Government of Karnataka

MATHEMATICS

9

Ninth Standard

Part-II

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Karnataka Textbook Society (R.)
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FOREWORD

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the national Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognize that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

This aims imply considerable change is school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather then a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Professor P. Sinclair of IGNOU, New Delhi for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organizations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
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New Delhi
20 December 2005

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CONTENTS

CHAPTER 8

HERON'S FORMULA

8.1 Introduction

You have studied in earlier classes about figures of different shapes such as squares, rectangles, triangles and quadrilaterals. You have also calculated perimeters and the areas of some of these figures like rectangle, square etc. For instance, you can find the area and the perimeter of the floor of your classroom.

Let us take a walk around the floor along its sides once; the distance we walk is its perimeter. The size of the floor of the room is its area.

So, if your classroom is rectangular with length 10 m and width 8 m, its perimeter would be 2(10 m + 8 m) = 36 m and its area would be 10 m × 8 m, i.e., 80 m^2.

Unit of measurement for length or breadth is taken as metre (m) or centimetre (cm) etc.

Unit of measurement for area of any plane figure is taken as square metre (m^2) or square centimetre (cm^2) etc.

Suppose that you are sitting in a triangular garden. How would you find its area? From Chapter 9 and from your earlier classes, you know that:

$$\boxed{\textbf{Area of a triangle} = \frac{1}{2} \times \textbf{base} \times \textbf{height}} \qquad \text{(I)}$$

We see that when the triangle is **right angled**, we can directly apply the formula by using two sides containing the right angle as base and height. For example, suppose that the sides of a right triangle ABC are 5 cm, 12 cm and 13 cm; we take base as 12 cm and height as 5 cm (see Fig. 8.1). Then the

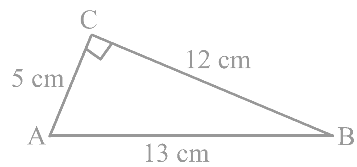


Fig. 8.1

area of Δ ABC is given by

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 5 \text{ cm}^2, \text{ i.e., } 30 \text{ cm}^2$$

Note that we could also take 5 cm as the base and 12 cm as height.

Now suppose we want to find the area of an **equilateral triangle** PQR with side 10cm (see Fig. 8.2). To find its area we need its height. Can you find the height of this triangle?

Let us recall how we find its height when we know its sides. This is possible in an equilateral triangle. Take the mid-point of QR as M and join it to P. We know that PMQ is a right triangle. Therefore, by using Pythagoras Theorem, we can find the length PM as shown below:

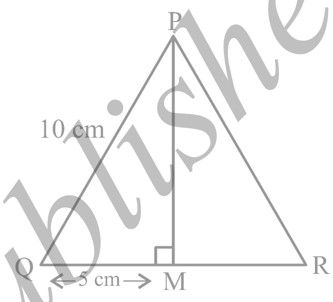


Fig. 8.2

$$PQ^2 = PM^2 + QM^2$$

i.e., $(10)^2 = PM^2 + (5)^2$, since QM = MR.

Therefore, we have $PM^2 = 75$

i.e., $PM = \sqrt{75}$ cm $= 5\sqrt{3}$ cm.

Then area of Δ PQR $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 10 \times 5\sqrt{3} \text{ cm}^2 = 25\sqrt{3} \text{ cm}^2$.

Let us see now whether we can calculate the area of an **isosceles triangle** also with the help of this formula. For example, we take a triangle XYZ with two equal sides XY and XZ as 5 cm each and unequal side YZ as 8 cm (see Fig. 8.3).

In this case also, we want to know the height of the triangle. So, from X we draw a perpendicular XP to side YZ. You can see that this perpendicular XP divides the base YZ of the triangle in two equal parts.

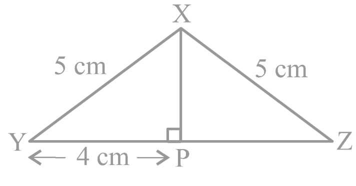


Fig. 8.3

Therefore, $YP = PZ = \frac{1}{2} YZ = 4$ cm

Then, by using Pythagoras theorem, we get

$$XP^2 = XY^2 - YP^2$$

$$= 5^2 - 4^2 = 25 - 16 = 9$$

So, $XP = 3$ cm

Now, area of Δ XYZ $= \frac{1}{2} \times \text{base YZ} \times \text{height XP}$

$$= \frac{1}{2} \times 8 \times 3 \text{ cm}^2 = 12 \text{ cm}^2.$$

Now suppose that we know the lengths of the sides of a scalene triangle and not the height. Can you still find its area? For instance, you have a triangular park whose sides are 40 m, 32 m, and 24 m. How will you calculate its area? Definitely if you want to apply the formula, you will have to calculate its height. But we do not have a clue to calculate the height. Try doing so. If you are not able to get it, then go to the next section.

8.2 Area of a Triangle — by Heron's Formula

Heron was born in about 10AD possibly in Alexandria in Egypt. He worked in applied mathematics. His works on mathematical and physical subjects are so numerous and varied that he is considered to be an encyclopedic writer in these fields. His geometrical works deal largely with problems on mensuration written in three books. Book I deals with the area of squares, rectangles, triangles, trapezoids (trapezia), various other specialised quadrilaterals, the regular polygons, circles, surfaces of cylinders, cones, spheres etc. In this book, Heron has derived the famous formula for the area of a triangle in terms of its three sides.

Heron (10 A.D. – 75 A.D.)

Fig. 8.4

The formula given by Heron about the area of a triangle, is also known as *Hero's formula*. It is stated as:

$$\boxed{\textbf{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}} \qquad \text{(II)}$$

where a, b and c are the sides of the triangle, and *s = semi-perimeter, i.e., half the perimeter of the triangle* = $\frac{a+b+c}{2}$,

This formula is helpful where it is not possible to find the height of the triangle easily. Let us apply it to calculate the area of the triangular park ABC, mentioned above (see Fig. 8.5).

Let us take $a = 40$ m, $b = 24$ m, $c = 32$ m,

so that we have $s = \frac{40+24+32}{2}$ m = 48 m.

$s - a = (48 - 40)$ m = 8 m,

$s - b = (48 - 24)$ m = 24 m,

$s - c = (48 - 32)$ m = 16 m.

Therefore, area of the park ABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48 \times 8 \times 24 \times 16}\ \text{m}^2 = 384\ \text{m}^2$

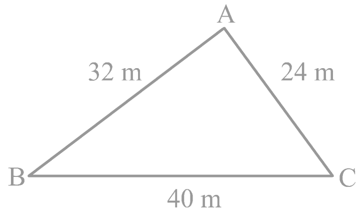


Fig. 8.5

We see that $32^2 + 24^2 = 1024 + 576 = 1600 = 40^2$. Therefore, the sides of the park make a right triangle. The largest side, i.e., BC which is 40 m will be the hypotenuse and the angle between the sides AB and AC will be 90°.

By using Formula I, we can check that the area of the park is $\frac{1}{2} \times 32 \times 24\ \text{m}^2 = 384\ \text{m}^2$.

We find that the area we have got is the same as we found by using Heron's formula.

Now using Heron's formula, you verify this fact by finding the areas of other triangles discussed earlier *viz*.,

(i) equilateral triangle with side 10 cm.

(ii) isosceles triangle with unequal side as 8 cm and each equal side as 5 cm.

You will see that

For (i), we have $s = \frac{10 + 10 + 10}{2}$ cm = 15 cm.

Area of triangle $= \sqrt{15(15-10)(15-10)(15-10)}\ \text{cm}^2$

$= \sqrt{15 \times 5 \times 5 \times 5}\ \text{cm}^2 = 25\sqrt{3}\ \text{cm}^2$

For (ii), we have $s = \frac{8 + 5 + 5}{2}$ cm = 9 cm.

Area of triangle $= \sqrt{9(9-8)(9-5)(9-5)}\ \text{cm}^2 = \sqrt{9 \times 1 \times 4 \times 4}\ \text{cm}^2 = 12\ \text{cm}^2$.

Let us now solve some more examples:

Example 1 : Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm (see Fig. 8.6).

Solution : Here we have perimeter of the triangle = 32 cm, a = 8 cm and b = 11 cm.

Third side c = 32 cm – (8 + 11) cm = 13 cm

So, $2s = 32$, i.e., s = 16 cm,

$s - a$ = (16 – 8) cm = 8 cm,

$s - b$ = (16 – 11) cm = 5 cm,

$s - c$ = (16 – 13) cm = 3 cm.

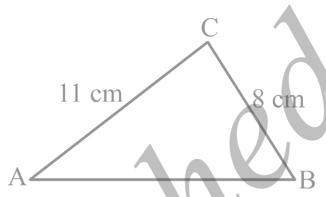


Fig. 8.6

Therefore, area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16 \times 8 \times 5 \times 3}\ \text{cm}^2 = 8\sqrt{30}\ \text{cm}^2$

Example 2 : A triangular park ABC has sides 120m, 80m and 50m (see Fig. 8.7). A gardener *Dhania* has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of Rs 20 per metre leaving a space 3m wide for a gate on one side.

Solution : For finding area of the park, we have

$2s$ = 50 m + 80 m + 120 m = 250 m.

i.e., s = 125 m

Now, $s - a$ = (125 – 120) m = 5 m,

$s - b$ = (125 – 80) m = 45 m,

$s - c$ = (125 – 50) m = 75 m.

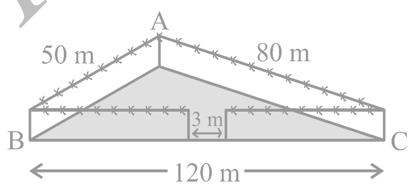


Fig. 8.7

Therefore, area of the park = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}\ \text{m}^2$

$= 375\sqrt{15}\ \text{m}^2$

Also, perimeter of the park = AB + BC + CA = 250 m

Therefore, length of the wire needed for fencing = 250 m – 3 m (to be left for gate)

= 247 m

And so the cost of fencing = Rs 20 × 247 = Rs 4940

Example 3 : The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Solution : Suppose that the sides, in metres, are $3x$, $5x$ and $7x$ (see Fig. 8.8).

Then, we know that $3x + 5x + 7x = 300$ (perimeter of the triangle)

Therefore, $15x = 300$, which gives $x = 20$.

So the sides of the triangle are 3×20 m, 5×20 m and 7×20 m

i.e., 60 m, 100 m and 140 m.

Can you now find the area [Using Heron's formula]?

Fig. 8.8

We have $s = \frac{60 + 100 + 140}{2}$ m = 150 m,

and area will be $\sqrt{150(150 - 60)(150 - 100)(150 - 140)}$ m^2

$$= \sqrt{150 \times 90 \times 50 \times 10} \text{ m}^2$$

$$= 1500\sqrt{3} \text{ m}^2$$

EXERCISE 8.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side '*a*'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?
2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 8.9). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?

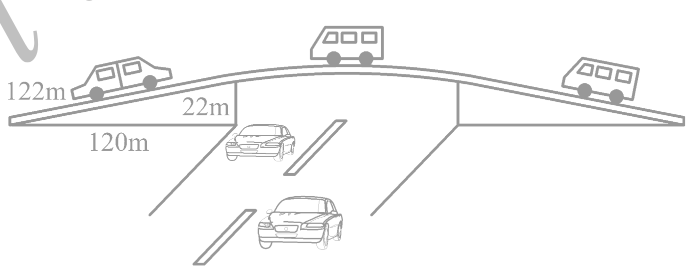


Fig. 8.9

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 8.10). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

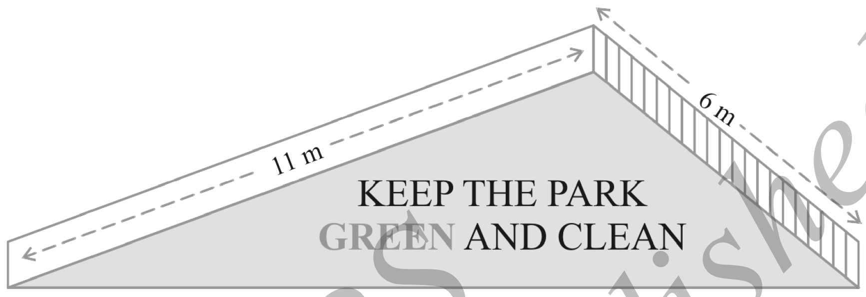


Fig. 8.10

4. Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.

6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

8.3 Application of Heron's Formula in Finding Areas of Quadrilaterals

Suppose that a farmer has a land to be cultivated and she employs some labourers for this purpose on the terms of wages calculated by area cultivated per square metre. How will she do this? Many a time, the fields are in the shape of quadrilaterals. We need to divide the quadrilateral in triangular parts and then use the formula for area of the triangle. Let us look at this problem:

Example 4 : Kamla has a triangular field with sides 240 m, 200 m, 360 m, where she grew wheat. In another triangular field with sides 240 m, 320 m, 400 m adjacent to the previous field, she wanted to grow potatoes and onions (see Fig. 8.11). She divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew patatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m^2)

Solution : Let ABC be the field where wheat is grown. Also let ACD be the field which has been divided in two parts by joining C to the mid-point E of AD. For the area of triangle ABC, we have

$$a = 200 \text{ m}, b = 240 \text{ m}, c = 360 \text{ m}$$

Therefore, $s = \dfrac{200 + 240 + 360}{2}$ m = 400 m.

So, area for growing wheat

$$= \sqrt{400(400-200)(400-240)(400-360)}\ \text{m}^2$$

$$= \sqrt{400 \times 200 \times 160 \times 40}\ \text{m}^2$$

$$= 16000\sqrt{2}\ \text{m}^2 = 1.6 \times \sqrt{2} \text{ hectares}$$

$$= 2.26 \text{ hectares (nearly)}$$

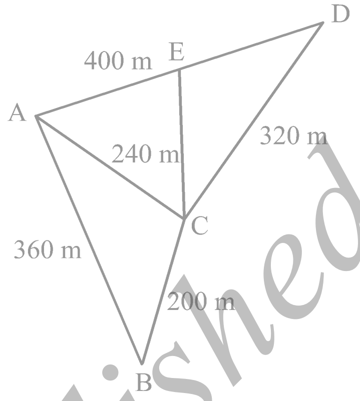


Fig. 8.11

Let us now calculate the area of triangle ACD.

