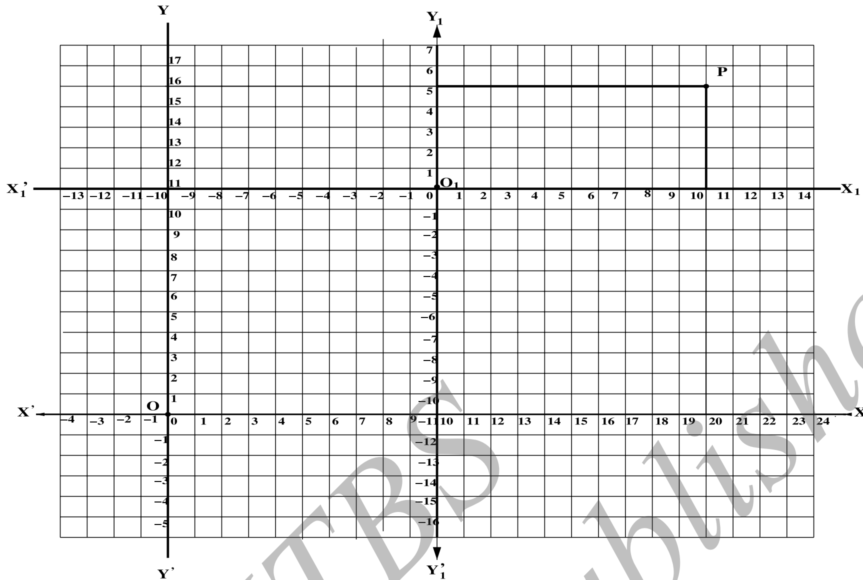
Suppose you are given a graph paper. You also fix up your coordinate system in the graph paper, say $X^{\prime}OX$ and $Y^{\prime}OY$ which are perpendicular to each other. How do you locate the point P with coordinates $(-4,6)$?



Starting from the origin O, move 4 units in the direction of the ray $\overrightarrow{OX'}$. Since $\overrightarrow{OX'}$ represents negative X-axis, when −4 is given as x-coordinate, you have to move 4 units in the direction of $\overrightarrow{OX'}$. You end up with the point Q on $\overrightarrow{OX'}$. Now move 6 units up in the direction parallel to $\overrightarrow{OY}$. Remember $\overrightarrow{OY}$ represents positive Y-axis. Hence the number 6 (with positive sign) tells you that you have to move 6 units upwards parallel to $\overrightarrow{OY}$. You then reach a point P whose coordinates are (−4,6).

Activity 2:

Take a sheet of graph paper. Now set up two rectangular coordinate axes: $X'OX \leftrightarrow Y'OY$ and $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$, where $X'OX \parallel X_1'O_1X_1$. Locate a point P with coordinates (10,5) with respect to the system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$. What are the coordinates of P with respect to the system $X'OX \leftrightarrow Y'OY$? Find the coordinates of O_1 with respect to the system $X'OX$– $Y'OY$. Now can you find the relation among the coordinates of P in the system $X'OX \leftrightarrow Y'OY$, the coordinates of O_1 in the system $X'OX \leftrightarrow Y'OY$ coordinates of P in the system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$?



In the above figure, you see that O_1 has coordinates (11,11) in the system $X'OX \leftrightarrow Y'OY$. The point P has the coordinates (21,16) in the system $X'OX \leftrightarrow Y'OY$ and (10, 5) in the system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$. Observe that

$$21 = 11+10, \quad 16 = 11+5.$$

We write this in the form (21,16) = (11,11) + (10,5). Repeat this with various positions of P and different systems $X'OX \leftrightarrow Y'OY$, $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$. What is the conclusion you draw from these activities?

Observe!

Suppose $X'OX \leftrightarrow Y'OY$ and $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ are two system of coordinates such that $X'OX \parallel X_1'O_1X_1$. Let a point P have coordinates (x,y)in the system $X'OX \leftrightarrow Y'OY$ and (x', y') in the second system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$. If O_1 has coordinates (a,b) with respect to the system $X'OX - YOY$, then $x = a + x'$ and $y = b + y'$.

Activity 3:

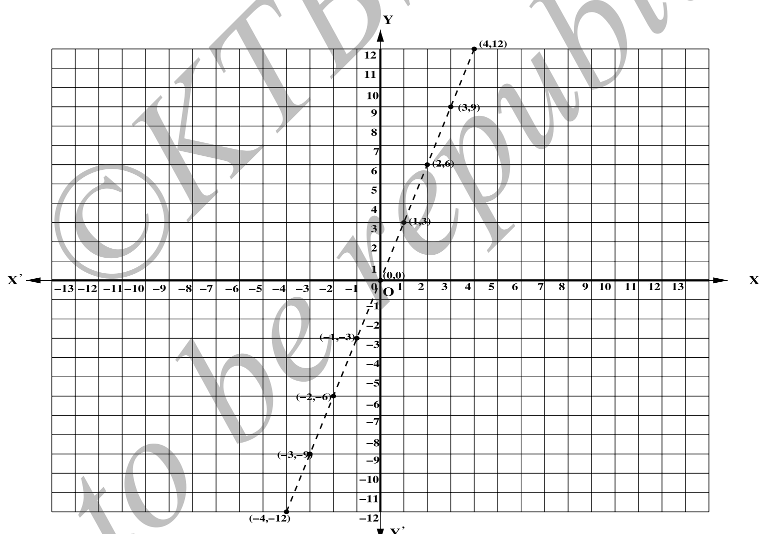
Take a sheet of graph paper and set up your coordinate system $X'OX \leftrightarrow Y'OY$. Locate a point P with coordinates (−5, 8). Trace the point with coordinates (8,−5). Do you get the same point P?

Example 1.

Consider the number 3. Let us tabulate the multiples of 3, some positive and some negative.

x	−3	−2	−1	0	1	2	3
	$3 \times (-3)$	$3 \times (-2)$	$3 \times (-1)$	3×0	3×1	3×2	3×3
y	−9	−6	−3	0	3	6	9
(x, y)	(−3,−9)	(−2,−6)	(−1,−3)	(0, 0)	(1, 3)	(2, 6)	(3, 9)

Take a sheet of graph paper, and set up your own coordinate system, $X'OX \leftrightarrow Y'OY$. Locate the points (x,y) tabulated in the last row. Can you see that all these lie on a straight-line?

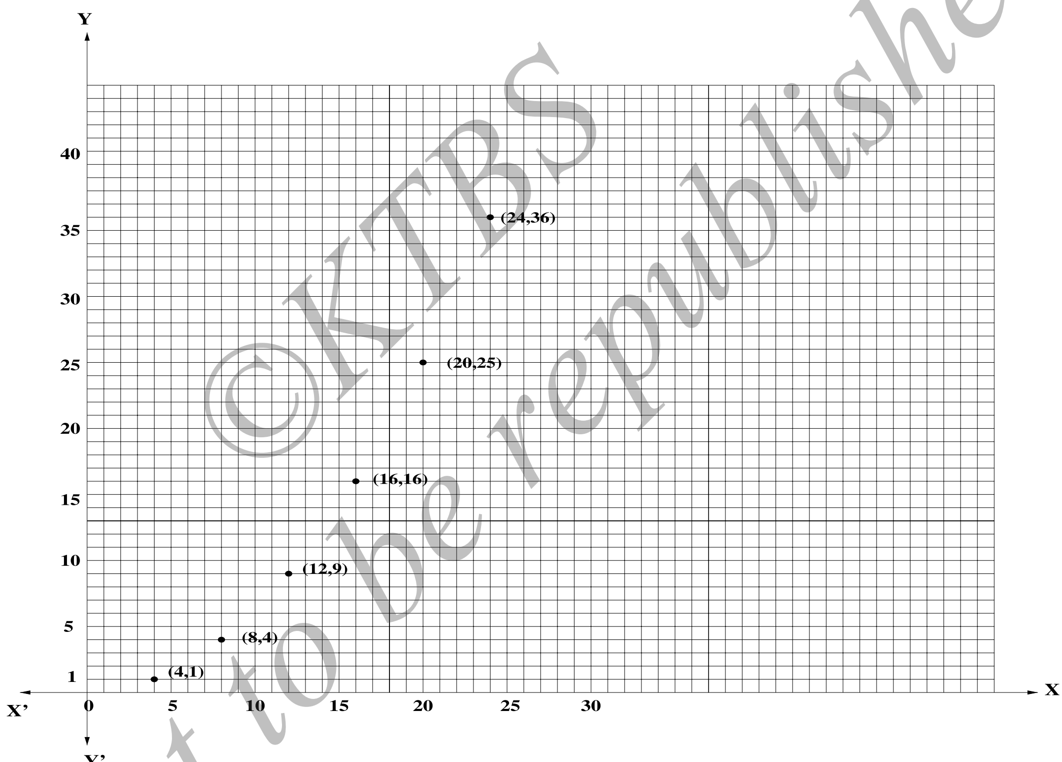


Example 2. (Area versus perimeter of a square)

Consider a square of side length 1 unit. What is its area? You know that the area of a square is l^2, where l is the length of the square. And its perimeter is $l+l+l+l = 4l$. Hence the area of a square of side length 1 unit is 1 square unit and its perimeter is 4 units. What are these for a square of length 2 units? You see that they are respectively 4 square units and 8 units. Let us tabulate the perimeter and area of squares of different side lengths.

l	1	2	3	4	5	6
x = 4l	4	8	12	16	20	24
$y = l^2$	1	4	9	16	25	36
(x, y)	(4,1)	(8,4)	(12,9)	(16,16)	(20,25)	(24,36)

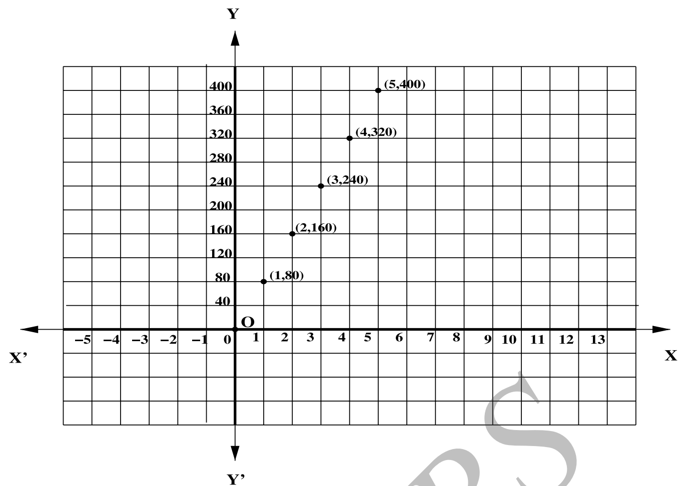
Again set up your own coordinate system on a graph paper and plot these points (x, y) on the graph paper. Do you think that these points lie on a straight line?



Example 3. (Simple interest versus number of years)

Suppose Shiva deposits ₹1,000 in a bank for 5 years for simple interest at the rate of 8% per annum. You can easily calculate what he earns as interest over the years and tabulate it.

Year = x	1	2	3	4	5
Interest = y	80	160	240	320	400
(x, y)	(1, 80)	(2, 160)	(3, 240)	(4, 320)	(5, 400)



Again you are required to plot these points on a graph paper. Here you face a practical problem. The interest Shiva obtains are big numbers and it is difficult to locate numbers like 240, 320, 400 on a graph paper, unless the graph paper you take is very huge. However, this difficulty may be avoided using different scaling. For example, you may use 1 unit = 40 along Y-axis. Here again you notice that all the points lie on a straight line.

Activity 4:

Suppose a car moves with a constant speed of 40 km per hour. Make a table of the distance covered by the car at the end of 1st, 2nd, 3rd, 4th, 5th, 6th, 7th and 8th hour. Trace the points on a graph paper where X-axis represents the time and Y-axis represents the distance travelled by the car.

Exercise 14.1

1. Fix up your own coordinate system on a graph paper and locate the following points on the sheet:

   (i) P (–3,5) (ii) Q (0,–8) (iii) R (4,0) (iv) S (–4,–9).

2. Suppose you are given a coordinate system. Determine the quadrant in which the following points lie:

   (i) A (4,5) (ii) B (–4,–5) (iii) C (4,–5).

3. Suppose P is a point with coordinates (–8,3) with respect to a coordinate system $X'OX \leftrightarrow Y'OY$. Let $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ be another system with $X'OX \parallel X_1'O_1X_1$ and suppose O_1 has coordinates (9,5) with respect to $X'OX \leftrightarrow Y'OY$. What are the coordinates of P in the system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$?

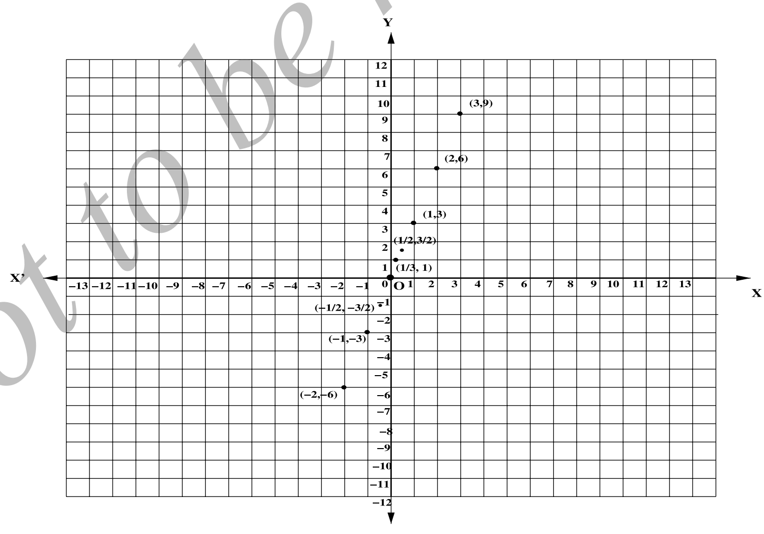
4. Suppose P has coordinates (10,2) in a coordinate system $X'OX-Y'OY$ and $(-3,-6)$ in another coordinate system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$? with $X'OX \parallel X_1'O_1X_1$. Determine the coordinates of O with respect to the system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$.

Linear graphs

Take a re-look at Example1, where you have plotted integral multiples of 3. Instead of taking integer multiples of 3, suppose you take real multiples of 3: for each real number x, consider $y = 3x$ (obtained by multiplying x by 3).

x	0	1	2	3	−1	−2	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{3}$
y	0	3	6	9	−3	−6	$\frac{3}{2}$	$\frac{-3}{2}$	1

As you assign different real values for x, you get back different real numbers $y = 3x$. Thus for each real number x, you get a point whose coordinates are $(x, 3x)$. Using a coordinate system, we can trace these points on a graph sheet.



If you take more and more values of x, you get more and more points $(x,3x)$. As you go on tracing them on graph paper, you see that these points get more and more clustered. Of course, you have to get finer and finer graph paper to trace these points. However, you see that all these points lie on a straight line. Now it is clear that as x exhaust all real numbers, the points (x,y), where $y = 3x$, trace a straight line in the plane. We say the equation $y = 3x$ represents a straight line on the **coordinate plane**.

Activity 5:

Use the scaling 1 unit = $\frac{1}{3}$ on the X-axis. Calculate more values of $y = 3x$ as x varies over real numbers and trace the points (x,y) on a graph sheet using the suggested scaling.

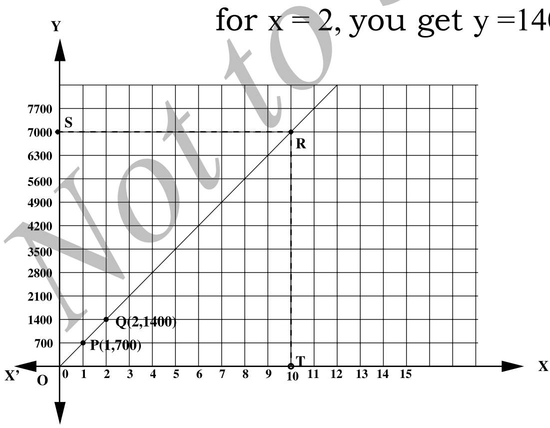
The relation $y = 3x$ is of the **first degree**. Such a relation always represents a straight line. The general form of the equation for a straight line is $y = ax + b$. (Here a and b are real numbers.) If $a = 0$, then $y = b$ is a straight line parallel to Y-axis.

Example 4. Suppose a person deposits ₹ 10,000 in a bank for simple interest at the rate of 7% per annum. Obtain a relation for accrued interest in terms of years. Draw the graph of this relation. Use this to read the interest at the end of 10 years.

Solution: Let y be the interest obtained in x years. The interest for x years is $\frac{7}{100} \times 10000 \times x = 700x$. Thus you get $y = 700x$ as the required relation. Now use the scaling: 1 unit= 700 along Y-axis. Note that

for $x = 1$, you get $y = 700$: $(x,y) = (1,700)$,

for $x = 2$, you get $y = 1400$: $(x,y) = (2,1400)$.



Take a sheet of graph paper and set up coordinate axes $X'OX \leftrightarrow Y'OY$. Locate points P(1,700) and Q (2,1400). Remember you are using the scale 1 unit = 700 along Y-axis. Now the equation is a straight line and hence P, Q determine this straight line.

Observe!

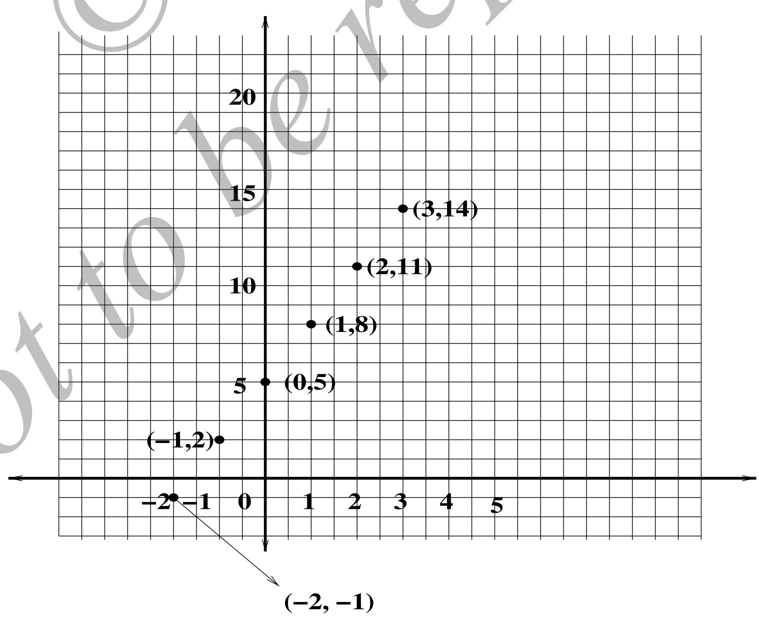
To draw a straight-line, it is enough to know only two points on that line; given any two distinct points in the plane, you can use the scale to draw the unique line passing through those two points.

Use a straight-edge to join PQ and extend it to possible extent. This straightline represents the **curve** of the interest versus years for this problem. Now look at the point 10 on X-axis, say T. Draw a perpendicular to X-axis at T to meet the straight line $y = 700x$ at R. From R draw a perpendicular to Y-axis. Record the point where it cuts Y-axis. That gives the interest at the end of 10 years, which you read as ₹ 7,000.

Example 5. Draw the graph of $y = 3x + 5$.

Solution: Give different values for x and get values for y. Tabulate them.

x	0	1	2	3	−1	−2
y	5	8	11	14	2	−1



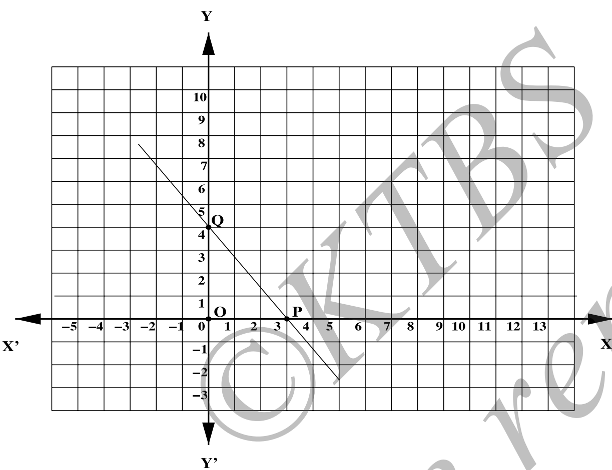
Let us trace these on a graph paper. Fix X and Y axes and locate the points

$$(x, y) = (0, 5), (1, 8), (2, 11), (3, 14), (-1, 2), (-2,-1).$$

Do you see that all these points lie on a straight line? Use a straight edge and join all these points by a straight line segment.

Activity 6: Draw the graphs of $y = x + 4$, $y = 2x - 3$, $y = 3$, $x = 2y + 1$, $x = 2$. All these are straight lines. Do you see that $y = ax + b$ is always a straight line?

Example 6. Determine the equation of the line in the following given graph.



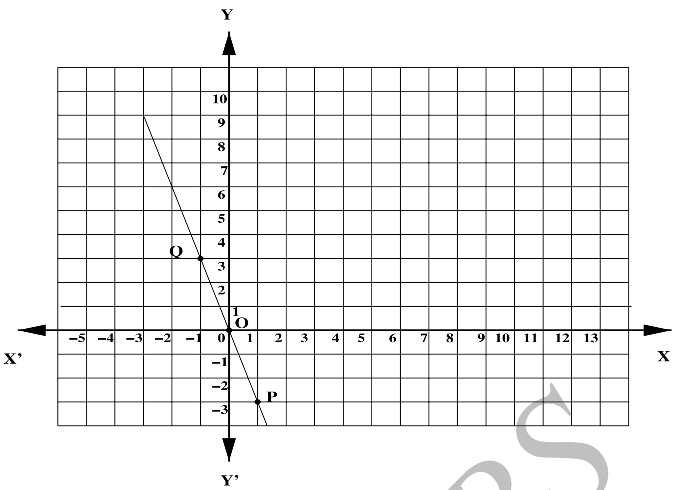
Solution: Recall that the general equation of a straight line is $y = ax + b$, where a, b are real constants. The given graph shows that $a \neq 0$. (If $a = 0$, then $y = b$ represents a straight line parallel to Y-axis.) If $x = 0$, then $y = b$. Thus $(0, b)$ is a point on the straight line.

Similarly, taking $y = 0$, you get $ax + b = 0$ or $x = \frac{b}{a}$. Thus $\left(-\frac{b}{a}, 0\right)$ is also a a point on the line. Looking at the graph, we see that it cuts Y-axis at (0,4) and the X-axis at (3,0). We conclude that

$$(0,b) = (0,4), \quad \left(-\frac{b}{a}, 0\right) = (3,0).$$

Thus $b = 4$ and $-\frac{b}{a} = 3$, which gives $a = -\frac{4}{3}$. Hence the equation of the given graph is $y = -\frac{4}{3}x + 4$. This may also be written as $3y + 4x = 12$ (why?)

Example 7. Determine the equation of the line in the following given graph.



Solution: We may take the equation in the form y = ax + b. Observe that the straight-line passes through (0,0). This means that y =0 when x =0. But the substitution x = 0 in the equation gives 0 = y = b. Thus b = 0 and so that y = ax.

We also observe that the line passes through (1,−3); that is y = −3, whenever x = 1. This gives −3 = a × 1 or a = −3. The equation of the line is therefore y = −3x.

Exercise 14.2

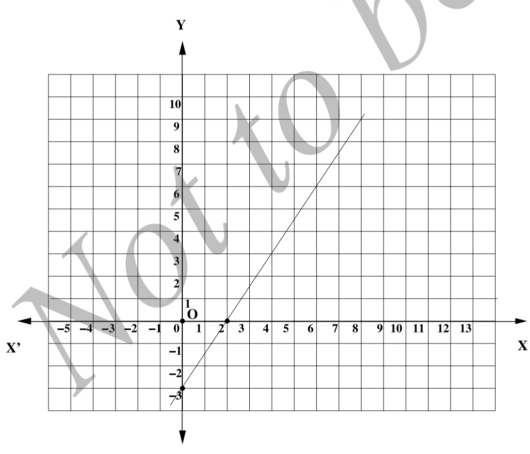
1. Draw the graphs of the following straight-lines:

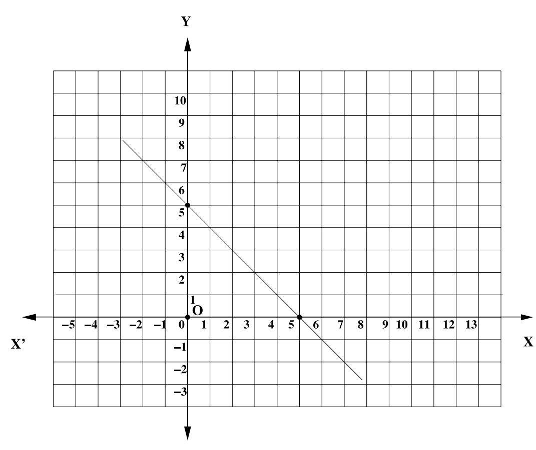
   (i) y=3-x (ii) y=x-3 (iii) y=3x-2 (iv) y=5-3x

   (v) 4y = −x +3 (vi) 3y =4x +1 (vii) x =4 (viii) 3y =1.

2. Draw the graph of $\dfrac{y}{2} = \dfrac{y+1}{x+2}$

3. Determine the equation of the line in each of the following graphs:





4. A boat is moving in a river, down stream, whose stream has speed 8 km per hour. The speed of the motor of the boat is 22 km per hour. Draw the graph of the distance covered by the boat versus hour.

5. Find the point of intersection of the straight lines $3y+4x=7$ and $4y+3x=7$, by drawing their graphs and looking for the point where they meet.

Glossary

Graph: a visual representation of numerical data.

Bar graph: graph in which data is represented by rectangular bars.

Pie chart: a graph in which data is represented by sectors of a circle.

Rectangular frame: two perpendicular lines in a plane which helps to locate points on the plane.

Rectangular coordinate system: a system which making use of rectangular frames locates points as an ordered pair of real numbers.

X-axis**:** the horizontal line in a rectangular coordinate system.

Y-axis**:** the vertical line in a rectangular coordinate system.

Quadrant: the four parts that a plane gets divided by a rectangular coordinate system.

Abscissa: the x-coordinate of a point.

Ordinate: the y-coordinate of a point.

Cartesian coordinate system: a coordinate system in which every point in the plane is determined by a pair of real numbers called the coordinates of the point.

Analytic geometry: the geometry in which all the geometrical concepts are studied using the coordinate system.

Points to remember

- You can set up your own coordinate system; there is no sacred coordinate system.
- The coordinates of a point depends on the coordinate system you choose.
- A rectangular coordinate system is only a convenient coordinate system; it is not necessary that one should always use a rectangular system.
- A graph is visual representation of the numerical data.

✸✸✸✸✸

Answers

Exercise 14.1

2. (i) first quadrant (ii) third quadrant (iii) fourth quadrant. **3.** $(-17,-2)$. **4.** $(-13,-8)$.

Exercise 14.2

3. (i) $3x - 2y = 6$ (ii) $x + y = 5$. **5.** $(1,1)$.

✡✡✡✡✡

UNIT 15
QUADRILATERALS

After studying this unit, you learn to:

- recognise quadrilaterals from a given list of figures.
- list out common properties of a quadrilateral.
- solve problems in properties of quadrilaterals.
- classify different types of quadrilaterals and recognise their distinct properties.
- transform the problems which occur in daily life related to quadrilaterals to numerical problems and solve them.
- construct squares and rectangles.

Introduction

You have learnt earlier that a triangle is a plane figure bounded by three sides. Triangles are classified based on the measures of sides and angles.

Activity 1: Name the triangles based on their sides. Name the triangles based on their angles.

Now let us take four points on the plane and see what we obtain on joining them in pairs in some order.

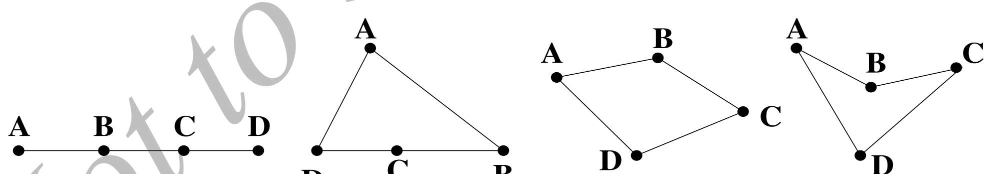
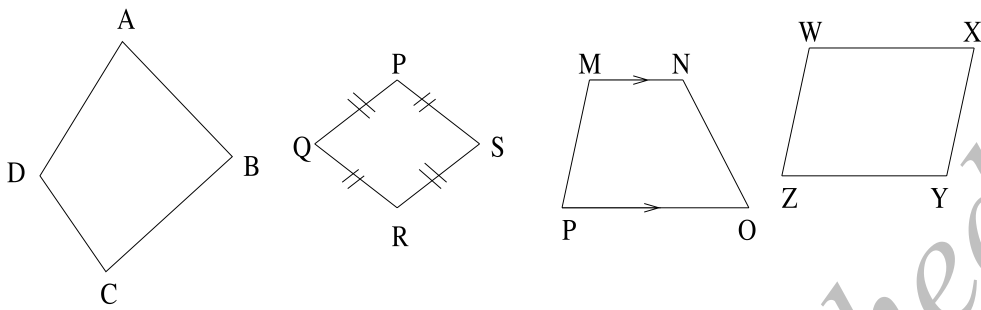


Fig 1 Fig 2 Fig 3 Fig 4

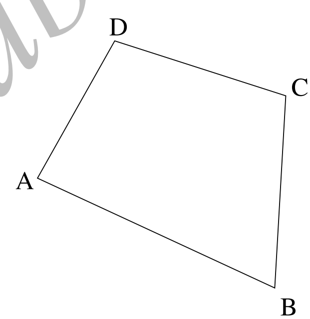
If all the points are collinear, that is they all lie on the same straight line, we obtain a line segment (Fig1). If three points out of the four are collinear, we get a triangle (Fig 2). If no three points out of four points are collinear, we get a closed figure with four sides (Fig 3 and Fig 4). Such a closed figure having four sides formed by joining four points, no three of which are collinear, in an order, is called **quadrilateral**.

Look at the following figures:



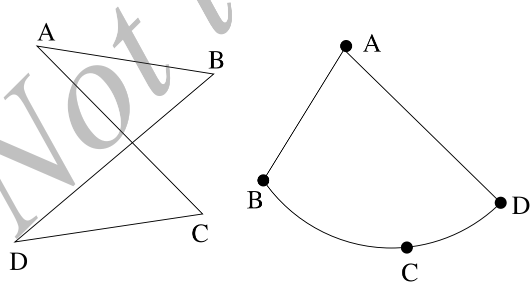
They are closed figures. Their boundary consists of four line segments. The common name to all the above plane figures is **quadrilateral**.

A quadrilateral is named by referring to its vertices in a particular order. In the adjoining figure, we can read it as ABCD or ADCB, we cannot read it ADBC and such names. You may keep in mind that the vertices should be read in such a way that if you join adjacent letters in the name, then there should not be any crossing of line segments.



For example in the name ADBC, we see that AC and BD cross each other.

Observe the following figures.



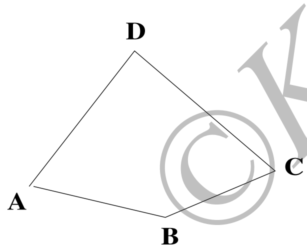
Are these two figures quadrilaterals? No they are not quadrilaterals; in the first, there is crossing of sides; in the second, the sides are not all line segments. So we may redefine the quadrilateral as: **A quadrilateral is the union of four line segments that join four coplanar points, no three of which are collinear and each segment meet exactly two other lines, each at their end point**.

Here again we do not distinguish between the closed curve which is the union of four line segments and the plane figure which is bounded by these four line segments. The context makes it clear which form is taken.