Here, we have $s = \dfrac{240 + 320 + 400}{2}$ m = 480 m.

So, area of Δ ACD $= \sqrt{480(480-240)(480-320)(480-400)}\ \text{m}^2$

$$= \sqrt{480 \times 240 \times 160 \times 80}\ \text{m}^2 = 38400 \text{ m}^2 = 3.84 \text{ hectares}$$

We notice that the line segment joining the mid-point E of AD to C divides the triangle ACD in two parts equal in area. Can you give the reason for this? In fact, they have the bases AE and ED equal and, of course, they have the same height.

Therefore, area for growing potatoes = area for growing onions

$$= (3.84 \div 2) \text{ hectares} = 1.92 \text{ hectares.}$$

Example 5 : Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA (see Fig. 8.12). Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m and $\angle$ B = 90º, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes).

Solution : Since AB = 9 m and BC = 40 m, $\angle$ B = 90°, we have:

$$\text{AC} = \sqrt{9^2 + 40^2}\ \text{m}$$

$$= \sqrt{81 + 1600}\ \text{m}$$

$$= \sqrt{1681}\ \text{m} = 41\text{m}$$

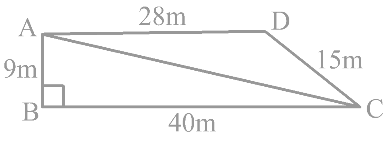


Fig. 8.12

Therefore, the first group has to clean the area of triangle ABC, which is right angled.

Area of Δ ABC $= \dfrac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times 40 \times 9 \text{ m}^2 = 180 \text{ m}^2$$

The second group has to clean the area of triangle ACD, which is scalene having sides 41 m, 15 m and 28 m.

Here, $$s = \frac{41 + 15 + 28}{2} \text{ m} = 42 \text{ m}$$

Therefore, area of Δ ACD = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42-41)(42-15)(42-28)} \text{ m}^2$$

$$= \sqrt{42 \times 1 \times 27 \times 14} \text{ m}^2 = 126 \text{ m}^2$$

So first group cleaned 180 m^2 which is (180 – 126) m^2, i.e., 54 m^2 more than the area cleaned by the second group.

Total area cleaned by all the students = (180 + 126) m^2 = 306 m^2.

Example 6 : Sanya has a piece of land which is in the shape of a rhombus (see Fig. 8.13). She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get for their crops?

Solution : Let ABCD be the field.

Perimeter = 400 m

So, each side = 400 m ÷ 4 = 100 m.

i.e. AB = AD = 100 m.

Let diagonal BD = 160 m.

Then semi-perimeter s of Δ ABD is given by

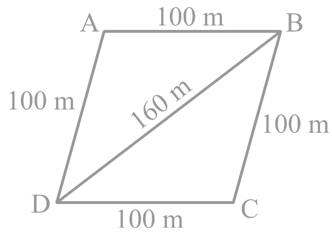


Fig. 8.13

$$s = \frac{100 + 100 + 160}{2} \text{ m} = 180 \text{ m}$$

Therefore, area of Δ ABD = $\sqrt{180(180-100)(180-100)(180-160)}$

$$= \sqrt{180 \times 80 \times 80 \times 20} \text{ m}^2 = 4800 \text{ m}^2$$

Therefore, each of them will get an area of 4800 m^2.

Alternative method : Draw CE $\perp$ BD (see Fig. 8.14).

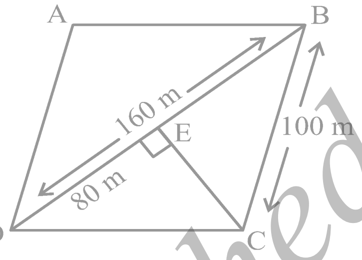


Fig. 8.14

As BD = 160 m, we have

DE = 160 m $\div$ 2 = 80 m

And, $DE^2 + CE^2 = DC^2$, which gives

$$CE = \sqrt{DC^2 - DE^2}$$

or, $$CE = \sqrt{100^2 - 80^2}\ \text{m} = 60\ \text{m}$$

Therefore, area of Δ BCD = $\frac{1}{2} \times 160 \times 60\ \text{m}^2 = 4800\ \text{m}^2$

EXERCISE 8.2

1. A park, in the shape of a quadrilateral ABCD, has $\angle$C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?
2. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.
3. Radha made a picture of an aeroplane with coloured paper as shown in Fig 8.15. Find the total area of the paper used.

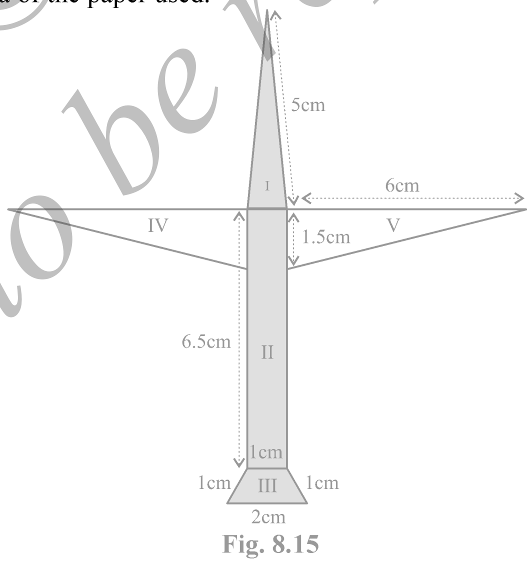


Fig. 8.15

4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig. 8.16), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. 8.17. How much paper of each shade has been used in it?

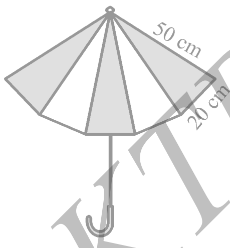


Fig. 8.16

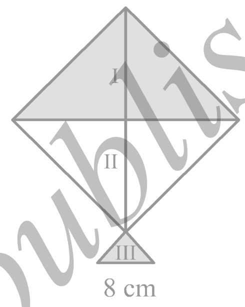


Fig. 8.17

8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig. 8.18). Find the cost of polishing the tiles at the rate of 50p per cm^2.

9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

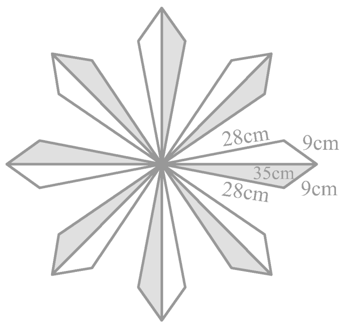


Fig. 8.18

8.4 Summary

In this chapter, you have studied the following points :

1. Area of a triangle with its sides as a, b and c is calculated by using Heron's formula, stated as

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $$s = \frac{a+b+c}{2}$$

2. Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using the Heron's formula.

CHAPTER 9

COORDINATE GEOMETRY

> What's the good of Mercator's North Poles and Equators, Tropics, Zones and Meridian Lines?' So the Bellman would cry; and crew would reply ' They are merely conventional signs!'

LEWIS CARROLL, *The Hunting of the Snark*

9.1 Introduction

You have already studied how to locate a point on a number line. You also know how to describe the position of a point on the line. There are many other situations, in which to find a point we are required to describe its position with reference to more than one line. For example, consider the following situations:

I. In Fig. 9.1, there is a main road running in the East-West direction and streets with numbering from West to East. Also, on each street, house numbers are marked. To look for a friend's house here, is it enough to know only one reference point? For instance, if we only know that she lives on Street 2, will we be able to find her house easily? Not as easily as when we know two pieces of information about it, namely, the number of the street on which it is situated, and the house number. If we want to reach the house which is situated in the 2nd street and has the number 5, first of all we would identify the 2nd street and then the house numbered 5 on it. In Fig. 9.1, H shows the location of the house. Similarly, P shows the location of the house corresponding to Street number 7 and House number 4.

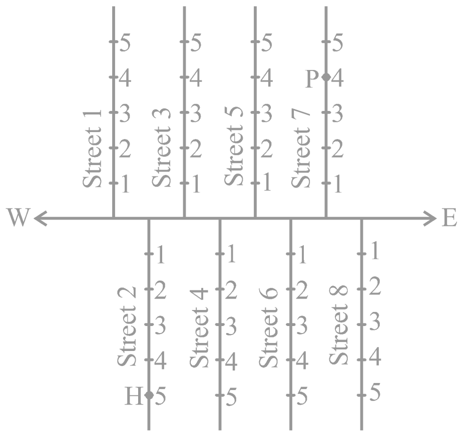
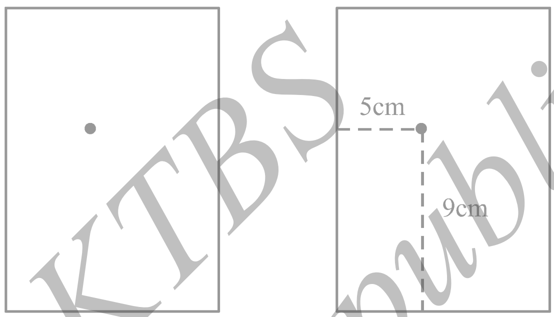


Fig. 9.1

II. Suppose you put a dot on a sheet of paper [Fig.9.2 (a)]. If we ask you to tell us the position of the dot on the paper, how will you do this? Perhaps you will try in some such manner: "The dot is in the upper half of the paper", or "It is near the left edge of the paper", or "It is very near the left hand upper corner of the sheet". Do any of these statements fix the position of the dot precisely? No! But, if you say " The dot is nearly 5 cm away from the left edge of the paper", it helps to give some idea but still does not fix the position of the dot. A little thought might enable you to say that the dot is also at a distance of 9 cm above the bottom line. We now know exactly where the dot is!



(a) (b)

Fig. 9.2

For this purpose, we fixed the position of the dot by specifying its distances from two fixed lines, the left edge of the paper and the bottom line of the paper [Fig.9.2 (b)]. In other words, we need **two** independent informations for finding the position of the dot.

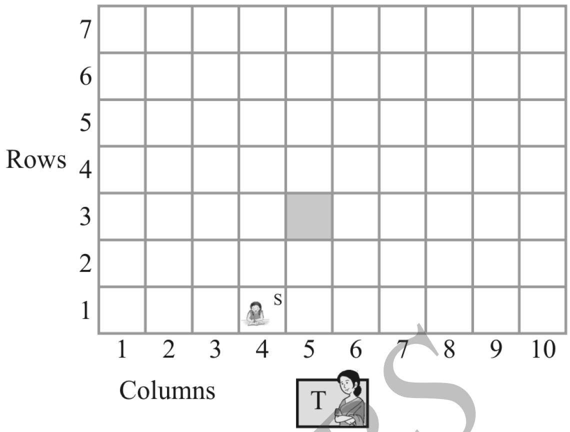
Now, perform the following classroom activity known as 'Seating Plan'.

Activity 1 (Seating Plan) : Draw a plan of the seating in your classroom, pushing all the desks together. Represent each desk by a square. In each square, write the name of the student occupying the desk, which the square represents. Position of each student in the classroom is described precisely by using two independent informations:

(i) the column in which she or he sits,

(ii) the row in which she or he sits.

If you are sitting on the desk lying in the 5^{th} column and 3^{rd} row (represented by the shaded square in Fig. 9.3), your position could be written as (5, 3), first writing the column number, and then the row number. Is this the same as (3, 5)? Write down the names and positions of other students in your class. For example, if Sonia is sitting in the 4^{th} column and 1^{st} row, write S(4,1). The teacher's desk is not part of your seating plan. We are treating the teacher just as an observer.

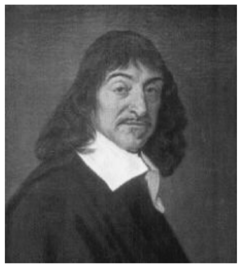


T shows teacher's desk
S shows Sonia's desk

Fig. 9.3

In the discussion above, you observe that position of any object lying in a plane can be represented with the help of two perpendicular lines. In case of 'dot', we require distance of the dot from bottom line as well as from left edge of the paper. In case of seating plan, we require the number of the column and that of the row. This simple idea has far reaching consequences, and has given rise to a very important branch of Mathematics known as *Coordinate Geometry*. In this chapter, we aim to introduce some basic concepts of coordinate geometry. You will study more about these in your higher classes. This study was initially developed by the French philosopher and mathematician *René Déscartes*.

René Déscartes, the great French mathematician of the seventeenth century, liked to lie in bed and think! One day, when resting in bed, he solved the problem of describing the position of a point in a plane. His method was a development of the older idea of latitude and longitude. In honour of Déscartes, the system used for describing the position of a point in a plane is also known as the *Cartesian system*.



René Déscartes (1596 -1650)
Fig. 9.4

EXERCISE 9.1

1. How will you describe the position of a table lamp on your study table to another person?
2. **(Street Plan) :** A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.

All the other streets of the city run parallel to these roads and are 200 m apart. There are 5 streets in each direction. Using 1cm = 200 m, draw a model of the city on your notebook. Represent the roads/streets by single lines.

There are many cross- streets in your model. A particular cross-street is made by two streets, one running in the North - South direction and another in the East - West direction. Each cross street is referred to in the following manner : If the 2^{nd} street running in the North - South direction and 5^{th} in the East - West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:

(i) how many cross - streets can be referred to as (4, 3).

(ii) how many cross - streets can be referred to as (3, 4).

9.2 Cartesian System

You have studied the *number line* in the chapter on 'Number System'. On the number line, distances from a fixed point are marked in equal units positively in one direction and negatively in the other. The point from which the distances are marked is called the *origin*. We use the number line to represent the numbers by marking points on a line at equal distances. If one unit distance represents the number '1', then 3 units distance represents the number '3', '0' being at the origin. The point in the positive direction at a distance r from the origin represents the number r. The point in the negative direction at a distance r from the origin represents the number $-r$. Locations of different numbers on the number line are shown in Fig. 9.5.