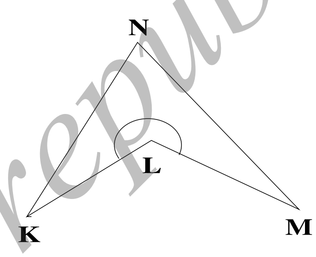
Observe that a quadrilateral consists of four sides and four **internal angles.** Based on these internal angles, we can classify quadrilaterals into two types: convex and concave quadrilaterals.

A quadrilateral is **convex** if each of the internal angle of the quadrilateral is less than 180^0. Otherwise it is called a **concave** quadrilateral.

Convex quadrilateral



Concave quadrilateral



Think it over:

Suppose you are given four sticks of different lengths. Can you put them together and make a quadrilateral?

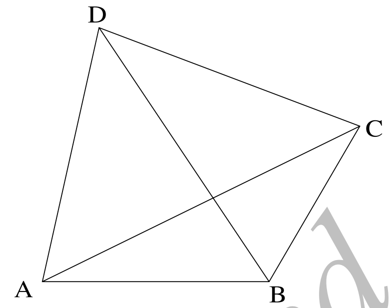
Like triangles, quadrilaterals also enjoy nice properties. We will study some of them in the coming sections.

3.5.2 Properties of quadrilaterals

Let ABCD be a quadrilateral.

1. The points A,B,C and D are called the vertices of the quadrilateral.
2. The segments AB, BC, CD and *DA* are the four sides of the quadrilateral.

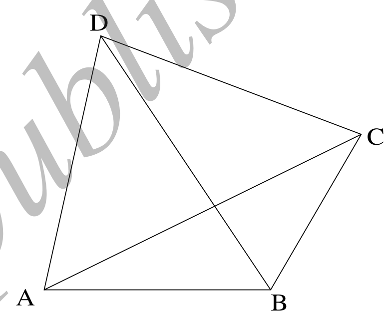
3. The angles $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle CDA$ are the **four angles** of the quadrilateral.
4. The segments AC and BD are called as the **diagonals** of the quadrilateral.



> **Note:** A quadrilateral has four sides, four angles and two diagonals. In all it has ten elements.

Adjacent sides and opposite sides

1. Two sides of a quadrilateral are said to be **adjacent sides** or **consecutive sides** if they have a common end point. In the adjoining figure, AB and AD are adjacent or consecutive sides. Identify the other pair of adjacent sides.



2. Two sides are said to be **opposite sides**, if they do not have a common end point. In the above figure AB and DC are opposite sides. Identify the other pair of opposite sides.

Adjacent angles and opposite angles.

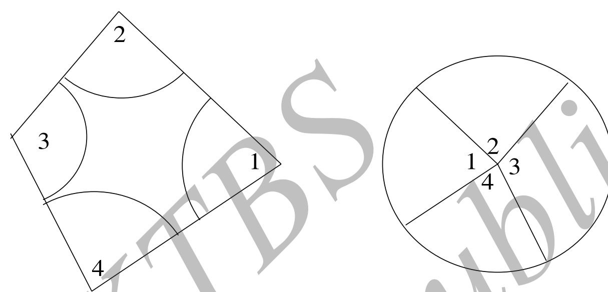
1. Two angles of a quadrilateral are **adjacent angles** or **consecutive angles**, if they have a side common to them. Thus $\angle DAB$ and $\angle ABC$ are adjacent or consecutive angles. Identify the other pair of adjacent angles.
2. Two angles of a quadrilateral are said to be **opposite angles**, if they do not contain a common side. Here $\angle DAB$ and $\angle BCD$ are opposite angles. Identify other pair of opposite angles.

Diagonal property

The diagonal AC divides the quadrilateral into two triangles, namely, triangle ABC and triangle ADC. Name the two triangles formed when the diagonal BD is drawn.

Angle sum property

Activity 2: Take a cut-out of a quadrilateral drawn on a card board. Cut it along the arms making each angle of the quadrilateral as in the figure below. The four pieces so obtained are numbered as 1,2,3 and 4. Arrange the cut outs as shown in next figure. Are they meeting at a point? What can you say about the sum of these four angles? The sum of the measures of the four angles is $360°$.



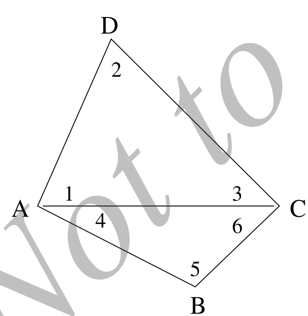
Note: Recall the sum of the angles at any point is $360°$.

Theorem 1. The sum of the angles of quadrilateral is $360°$.

Given: ABCD is a quadrilateral.

To prove: $\angle A + \angle B + \angle C + \angle D = 360°$.

Construction: Draw the diagonal AC.



Proof: In triangle ADC,

$\angle 1 + \angle 2 + \angle 3 = 180°$; (angle sum property.)
In triangle ABC, $\angle 4 + \angle 5 + \angle 6 = 180°$.

(again angle sum property.)

Adding these,

$\angle 1 + \angle 4 + \angle 2 + \angle 5 + \angle 3 + \angle 6 = 360°$.

But $\angle 1 + \angle 4 = \angle A$ and $\angle 3 + \angle 6 = \angle C$. Therefore $\angle A + \angle D + \angle B + \angle C = 360°$.

Thus the sum of the angles of the quadrilateral is $360°$.

Example 1. The four angles of a quadrilateral are in the ratio 2:3:4:6. Find the measures of each angle.

Solution:

Given: The ratio of the angles as 2:3:4:6.

To find: The measure of each angle.

Observe that 2+3+4+6 =15 (sum of the terms of the ratio). Thus 15 parts accounts for 360°. Hence

$$15 \text{ parts} \rightarrow 360^\circ;$$

$$2 \text{ parts} \rightarrow \frac{360^\circ}{15} \times 2 = 48^\circ;$$

$$3 \text{ parts} \rightarrow \frac{360^\circ}{15} \times 3 = 72^\circ;$$

$$4 \text{ parts} \rightarrow \frac{360^\circ}{15} \times 4 = 96^\circ;$$

$$6 \text{ parts} \rightarrow \frac{360^\circ}{15} \times 6 = 144^\circ.$$

Thus the angles are 48°, 72°, 96° and 144°. Can you see that their sum is 360° ?

Example 2. In a quadrilateral ABCD, $\angle A$ and $\angle C$ are of of equal measure; $\angle B$ is supplementary to $\angle D$. Find the measure of $\angle A$ and $\angle C$.

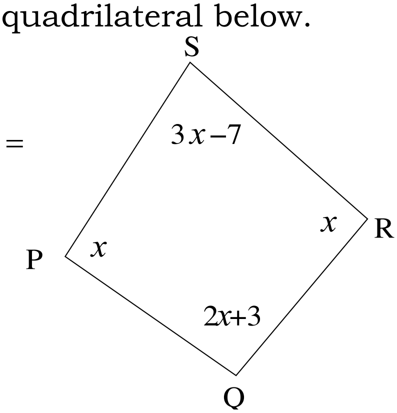
Solution: We are given $\angle B + \angle D = 180^\circ$. Using angle-sum property of a quadrilateral, we get

$$\angle A + \angle C = 360^\circ - 180^\circ = 180^\circ.$$

Since $\angle A$ and $\angle C$ are of equal measure, we obtain $\angle A = \angle C = \frac{180^\circ}{2} = 90^\circ$

Can you draw a quadrilateral in which $\angle A = \angle C = 90^\circ$ and $\angle B$, $\angle D$ are complementary ?

Example 3. Find all the angles in the given quadrilateral below.



Solution: We know that $\angle P + \angle Q + \angle R + \angle S = 360^\circ$ (angle sum property). Hence

$x + 2x + 3 + x + 3x - 7 = 360^\circ$.

This gives $7x - 4 = 360^\circ$ or $7x = 364^\circ$.

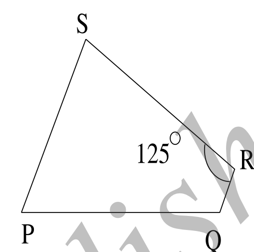
Therefore $x = \frac{364}{7} = 52^\circ$.

We obtain, $\angle P = 52°$ $\angle R = 52°$; $\angle Q = 2x + 3 = 2 \times 52 + 3 = (104+3) = 107°$; $\angle S = 3 \times 52° - 7° = 156° - 7° = 149°$. Check that $\angle P + \angle Q + \angle R + \angle S = 360°$.

Exercise 15.1

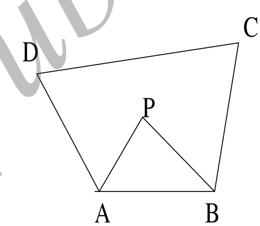
1. Two angles of a quadrilateral are $70°$ and $130°$ and the other two angles are equal. Find the measure of these two angles.

2. In the fig, suppose $\angle P$ and $\angle Q$ are supplementary angles and $\angle R = 125°$. Find the measures of $\angle S$.



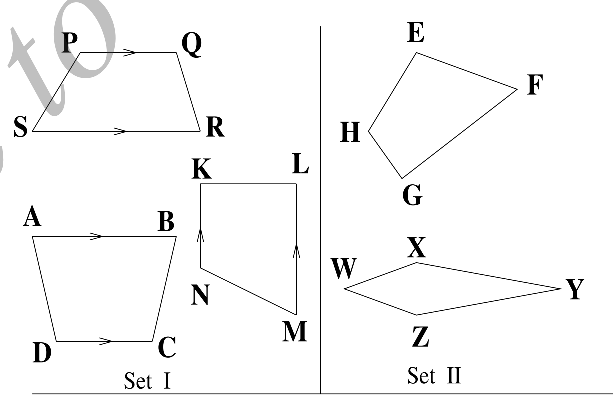
3. Three angles of a quadrilateral are in the ratio 2:3:5 and the fourth angle is $90°$. Find the measures of the other three angles.

4. In the adjoining figure, ABCD is a quadrilateral such that $\angle D + \angle C = 100°$. The bisectors of $\angle A$ and $\angle B$ meet at $\angle P$. Determine $\angle APB$.



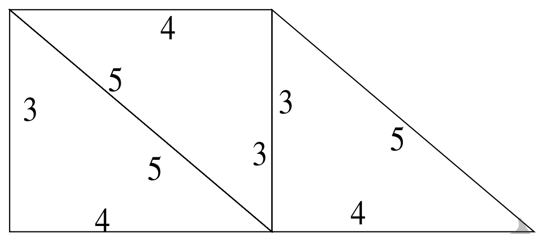
3.5.3 Trapezium

Based on the nature of the sides or angles, a quadrilateral gets special name.



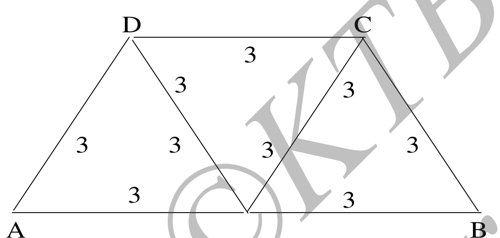
Observe the above figures given in two sets. Discuss with your friends what is the difference you observe in the first set and the second set of quadrilaterals. [Note: the arrow mark indicates parallel lines].

The first set of quadrilaterals have a pair of opposite sides which are parallel. Such a quadrilateral is known as **trapezium**. In the first set each quadrilateral is a trapezium. In the second set no quadrilateral is a trapezium.



Activity 3:

Take identical cut-outs of congruent triangles of sides 3 cm, 4 cm and 5 cm. Arrange them as shown in the figure. Which figure do you get? It is a trapezium. Which are the parallel sides? Can you get a trapezium in which non parallel sides are equal?



Activity 4:

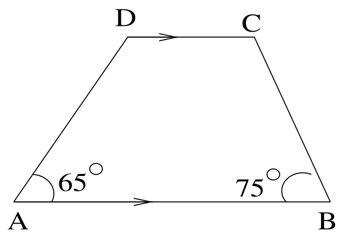
Cut three identical equilateral triangles and place them as shown in the figure. Measure AD and BC. Are they equal? Measure angles A and B. Are they equal? Measure angles D and C. Are they equal? Measure AC and BD. Are they equal?

The special name given to the above trapezium is **isosceles trapezium**. You see that in an isosceles trapezium;

1. the non parallel sides are equal
2. the base angles are equal
3. the adjacent angles corresponding to parallel sides are supplementary
4. the diagonals are equal.

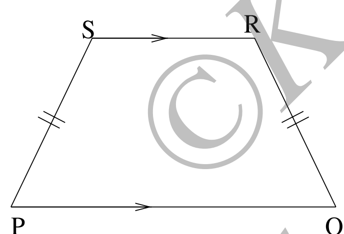
> **Think it over!**
>
> **Is there a trapezium whose all angles are equal?**

Example 4. In the figure ABCD, suppose AB $\parallel$ CD; $\angle A = 65^\circ$ and $\angle B = 75^\circ$. What is the measure of $\angle D$ and $\angle C$?



Solution: Observe that, $\angle A + \angle D = 180°$ (a pair of adjacent angles of a trapezium is supplementary). Thus $65° + \angle D = 180°$. This gives $\angle D = 180° - 65° = 115°$. Similarly $\angle B + \angle C = 180°$, which gives $75° + \angle C = 180°$. Hence $\angle C = 180° - 75° = 105°$.

Example 5. In an isosceles trapezium PQRS, $\angle P$ and $\angle S$ are in the ratio 1:2. Find the measure of all the angles.



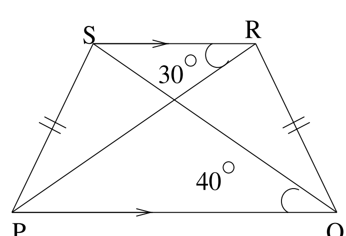
Solution: In an isosceles trapezium base angles are equal; $\angle P = \angle Q$ Let $\angle P = x°$ and $\angle S = 2x°$. Since $\angle P + \angle S = 180°$ (one pair of adjacent angles of a trapezium is supplementary), we get

$$x° + 2x° = 180°.$$

Thus $3x = 180°$ or $x = 180°/3 = 60°$. Hence $\angle P = 60°$ and $\angle S = 2 \times 60° = 120°$. Since, $\angle P = \angle Q$, we get $\angle Q = 60°$. But, $\angle Q + \angle R = 180°$ (one pair of adjacent angles are supplementary). Hence we also get $\angle R = 180° - 60° = 120°$.

Exercise 15.2

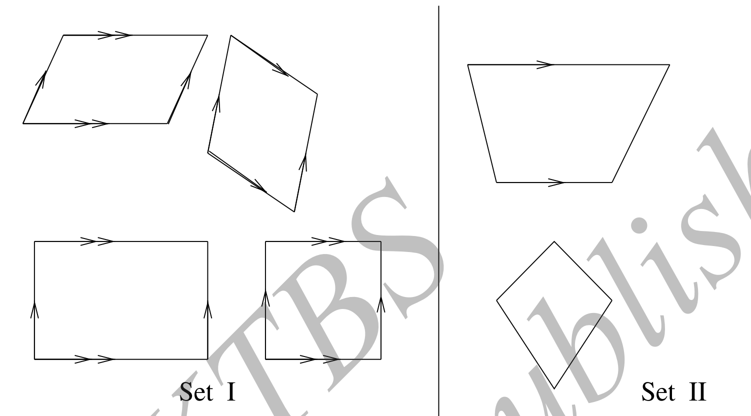
1. In a trapezium PQRS, PQ $\parallel$ RS; and $\angle P = 70°$ and $\angle Q = 80°$. Calculate the measure of $\angle S$ and $\angle R$.
2. In a trapezium ABCD with AB $\parallel$ CD, it is given that AD is not parallel to BC. Is $\Delta ABC \cong \Delta ADC$? Give reasons.



3. In the figure, PQRS is an isosceles trapezium; $\angle SRP = 30°$, and $\angle PQS = 40°$. Calculate the angles $\angle RPQ$ and $\angle RSQ$.

Parallelograms

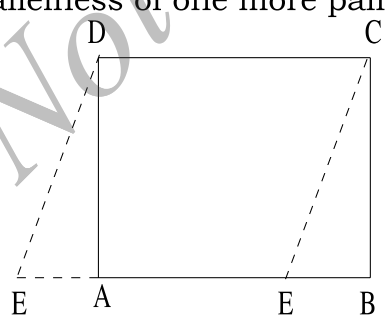
Look at the following sets of quadrilaterals.



How many pairs of parallel sides do you see in each of the quadrilaterals in set I ? How many pairs of parallel sides do you see in each of the quadrilaterals in set II ? What is your conclusion?

A quadrilateral in which both the pairs of opposite sides are parallel is called a **parallelogram.**

The quadrilaterals in set II are not parallelogram, while all the quadrilaterals in set I are parallelograms. Can you see that a parallelogram is a particular type of trapezium? Hence, whatever properties are true for trapezium also hold good for parallelograms. You will see that additional properties are also true because the parallelness of one more pair of sides.



Activity 5:

Take a rectangular card board, ABCD and mark a point E on AB (as shown in the fig). Join CE by dotted line and cut the card board along CE. Place the triangular part EBC to the left of the rectangle such that BC coincides with AD to get a quadrilateral.

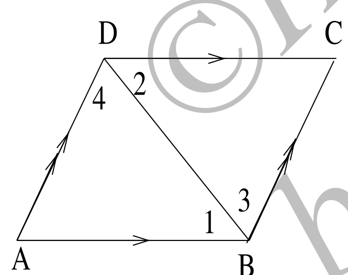
Which type of quadrilateral is this? It is a parallelogram.

Trace the above cut-out card board in your book and measure the opposite sides, opposite angles, Repeat the same activity with two more parallelograms and mark them as PQRS and KLMN and tabulate the results as follows.

Parallelogram	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
ABCD	$\angle A =$	$\angle B =$	$\angle C =$	$\angle D =$	AB =	BC =	CD =	DA =
PQRS	$\angle P =$	$\angle Q =$	$\angle R =$	$\angle S =$	PQ=	QR =	RS =	SP =
KLMN	$\angle K =$	$\angle L =$	$\angle M =$	$\angle N =$	KL =	LM =	MN =	NK =

What relations are there among angles? What relations are there among sides? Do you observe that opposite angles are equal and opposite sides are also equal? These observations can be proved logically.

Proposition 1. In a parallelogram, opposite sides are equal and opposite angel are equal



Proof: Let ABCD is a parallelogram. Join BD. Then $\angle 1 = \angle 2$, and $\angle 3 = \angle 4$. (Why ? See figure.) In triangles ABD and CBD, we observe $\angle 1 = \angle 2$, $\angle 4 = \angle 3$, BD (common).

Hence $\Delta ABD \cong \Delta CDB$ (ASA postulate). It follows that

$$AB = DC, AD = BC \text{ and } \angle A = \angle C.$$

Similarly join AC, and you can prove that $\Delta ADC \cong \Delta CBA$. Hence $\angle D = \angle B$.

Activity 6:

Trace a parallelogram as in the previous case on a card board; draw the diagonals and mark the point of intersection. Measure the intercepts formed by the point of intersection. What is the inference drawn from this activity?

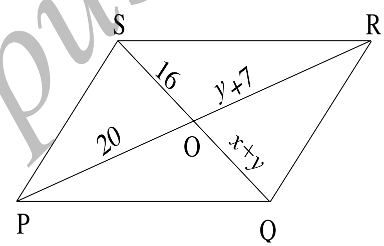
The diagonals bisect each other. Thus you may describe the properties of parallelogram as:

1. The opposite sides are equal and parallel.
2. The opposite angles are equal.
3. The adjacent angles are supplementary
4. The diagonals bisect each other.
5. Each diagonal bisects the parallelogram into two congruent triangles.

Example 6. The ratio of two sides of parallelogram is 3:4 and its perimeter is 42 cm. Find the measures of all sides of the parallelogram.

Solution: Let the sides be 3x and 4x. Then the perimeter of the parallelogram is $2(3x + 4x) = 2 \times 7x = 14x$. The given data implies that $42 = 14x$, so that $x = 42/14 = 3$. Hence the sides of the parallelogram are $3 \times 3 = 9$ cm and $3 \times 4 = 12$ cm.

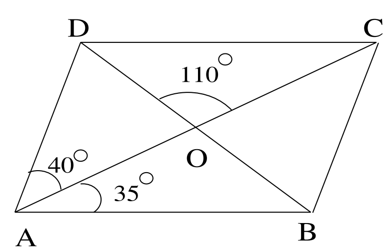
Example 7. In the adjoining figure, PQRS is a parallelogram. Find x and y in cm.



Solution: In a parallelogram, we know that the diagonals bisect each other. Therefore SO = OQ. This gives $16 = x + y$. Similarly, PO = OR, so that $20 = y + 7$. We obtain $y = 20 - 7 = 13$cm. Substituting the value of y in the first relation, we get $16 = x + 13$. Hence $x = 3$ cm.

Exercise 15.3

1. The adjacent angles of a parallelogram are in the ratio 2:1. Find the measures of all the angles.
2. A field is in the form of a parallelogram, whose perimeter is 450 m and one of its sides is larger than the other by 75 m. Find the lengths of all sides.
3. In the figure, ABCD is a parallelogram. The diagonals AC and BD intersect at O; and $\angle DAC = 40°$, $\angle CAB = 35°$; and $\angle DOC = 110°$. Calculate the measure of $\angle ABO$, $\angle ADC$, $\angle ACB$, and $\angle CBD$.



4. In a parallelogram ABCD, the side DC is produced to E and $\angle BCE = 105°$. Calculate $\angle A$, $\angle B$, $\angle C$, and $\angle D$.
5. In a parallelogram KLMN, $\angle K = 60°$. Find the measures of all the angles.

Special kinds of parallelograms

There are special kind of parallelograms which enjoy different types of properties. We study them here.

Rectangle

Activity 7:

Take a sheet from your note book and paste it on a card board; cut the cardboard along the boundary, measure all the sides, all the angles. Write down the observations in the following chart. Repeat the activity with sheets of different sizes.

Parallelogram	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
ABCD	$\angle A =$	$\angle B =$	$\angle C =$	$\angle D =$	AB =	BC =	CD =	DA =
PQRS	$\angle P =$	$\angle Q =$	$\angle R =$	$\angle S =$	PQ=	QR =	RS =	SP =

From the above activity, you can infer that:

- all angles are equal to $90°$;
- opposite sides are equal;
- opposite sides are parallel;

The special name given to such a parallelogram is **rectangle**. A rectangle is a parallelogram whose all angles are right angles.

Activity 8:

Take a rectangular sheet of paper and name it as ABCD. Fold it along the diagonals. Mark the point of intersection as O. Answer the following questions:

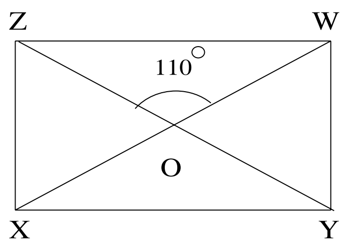
When you fold along any of the diagonals, are the two triangles formed congruent? Measure the length of the diagonals. Are they equal? Measure the line segments OA, OC, OB and OD. Do you find any relation among the length of these segments?

Diagonal properties of a rectangle

(i) The diagonals of a rectangle are equal.

(ii) The diagonals of a rectangle bisect each other.

Example 8. In a rectangle XYWZ, suppose O is the point of intersection of its diagonals. If $\angle ZOW = 110°$, calculate the measure of $\angle OYW$.

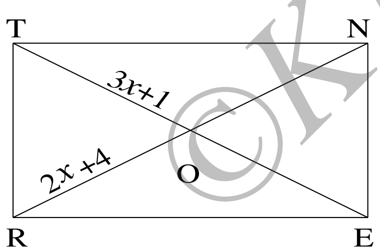


Solution:-We know $\angle ZOW = 110°$.

Hence, $\angle WOY = 180° - 110° = 70°$ (supplementary angles). Now OYW is an isosceles triangle, as OY = OW. Hence $\angle OYW = \angle OWY = \frac{180°-70°}{2} = \frac{110°}{2} = 55°$.

(Can you suggest an alternate method?)

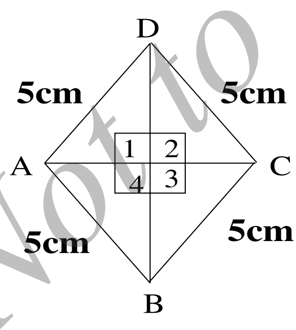
Example 9. In a rectangle RENT, the diagonals meet at O. If OR = 2x + 4 and OT = 3x + 1, find x.



Solution: Observe that OR = OT (diagonal bisect each other and they are equal in a rectangle). Hence $2x + 4 = 3x + 1$.

This implies that $4 - 1 = 3x - 2x$
Hence $x = 3$.

Rhombus



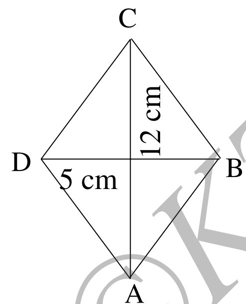
Activity 9:

Construct four identical right angle triangles with measures 3 cm, 4 cm and 5 cm using card board. Arrange them as shown in the figure, on a plane sheet. Draw the boundary of the figure. Measure the sides and angles of the figure. Tabulate them. Repeat this with right triangles of different dimensions. What do you observe?

Can you conclude that $\angle A = \angle C$, $\angle B = \angle D$; AB = BC = CD = DA? Such a figure is called a **rhombus**. **A rhombus is a parallelogram in which all the four sides are equal**. Being a parallelogram, a rhombus has all the properties of a parallelogram and more:

(i) all the sides of a rhombus are equal
(ii) opposite sides are parallel
(iii) the diagonals bisect each other at right angles
(iv) the two diagonals divide the rhombus into four congruent right angled triangles
(v) angles are also bisected by the diagonals.

Example10. The diagonals of a rhombus are 24cm and 10cm. Calculate the area of the rhombus.



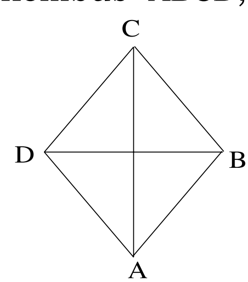
Solution: We are given that AC = 24 cm; BD = 10 cm. We know that the diagonals of a rhombus bisect each other at right angles. Let O be the point of intersection of these diagonals. Then we have AO = CO =12cm and BO = DO = 5cm. We also know that AOD is a right angled triangle. Hence the area of triangle AOD is

$\frac{1}{2} \times OA \times OD = \frac{1}{2} \times 12 \times 5 = 30cm^2$.

Since a rhombus has four congruent right triangles, is area is $4 \times 30 = 120\ cm^2$.

Example 11. In a rhombus ABCD, $\angle BAC = 38°$. Find (i) $\angle ACD$,(ii) $\angle DAC$ and(iii) $\angle ADC$.

Solution: We know that $\angle BAC = 38°$. But in a rhombus ABCD, since ABC is an isosceles triangle, we see that $\angle BAC = \angle ACB = 38°$ Moreover $\angle DAC = 38°$, since the diagonal AC bisect $\angle A$. Since ADC is also an isosceles triangle, we get $\angle ACD = \angle DAC = 38°$. Finally,



$$
\begin{aligned}
\angle ADC &= 180° - (\angle DAC + \angle DCA) \\
&= 180° - (38° + 38°) \\
&= 180° - 76° \\
&= 104°
\end{aligned}
$$

Square

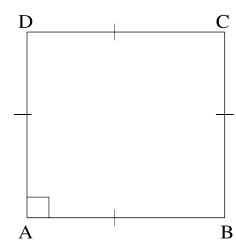
There is a type of parallelogram which is simultaneously a rectangle and a rhombus. All its angles are equal and all its sides are equal. Recall what you have for triangles: triangle which is at the same time have all angles equal and all sides equal. These are equilateral triangles. In the case of triangles, you have seen that whenever all the angles are equal, all the sides are also equal. Conversely,if all the sides of a triangle are equal, then all the angles are also equal. But when you move to quadrilaterals, you do not have such a nice property. A rectangle is a quadrilateral in which all the angles are equal, but the sides need not be equal. On the other hand a rhombus is a quadrilateral in which all the sides are equal, but all the angles need not be equal. A quadrilateral in which all the angles are equal and all the sides are equal is given a special name; a **square**. Thus a square is a quadrilateral in which all the nice properties come together. A square is a parallelogram in which

(i) all the sides are equal

(ii) each angle is a right angle

(iii) diagonals are equal

(iv) diagonals bisect at right angles.



Think it over!

(iii) and (iv) are consequences of (i) and (ii). Can you prove them?

A square can also be defined as:

(a) a rectangle in which adjacent sides are equal

(b) a rhombus in which each angle is 90°.

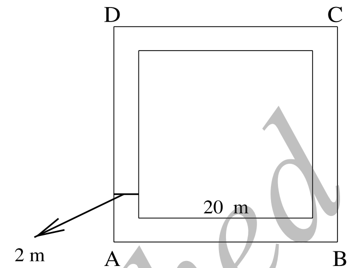
Think it over!

Suppose the perimeters of a square and a rhombus are equal. Do they have equal area?

Example 12. A field is in the shape of a square with side 20m. A pathway of 2m width is surrounding it. Find the outer perimeter of the pathway.

Solution: Width of the pathway is 2m. Length of the side of the outer square = (20+2+2) = 24m.

Hence perimeter = 4 × 24 = 96m.



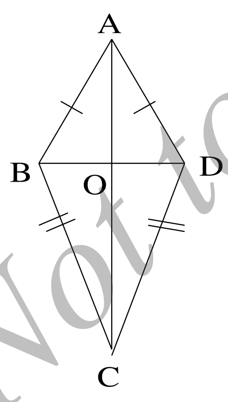
Example 13. The square field has area 196 m^2. Find the length of the wire required to fence it around 3 times.

Solution: Suppose S is the side-length of a square. Then its area is S^2 sq. units. We are given that $S^2 = 196m^2$. Therefore S = 14m. Thus perimeter = 4 × S = 4 × 14 = 56 m.

The length of the wire required to fence around it 3 times is 56 × 3 = 168 m.

Kite

You have seen that in a rhombus, a diagonal divides the rhombus into two congruent isosceles triangles. Suppose we take two isosceles triangles whose bases are of equal length and glue them together to get a quadrilateral. You get a special type of quadrilateral. Such a quadrilateral is called a **kite**.



Kite is a quadrilateral in which two isosceles triangles are joined along the common base. In the adjoining figure AB = AD, BC = CD, and BD is the common base. It is important to observe that triangles ABC and ADC are congruent, but the triangles ABD and CBD need not be congruent. Can you see that if ABD and CBD are also congruent, the quadrilateral ABCD reduces to a rombhus?

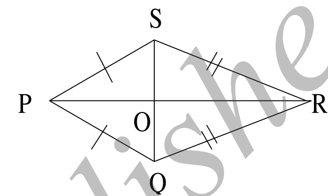
Properties of kite

Like rhombus, kite has some properties which we record here:

1. There are two pairs of equal sides; AB = AD and CB = CD in the previous diagram.

2. One of the diagonals bisect the other diagonal perpendicularly the diagonal BD is bisected perpendicularly by AC in the previous figure.
3. One of the diagonals bisect the apex angles; the diagonal AC bisects the apex angles $\angle A$ and $\angle C$.

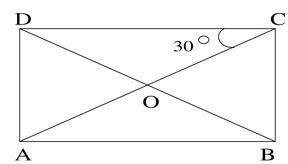
Example 14. In the figure PQRS is a kite; PQ = 3 cm and QR = 6 cm. Find the perimeter of PQRS.



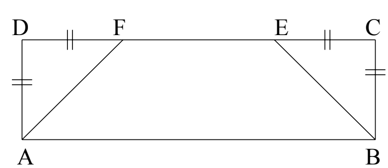
Solution: We have PQ = PS = 3 cm, QR = SR = 6 cm. Hence the perimeter = PQ + QR + RS + PS = 3 + 6 + 6 + 3 = 18 cm.

Exercise 15.4

1. The sides of the rectangle are in the ratio 2:1.The perimeter is 30 cm. Calculate the measure of all the sides.



2. In the adjacent rectangle ABCD, $\angle OCD = 30°$. Calculate $\angle BOC$. What type of triangle is BOC?
3. All rectangles are parallelograms, but all parallelograms are not rectangles. Justify this statement.
4. The sides of a rectangular park are in the ratio 4:3.If the area is1728 m^2, find the cost of fencing it at the rate of Rs. 2.50/m.