Fig. 9.5

Descartes invented the idea of placing two such lines perpendicular to each other on a plane, and locating points on the plane by referring them to these lines. The perpendicular lines may be in any direction such as in Fig.9.6. But, when we choose

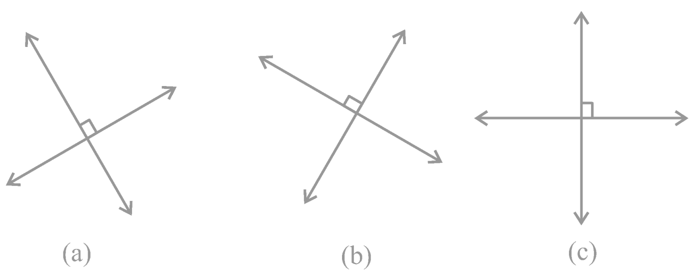
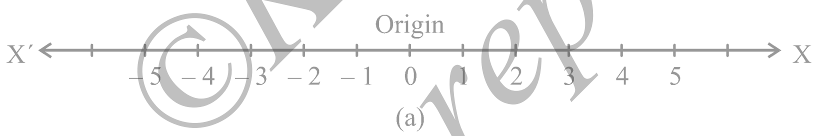


Fig. 9.6

these two lines to locate a point in a plane *in this chapter*, one line will be horizontal and the other will be vertical, as in Fig. 9.6(c).

These lines are actually obtained as follows : Take two number lines, calling them X′X and Y′Y. Place X′X horizontal [as in Fig. 9.7(a)] and write the numbers on it just as written on the number line. We do the same thing with Y′Y except that Y′Y is vertical, not horizontal [Fig. 9.7(b)].



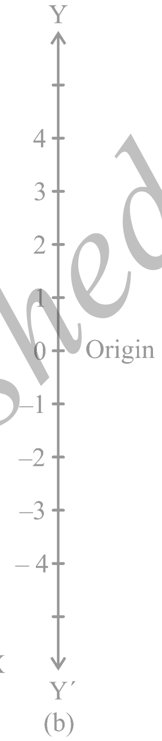


Fig. 9.7

Combine both the lines in such a way that the two lines cross each other at their zeroes, or origins (Fig. 9.8). The horizontal line X′X is called the x - axis and the vertical line Y′Y is called the y - axis. The point where X′X and Y′Y cross is called the **origin**, and is denoted by O. Since the positive numbers lie on the directions OX and OY, OX and OY are called the **positive directions** of the x - axis and the y - axis, respectively. Similarly, OX′ and OY′ are called the **negative directions** of the x - axis and the y - axis, respectively.

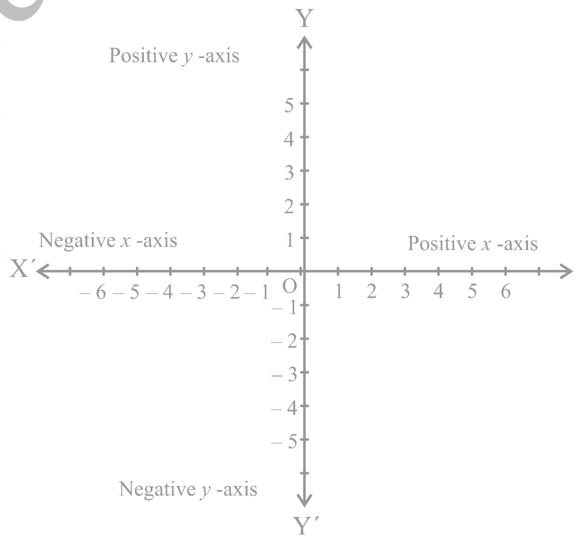


Fig. 9.8

You observe that the axes (plural of the word 'axis') divide the plane into four parts. These four parts are called the *quadrants* (one fourth part), numbered I, II, III and IV anticlockwise from OX (see Fig.9.9). So, the plane consists of the axes and these quadrants. We call the plane, the *Cartesian plane,* or the *coordinate plane,* or the *xy-plane*. The axes are called the *coordinate axes*.

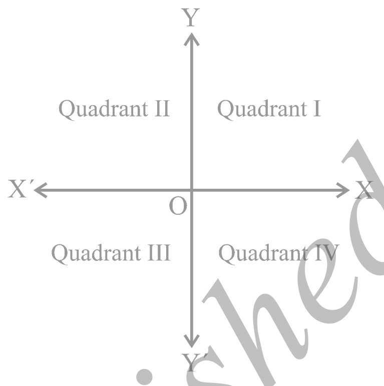


Fig. 9.9

Now, let us see why this system is so basic to mathematics, and how it is useful. Consider the following diagram where the axes are drawn on graph paper. Let us see the distances of the points P and Q from the axes. For this, we draw perpendiculars PM on the x - axis and PN on the y - axis. Similarly, we draw perpendiculars QR and QS as shown in Fig. 9.10.

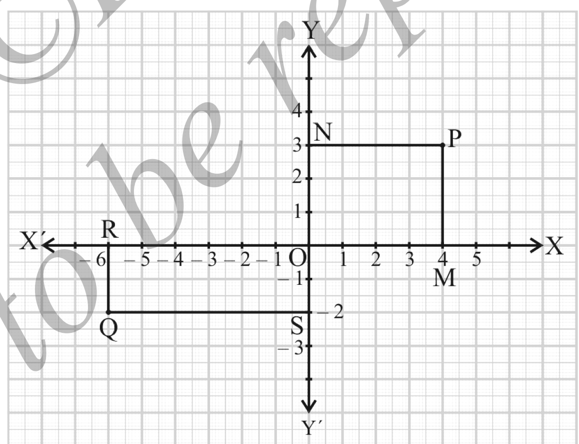


Fig.9.10

You find that

(i) The perpendicular distance of the point P from the y - axis measured along the positive direction of the x - axis is PN = OM = 4 units.

(ii) The perpendicular distance of the point P from the x - axis measured along the positive direction of the y - axis is PM = ON = 3 units.

(iii) The perpendicular distance of the point Q from the y - axis measured along the negative direction of the x - axis is OR = SQ = 6 units.

(iv) The perpendicular distance of the point Q from the x - axis measured along the negative direction of the y - axis is OS = RQ = 2 units.

Now, using these distances, how can we describe the points so that there is no confusion?

We write the coordinates of a point, using the following conventions:

(i) The x - *coordinate* of a point is its perpendicular distance from the y - axis measured along the x -axis (positive along the positive direction of the x - axis and negative along the negative direction of the x - axis). For the point P, it is + 4 and for Q, it is – 6. The x - coordinate is also called the *abscissa*.

(ii) The y - *coordinate* of a point is its perpendicular distance from the x - axis measured along the y - axis (positive along the positive direction of the y - axis and negative along the negative direction of the y - axis). For the point P, it is + 3 and for Q, it is –2. The y - coordinate is also called the *ordinate*.

(iii) In stating the coordinates of a point in the coordinate plane, the x - coordinate comes first, and then the y - coordinate. We place the coordinates in brackets.

Hence, the coordinates of P are (4, 3) and the coordinates of Q are (– 6, – 2).

Note that the coordinates describe a point in the plane *uniquely.* (3, 4) is not the same as (4, 3).

Example 1 : See Fig. 9.11 and complete the following statements:

(i) The abscissa and the ordinate of the point B are _ _ _ and _ _ _, respectively. Hence, the coordinates of B are (_ _, _ _).

(ii) The x-coordinate and the y-coordinate of the point M are _ _ _ and _ _ _, respectively. Hence, the coordinates of M are (_ _, _ _).

(iii) The x-coordinate and the y-coordinate of the point L are _ _ _ and _ _ _, respectively. Hence, the coordinates of L are (_ _, _ _).

(iv) The x-coordinate and the y-coordinate of the point S are _ _ _ and _ _ _, respectively. Hence, the coordinates of S are (_ _, _ _).

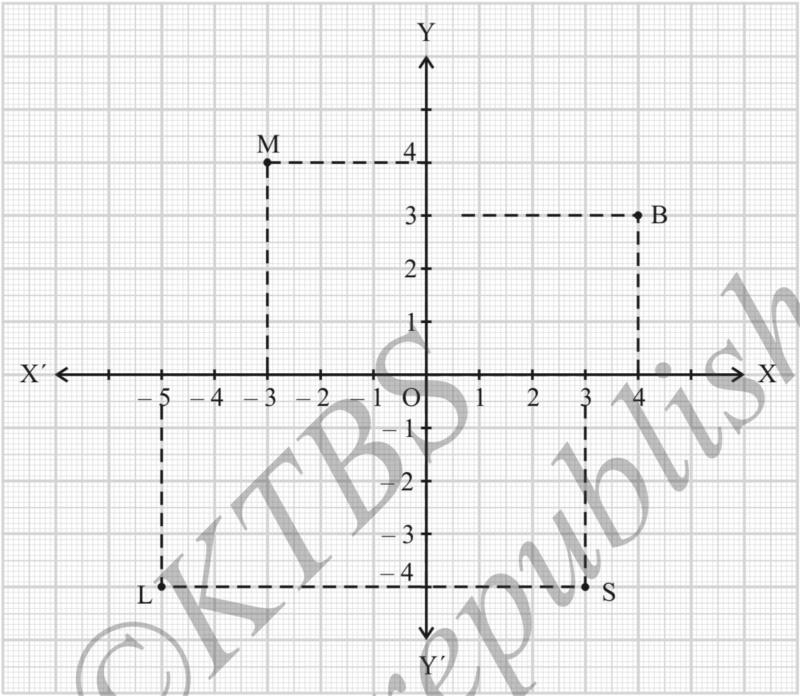


Fig. 9.11

Solution : (i) Since the distance of the point B from the y - axis is 4 units, the x - coordinate or abscissa of the point B is 4. The distance of the point B from the x - axis is 3 units; therefore, the y - coordinate, i.e., the ordinate, of the point B is 3. Hence, the coordinates of the point B are (4, 3).

As in (i) above :

(ii) The x - coordinate and the y - coordinate of the point M are –3 and 4, respectively. Hence, the coordinates of the point M are (–3, 4).

(iii) The x - coordinate and the y - coordinate of the point L are –5 and – 4, respectively. Hence, the coordinates of the point L are (–5, – 4).

(iv) The x - coordinate and the y- coordinate of the point S are 3 and – 4, respectively. Hence, the coordinates of the point S are (3, – 4).

Example 2 : Write the coordinates of the points marked on the axes in Fig. 9.12.

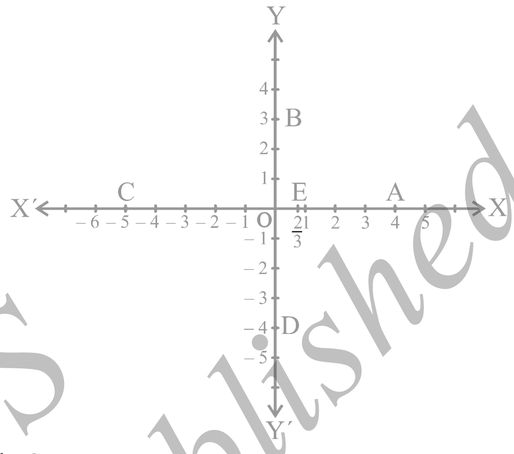


Fig. 9.12

Solution : You can see that :

(i) The point A is at a distance of + 4 units from the y - axis and at a distance zero from the x - axis. Therefore, the x - coordinate of A is 4 and the y - coordinate is 0. Hence, the coordinates of A are (4, 0).

(ii) The coordinates of B are (0, 3). Why?

(iii) The coordinates of C are (– 5, 0). Why?

(iv) The coordinates of D are (0, – 4). Why?

(v) The coordinates of E are $\left(\frac{2}{3}, 0\right)$. Why?

Since every point on the x - axis has no distance (zero distance) from the x - axis, therefore, the y - coordinate of every point lying on the x - axis is always zero. Thus, the coordinates of any point on the x - axis are of the form (x, 0), where x is the distance of the point from the y - axis. Similarly, the coordinates of any point on the y - axis are of the form (0, y), where y is the distance of the point from the x - axis. Why?

What are the coordinates of the ***origin*** **O**? *It has zero distance from both the axes so that its abscissa and ordinate are both zero. Therefore, the coordinates of the origin are* **(0, 0)**.

In the examples above, you may have observed the following relationship between the signs of the coordinates of a point and the quadrant of a point in which it lies.

(i) If a point is in the 1st quadrant, then the point will be in the form (+, +), since the 1st quadrant is enclosed by the positive x - axis and the positive y - axis.

(ii) If a point is in the 2nd quadrant, then the point will be in the form (–, +), since the 2nd quadrant is enclosed by the negative x - axis and the positive y - axis.

(iii) If a point is in the 3rd quadrant, then the point will be in the form (–, –), since the 3rd quadrant is enclosed by the negative x - axis and the negative y - axis.

(iv) If a point is in the 4th quadrant, then the point will be in the form (+, –), since the 4th quadrant is enclosed by the positive x - axis and the negative y - axis (see Fig. 9.13).

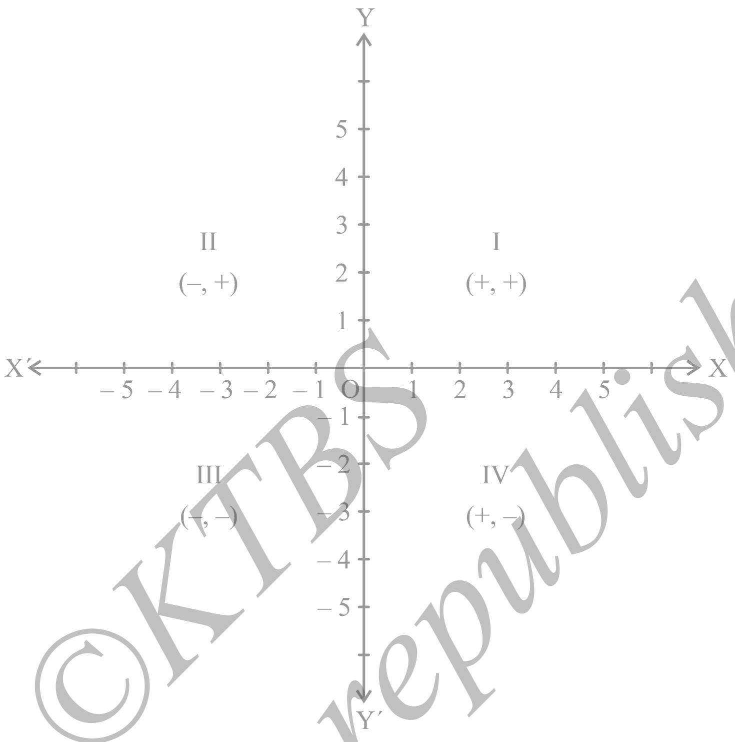


Fig. 9.13

Remark : The system we have discussed above for describing a point in a plane is only a convention, which is accepted all over the world. The system could also have been, for example, the ordinate first, and the abscissa second. However, the whole world sticks to the system we have described to avoid any confusion.