5. A rectangular yard contains two flower beds in the shape of congruent isosceles right triangle. The remaining portion is a yard of trapezoidal shape (see fig). whose parallel sides have lengths 15 m and 25 m. What fraction of the yard is occupied by the flower bed?
6. In a rhombus ABCD, $\angle C = 70°$. Find the other angles of the rhombus.
7. In a rhombus PQRS, if $PQ = 3x - 7$ and $QR = x + 3$, find PS.

8. Rhombus is a parallelogram. Justify.

9. In a given square ABCD, if the area of triangle ABD is $36cm^2$, find (i) the area of triangle BCD; (ii) the area of the square ABCD

10. The side of a square ABCD is 5cm and another square PQRS has perimeter equal to 40cm. Find the ratio of the perimeter of ABCD to perimeter of PQRS. Find the ratio of the area ABCD to the area of PQRS.

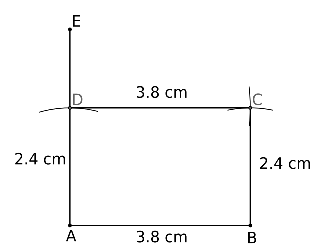
11. A square field has side 20 m. Find the length of the wire required to fence it four times.

12. List out the differences between square and rhombus.

Construction of rectangles

Example 15. Construct a rectangle whose adjacent sides are 3.8 cm and 2.4 cm. Given: AB = 3.8 cm and AD = 2.4 cm.

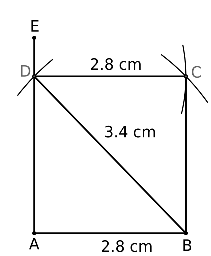
Steps:



1. Draw a line segment AB = 3.8 cm.
2. Construct a perpendicular AE to AB at A such that AE is greater than 2.4 cm.
3. With A as centre and radius 2.4 cm draw an arc so as to cut AE at D.
4. With D as centre and radius 3.8 cm, draw an arc.
5. With B as centre and radius 2.4 cm, draw another arc so as to intersect previous arc at C.
6. Join DC and BC. Then ABCD is the required rectangle.

Example 16. Construct a rectangle given that a diagonal is 3.4 cm and one side is 2.8 cm

Given: AB = 2.8 cm and BD = 3.4 cm.



Steps:

1. Draw a line segment AB = 2.8 cm.
2. Construct a perpendicular AE to AB at A.

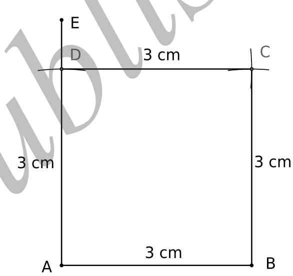
3. With B as centre and radius 3.4 cm draw an arc to cut AE in D.
4. With A as centre and radius 3.4 cm draw an arc.
5. With D as centre and radius 2.8 cm draw an arc so as to intersect the previous arc in C.
6. Join DC and BC. You get the rectangle ABCD.

Construction of a square

Example 17. Construct a square of side 3 cm.

Note that a square is a special type of rectangle; a rectangle with all sides equal. We use this to construct a square using this special property.

Steps:

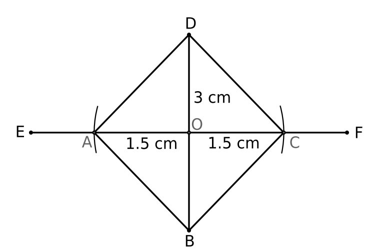


1. Draw a line segment AB = 3 cm.
2. At A, construct a perpendicular AE to AB.
3. With A as centre and radius 3 cm. draw an arc to cut AE in D.
4. With D as centre and radius 3 cm, draw an arc.
5. Draw B as centre and radius 3 cm, draw an arc so as to intersect the previous arc in C.
6. Join BC and DC. Then ABCD is the required square.

Example 18. Construct a square having a diagonal of length 3 cm.

Note that a square has equal diagonals and they bisect each other perpendicularly. These properties help us to construct a square.

Steps:



1. Draw a line segment BD = 3 cm.
2. Draw the perpendicular bisector EOF of BD intersecting at 0.
3. With 0 as centre and radius 1.5 cm, draw an arc to cut OE at A and OF at C
4. Join AB, AD, CB and CD. Then ABCD is the required square.

Exercise 15.5

1. Construct rectangle ABCD with the following data.
   (a) AB = 4 cm, BC = 6 cm; (b) AB = 6 cm, AC = 7.2 cm.
2. Construct square ABCD:
   (a) which has side-length 2 cm; (b) which has diagonal 6 cm.

Glossary

Quadrilateral: a linear figure on a plane consisting of four line segments, which are placed in an ordered way such that the adjacent segments meet only at their end points.

Convex quadrilaterals: a quadrilateral in which each interior angle is less than $180°$.

Concave quadrilateral: a quadrilateral in which some angle exceeds $180°$.

Diagonal: the line segment joining opposite vertices of a quadrilateral.

Trapezium: a quadrilateral in which a pair of sides are parallel.

Parallelogram: a quadrilateral in which two pairs of sides are parallel.

Rhombus: a quadrilateral in which all the sides are equal.

Square: a quadrilateral in which all the sides are equal and all the angles are equal to $90°$.

Rectangle: a quadrilateral in which all the angles are equal to $90°$.

Kite: a quadrilateral formed by a pair of isosceles triangle glued along a common side.

Points to remember

- The sum of all the four angles of a quadrilateral is $360°$.
- For a quadrilateral, equiangularity is not the same as equilaterality, unlike for triangles.
- The diagonals intersect perpendicularly in a rhombus and a kite.
- Any quadrilateral in which diagonals bisect each other is a parallelogram.

*****

Answers

Exercise 15.1

1. $80°$ each. **2.** $55°$. **3.** $54°$, $81°$ and $108°$. **4.** $50°$

Exercise 15.2

1. $\angle S = 110°$ and $\angle R = 100°$. **3.** $\angle RPQ = 30°$ and $\angle RSQ = 40°$.

Exercise 15.3

1. $120°$, $60°$, $120°$, $60°$.

2. 150 m and 75 m.

3. $\angle ABO = 35°$, $\angle ADC = 105°$, $\angle ACB = 40°$, $\angle CBD = 70°$.

4. $\angle A = \angle C = 75°$, $\angle B = \angle D = 105°$.

6. $60°$, $120°$, $60°$, $120°$.

Exercise 15.4

1. 10 cm, 5 cm, 10 cm, 5 cm. **2.** $\angle BOC = 60°$; ΔBOC is equilateral. **4.** ₹. 420. **5.** $\frac{1}{5}$ **6.** $110°$, $70°$, $110°$. **7.** 8. **9.** (i)36 cm^2; (ii) 72 cm^2. **10.** (i)1:2 (ii) 1:4. **11** 320 m.

✡✡✡✡✡

UNIT 16
MENSURATION

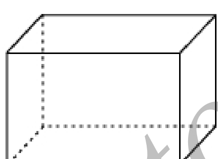
After studying this unit you learn to:

- recognise the cubes and cuboidal objects used in our day to day life.
- list out the properties of cubes and cuboids.
- substitute the data in the given formula and solve the problems.

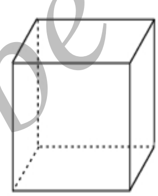
Introduction

An empty box, an empty bowl and an empty container, it has some space and we can keep things in that empty space. A class room has space for the students to sit in.

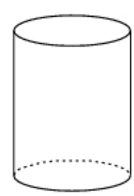
A solid occupies fixed amount of space. Solids occur in different shapes. Observe the following diagrams. These shapes (cuboid, cube, cylinder, sphere, cone, triangular prism etc) are known as three dimensional objects.



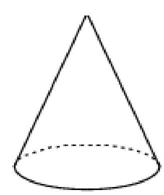
Cuboid



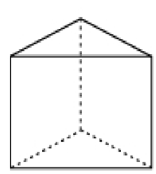
Cube



Cylinder



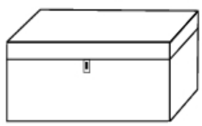
Cone



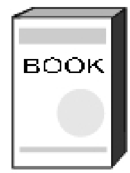
Triangular prism

Do you see that each solid occupies some space. Each solid also has some surface and hence has associated surface area. Since each occupies space, it has some volume. We shall study these aspects corresponding to some simple figures.

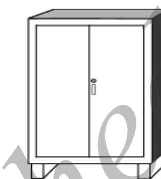
Observe the following figures


wooden box


match box



book case


Almirah

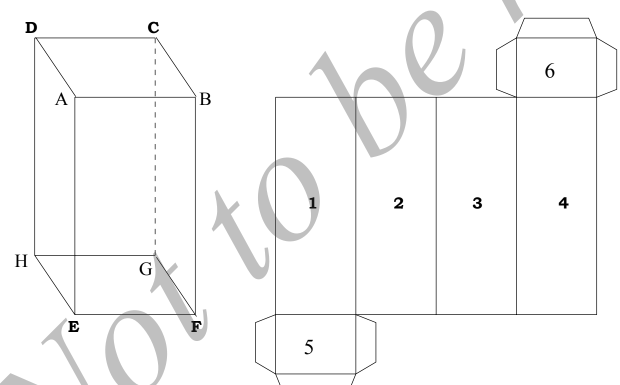
These are in the shape of cuboids.

Surface area of a cuboid

Let us understand the cuboids by doing an activity.

Activity 1:

Take a box in the shape of cuboid and cut it open along one edge, open up the lids and spread it over a sheet of paper and fasten it with pins. (See the figure)



How many **faces** does a cuboid has? Find the number of **edges** and **vertices**. A cuboid has 6 faces, 12 edges and 8 vertices. Do you see that a cuboid has rectangular faces?.

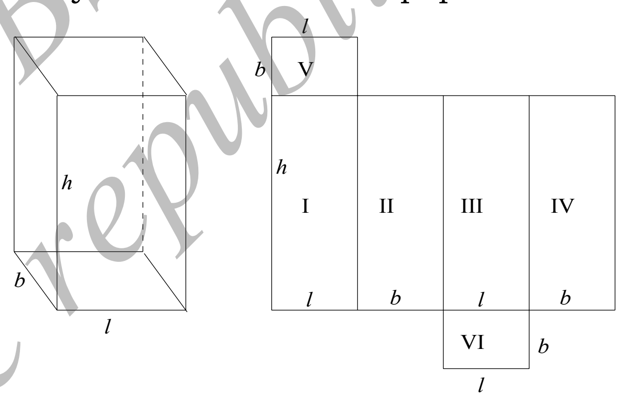
Any face of a cuboid may be called its **base** (can you give the reason?). The four faces which meet the base are called the **lateral faces** of a cuboid.

In the given figure above, the cuboid has 6 faces. They are ABCD, EFGH, EFBA, HGCD, EHDA and FGCB. Any two adjacent faces of a cuboid meet in a line segment, called an **edge** of the cuboid. The 12 edges are AB, BC, CD, DA, EF, FG, GH, EH, AE, DH, GC and BF. The point of intersection of three edges of a cuboid is a **vertex** of the cuboid. The eight vertices are A, B, C, D, E, F, G and H.

Activity 2:

Take any cuboidal box. Fix a base (observe any face can be taken as base). Place it vertically and wrap a thick sheet of paper such that it just fits around the surface. Remove the paper and measure the area of the paper. It is the **lateral surface area** (L.S.A) of the cuboid. Do this with different cuboids.

Cut open the cuboidal box b and lay it flat on a sheet of paper. Let the length of the base be l; breadth of the base be b; and the height of the cuboid corresponding to this base be h units.



Compute 2(lh + bh) and compare this with the area of the lateral surface you have measured. Do they match? What is 2(lh+ hb+ bl)?

If you change the base, you may observe that the units l, b, h do not change; only their representation as length, breadth and height may change.

There are 6 faces in a cuboid. All the faces are rectangular in shape. The sum of the areas of all the six surfaces is called the **total surface area** (T.S.A) of the cuboid. Let us find a formula for total surface area and lateral surface area of a cuboid. The total surface area of cuboid is

Area of I + area of II + area of III + area of IV + area of V + area of VI (see fig)

Hence

$$A = (l \times h) + (l \times b) + (b \times h) + (l \times h) + (b \times h) + (l \times b) \text{ sq. units.}$$

Thus

$$A = 2lh + 2lb + 2bh = 2\,(lh + lb + bh) \text{ sq. units.}$$

The lateral surface area of a cuboid is

area I + area II + area III + area IV.

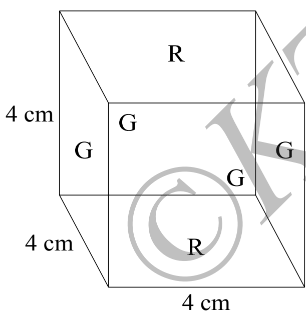
Hence

$$\text{L.S.A} = (l \times h) + (b \times h) + (l \times h) + b \times h = 2h(l + b) \text{ sq.units.}$$

Here you may observe that the lateral surface area of a cuboid depends on the base you choose for the cuboid. If you change the base the lateral surface area might change. However, the total surface area of the cuboid remains the same.

Cube

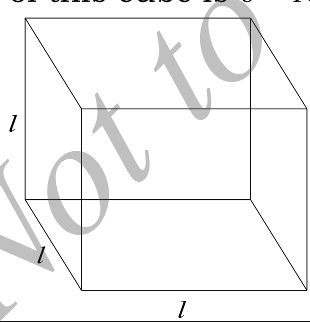
A cuboid whose length, breadth and height are all equal is called a **cube**. Ice cubes, sugar cubes, dice are some examples of cubes.



Activity 3:

Construct a cube of side 4 cm by using card board. Paint any two opposite sides with red and remaining with green. Place the cube on table with one of the red faces as base. Identify the number of green faces bounding the red faces.

The green faces are called **lateral faces** of the cube. What is the area of the lateral faces? Do you see that: L.S.A of this cube is $4 \times 16 = 64$ cm^2; T.S.A of this cube is $6 \times 16 = 96$ cm^2.



Observe the adjoining figure of a cube of side l units. It has six *square faces*. Area of each face of the cube is l^2 units. Area of 6 faces is therefore $6l^2$ units. And the area of 4 lateral faces is $4l^2$ units.

Think it over!

What should be the maximum length of the ladder which can be placed from the bottom of the floor to reach the opposite corner of the roof of a room which is in the shape of a cube?

Example1. Find the lateral surface area and total surface area of a cuboid which is 8 m long, 5 m broad and 3.5 m high.

Solution: We are given $l = 8$ m, $b = 6$ m, $h = 3.5$ m. We know that

$$\text{L.S.A} = 2h\,(l + b) = 2 \times 3.5\,(8 + 6)$$
$$= 7 \times 14 = 98\ \text{m}^2.$$

Similarly,

$$\text{T.S.A} = 2\,(lb + bh + lh) = 2\,(8 \times 6 + 6 \times 3.5 + 8 \times 3.5)$$
$$= 2\,(48 + 21 + 28) = 2 \times (97) = 194\ \text{m}^2.$$

Example 2. How many tiles each of 30 cm × 20 cm are required to cover the floor of hall of dimension 15 m by 12 m?

Solution: Since the tiles are in cm², we have to convert the dimensions of the hall to cm first. It will be 1500 cm by 1200 cm. Hence the area of the floor is

$$1500 \times 1200 = 1800000\ \text{cm}^z.$$

Area of each tile is $30 \times 20 = 600\ \text{cm}^2$. Hence the number of tiles required to cover the floor is: $\dfrac{1800000}{600} = 3000$

Example 3. Find the length of each side of a cube having the total surface area is 294 cm².

Solution: Given T.S.A of a cube as 294 cm², we have to find its length l. We know T.S.A of a cube is equal to $6l^2$. Thus $6l^2 = 294$ or $l^2 = \dfrac{294}{6} = 49$. Hence $l = 7$ cm.

Example 4. The total surface area of a cube is 600 cm². Find the lateral surface area of the cube.

Solution: Given T.S.A of a cube as 600 cm², we have to find its L.S.A. But we know T.S.A = $6l^2$, where l is the length of the cube. Hence $600 = 6l^2$ or $l^2 = 100$ units. Taking square-root, we get $l = 10$ cm. But we also know that L.S.A. = $4l^2$. Hence

$$\text{L.S.A.} = 4 \times 10 \times 10 = 400\ \text{cm}^2.$$

Example 5. Find the area of a metal sheet required to make a cube of length 2 m. Find the cost of metal sheet required at the rate of ₹ 8/ m² to make the cube.

Solution: We know that $l = 2$ m. We have to find T.S.A. of the cube. But

$$\text{T.S.A.} = 6l^2 = 6 \times 2 \times 2 = 24\ \text{m}^2.$$

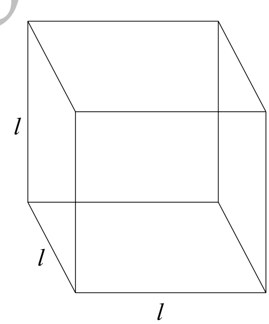
The cost of the metal sheet required is therefore $24 \times 8 = 192$ rupees.

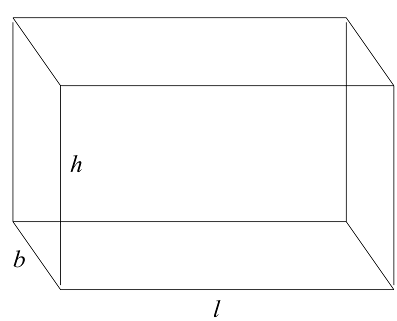
Exercise 16.1

1. Find the total surface area of the cuboid with l =4 m, b =3 m and h = 1.5 m.
2. Find the area of four walls of a room whose length 3.5 m, breadth 2.5 m and height 3 m.
3. The dimensions of a room are l =8 m, b =5 m, h =4 m. Find the cost of distempering its four walls at the rate of ₹ 40/ m^2.
4. A room is 4.8 m long, 3.6 m broad and 2 m high. Find the cost of laying tiles on its floor and its four walls at the rate of ₹ 100/ m^2.
5. A closed box is 40 cm long, 50 cm wide and 60 cm deep. Find the area of the foil needed for covering it.
6. The total surface area of a cube is 384 cm. Calculate the side of the cube.
7. The L.S.A of a cube is 64 cm. Calculate the side of the cube.
8. Find the cost of white washing the four walls of a cubical room of side 4 m at the rate of ₹ 20/ m^2.
9. A cubical box has edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.
   (i) Which box has smaller total surface area?
   (ii) If each edge of the cube is doubled, how many times will its T.S.A increase?

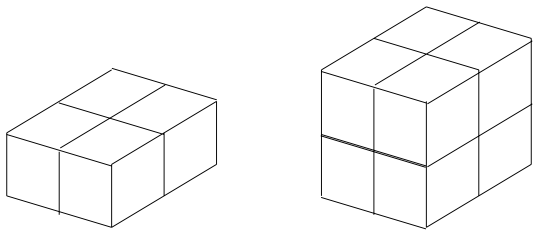
Volume of cubes and cuboids

Amount of space occupied by a three dimensional object is called its volume. Volume of a room is bigger than the volume of a brick or a shoe box. Remember we use square units to measure the area of a region or a surface. Similarly, we use cubic units to measure the volume of a solid, as solids are three dimensional objects.





The figures above are cube and cuboid which occupy space in three dimension and hence possess volume. We are interested in finding their volume.



Activity 4:

Take four cubes each of length 1 unit and arrange as shown in figure. Take another 4 cubes of same size and arrange them on the top of the cubes arranged earlier, as shown in the next figure.

Do you get another cube? You have used 8 cubes, i.e, the volume of the new cube is 8 cubic units. Here l = b = h = 2 units.

Volume of this cube = 2 units × 2 units × 2 units = 8 cubic units.

In general volume of a cube = side × side × side. Thus

$$V = l \times l \times l = l^3$$

cubic units. Cubic units are used to measure volume:

$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$;

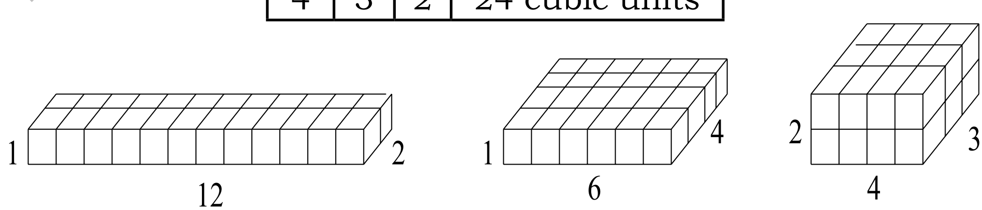
$1\ m^3 = 1\ m \times 1m \times 1m = 100\ cm \times 100\ cm \times 100\ cm = 10^6 cm^3$.

<u>Volume of a cuboid</u>

Activity 5:

Take 24 cubes of equal size. Arrange them to form a cuboid. You can arrange them in many ways to get a cuboid. Observe the following table:

l	*b*	*h*	*l* × *b* × *h*
12	2	1	24 cubic units
6	4	1	24 cubic units
4	3	2	24 cubic units



Since we have used 24 cubes in making these cuboids, volume of each cuboid is 24 cubic units. Thus we may arrive at the conclusion that the volume of each cuboid is equal to the product of its length, breadth and height: volume of the cuboid = $l \times b \times h$. Since $l \times b$ = area of the base, we can also write that

volume of the cuboid = area of the base × height.

Think it over!

There are 36 cubes having of side-length 1cm. How many cuboids of different dimensions can be prepared by using all of them.

Example 6 A match box measures 4 cm, 2.5 cm and 1.5 cm. What will be the volume of the packet containing 12 such boxes.

Solution: The volume of each box is $4 \times 2.5 \times 1.4 = 15$ cm^3. Volume of a packet containing 12 such boxes is $15 \times 12 = 180$ cm^3.

Example 7 How many 3 meter cubes can be cut from a cuboid measuring 18 m × 12 m × 9 m.

Solution: Volume of the cuboid = $18 \times 12 \times 9$ m^3. Volume of each cube to be cut is = $3 \times 3 \times 3$ m^3. Hence number of cubes that can be cut out from the cuboid is

$$\frac{18 \times 12 \times 9}{3 \times 3 \times 3} = 72.$$

Exercise 16.2

1. Find the total surface area and volume of a cube whose length is 12 cm.
2. Find the volume of a cube whose surface area is 486 cm^2.
3. A tank, which is cuboidal in shape, has volume 6.4 m^3. The length and breadth of the base are 2 m and 1.6 m respectively. Find the depth of the tank.
4. How many m^3 of soil has to be excavated from a rectangular well 28 m deep and whose base dimensions are 10 m and 8 m. Also find the cost of plastering its vertical walls at the rate of ₹ 15/m^2.

5. A solid cubical box of fine wood costs ₹ 256 at the rate ₹ 500/m^3. Find its volume and length of each side.

Glossary

Mensuration: finding the area of a planar objects or volume of a three dimensional objects.

Solids: objects which occupy space in three dimensions.

Cube: a cuboid having equal length, breadth and height.

Lateral surface: the surface of a cuboid which is neither a base nor the top surface.

Edges: the line along which surfaces meet.

Surface area: the area of the faces of a cuboid.

Volume: measure of the space occupied by a solid.

Points to remember

- A solid is three dimensional figure; it occupies space(in three dimensions).
- A solid has two quantities associated with it; surface area and volume.
- Area is measured in square units whereas volume is measured in cubic units.

Answers

Exercise 16.1

1. 45 m^2. **2.** 36 m^2. **3.** ₹ 4160. **4.** ₹ 5088. **5.** 14,800 cm^2. **6.** 8 cm. **7.** 4 cm. **8.** 1280. **9.** (i) cube. (ii) 4 times.

Exercise 16.2

1. (i) 864cm^2 , 1728cm^3 **2.** 729cm^2 **3.** 2m **4.** 2240 m3 and 15,120.

5. volume = 0.512 m3;

side-length= 80 cm.

ADDITIONAL PROBLEMS

9. Commercial Arithmetic

1. Four alternative options are given for each of the following statements. Select the correct option.

(a) Nine percent of ₹ 700 is:

A. ₹ 63 **B.** ₹ 630 **C.** ₹ 6.3 **D.** ₹ 0.63

(b) What percent of 50 metres is 12 metres?

A. 20% **B.** 60% **C.** 24% **D.** 32%

(c) The number whose 8% is 12 is:

A. 120 **B.** 150 **C.** 130 **D.** 140

(d) An article costing ₹ 600 is sold for ₹ 750. The gain percentage is:

A. 20 **B.** 25 **C.** 30 **D.** 35

(e) By selling note book for ₹ 22 a shopkeeper gains 10%. The cost price of the book is:

A. 18 **B.** 30 **C.** 20 **D.** 22

(f) The percentage of loss, when an article worth ₹ 10,000 was sold for ₹ 9,000 is:

A. 10 **B.** 20 **C.** 15 **D.** 25

(g) A radio marked 1000 is given away for ₹ 850. The discount is:

A. ₹ 50 **B.** 100 **C.** 150 **D.** 200

(h) A book marked ₹ 250 was sold for ₹ 200 after discount. The percentage of discount is:

A. 10 **B.** 30 **C.** 20 **D.** 25

(i) The marked price of an article is ₹ 200. If 15% of discount is allowed on it, its selling price is:

A. ₹ 185 **B.** ₹ 170 **C.** ₹ 215 **D.** ₹ 175

(j) One sells his bike through a broker by paying ₹ 200 brokerage. The rate of brokerage is 2%.The selling price of the bike is:

A. ₹ 12,000 **B.** ₹ 10,000 **C.** ₹ 14,000 **D.** ₹ 12,500

(k) The brokerage amount for a deal of ₹25,000 at 2% rate of commission is:

A. ₹ 500 **B.** ₹ 250 **C.** ₹ 5,000 **D.** ₹ 2,500

(l) If ₹ 1,600 is the commission at 8% for goods sold through a broker, the selling price of the goods is:

A. ₹ 18,000 **B.** ₹ 20,000 **C.** ₹ 22,000 **D.** ₹ 24,000

(m) The simple interest on ₹ 5,000 at 2% per month for 3 months is:

A. ₹ 100 **B.** ₹ 200 **C.** ₹ 300 **D.** ₹ 400

(n) The time in which simple interest on a certain sum be 0.15 times the principal at 10% per annum is:

A. 1.5 years **B.** 1 year **C.** 2 years **D.** 2.5 years

(o) The principal that yields a simple interest of ₹ 1,280 at 16% per annum for 8 months is:

A. ₹ 10,000 **B.** ₹ 12,000 **C.** ₹ 12,800 **D.** 14,000

2. A time interval of 3 minutes and 20 seconds is wrongly measured as 3 minutes and 25 seconds. What is the percentage error?

3. Hari reads 22% of the pages of a book on the first day, 53% on the second day and 15% on the third day. If the number of pages remaining to be read is 30, find the total number of pages in the book.

4. If 55% of students in a school are girls and the number of boys is 270, find the number of girls in the school.

5. By selling an article for ₹ 920, a shopkeeper gains15%. Find the cost price of the article.

6. Amit sells a watch at 20% gain. Had he sold it for ₹ 36 more, he would have gained 23%. Find the cost price of the watch.

7. On selling apples at 40 per Kg, a vendor incurs 10% loss. If he incurs a total loss of ₹ 120, calculate the quantity (in Kg) of apples he sold.

8. A dealer allows a discount of 20% and still gains 20%. Find the marked price of an article which costs the dealer ₹ 720.

9. A shopkeeper buys an article for ₹ 600 and marks 25% above the cost price. Find (i) the selling price if he sells the article at 10% discount; (ii) the percentage of discount if it is sold for ₹ 690.

10. A retailer purchases goods worth ₹ 33,600 and gets a discount of 14% from a whole seller. For paying in cash, the whole seller gives an additional discount of 1.5% on the amount to be paid after the first discount. What is the net amount the retailer has to pay?
11. An old car was disposed through a broker for ₹ 42,000. If the broker age is $2\frac{1}{2}$%, find the amount the owner gets.
12. A milk-man sells 20 litres of milk everyday at ₹ 22. He receives a commission of 4% for every litre. Find the total commission he receives in a month of 30 days.
13. A bike was sold for ₹ 48,000 and a commission of ₹ 8,640 was received by the dealer. Find the rate of commission.
14. In how many years will a sum of money becomes three times at the rate of interest 10% per annum?
15. In what time will the simple interest on a certain sum be 0.24 times the principal at 12% per annum?
16. Find the amount of ₹ 30,000 from 15th January, 2010 to 10th August, 2010 at 12% per annum.
17. A person purchases electronic items worth ₹ 2,50,000. The shopkeeper charges him a sales tax of 21% instead of 12%. The consumer does not realise that he has over paid. But after some time he finds that he has paid excess and asks the shopkeeper to return the excess money. The shopkeeper refuses and the consumer moves the consumer court. The court with due hearing orders the shopkeeper to pay the consumer the excess money paid by the way of sales tax, with an interest of 12% per annum. If the whole deliberation takes 8 months, what is the money that the consumer gets back?

Answers

1. (a) **A.** (b) **C.** (c) **B.** (d) **B.** (e) **C.** (f) **A.** (g) **C.** (h) **C.** (i) **B.** (j) **B.** (k) **A.** (l) **B.** (m) **C.** (n) **A.** (o) **B.**.

2. $2\,^1/_2$ **3.** 325. **4.** 330. **5.** ₹ 800. **6.** ₹ 1,200. **7.** 27 kg.

8. ₹ 1,080. **9.** (i) ₹ 675; (ii) 8%. **10** ₹ 28,462.56. **11.** ₹ 41,050.

12. ₹ 528. **13.** 18%. **14.** 20 years. **15.** 2 years. **16.** ₹ 32,041.65.

17. ₹ 24,300.

10. Exponents

1. Mark the correct option:

   (a) The value of $(3^m)^n$, for every pair of integers (m,n), is

   A. 3^{m+n} **B.** 3^{mn} **C.** 3^{mn} **D.** 3^m+3^n

   (b) If x,y, $2x+\frac{y}{2}$ are nonzero real numbers, then

   $$\left(2x+\frac{y}{2}\right)^{-1}\left\{(2x)^{-1}+\left(\frac{y}{2}\right)^{-1}\right\}$$

   equals

   A. 1 **B.** x · y **C.** x· y **D.** x· y

   (c) If $2^x - 2^{x-2} = 192$, the value of x is

   A. 5 **B.** 6 **C.** 7 **D.** 8

   (d) The number $\left(6^{\left(6^6\right)}\right)^{1/6}$ is equal to

   A. 6^6 **B.** 6^{6^6-1} **C.** $6^{(6^5)}$ **D.** $6^{(5^6)}$

   (e) The number of pairs positive integers (m,n)such that $m^n=25$ is

   A. 0 **B.** 1 **C.** 2 **D.** more than 2

2. The diameter of the Sun is 1.4×10^9 meters and that of the Earth is about 1.2768×10^7 meters. Find the approximate ratio of the diameter of the Sun to that of the Earth.

3. Find the value of each of the following expressions:

   (a) $(-0.75)^3+(0.3)^{-3}-\left(-\frac{3}{2}\right)^{-3}$;

   (b) $\dfrac{\left(8\times(4^2)^4\times 3^3\times 27^2\right)+\left(9\times 6^3\times 4^7\times(3^2)^3\right)}{\left(24\times(6^2)^4\times(2^4)^2\right)+\left(144\times(2^3)^4\times(9^2)^2\times 4^2\right)}$;

   (c) $\dfrac{(2^{19}\times 27^3)+(15\times 4^9\times 9^4)}{(6^9\times 2^{10})+12^{10}}$.