EXERCISE 9.2

1. Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

(ii) What is the name of each part of the plane formed by these two lines?

(iii) Write the name of the point where these two lines intersect.

2. See Fig.9.14, and write the following:

(i) The coordinates of B.

(ii) The coordinates of C.

(iii) The point identified by the coordinates (–3, –5).

(iv) The point identified by the coordinates (2, – 4).

(v) The abscissa of the point D.

(vi) The ordinate of the point H.

(vii) The coordinates of the point L.

(viii) The coordinates of the point M.

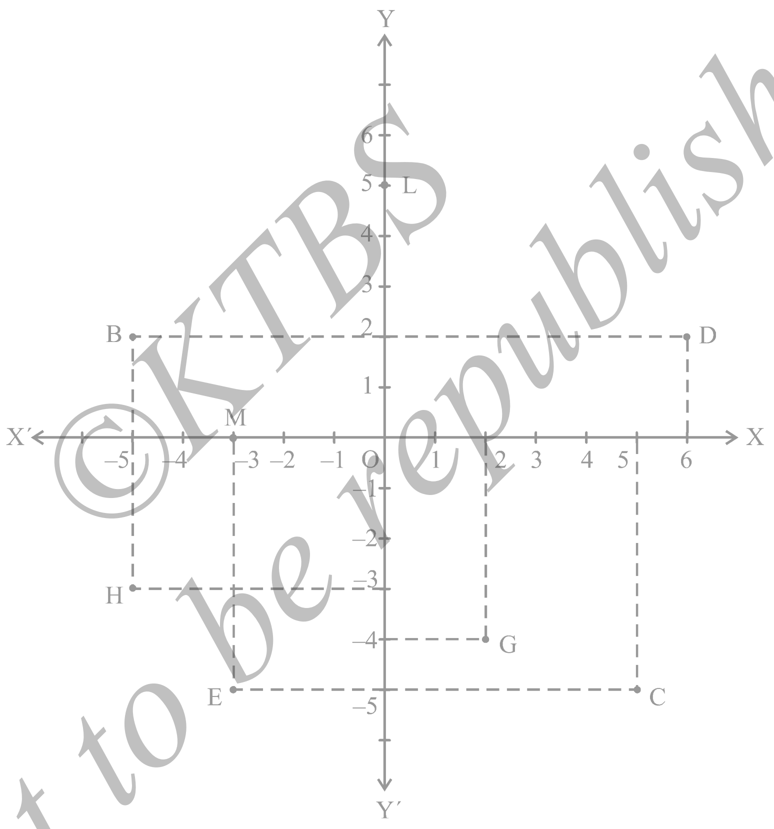


Fig. 9.14

9.3 Plotting a Point in the Plane if its Coordinates are Given

Uptil now we have drawn the points for you, and asked you to give their coordinates. Now we will show you how we place these points in the plane if we know its coordinates. We call this process "plotting the point".

Let the coordinates of a point be (3, 5). We want to plot this point in the coordinate plane. We draw the coordinate axes, and choose our units such that one centimetre represents one unit on both the axes. The coordinates of the point (3, 5) tell us that the distance of this point from the y - axis along the positive x - axis is 3 units and the distance of the point from the x - axis along the positive y - axis is 5 units. Starting from the origin O, we count 3 units on the positive x - axis and mark the corresponding point as A. Now, starting from A, we move in the positive direction of the y - axis and count 5 units and mark the corresponding point as P (see Fig.9.15). You see that the distance of P from the y - axis is 3 units and from the x - axis is 5 units. Hence, P is the position of the point. Note that P lies in the 1st quadrant, since both the coordinates of P are positive. Similarly, you can plot the point Q (5, – 4) in the coordinate plane. The distance of Q from the x - axis is 4 units along the negative y - axis, so that its y - coordinate is – 4 (see Fig.9.15). The point Q lies in the 4th quadrant. Why?

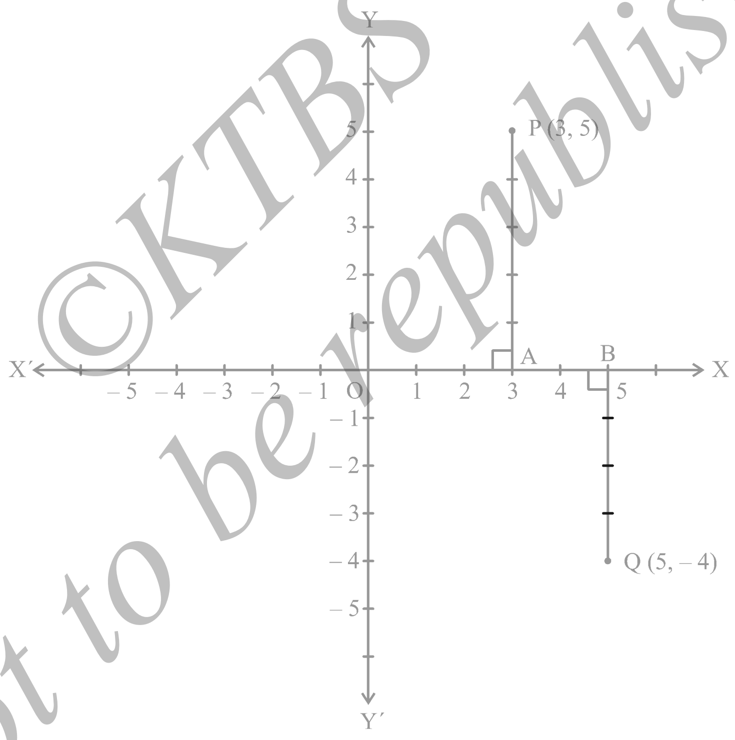


Fig. 9.15

Example 3 : Locate the points (5, 0), (0, 5), (2, 5), (5, 2), (–3, 5), (–3, –5), (5, –3) and (6, 1) in the Cartesian plane.

Solution : Taking 1cm = 1unit, we draw the x - axis and the y - axis. The positions of the points are shown by dots in Fig.9.16.

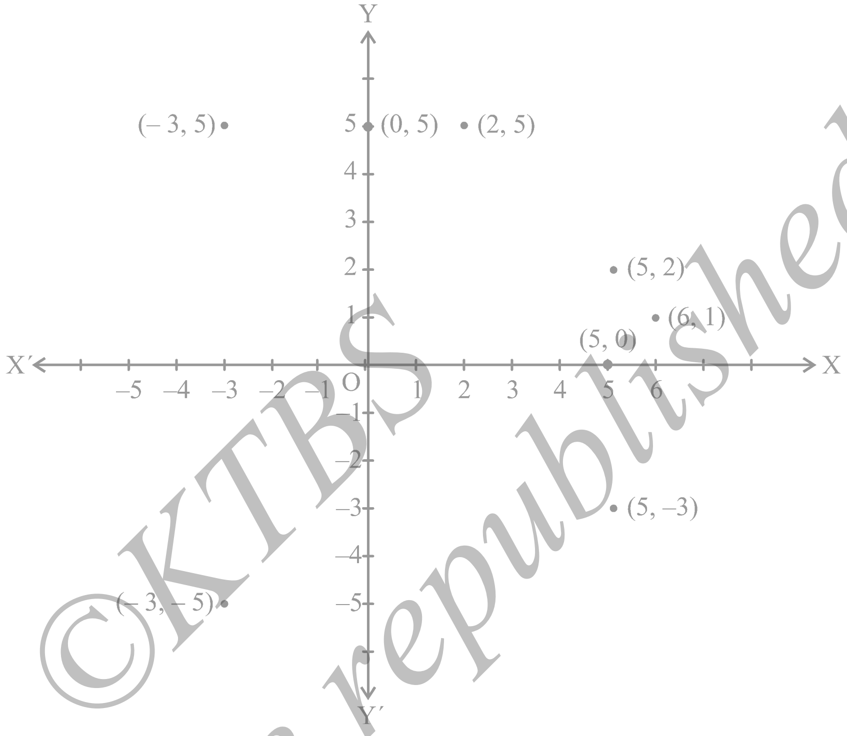


Fig. 9.16

Note : In the example above, you see that (5, 0) and (0, 5) are not at the same position. Similarly, (5, 2) and (2, 5) are at different positions. Also, (–3, 5) and (5, –3) are at different positions. By taking several such examples, you will find that, if $x \neq y$, **then the position of (x, y) in the Cartesian plane is different from the position of (y, x)**. So, if we interchange the coordinates x and y, the position of (y, x) will differ from the position of (x, y). This means that the order of x and y is important in (x, y). Therefore, (x, y) is called an ordered pair. The ordered pair (x, y) $\neq$ ordered pair (y, x), if $x \neq y$. Also (x, y) = (y, x), if $x = y$.

Example 4 : Plot the following ordered pairs (x, y) of numbers as points in the Cartesian plane. Use the scale 1cm = 1 unit on the axes.

x	– 3	0	– 1	4	2
y	7	–3.5	– 3	4	– 3

Solution : The pairs of numbers given in the table can be represented by the points $(-3, 7)$, $(0, -3.5)$, $(-1, -3)$, $(4, 4)$ and $(2, -3)$. The locations of the points are shown by dots in Fig.9.17.

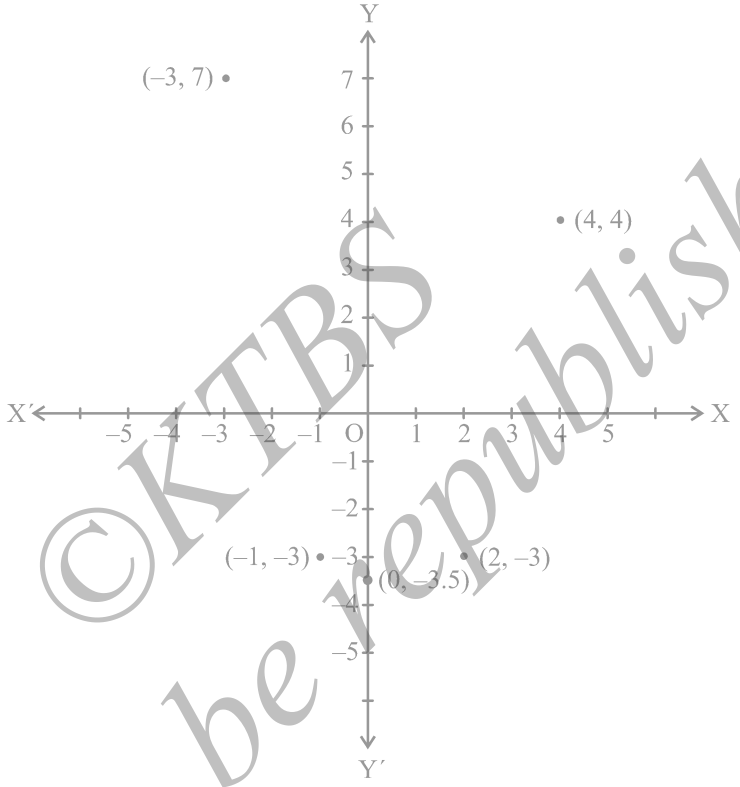


Fig. 9.17

Activity 2 : *A game for two persons* (Requirements: two counters or coins, graph paper, two dice of different colours, say red and green):

Place each counter at (0, 0). Each player throws two dice simultaneously. When the first player does so, suppose the red die shows 3 and the green one shows 1. So, she moves her counter to (3, 1). Similarly, if the second player throws 2 on the red and 4 on the green, she moves her counter to (2, 4). On the second throw, if the first player throws 1 on the red and 4 on the green, she moves her counter from (3, 1) to (3 + 1, 1 + 4), that is, adding 1 to the x - coordinate and 4 to the y - coordinate of (3, 1).

The purpose of the game is to arrive first at (10, 10) without overshooting, i.e., neither the abscissa nor the ordinate can be greater than 10. Also, a counter should not coincide with the position held by another counter. For example, if the first player's counter moves on to a point already occupied by the counter of the second player, then the second player's counter goes to (0, 0). If a move is not possible without overshooting, the player misses that turn. You can extend this game to play with more friends.

Remark : Plotting of points in the Cartesian plane can be compared to some extent with drawing of graphs in different situations such as Time-Distance Graph, Side-Perimeter Graph, etc which you have come across in earlier classes. In such situations, we may call the axes, t-axis, d-axis, s-axis or p-axis, etc. in place of the x and y axes.

EXERCISE 9.3

1. In which quadrant or on which axis do each of the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5) lie? Verify your answer by locating them on the Cartesian plane.
2. Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

x	– 2	–1	0	1	3
y	8	7	– 1.25	3	– 1

9.4 Summary

In this chapter, you have studied the following points :

1. To locate the position of an object or a point in a plane, we require two perpendicular lines. One of them is horizontal, and the other is vertical.
2. The plane is called the Cartesian, or coordinate plane and the lines are called the coordinate axes.
3. The horizontal line is called the x -axis, and the vertical line is called the y - axis.
4. The coordinate axes divide the plane into four parts called quadrants.
5. The point of intersection of the axes is called the origin.
6. The distance of a point from the y - axis is called its x-coordinate, or abscissa, and the distance of the point from the x-axis is called its y-coordinate, or ordinate.
7. If the abscissa of a point is x and the ordinate is y, then (x, y) are called the coordinates of the point.
8. The coordinates of a point on the x-axis are of the form $(x, 0)$ and that of the point on the y-axis are $(0, y)$.
9. The coordinates of the origin are (0, 0).
10. The coordinates of a point are of the form (+ , +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant, where + denotes a positive real number and – denotes a negative real number.
11. If $x \neq y$, then $(x, y) \neq (y, x)$, and $(x, y) = (y, x)$, if $x = y$.