4. How many digits are there in the number $2^3\times 5^4\times 20^5$?

5. If $a^7 = 3$, find the value of $\dfrac{(a^{-2})^{-3} \times (a^3)^4 \times (a^{-17})^{-1}}{a^7}$.

6. If $2^m \times a^2 = 2^8$, where a,m are positive integers, find all possible values of $a + m$.

7. Suppose $3^k \times b^2 = 6^4$ for some positive integers k,b. Find all possible values of $k + b$.

8. Find the value of

$$\frac{(625)^{6.25} \times (25)^{2.60}}{(625)^{7.25} \times (5)^{1.20}}.$$

9. A person had some rupees which is a power of 5. He gave a part of it to his friend which is also a power of 5. He was left with ₹ 500. How much did money he have?

Answers

1. (i) **B.** (ii) **D.** (iii) **D.** (iv) **C.** (v) **B.**

2. 109.65. **3.** (a) $\dfrac{944}{27}$ (b) $\dfrac{1}{2}$.

4. 11. **5.** 81. **6.** $a + m = 8$. **7.** $k + 2 = 8$ or 14. **8.** 1. **9.** 625.

✡✡✡✡✡

11. Congruency of triangles

1. Fill in the blanks to make the statements true .

   (a) In right triangle the hypotenuse is the __________ side.

   (b) The sum of three altitudes of a triangle is __________ than its perimeter.

   (c) The sum of any two sides of a triangle is __________-than the third side.

   (d) If two angles of a triangle are unequal , then the smaller angle has the __________ side opposite to it.

   (e) Difference of any two sides of a triangle is __________-than the third side.

   (f) If two sides of a triangle are unequal, then the larger side has __________-angle opposite to it.

2. Justify the following statements with reasons:

   (a) The sum of three sides of a triangle is more than the sum of its altitudes.

   (b) The sum of any two sides of a triangle is greater than twice the median drawn to the third side.

   (c) Difference of any two sides of triangle is less than the third side.

3. Two triangles ABC and DBC have common base BC. Suppose $AB = DC$ and $\angle ABC = \angle BCD$. Prove that $AC = BD$.

4. Let AB and CD be two line segments such that AD and BC intersect at O. Suppose $AO = OC$ and $BO = OD$. Prove that $AB = CD$.

5. Let ABC be a triangle. Draw a triangle BDC externally on BC such that $AB = BD$ and $AC = CD$. Prove that $\Delta ABC \cong \Delta DBC$.

6. Let ABCD be a square and let points P on AB and Q on DC be such that $DP = AQ$. Prove that $BP = CQ$.

7. In a triangle ABC, $AB = AC$. Suppose P is point on AB and Q is a point on AC such that $AP = AQ$. = Prove that $\Delta APC \cong \Delta AQB$.

8. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

9. If the bisector of the vertical angle of a triangle bisects the base, show that the triangle is isosceles.

10. Suppose ABC is an isosceles triangle with $AB = AC$. Side BA is produced to D such that $BA = AD$. Prove that $\angle BCD$ is a right angle.

11. Let AB, CD be two line segments such that $AB \parallel CD$ and $AD \parallel BC$. Let E be the midpoint of BC and let DE extended meet AB in F. Prove that $AB = BF$.

Answers

1. (a) largest (b) less (c) larger (d) smaller (e) less (f) larger

8. 30°, 30°, 120°.

12. Constructions of triangles

1. Construct a triangle ABC in which AB = 5 cm, BC = 4.7 cm and AC = 4.3 cm.

2. Construct a triangle ABC in which AB = 5 cm, BC = 5 cm and AC = 4.3 cm.
3. Construct a triangle PQR in which PQ = 4 cm, QR = 4.5 cm and $\angle Q = 60^\circ$.
4. Construct a triangle PQR in which PQ = 4 cm, $\angle P = 60^\circ$ and $\angle Q = 60^\circ$
5. Construct a triangle ABC in which AB = 3.5 cm, AC = 4 cm and length of the perpendicular from A to BC is 3 cm.
6. Construct an isosceles triangle ABC in which base BC = 4.5 cm and altitude from A on BC is 3.8 cm.
7. Construct an isosceles triangle whose altitude is 5 cm and whose vertex angle is 70°.
8. Construct an isosceles triangle whose altitude is 5 cm and whose vertex angle is 80°.
9. Construct an equilateral triangle of height 3.5 cm.
10. Construct an equilateral triangle of height 4.3 cm.
11. Construct right angle triangle LMN in which $\angle M = 90^\circ$, MN = 4.5 cm and LN = 5.6 cm.
12. Construct right angle triangle PQR in which $\angle Q = 90^\circ$, QR = 4.5 cm and $\angle R = 50^\circ$.
13. Construct a triangle PQR, whose perimeter is13 cm and whose sides are in the ratio 2:3:4.
14. Construct a triangle PQR, whose perimeter is15 cm and whose sides are in the ratio 3:4:6.
15. Construct a triangle ABC, whose perimeter is 13.5 cm and whose base angles are 60° and 75°.
16. Construct a triangle ABC, whose perimeter is 12.5 cm and whose base angles are 50° and 80°.
17. Construct a triangle XYZ in which YZ = 4.5 cm, $\angle Y = 60^\circ$ and sum of other two sides is 7.5 cm.
18. Construct a triangle ABC whose perimeter is 9 cm and the angles are in the ratio 3:4:5.

19. Construct a triangle ABC whose perimeter is 12 cm and the angles are in the ratio 2:3:5.
20. Construct a triangle ABC in which BC = 4.5 cm, $\angle B = 35^\circ$ and difference between the other two sides is 2.8 cm.

13. Statistics

1. Four alternative options are given for each of the following statements. Select the correct option.

(a) The size or width of the Class interval (0 - 4) is :

A. 4 **B.** 5 **C.** 3 **D.** 0

(b) The midpoint of class interval (10 -19) is:

A. 10 **B.** 14 **C.** 15 **D.** 14.5

(c) The difference between highest and lowest score of a distribution gives:

A. class interval **B.** class width **C.** range **D.** class limit

(d) The number of times a particular observation (score) occurs in a data is called its:

A. frequency **B.** range **C.** class interval **D.** class limit

(e) In inclusive form, the actual upper limit and lower limit of class interval (0-4) are:

A. -0.5 & 3.5 **B.** 0.5 & 4.5 **C.** -1& 5 **D.** 1 & 5

(f) The height of a rectangle in a histogram represents:

A. class interval **B.** midpoint **C.** frequency density **D.** frequency

(g) In a histogram, the width of the rectangle indicates:

A. class interval **B.** midpoint **C.** frequency density **D.** frequency

(h) The mean of scores 10, 15, 12, 15, 15 is:

A. 15 **B.** 13 **C.** 13.4 **D.** 14.3

(i) Class interval grouping of data is done when:

A. the range of data is small **B.** the range of data is large

C. the class intervals are small **D.** class intervals are large

(j) The mean of 6, 4, 7, x and 10 is 8. The value of x is:

A. 10 **B.** 12 **C.** 14 **D.** 13

(k) If n =10 and Mean = 12, then $\sum$ fx is:

A. 120 **B.** 1200 **C.** 12 **D.** 13

(l) The mean of first three multiples of 5 is :

A. 5 **B.** 10 **C.** 15 **D.** 30

(m) The median of 37, 83, 70, 29, 32, 42, 40 is:

A. 29 **B.** 30 **C.** 40 **D.** 42

(n) In an inclusive class interval (10 -14), the lower real limit is:

A. 9.5 **B.** 10.5 **C.** 13.5 **D.** 14.5

(o) In an exclusive class interval (10 -20), the lower real limit is:

A. 20 **B.** 10 **C.** 10.5 **D.** 20.5

(p) The mode of 2, 3, 3, 5, 3, 5, 7, 3,5is:

A. 3 **B.** 5 **C.** 3 and 5 **D.** 3,5,7

(q) For given two values of x, 16, 18 the frequencies are respectively 12 and 20. Then the mode is:

A. 16 **B.** 18 **C.** 12 **D.** 20

(r) A collection of data having more than 3 modes is said to be:

A. uni-mode **B.** bi-mode **C.** tri-mode **D.** multi-mode

2. Prepare a frequency distribution table for the scores given:

42,22,55,18,50,10,33,29,17,29,29,27,34,15,40,42,40,41,35,27,
44,31,38,19,54,55,38,19,20,30,42,59,15,19,27,23,40,32,28,51.

Take the class intervals as 10-20, 20-30, 30-40, 40-50, 50-60. From the frequency distribution table answer the following questions:

(i) What does the frequency corresponding to the class interval 20-30 indicate?

(ii) In which class intervals are the scores 10, 20 and 30 included?

(iii) Find the range of the scores.

3. The following are the marks scored in a unit test (out of 25). Prepare a frequency distribution table, taking the class intervals as 0-4, 5-9, 10-14, 15-19, 20-24:

21,14,3,7,23,18,24,16,18,17,20,10,17,18,21,23,19,12,14,9,16,1
8,12,14,11.

From the table (i) find the mid-points of each class interval; (ii) find the class interval having maximum frequency; (iii) find the range of the scores.

4. Draw a histogram for the following frequency distribution.

Class - Interval	Frequency
5-15	2
15-25	8
25-35	14
35-45	14
45-55	12

5. Draw a histogram for the following frequency distribution.

Class - Interval	Frequency
0-10	4
11-20	18
21-30	12
31-40	6
41-50	20
51-60	10

6. The marks obtained by 12 students in a mathematics examination are given below.

48,78,93,90,66,54,83,58,60,75,89,84.

Find (i) the mean of the marks; (ii) the mean mark of the students if each student is given 4 grace marks.

7. If the mean of 8,12,21,42, x is 20, find the value of x.

8. Find the mean for the following distribution: 12,14,10,12,15,12,18,10,15,11,19,20,12,15,19,10,18,16,20,17.

13. Answers

1. (a) **B.** (b) **D.** (c) **B.** (d) **A.** (e) **A.** (f) **D.** (g) **A.** (h) **C.** (i) **B.** (j) **D.** (k) **A.** (l) **B.** (m) **C.** (n) **A.** (o) **B.** (p) **A.** (q) **D.** (r) **B.**

2.

Class Interval	Tally marks	Frequency
10 - 20	卌 , III	8
20 - 30	卌 , 卌	10
30 - 40	卌 , III	8
40 - 50	卌 , III	8
50 - 60	卌 , I	6
Total	40	40

(i) The largest number of scores lie in the class interval 20-30.

(ii) 10,20,30 are included respectively in the class intervals 10-20, 20-30, 30-40.

(iii) The range of the scores is 59-10=49.

3.

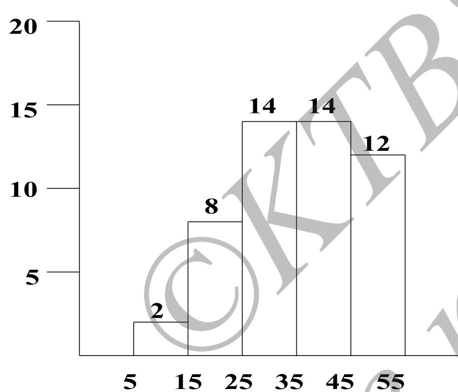
Class Interval	Tally marks	Frequency
0 - 4	I	1
5 - 9	II	2
10 - 14	~~IIII~~ , II	7
15 - 19	~~IIII~~ , IIII	9
20 - 24	~~IIII~~ , I	6
Total	25	25

(i) Mid-points of 0-4, 5-9, 1014, 15-19, 20-24 are respectively 2,7,12,17,22.

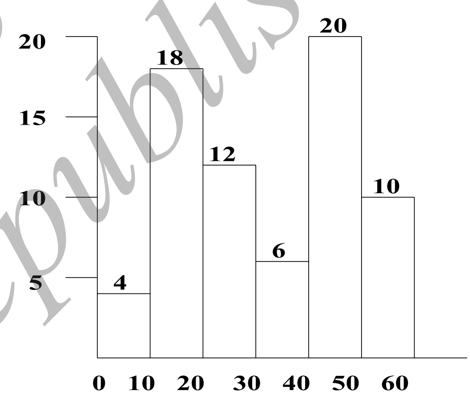
(ii) The class interval having maximum frequency is 15-19.

(iii) The range of the scores is 24-3=21.

4.



5.



6. (i) 73.17 (approximately); (i) 77.17 (approximately). **7.** 17. **8.** 14.75

<u>14. Introduction to graphs</u>

1. Choose the correct option:

   (a) The point (4,0) lie on the line ____________

   A. y− x = 0 **B.** y =0 **C.** x = 0 **D.** y+ x =0

   (b) The point (−5,4) lie in ____________

   A. the first quadrant **B.** the second quadrant

   C. the third quadrant **D.** the fourth quadrant

   (c) If a straight-line pass through (0,0) and (1,5), then its equation is ____________

   A. y = x **B.** y =5x **C.** 5y = x **D.** y = x +5

(d) If a point P has coordinates (3,4) in a coordinate system $X'OX \leftrightarrow Y'OY$, and if O has coordinates (4,3) in another system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ with $X'OX \parallel X'O_1X_1$, then the coordinates of P in the new system $X_1'O_1X_1 \leftrightarrow Y_1'O_1Y_1$ is ———

A. (3,4) **B.** (1,−1) **C.** (7,7) **D.** (−1,1)

(e) The coordinates of a point P in a system $X'OX \leftrightarrow Y'OY$ are (5,8). The coordinates of the same point in the system $Y'OY \leftrightarrow XOX'$ are ______________

A. (−8,5) **B.** (8,5) **C.** (8,−5) **D.** (−8,−5)

(f) The signs of the coordinates of a point in the third quadrant are ______________

A. (+,−) **B.** (−,+) **C.** (+,+) **D.** (−,−)

(g) If a person moves either 1 unit in the direction of positive x-axis or 1 unit in the direction of positive y-axis per step, then the number of steps he requires to reach (10,12) starting from the origin (0,0) is ______________

A. 10 **B.** 12 **C.** 22 **D.** 120

(h) The y-coordinate of the point of intersection of the line $y = 3x + 4$ with $x = 3$ is ______________

A. 4 **B.** 7 **C.** 10 **D.** 13

(i) The equation of the line which passes through (0,0) and (1,1) is ______________

A. $y = x$ **B.** $y = -x$ **C.** $y = 1$ **D.** $x = 1$

2. Find the quadrant in which the following points lie:

(i) (5,10); (ii) (−8,9); (iii) (−800,−3000); (iv) (8,−100).

3. Match the following:

(A) On the x-axis	(i) x coordinate is negative
(B) In the second quadrant	(ii) cuts the y-axis at (0,4)
(C) The line $y = 3x + 4$	(iii) coordinates of a point are of the form (a,0).

4. Fill in the blanks:

   (a) The y-coordinate of a point on the X-axis is ——————.

   b) The x-coordinate is called as ——————.

   (c) The X-axis and Y-axis intersect at ——————.

   (d) If a point $(x,y) \neq (0,0)$ is in the third quadrant, then $x + y$ has —————— sign.

   (e) If a point (x,y) lies above horizontal axis, then y is always ——————.

   (f) The point of intersection of $x = y$ and $x = -y$ is ——————.

   (g) The line $y = 4x + 5$ intersects y-axis at the point ——————.

5. True or false?

   (a) The equation of the X-axis is $x = 0$.

   (b) The line $x = 4$ is parallel to Y-axis.

   (c) The line $y = 8$ is perpendicular to X-axis.

   (d) The lines $x = y$ and $x = -y$ are perpendicular to each other.

   (e) The lines $x = 9$ and $y = 9$ are perpendicular to each other.

   (f) The graph of $y = x^2$ is a straight line.

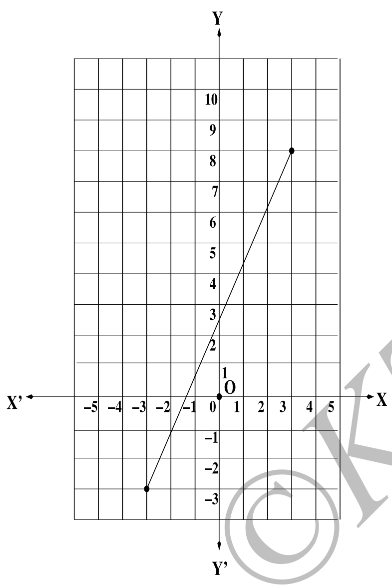
   (g) The line $y = 3x + 4$ does not intersect x-axis.

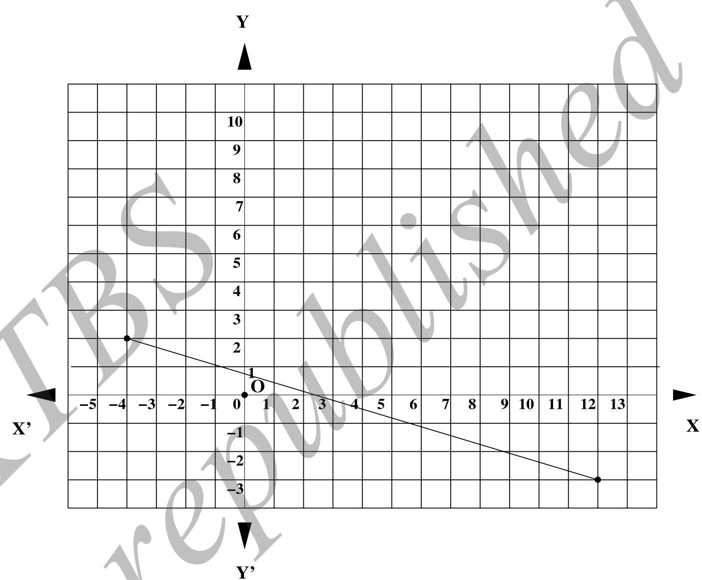
   (h) In a rectangular coordinate system, the coordinate axes are chosen such that they form a pair of perpendicular lines.

6. Determine the equation of the line which passes through the points $(0,-8)$ and $(7,0)$.

7. Determine the equation of the line in each of the following graph:

(i) (ii)





8. A point P has coordinates (7,10) in a coordinate system $X'OX \leftrightarrow Y'OY$. Suppose it has coordinates (10,7) in another coordinate system $X'_1O_1X_1 \leftrightarrow Y'_1O_1Y_1$ with $X'OX \parallel X'_1O_1X_1$. Find the coordinates of O_1 in the system $X'OX \leftrightarrow Y'OY$.

9. Sketch the region $\{(x, y): x \geq 0, y \geq 0, x + 2y \leq 4\}$ in a coordinate system set up by you.

10. Draw the graphs of lines $3y = 4x - 4$ and $2x = 3y + 4$ and determine the point at which these lines meet.

11. If $a \star b = ab + a + b$, draw the graph of $y = 3 \star x + 1 \star 2$.

Answers

1. (a) **B.** (b) **B.** (c) **C.** (d) **C.** (e) **C.** (f) **D.** (g) C. (h) **B.** (i) **A.**

2. (a) First (b) second (c) third (d) fourth.

3. (A) → (iii) (B) → (i) (C) → (ii).

4. (a) zero (b) abscissa (c) (0,0) (d) negative (e) positive (f) (0,0) (g) 0,5).

5. (a) false (b) true (c) false (d) true (e) true (f) false (g) false (h)true.

6. $7y = 8(x - 7)$ $7y = 8x$. **7.** (i) $6y = 11x + 15$ (ii) $16y = -5x + 12$.

8. $(-3,3)$.

15. Quadrilaterals

1. Complete the following:
   (a) A quadrilateral has ________________ sides.
   (b) A quadrilateral has ________________ diagonals.
   (c) A quadrilateral in which one pair of sides are parallel to each other is ________________.
   (d) In an isosceles trapezium, the base angles are ________ _________.
   (e) In a rhombus, the diagonals bisect each other in _________ ________ angles.
   (f) In a square, all the sides are ________________.
2. Let ABCD be a parallelogram. What special name will you give it:
   (a) if AB = BC?
   (b) if $\angle BAD = 90°$?
   (c) if AB = AD and $\angle BAD = 90°$?
3. A quadrilateral has three acute angles each measuring 70°. What is the measure of the fourth angle?
4. The difference between the two adjacent angles of a parallelogram is 20°. Find measures of all the angles of the parallelogram.
5. The angles of a quadrilateral are in the ratio 1:2:3:4. Find all the angles of the quadrilateral.

6. Let PQRS be a parallelogram with PQ = 10 cm and QR = 6 cm. Calculate the measures of the other two sides and the perimeter of PQRS.

7. The perimeter of a square is 60 cm. Find its side-length.

8. Let ABCD be a square and let AC = BD = 10 cm. Let AC and BD intersect in O. Find OC and OD.

9. Let PQRS be a rhombus, with PR = 15 cm and QS =8 cm. Find the area of the rhombus.

10. Let ABCD be a parallelogram and suppose the bisectors of $\angle A$ and $\angle B$ meet at P. Prove that $\angle APB = 90^\circ$.

11. Let ABCD be a square. Locate points P, Q, R, S on the sides AB, BC, CD, DA respectively such that AP = BQ = CR = DS. Prove that PQRS is a square.

12. Let ABCD be a rectangle and let P,Q,R,S be the mid-points of AB, BC, CD, DA respectively. Prove that PQRS is a rhombus.

13. Let ABCD be a quadrilateral in which the diagonals intersect at O perpendicularly. Prove that $AB + BC + CD + DA > AC + BD$.

14. Let ABCD be a quadrilateral with diagonals AC and BD. Prove the following statements (Compare these with the previous problem);

    (a) $AB + BC + CD > AD$; (b) $AB + BC + CD + DA > 2AC$;

    (c) $AB + BC + CD + DA > 2BD$; (d) $AB + BC + CD + DA > AC + BD$.

15 Let PQRS be a kite such that PQ >PS. Prove that $\angle PQR > \angle PSR$. (Hint: Join QS.)

16. Let ABCD beaquadrilateralin which AB is the smallest side and CD is the largest side. Prove that $\angle A > \angle C$ and $\angle B > \angle D$.(Hint: Join AC and BD.)

17. In a triangle ABC, let D be the mid-point of BC. Prove that $AB + AC > 2AD$. (What property of quadrilateralis needed here?)

18. Let ABCD be a quadrilateral and let P,Q,R,S be the mid-points of AB, BC, CD, DA respectively. Prove that PQRS is a parallelogram. (What extra result you need to prove this?)

19. Prove that the base angles of an isosceles trapezium are equal.

20. Suppose in a quadrilateral ABCD, AC = BD and AD = BC. Prove that ABCD is a trapezium.

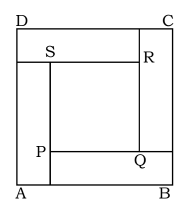
21. Prove logically the diagonals of a parallelogram bisect each other. Show conversely that a quadrilateral in which diagonals bisect each other is a parallelogram.

22. Prove logically that the diagonal of a rectangle are equal.

23. In a rhombus PQRS, $\angle SQR = 40^0$ and PQ = 3 cm. Find $\angle SPQ$, $\angle QSR$ and the perimeter of the rhombus.

24. Let *ABCD* be a rhombus and $\angle ABC = 124^0$. Calculate. $\angle A$, $\angle D$ and $\angle C$

25. Four congruent rectangles are place as shown in the figure. Area of the outer square is 4 times that of the inner square is 4 times that of the inner square. What is the ratio of length to breadth of the congruent rectangles?



26. Let ABCD be a quadrilateral in which $\angle A = \angle C$, and $\angle B = \angle D$. Prove that ABCD is a parallelogram.

27. In a quadrilateral ABCD, suppose AB = CD and AD = BC. Prove that ABCD is a parallelogram.

Answers

1. (a) four (b) two (c) a trapezium (d) equal (e) right (f) equal.

2. (a) rhombus (b) rectangle (c) square.

3. $150°$. **4.** $80°$, $100°$, $80°$, $100°$. **5.** $36°$, $72°$, $108°$, $144°$. **6.** RS = 10 cm, SP = 6 cm perimeter is 32 cm. **7.** 15 cm. **8.** OC = OD =5 cm. **9.** 120 cm^2.

16. Mensuration

1. Three metal cubes whose edges measure 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. Find (i) side-length (ii) total surface area of the new cube. What is the diifernece between the total surface area of the new cube and the sum of total surface areas of the original three cubes?

2. Two cubes, each of volume 512 cm^3 are joined end to end. Find the lateral and total surface areas of the resulting cuboid.

3. The length, breadth and height of a cuboid are in the ratio 6:5:3. If the total surface area is 504 cm^2, find its dimension. Also find the volume of the cuboid.

4. Find the area of four walls of a room having length, breadth and height as 8 m, 5 m and 3 m respectively. Find the cost of white washing the walls at the rate of Rs. $15/m^2$.

5. A room is 6 m long, 4 m broad and 3 m high. Find the cost of laying tiles on its floor and four walls at the cost of Rs. $80/m^2$.

6. The length, breadth and height of a cuboid are in the ratio 5:3:2. If its volume is 35.937 m^3, find its dimension. Also find the total surface area of the cuboid.

7. Suppose the perimeter of one face of a cube is 24 cm. What is its volume?

8. A wooden box has inner dimensions $l = 6$ m, $b = 8$ m and $h = 9$ m and it has uniform thickness of 10 cm. The lateral surface of the outer side has to be painted at the rate of Rs. $50/\ m^2$. What is the cost of painting?

9. Each edge of a cube is increased by 20%. What is the percentage increase in the volume of the cube?

10. Suppose the length of a cube is increased by 10% and its breadth is decreased by 10%. Will the volume of the new cuboid be the same as that of the cube? What about the total surface areas? If they change, what would be the percentage change in both the cases?

Answers

1. (i) 6cm (ii) 216cm^2 (iii) 84cm^2.

2. (i) L.S.A = 384cm^2 (ii) T.S.A = 64cm^2.

3. length = 12cm, breadth = 10cm, height = 6cm; volume = 720cm^3.

4. 78m^2 and ₹ 1,170.

5. ₹ 6,720.

6. 1.65m, 0.99m, 0.66m; T.S.A = 6.7918 m^2.

7. 216 cm^3.

8. ₹ 8,931.5

9. 1.728 %

10. Volume decreases by 1% and T.S.A decreases by 2%.

✡✡✡✡✡

OPTIONAL PROBLEMS

These problems are included to pose challenge to those students who are looking for it. These are neither for class room discussion nor for examination.

1. Find a proper fraction greater than $\frac{1}{3}$, given that the fraction does not change if the numerator is increased by a positive integer and the denominator is multiplied by the same positive integer.

2. Find all rational numbers p/q such that

$$\frac{p}{q} = \frac{p^2+30}{q^2+30}.$$

3. Show that the number of distinct remainders which occur when a perfect square is divided by an odd prime p is $\frac{(p+1)}{2}$.

4. Find the number of positive divisors of 2^2, 3^2, 4^2, 5^2, 10^2. Do you see that the number of divisors is odd? Prove the proposition that the number of positive divisors of a perfect square is always an odd number.

5. Find all odd natural numbers n for which there is a unique perfect square strictly between n^2 and $2n^2$.

6. A person was born in 19-th century. His age was x years in the year x^2. If he passed away in 1975, what was his age at the time of his demise?

7. There is a unique 4-digit number $n = \overline{abcd}$ such that n^2 also ends in abcd. Find this number.

14	**11**	**5**	**A**
	8		
12		**3**	
			B

8. In the 20 15. The numbers 1 to 20 are written on a black board in a row. Two players take turns and put either + sign or − sign between these numbers according to their choice, one at a time. After all the signs are put, the sum is evaluated. The first player wins if the number obtained is even and the second wins if the number is odd. Who wins?

9. Suppose a natural number n is such that $m<n<(m+1)^2$ for some natural number m. If $n-l=m^2$ and $n+k=(m+1)^2$, prove that $n-kl$ is a perfect square.

10. If x, y, z are integers such that $x^2+y=z^2$, prove that one of x, y is divisible by 3.

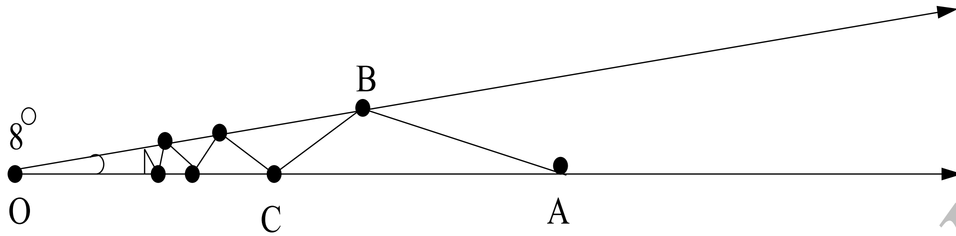
11. (**Hard**) Suppose x, y, z are three natural numbers such that they do not have any common factor and $x^2+y^2=z^2$. Prove that $x\,y\,z$ is divisible by 60.

12. For any x, suppose [x] denotes the largest integer not exceeding x; for example $[2.5]=2$ and $[-1.6]=-2$. Find all positive real numbers a such that $a^{[a]}=8$.

13. How many positive integers less than 1000 are 6 times the sum of their digits?

14. How many palindromes between 1000 and 10000 are there which are divisible by 7?

15. (**Hard**) Suppose there are two mirrors inclined at an angle 8°. A ray of light starts at the point A gets reflected in B and then gets reflected in C and so on n times in successive mirrors until it hits a mirror at right angle and then traces back the same path. What is the largest possible value of n? (The following figure is an example of 7 reflections.)



16. Show that the shortest distance from a point to a line is the perpendicular distance.

17. (**Hard**) Let P be any point inside a triangle ABC. Prove that

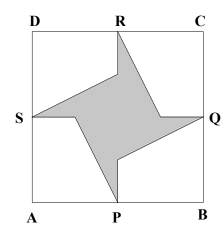
$$\frac{BC + CA + AB}{2} < PA + PB + PC < BC + CA + AB$$

18. Let D be the mid-point of the side BC in a triangle ABC. Prove that $AB + AC > 2AD$.

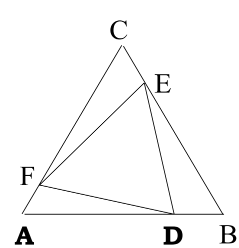
19. Suppose AD and BE are respectively the medians drawn from A and B on to the opposite sides in a triangle ABC. If $BC > CA$, prove that $BE > AD$.

20. Let ABCD be a quadrilateral in which AB is the smallest side and CD is the largest side. Prove that $\angle A > \angle C$ and $\angle B > \angle D$.

21. In the adjoining figure, a corner of a shaded star is at the mid-point of each side of the large square. What fraction of the large square is shaded?



22. In the adjoining figure, the outer equilateral triangle ABC has area 1



and the points D,E,F are such that DB = EC = FA and each equal to one-fourth the side of the triangle ABC. What is the area of DEF.

23. In the quadrilateral ABCD, AB =5, BC = 17, CD =5 and DA =9. It is known that BD is an integer. What is BD?

24. In a triangle ABC, AB =2AC. Let D,E be points respectively on the segments AB,BC such that $\angle BAE = \angle ACD$. Let F be the point of intersection of AE and CD. Suppose CFE is an equilateral triangle. What is $\angle ACB$?

25. Let ABC be a triangle and let AD be the bisector of $\angle A$ with D on BC. Prove that $\frac{AB}{AC} = \frac{BD}{DC}$.

26. Prove that the sum of the interior angles of pentagon is 540°. What do you expect for a hexagon? What about an octogon? Can you generalise this to a general n-gon? Can you prove your guess? What tools you need?

27. Suppose ABCD is a parallelogram. Equilateral triangles CBX and DCY are constructed externally respectvely on the sides DC and CB. Prove that AXY is an equilateral triangle.

28. One dimension of a cube is increased by 1, another is decreased by 1 and the third remains as it is. The volume of the resulting cuboid is 5 less than that of the original cube. What is the volume of the original cube?

29. A solid cube has side length 3 units.A 2×2 square hole is cut into the centre of each face. The edge of each square is parallel to the sides of the cube and each cut goes all the way through the cube. What is the volume of the resulting solid?

30. Let ABCD be a trapezium in which AB $\parallel$ CD and AD $\perp$ DC. Suppose AB >BC and draw CM $\perp$ AB. Suppose BC = 5 cm, MB = 3 cm and DC =8 cm. Find the perimeter of ABCD. What is the area of ABCD?

31. The diagonals of a rhombus are 24 cm and 10 cm respectively. Find its sides.

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