CHAPTER 10

LINEAR EQUATIONS IN TWO VARIABLES

The principal use of the Analytic Art is to bring Mathematical Problems to Equations and to exhibit those Equations in the most simple terms that can be.

—Edmund Halley

10.1 Introduction

In earlier classes, you have studied linear equations in one variable. Can you write down a linear equation in one variable? You may say that $x + 1 = 0$, $x + \sqrt{2} = 0$ and $\sqrt{2}\,y + \sqrt{3} = 0$ are examples of linear equations in one variable. You also know that such equations have a unique (i.e., one and only one) solution. You may also remember how to represent the solution on a number line. In this chapter, the knowledge of linear equations in one variable shall be recalled and extended to that of two variables. You will be considering questions like: Does a linear equation in two variables have a solution? If yes, is it unique? What does the solution look like on the Cartesian plane? You shall also use the concepts you studied in Chapter 3 to answer these questions.

10.2 Linear Equations

Let us first recall what you have studied so far. Consider the following equation:

$$2x + 5 = 0$$

Its solution, i.e., the root of the equation, is $-\frac{5}{2}$. This can be represented on the number line as shown below:

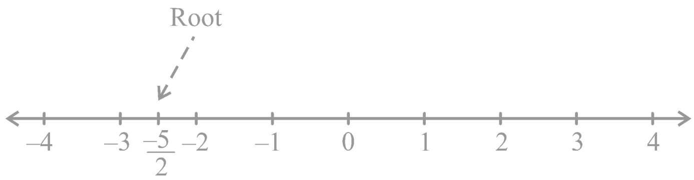


Fig. 10.1

While solving an equation, you must always keep the following points in mind:

The solution of a linear equation is not affected when:

(i) the same number is added to (or subtracted from) both the sides of the equation.

(ii) you multiply or divide both the sides of the equation by the same non-zero number.

Let us now consider the following situation:

In a One-day International Cricket match between India and Sri Lanka played in Nagpur, two Indian batsmen together scored 176 runs. Express this information in the form of an equation.

Here, you can see that the score of neither of them is known, i.e., there are two unknown quantities. Let us use x and y to denote them. So, the number of runs scored by one of the batsmen is x, and the number of runs scored by the other is y. We know that

$$x + y = 176,$$

which is the required equation.

This is an example of a linear equation in two variables. It is customary to denote the variables in such equations by x and y, but other letters may also be used. Some examples of linear equations in two variables are:

$$1.2s + 3t = 5,\ p + 4q = 7,\ \pi u + 5v = 9 \text{ and } 3 = \sqrt{2}\,x - 7y.$$

Note that you can put these equations in the form $1.2s + 3t - 5 = 0$, $p + 4q - 7 = 0$, $\pi u + 5v - 9 = 0$ and $\sqrt{2}\,x - 7y - 3 = 0$, respectively.

So, any equation which can be put in the form $ax + by + c = 0$, where a, b and c are real numbers, and a and b are not both zero, is called a *linear equation in two variables*. This means that you can think of many many such equations.

Example 1 : Write each of the following equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case:

(i) $2x + 3y = 4.37$ (ii) $x - 4 = \sqrt{3}\,y$ (iii) $4 = 5x - 3y$ (iv) $2x = y$

Solution : (i) $2x + 3y = 4.37$ can be written as $2x + 3y - 4.37 = 0$. Here $a = 2$, $b = 3$ and $c = -4.37$.

(ii) The equation $x - 4 = \sqrt{3}\,y$ can be written as $x - \sqrt{3}\,y - 4 = 0$. Here $a = 1$, $b = -\sqrt{3}$ and $c = -4$.

(iii) The equation $4 = 5x - 3y$ can be written as $5x - 3y - 4 = 0$. Here $a = 5$, $b = -3$ and $c = -4$. Do you agree that it can also be written as $-5x + 3y + 4 = 0$? In this case $a = -5$, $b = 3$ and $c = 4$.

(iv) The equation $2x = y$ can be written as $2x - y + 0 = 0$. Here $a = 2$, $b = -1$ and $c = 0$.

Equations of the type $ax + b = 0$ are also examples of linear equations in two variables because they can be expressed as

$$ax + 0.y + b = 0$$

For example, $4 - 3x = 0$ can be written as $-3x + 0.y + 4 = 0$.

Example 2 : Write each of the following as an equation in two variables:

(i) $x = -5$ (ii) $y = 2$ (iii) $2x = 3$ (iv) $5y = 2$

Solution : (i) $x = -5$ can be written as $1.x + 0.y = -5$, or $1.x + 0.y + 5 = 0$.

(ii) $y = 2$ can be written as $0.x + 1.y = 2$, or $0.x + 1.y - 2 = 0$.

(iii) $2x = 3$ can be written as $2x + 0.y - 3 = 0$.

(iv) $5y = 2$ can be written as $0.x + 5y - 2 = 0$.

EXERCISE 10.1

1. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

   (Take the cost of a notebook to be ₹ x and that of a pen to be ₹ y).

2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case:

   (i) $2x + 3y = 9.3\overline{5}$ (ii) $x - \frac{y}{5} - 10 = 0$ (iii) $-2x + 3y = 6$ (iv) $x = 3y$

   (v) $2x = -5y$ (vi) $3x + 2 = 0$ (vii) $y - 2 = 0$ (viii) $5 = 2x$

10.3 Solution of a Linear Equation

You have seen that every linear equation in one variable has a unique solution. What can you say about the solution of a linear equation involving two variables? As there are two variables in the equation, a solution means a pair of values, one for x and one for y which satisfy the given equation. Let us consider the equation $2x + 3y = 12$. Here, $x = 3$ and $y = 2$ is a solution because when you substitute $x = 3$ and $y = 2$ in the equation above, you find that

$$2x + 3y = (2 \times 3) + (3 \times 2) = 12$$

This solution is written as an ordered pair (3, 2), first writing the value for x and then the value for y. Similarly, (0, 4) is also a solution for the equation above.

On the other hand, (1, 4) is not a solution of $2x + 3y = 12$, because on putting $x = 1$ and $y = 4$ we get $2x + 3y = 14$, which is not 12. Note that (0, 4) is a solution but not (4, 0).

You have seen at least two solutions for $2x + 3y = 12$, i.e., (3, 2) and (0, 4). Can you find any other solution? Do you agree that (6, 0) is another solution? Verify the same. In fact, we can get many many solutions in the following way. Pick a value of your choice for x (say $x = 2$) in $2x + 3y = 12$. Then the equation reduces to $4 + 3y = 12$, which is a linear equation in one variable. On solving this, you get $y = \frac{8}{3}$. So $\left(2, \frac{8}{3}\right)$ is another solution of $2x + 3y = 12$. Similarly, choosing $x = -5$, you find that the equation becomes $-10 + 3y = 12$. This gives $y = \frac{22}{3}$. So, $\left(-5, \frac{22}{3}\right)$ is another solution of $2x + 3y = 12$. So there is no end to different solutions of a linear equation in two variables. That is, *a linear equation in two variables has infinitely many solutions.*

Example 3 : Find four different solutions of the equation $x + 2y = 6$.

Solution : By inspection, $x = 2$, $y = 2$ is a solution because for $x = 2$, $y = 2$

$$x + 2y = 2 + 4 = 6$$

Now, let us choose $x = 0$. With this value of x, the given equation reduces to $2y = 6$ which has the unique solution $y = 3$. So $x = 0$, $y = 3$ is also a solution of $x + 2y = 6$. Similarly, taking $y = 0$, the given equation reduces to $x = 6$. So, $x = 6$, $y = 0$ is a solution of $x + 2y = 6$ as well. Finally, let us take $y = 1$. The given equation now reduces to $x + 2 = 6$, whose solution is given by $x = 4$. Therefore, (4, 1) is also a solution of the given equation. So four of the infinitely many solutions of the given equation are:

$$(2, 2), (0, 3), (6, 0) \text{ and } (4, 1).$$

Remark : Note that an easy way of getting a solution is to take $x = 0$ and get the corresponding value of y. Similarly, we can put $y = 0$ and obtain the corresponding value of x.

Example 4 : Find two solutions for each of the following equations:

(i) $4x + 3y = 12$

(ii) $2x + 5y = 0$

(iii) $3y + 4 = 0$

Solution : (i) Taking $x = 0$, we get $3y = 12$, i.e., $y = 4$. So, (0, 4) is a solution of the given equation. Similarly, by taking $y = 0$, we get $x = 3$. Thus, (3, 0) is also a solution.

(ii) Taking $x = 0$, we get $5y = 0$, i.e., $y = 0$. So (0, 0) is a solution of the given equation.

Now, if you take $y = 0$, you again get (0, 0) as a solution, which is the same as the earlier one. To get another solution, take $x = 1$, say. Then you can check that the corresponding value of y is $-\frac{2}{5}$. So $\left(1, -\frac{2}{5}\right)$ is another solution of $2x + 5y = 0$.

(iii) Writing the equation $3y + 4 = 0$ as $0.x + 3y + 4 = 0$, you will find that $y = -\frac{4}{3}$ for any value of x. Thus, two solutions can be given as $\left(0, -\frac{4}{3}\right)$ and $\left(1, -\frac{4}{3}\right)$.

EXERCISE 10.2

1. Which one of the following options is true, and why?

$y = 3x + 5$ has

(i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions

2. Write four solutions for each of the following equations:

(i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$

3. Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:

(i) (0, 2) (ii) (2, 0) (iii) (4, 0)

(iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$ (v) (1, 1)

4. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.

10.4 Graph of a Linear Equation in Two Variables

So far, you have obtained the solutions of a linear equation in two variables algebraically. Now, let us look at their geometric representation. You know that each such equation has infinitely many solutions. How can we show them in the coordinate plane? You may have got some indication in which we write the solution as pairs of values. The solutions of the linear equation in Example 3, namely,

$$x + 2y = 6 \tag{1}$$

can be expressed in the form of a table as follows by writing the values of y below the corresponding values of x :

Table 1

x	0	2	4	6	. . .
y	3	2	1	0	. . .

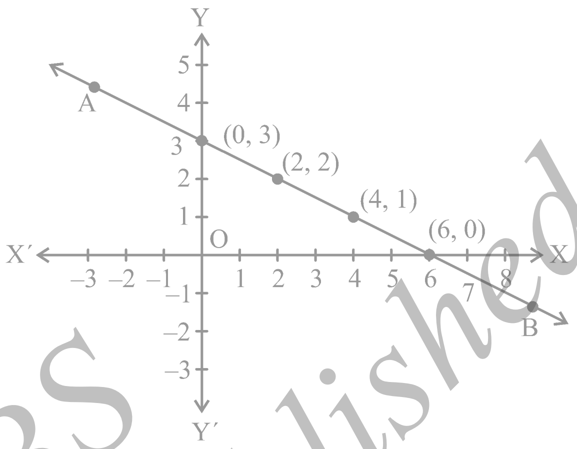


Fig. 10.2

In the previous chapter, you studied how to plot the points on a graph paper. Let us plot the points (0, 3), (2, 2), (4, 1) and (6, 0) on a graph paper. Now join any two of these points and obtain a line. Let us call this as line AB (see Fig. 10.2).

Do you see that the other two points also lie on the line AB? Now, pick another point on this line, say (8, –1). Is this a solution? In fact, $8 + 2(-1) = 6$. So, (8, –1) is a solution. Pick any other point on this line AB and verify whether its coordinates satisfy the equation or not. Now, take any point not lying on the line AB, say (2, 0). Do its coordinates satisfy the equation? Check, and see that they do not.

Let us list our observations:

1. Every point whose coordinates satisfy Equation (1) lies on the line AB.
2. Every point (a, b) on the line AB gives a solution $x = a$, $y = b$ of Equation (1).
3. Any point, which does not lie on the line AB, is not a solution of Equation (1).

So, you can conclude that every point on the line satisfies the equation of the line and every solution of the equation is a point on the line. In fact, a linear equation in two variables is represented geometrically by a line whose points make up the collection of solutions of the equation. This is called the *graph* of the linear equation. So, to obtain the graph of a linear equation in two variables, it is enough to plot two points corresponding to two solutions and join them by a line. However, it is advisable to plot more than two such points so that you can immediately check the correctness of the graph.

Remark : The reason that a, degree one, polynomial equation $ax + by + c = 0$ is called a *linear* equation is that its geometrical representation is a straight line.

Example 5 : Given the point (1, 2), find the equation of a line on which it lies. How many such equations are there?

Solution : Here (1, 2) is a solution of a linear equation you are looking for. So, you are looking for any line passing through the point (1, 2). One example of such a linear equation is $x + y = 3$. Others are $y - x = 1$, $y = 2x$, since they are also satisfied by the coordinates of the point (1, 2). In fact, there are infinitely many linear equations which are satisfied by the coordinates of the point (1, 2). Can you see this pictorially?

Example 6 : Draw the graph of $x + y = 7$.

Solution : To draw the graph, we need at least two solutions of the equation. You can check that $x = 0$, $y = 7$, and $x = 7$, $y = 0$ are solutions of the given equation. So, you can use the following table to draw the graph:

Table 2

x	0	7
y	7	0

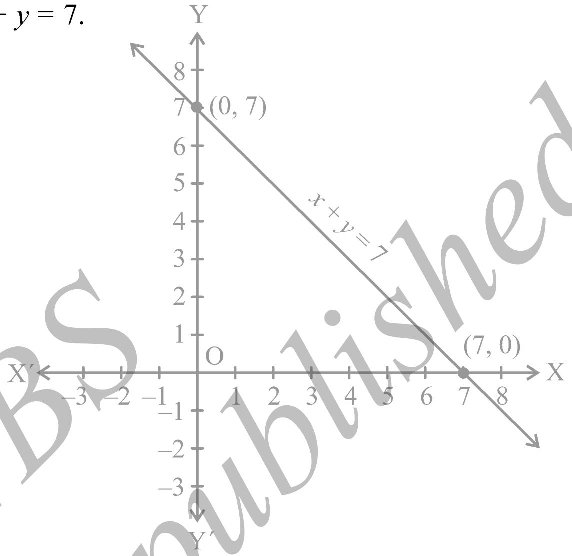


Fig. 10.3

Draw the graph by plotting the two points from Table 2 and then by joining the same by a line (see Fig. 10.3).

Example 7 : You know that the force applied on a body is directly proportional to the acceleration produced in the body. Write an equation to express this situation and plot the graph of the equation.

Solution : Here the variables involved are force and acceleration. Let the force applied be y units and the acceleration produced be x units. From ratio and proportion, you can express this fact as $y = kx$, where k is a constant. (From your study of science, you know that k is actually the mass of the body.)

Now, since we do not know what k is, we cannot draw the precise graph of $y = kx$. However, if we give a certain value to k, then we can draw the graph. Let us take $k = 3$, i.e., we draw the line representing $y = 3x$.

For this we find two of its solutions, say (0, 0) and (2, 6) (see Fig. 10.4).

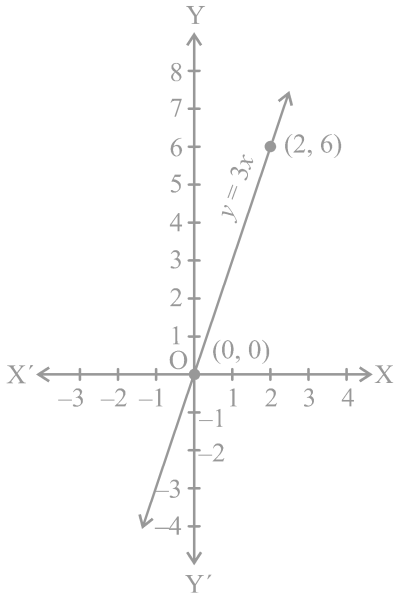


Fig. 10.4

From the graph, you can see that when the force applied is 3 units, the acceleration produced is 1 unit. Also, note that (0, 0) lies on the graph which means the acceleration produced is 0 units, when the force applied is 0 units.

Remark : The graph of the equation of the form $y = kx$ is a line which always passes through the origin.

Example 8 : For each of the graphs given in Fig. 10.5 select the equation whose graph it is from the choices given below:

(a) For Fig. 10.5 (i),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = x$ (iv) $y = 2x + 1$

(b) For Fig. 10.5 (ii),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 4$ (iv) $y = x - 4$

(c) For Fig. 10.5 (iii),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 1$ (iv) $y = 2x - 4$

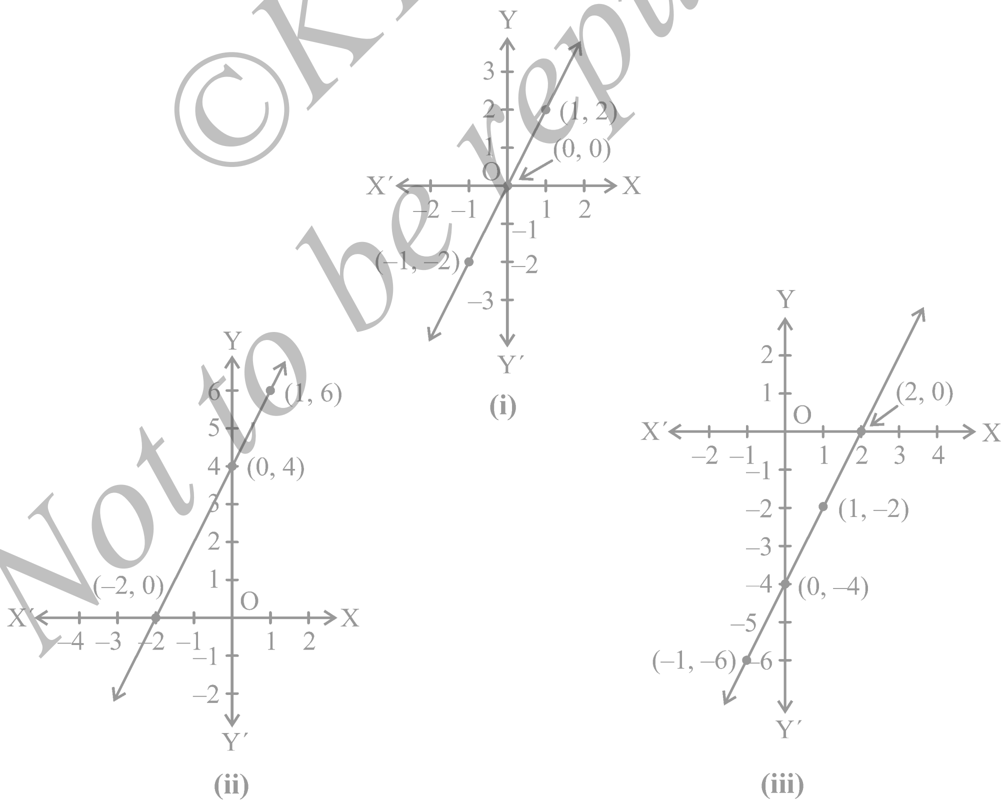


Fig. 10.5

Solution : (a) In Fig. 10.5 (i), the points on the line are (–1, –2), (0, 0), (1, 2). By inspection, $y = 2x$ is the equation corresponding to this graph. You can find that the y-coordinate in each case is double that of the x-coordinate.

(b) In Fig. 10.5 (ii), the points on the line are (–2, 0), (0, 4), (1, 6). You know that the coordinates of the points of the graph (line) satisfy the equation $y = 2x + 4$. So, $y = 2x + 4$ is the equation corresponding to the graph in Fig. 10.5 (ii).

(c) In Fig. 10.5 (iii), the points on the line are (–1, –6), (0, –4), (1, –2), (2, 0). By inspection, you can see that $y = 2x – 4$ is the equation corresponding to the given graph (line).

EXERCISE 4.3

1. Draw the graph of each of the following linear equations in two variables:
   (i) $x + y = 4$ (ii) $x – y = 2$ (iii) $y = 3x$ (iv) $3 = 2x + y$
2. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?
3. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.
4. The taxi fare in a city is as follows: For the first kilometre, the fare is ₹ 8 and for the subsequent distance it is ₹ 5 per km. Taking the distance covered as x km and total fare as Rs y, write a linear equation for this information, and draw its graph.
5. From the choices given below, choose the equation whose graphs are given in Fig. 10.6 and Fig. 10.7.

For Fig. 10. 6

(i) $y = x$

(ii) $x + y = 0$

(iii) $y = 2x$

(iv) $2 + 3y = 7x$

For Fig. 10.7

(i) $y = x + 2$

(ii) $y = x – 2$

(iii) $y = –x + 2$

(iv) $x + 2y = 6$

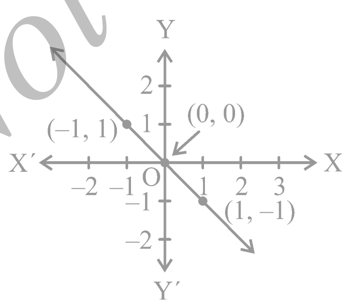


Fig. 10.6

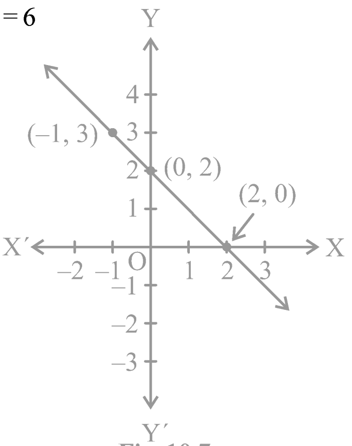


Fig. 10.7

6. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is

(i) 2 units (ii) 0 unit

7. Yamini and Fatima, two students of Class IX of a school, together contributed ₹ 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹ x and ₹ y.) Draw the graph of the same.

8. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

10.5 Equations of Lines Parallel to the x-axis and y-axis

You have studied how to write the coordinates of a given point in the Cartesian plane. Do you know where the points (2, 0), (–3, 0), (4, 0) and (n, 0), for any real number n, lie in the Cartesian plane? Yes, they all lie on the x-axis. But do you know why? Because on the x-axis, the y-coordinate of each point is 0. In fact, every point on the x-axis is of the form (x, 0). Can you now guess the equation of the x-axis? It is given by $y = 0$. Note that $y = 0$ can be expressed as $0.x + 1.y = 0$. Similarly, observe that the equation of the y-axis is given by $x = 0$.

Now, consider the equation $x - 2 = 0$. If this is treated as an equation in one variable x only, then it has the unique solution $x = 2$, which is a point on the number line. However, when treated as an equation in two variables, it can be expressed as

$x + 0.y - 2 = 0$. This has infinitely many solutions. In fact, they are all of the form $(2, r)$, where r is any real number. Also, you can check that every point of the form $(2, r)$ is a solution of this equation. So as, an equation in two variables, $x - 2 = 0$ is represented by the line AB in the graph in Fig. 10.8.

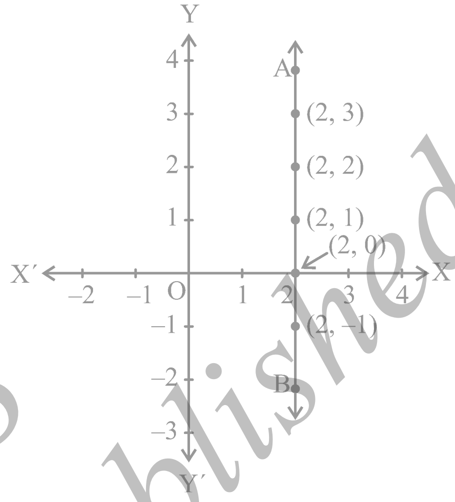


Fig. 10.8

Example 9 : Solve the equation $2x + 1 = x - 3$, and represent the solution(s) on (i) the number line,

(ii) the Cartesian plane.

Solution : We solve $2x + 1 = x - 3$, to get

$$2x - x = -3 - 1$$

i.e.,
$$x = -4$$

(i) The representation of the solution on the number line is shown in Fig. 10.9, where $x = -4$ is treated as an equation in one variable.

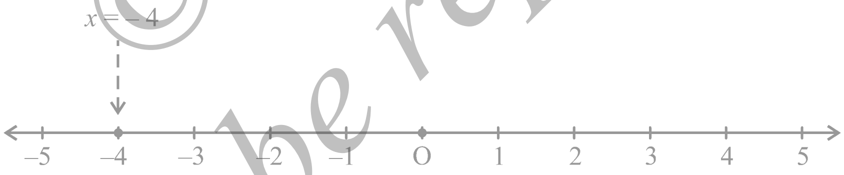


Fig. 10.9

(ii) We know that $x = -4$ can be written as $x + 0.y = -4$

which is a linear equation in the variables x and y. This is represented by a line. Now all the values of y are permissible because $0.y$ is always 0. However, x must satisfy the equation $x = -4$. Hence, two solutions of the given equation are $x = -4$, $y = 0$ and $x = -4$, $y = 2$.

Note that the graph AB is a line parallel to the y-axis and at a distance of 4 units to the left of it (see Fig. 10.10).

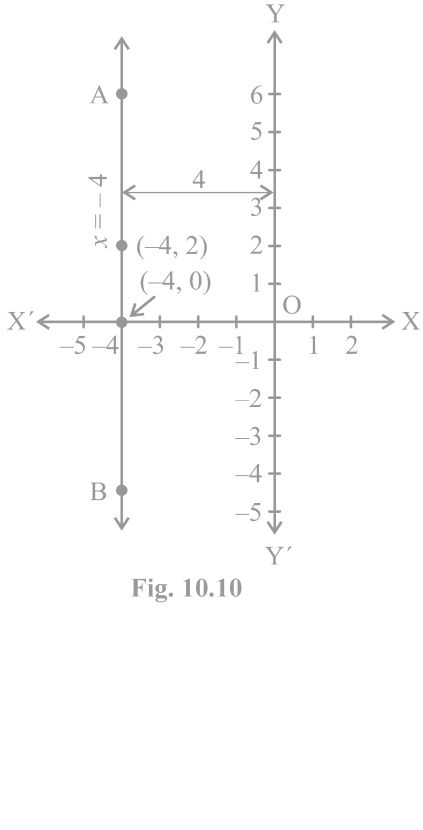


Fig. 10.10

Similarly, you can obtain a line parallel to the x-axis corresponding to equations of the type

$$y = 3 \quad \text{or} \quad 0.x + 1.y = 3$$

EXERCISE 10.4

1. Give the geometric representations of $y = 3$ as an equation

(i) in one variable

(ii) in two variables

2. Give the geometric representations of $2x + 9 = 0$ as an equation

(i) in one variable

(ii) in two variables

10.6 Summary

In this chapter, you have studied the following points:

1. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are not both zero, is called a linear equation in two variables.

2. A linear equation in two variables has infinitely many solutions.

3. The graph of every linear equation in two variables is a straight line.

4. $x = 0$ is the equation of the y-axis and $y = 0$ is the equation of the x-axis.

5. The graph of $x = a$ is a straight line parallel to the y-axis.

6. The graph of $y = a$ is a straight line parallel to the x-axis.

7. An equation of the type $y = mx$ represents a line passing through the origin.

8. Every point on the graph of a linear equation in two variables is a solution of the linear equation. Moreover, every solution of the linear equation is a point on the graph of the linear equation.

CHAPTER 11

AREAS OF PARALLELOGRAMS AND TRIANGLES

11.1 Introduction

In Chapter 2, you have seen that the study of Geometry, originated with the measurement of earth (lands) in the process of recasting boundaries of the fields and dividing them into appropriate parts. For example, a farmer *Budhia* had a triangular field and she wanted to divide it equally among her two daughters and one son. Without actually calculating the area of the field, she just divided one side of the triangular field into three equal parts and joined the two points of division to the opposite vertex. In this way, the field was divided into three parts and she gave one part to each of her children. Do you think that all the three parts so obtained by her were, in fact, equal in area? To get answers to this type of questions and other related problems, there is a need to have a relook at areas of plane figures, which you have already studied in earlier classes.

You may recall that the part of the plane enclosed by a simple closed figure is called a *planar region* corresponding to that figure. The magnitude or measure of this planar region is called its *area*. This magnitude or measure is always expressed with the help of a number (in some unit) such as 5 cm^2, 8 m^2, 3 hectares etc. So, we can say that area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

We are also familiar with the concept of congruent figures from earlier classes and from Chapter 5. *Two figures are called congruent, if they have the same shape and the same size*. In other words, if two figures A and B are congruent (see Fig. 11.1), then using a tracing paper,

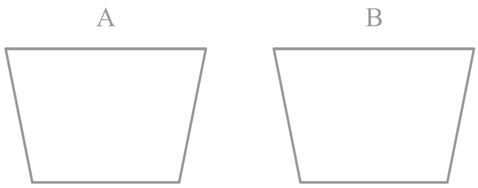


Fig. 11.1

you can superpose one figure over the other such that it will cover the other completely. So *if two figures A and B are congruent, they must have equal areas*. However, the converse of this statement is *not true*. In other words, *two figures having equal areas need not be congruent*. For example, in Fig. 11.2, rectangles ABCD and EFGH have equal areas (9×4 cm^2 and 6×6 cm^2) but clearly they are not congruent. (Why?)

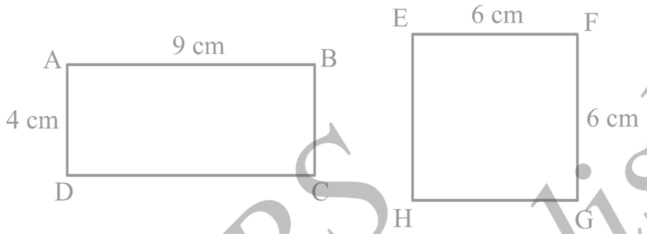


Fig. 11.2

Now let us look at Fig. 11.3 given below:

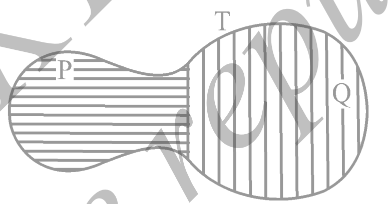


Fig. 11.3

You may observe that planar region formed by figure T is made up of two planar regions formed by figures P and Q. You can easily see that

Area of figure T = Area of figure P + Area of figure Q.

You may denote the area of figure A as ar(A), area of figure B as ar(B), area of figure T as ar(T), and so on. Now you can say that *area of a figure is a number* (*in some unit*) *associated with the part of the plane enclosed by the figure with the following two properties:*

(1) *If* A *and* B *are two congruent figures, then ar*(A) = ar(B);

and (2) *if a planar region formed by a figure* T *is made up of two non-overlapping planar regions formed by figures* P *and* Q*, then* ar(T) = ar(P) + ar(Q).

You are also aware of some formulae for finding the areas of different figures such as rectangle, square, parallelogram, triangle etc., from your earlier classes. In this chapter, attempt shall be made to consolidate the knowledge about these formulae by studying some relationship between the areas of these geometric figures under the condition when they lie on the same base and between the same parallels. This study will also be useful in the understanding of some results on 'similarity of triangles'.

11.2 Figures on the Same Base and Between the Same Parallels

Look at the following figures:

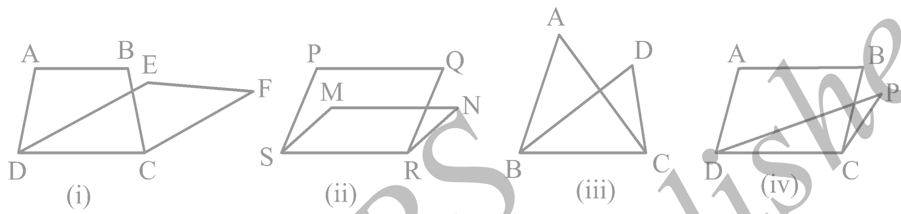


Fig. 11.4

In Fig. 11.4(i), trapezium ABCD and parallelogram EFCD have a common side DC. We say that trapezium ABCD and parallelogram EFCD *are on the same base* DC. Similarly, in Fig. 11.4 (ii), parallelograms PQRS and MNRS are on the same base SR; in Fig. 11.4(iii), triangles ABC and DBC are on the same base BC and in Fig. 11.4(iv), parallelogram ABCD and triangle PDC are on the same base DC.

Now look at the following figures:

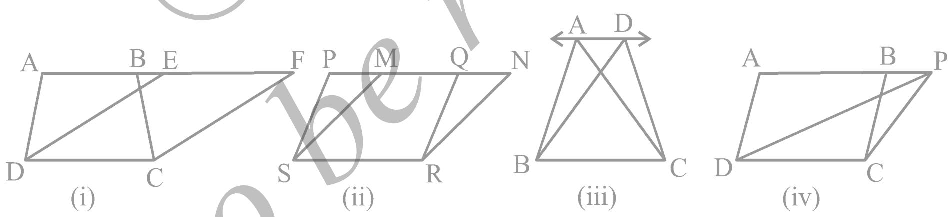


Fig. 11.5

In Fig. 11.5(i), clearly trapezium ABCD and parallelogram EFCD are on the *same base* DC. In addition to the above, the vertices A and B (of trapezium ABCD) opposite to base DC and the vertices E and F (of parallelogram EFCD) opposite to base DC lie on a line AF parallel to DC. We say that trapezium ABCD and parallelogram EFCD are on *the same base* DC *and between the same parallels* AF and DC. Similarly, parallelograms PQRS and MNRS are on the same base SR and between the same parallels PN and SR [see Fig.11.5 (ii)] as vertices P and Q of PQRS and vertices M and N of MNRS lie on a line PN parallel to base SR.In the same way, triangles ABC and DBC lie on the same base BC and between the same parallels AD and BC [see Fig. 11.5 (iii)] and parallelogram ABCD and triangle PCD lie on the same base DC and between the same parallels AP and DC [see Fig. 11.5(iv)].

So, *two figures are said to be on the same base and between the same parallels, if they have a common base* (*side*) *and the vertices* (*or the vertex*) *opposite to the common base of each figure lie on a line parallel to the base.*

Keeping in view the above statement, you cannot say that Δ PQR and Δ DQR of Fig. 11.6(i) lie between the same parallels *l* and QR. Similarly, you cannot say that

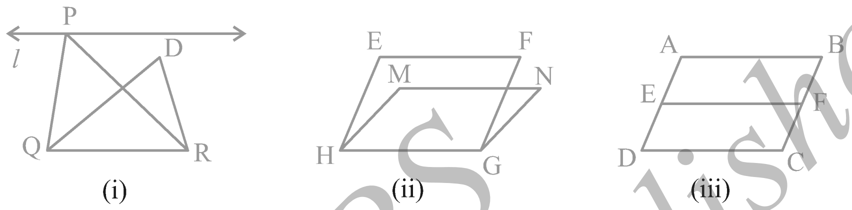


Fig. 11.6

parallelograms EFGH and MNGH of Fig. 11.6(ii) lie between the same parallels EF and HG and that parallelograms ABCD and EFCD of Fig. 11.6(iii) lie between the same parallels AB and DC (even though they have a common base DC and lie between the parallels AD and BC). So, it should clearly be noted that *out of the two parallels, one must be the line containing the common base.* Note that ΔABC and ΔDBE of Fig. 11.7(i) are not on the common base. Similarly, ΔABC and parallelogram PQRS of Fig. 11.7(ii) are also not on the same base.

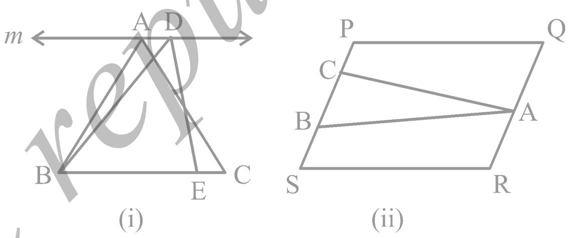


Fig. 11.7

EXERCISE 11.1

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

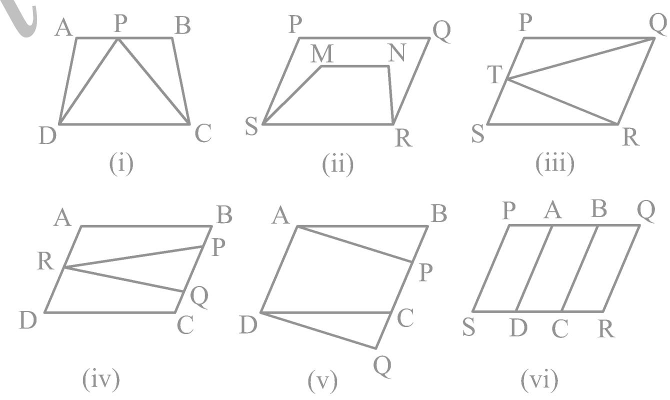


Fig. 11.8

11.3 Parallelograms on the same Base and Between the same Parallels

Now let us try to find a relation, if any, between the areas of two parallelograms on the same base and between the same parallels. For this, let us perform the following activities:

Activity 1 : Let us take a graph sheet and draw two parallelograms ABCD and PQCD on it as shown in Fig. 11.9.

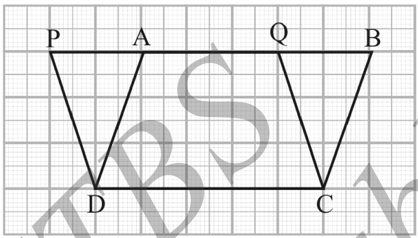


Fig. 11.9

The above two parallelograms are on the same base DC and between the same parallels PB and DC. You may recall the method of finding the areas of these two parallelograms by counting the squares.

In this method, the area is found by counting the number of complete squares enclosed by the figure, the number of squares a having more than half their parts enclosed by the figure and the number of squares having half their parts enclosed by the figure. The squares whose less than half parts are enclosed by the figure are ignored. You will find that areas of both the parallelograms are (approximately) 15cm^2. Repeat this activity* by drawing some more pairs of parallelograms on the graph sheet. What do you observe? Are the areas of the two parallelograms different or equal? If fact, they are equal. So, this may lead you to conclude that *parallelograms on the same base and between the same parallels are equal in area*. However, remember that this is just a verification.

Activity 2 : Draw a parallelogram ABCD on a thick sheet of paper or on a cardboard sheet. Now, draw a line-segment DE as shown in Fig. 11.10.

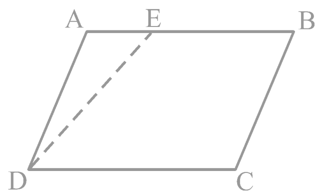


Fig. 11.10

*This activity can also be performed by using a Geoboard.

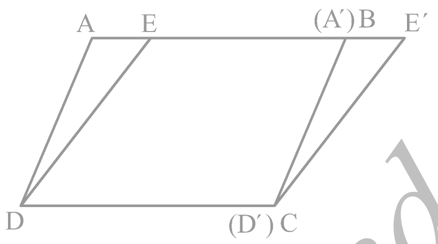


Fig. 11.11

Next, cut a triangle A′ D′ E′ congruent to triangle ADE on a separate sheet with the help of a tracing paper and place Δ A′D′E′ in such a way that A′D′ coincides with BC as shown in Fig 11.11.Note that there are two parallelograms ABCD and EE′CD on the same base DC and between the same parallels AE′ and DC. What can you say about their areas?

As Δ ADE $\cong$ Δ A′D′E′

Therefore ar (ADE) = ar (A′D′E′)

Also ar (ABCD) = ar (ADE) + ar (EBCD)

= ar (A′D′E′) + ar (EBCD)

= ar (EE′CD)

So, the two parallelograms are equal in area.

Let us now try to prove this relation between the two such parallelograms.

Theorem 11.1 : *Parallelograms on the same base and between the same parallels are equal in area.*

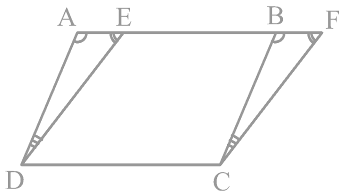


Fig. 11.12

Proof : Two parallelograms ABCD and EFCD, on the same base DC and between the same parallels AF and DC are given (see Fig.11.12).

We need to prove that ar (ABCD) = ar (EFCD).

In Δ ADE and Δ BCF,

$\angle$DAE = $\angle$CBF (Corresponding angles from AD || BC and transversal AF) (1)

$\angle$AED = $\angle$BFC (Corresponding angles from ED || FC and transversal AF) (2)

Therefore, $\angle$ADE = $\angle$BCF (Angle sum property of a triangle) (3)

Also, AD = BC (Opposite sides of the parallelogram ABCD) (4)

So, Δ ADE $\cong$ Δ BCF [By ASA rule, using (1), (3), and (4)]

Therefore, ar (ADE) = ar (BCF) (Congruent figures have equal areas) (5)

Now, ar (ABCD) = ar (ADE) + ar (EDCB)

= ar (BCF) + ar (EDCB) [From(5)]

= ar (EFCD)

So, parallelograms ABCD and EFCD are equal in area.

Let us now take some examples to illustrate the use of the above theorem.

Example 1 : In Fig. 11.13, ABCD is a parallelogram and EFCD is a rectangle.

Also, AL $\perp$ DC. Prove that

(i) ar (ABCD) = ar (EFCD)

(ii) ar (ABCD) = DC × AL

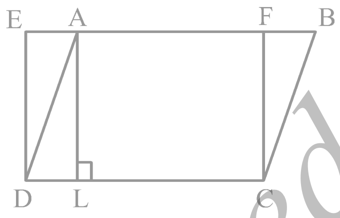


Fig. 11.13

Solution : (i) As a rectangle is also a parallelogram,

therefore, ar (ABCD) = ar (EFCD) (Theorem 11.1)

(ii) From above result,

ar (ABCD) = DC × FC (Area of the rectangle = length × breadth) (1)

As AL $\perp$ DC, therefore, AFCL is also a rectangle

So, AL = FC (2)

Therefore, ar (ABCD) = DC × AL [From (1) and (2)]

Can you see from the Result (ii) above *that area of a parallelogram is the product of its any side and the coresponding altitude*. Do you remember that you have studied this formula for area of a parallelogram in Class VII. On the basis of this formula, Theorem 11.1 can be rewritten as *parallelograms on the same base or equal bases and between the same parallels are equal in area*.

Can you write the converse of the above statement? It is as follows: *Parallelograms on the same base* (*or equal bases*) *and having equal areas lie between the same parallels.* Is the converse true? Prove the converse using the formula for area of the parallelogram.

Example 2 : If a triangle and a parallelogram are on the same base and between the same parallels, then prove that the area of the triangle is equal to half the area of the parallelogram.

Solution : Let Δ ABP and parallelogram ABCD be on the same base AB and between the same parallels AB and PC (see Fig. 11.14).

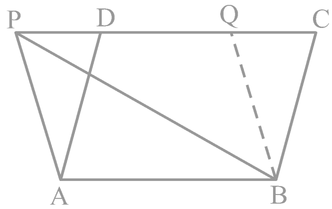


Fig. 11.14

You wish to prove that ar (PAB) = $\frac{1}{2}$ ar (ABCD)

Draw BQ || AP to obtain another parallelogram ABQP. Now parallelograms ABQP and ABCD are on the same base AB and between the same parallels AB and PC.

Therefore, ar (ABQP) = ar (ABCD) (By Theorem 9.1) (1)

But Δ PAB $\cong$ Δ BQP (Diagonal PB divides parallelogram ABQP into two congruent triangles.)

So, ar (PAB) = ar (BQP) (2)

Therefore, ar (PAB) = $\frac{1}{2}$ ar (ABQP) [From (2)] (3)

This gives ar (PAB) = $\frac{1}{2}$ ar (ABCD) [From (1) and (3)]

EXERCISE 11.2

1. In Fig. 9.15, ABCD is a parallelogram, AE $\perp$ DC and CF $\perp$ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

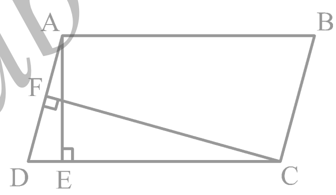


Fig. 11.15

2. If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).

3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

4. In Fig. 11.16, P is a point in the interior of a parallelogram ABCD. Show that

   (i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

   (ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

   [*Hint* : Through P, draw a line parallel to AB.]

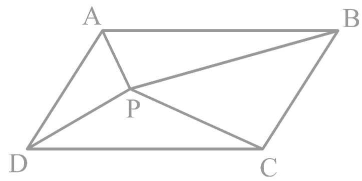


Fig. 11.16

5. In Fig. 11.17, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

   (i) ar (PQRS) = ar (ABRS)

   (ii) ar (AX S) = $\frac{1}{2}$ ar (PQRS)

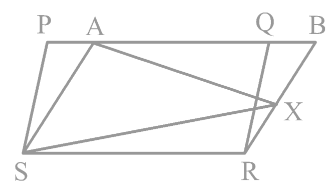


Fig. 11.17

6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

11.4 Triangles on the same Base and between the same Parallels

Let us look at Fig. 11.18. In it, you have two triangles ABC and PBC on the same base BC and between the same parallels BC and AP. What can you say about the areas of such triangles? To answer this question, you may perform the activity of drawing several pairs of triangles on the same base and between the same parallels on the graph sheet and find their areas by the method of counting the squares. Each time, you will find that the areas of the two triangles are (approximately) equal. This activity can be performed using a geoboard also. You will again find that the two areas are (approximately) equal.

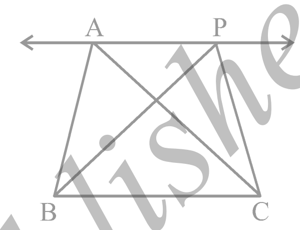


Fig. 11.18

To obtain a logical answer to the above question, you may proceed as follows:

In Fig. 11.18, draw CD || BA and CR || BP such that D and R lie on line AP(see Fig.11.19).

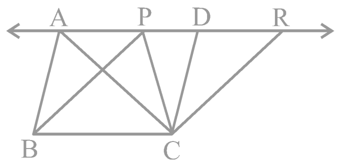


Fig. 11.19

From this, you obtain two parallelograms PBCR and ABCD on the same base BC and between the same parallels BC and AR.

Therefore, ar (ABCD) = ar (PBCR) (Why?)

Now Δ ABC $\cong$ Δ CDA and Δ PBC $\cong$ Δ CRP (Why?)

So, ar (ABC) = $\frac{1}{2}$ ar (ABCD) and ar (PBC) = $\frac{1}{2}$ ar (PBCR) (Why?)

Therefore, ar (ABC) = ar (PBC)

In this way, you have arrived at the following theorem:

Theorem 11.2 : *Two triangles on the same base (or equal bases) and between the same parallels are equal in area.*

Now, suppose ABCD is a parallelogram whose one of the diagonals is AC (see Fig. 11.20). Let $AN \perp DC$. Note that

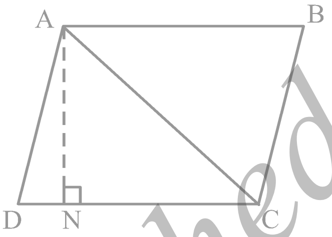


Fig. 11.20

$$\Delta\, ADC \cong \Delta\, CBA \qquad \text{(Why?)}$$

So, $\quad ar\,(ADC) = ar\,(CBA) \qquad$ (Why?)

Therefore, $ar\,(ADC) = \frac{1}{2}\, ar\,(ABCD)$

$$= \frac{1}{2}\,(DC \times AN) \qquad \text{(Why?)}$$

So, $\quad$ area of $\Delta\, ADC = \frac{1}{2} \times$ base DC $\times$ corresponding altitude AN

In other words, *area of a triangle is half the product of its base* (*or any side*) *and the corresponding altitude* (*or height*). Do you remember that you have learnt this formula for area of a triangle in Class VII ? From this formula, you can see that *two triangles with same base* (*or equal bases*) *and equal areas will have equal corresponding altitudes.*

For having equal corresponding altitudes, the triangles must lie between the same parallels. From this, you arrive at the following converse of Theorem 11.2 .

Theorem 11.3 : *Two triangles having the same base* (*or equal bases*) *and equal areas lie between the same parallels.*

Let us now take some examples to illustrate the use of the above results.

Example 3 : Show that a median of a triangle divides it into two triangles of equal areas.

Solution : Let ABC be a triangle and let AD be one of its medians (see Fig. 11.21). You wish to show that

$$ar\,(ABD) = ar\,(ACD).$$

Since the formula for area involves altitude, let us draw $AN \perp BC$.

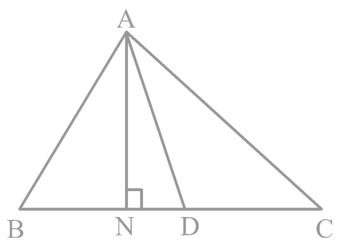


Fig. 11.21

Now $\quad ar(ABD) = \frac{1}{2} \times$ base $\times$ altitude (of Δ ABD)

$$= \frac{1}{2} \times BD \times AN$$

$$= \frac{1}{2} \times CD \times AN \qquad \text{(As BD = CD)}$$

$$= \frac{1}{2} \times \text{base} \times \text{altitude (of } \Delta\, ACD)$$

$$= ar(ACD)$$

Example 4 : In Fig. 11.22, ABCD is a quadrilateral and BE || AC and also BE meets DC produced at E. Show that area of Δ ADE is equal to the area of the quadrilateral ABCD.

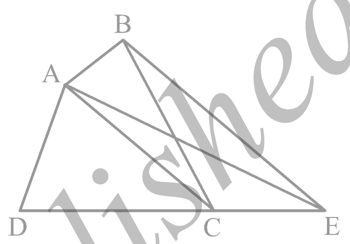


Fig. 11.22

Solution : Observe the figure carefully .

Δ BAC and Δ EAC lie on the same base AC and between the same parallels AC and BE.

Therefore, ar(BAC) = ar(EAC) (By Theorem 11.2)

So, ar(BAC) + ar(ADC) = ar(EAC) + ar(ADC) (Adding same areas on both sides)

or ar(ABCD) = ar(ADE)

EXERCISE 11.3

1. In Fig.11.23, E is any point on median AD of a Δ ABC. Show that ar (ABE) = ar (ACE).
2. In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar(ABC) .
3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.
4. In Fig. 11.24, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that ar(ABC) = ar (ABD).

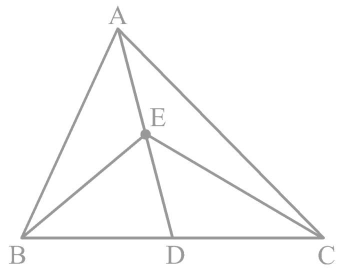


Fig. 11.23

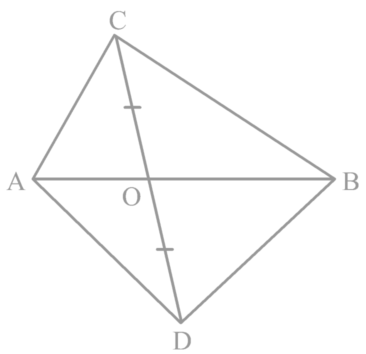


Fig. 11.24

5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a Δ ABC. Show that

(i) BDEF is a parallelogram. (ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

6. In Fig. 11.25, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

[*Hint* : From D and B, draw perpendiculars to AC.]

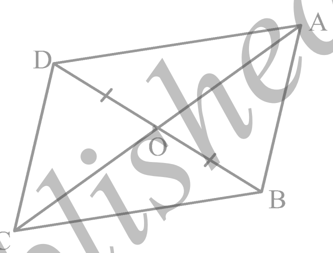


Fig. 11.25

7. D and E are points on sides AB and AC respectively of Δ ABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

8. XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that

ar (ABE) = ar (ACF)

9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see Fig. 11.26). Show that ar (ABCD) = ar (PBQR).

[*Hint* : Join AC and PQ. Now compare ar (ACQ) and ar (APQ).]

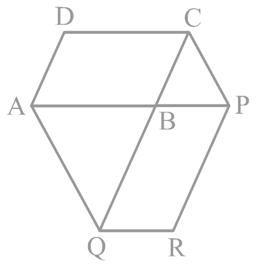


Fig. 11.26

10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

11. In Fig. 11.27, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

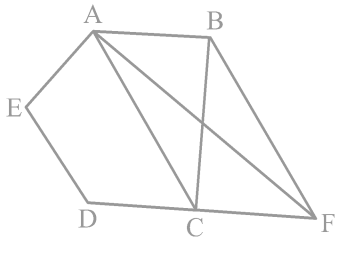


Fig. 11.27

12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

[*Hint* : Join CX.]

14. In Fig.11.28, AP || BQ || CR. Prove that

ar (AQC) = ar (PBR).

15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

16. In Fig.11.29, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

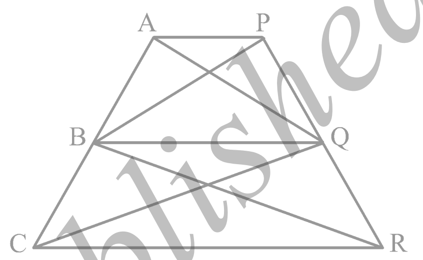


Fig. 11.28

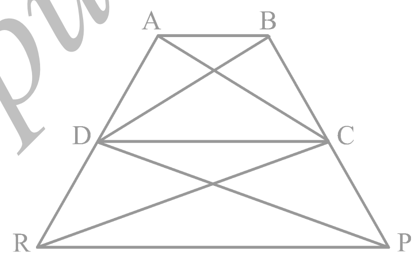


Fig. 11.29

EXERCISE 11.4 (Optional)*

1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

2. In Fig. 11.30, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).

Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of *Budhia* has been actually divided into three parts of equal area?

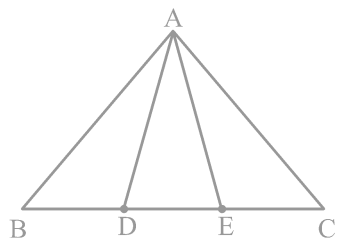


Fig. 11.30

*These exercises are not from examination point of view.

[**Remark:** Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into *n* equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into *n* triangles of equal areas.]

3. In Fig. 11.31, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).

4. In Fig. 11.32, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ).

[*Hint* : Join AC.]

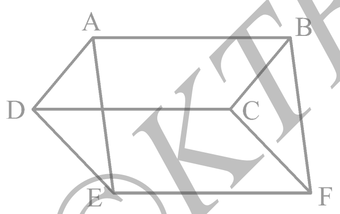


Fig. 11.31

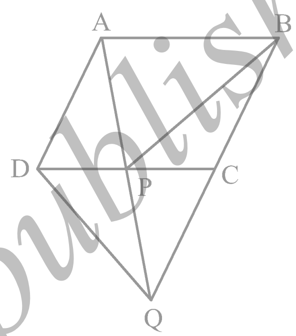


Fig. 11.32

5. In Fig.11.33, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that

(i) ar (BDE) = $\frac{1}{4}$ ar (ABC)

(ii) ar (BDE) = $\frac{1}{2}$ ar (BAE)

(iii) ar (ABC) = 2 ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ ar (AFC)

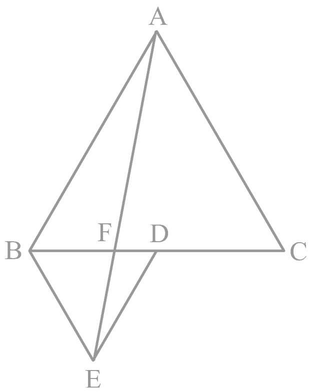


Fig. 11.33

[*Hint* : Join EC and AD. Show that BE || AC and DE || AB, etc.]

6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

$$\text{ar (APB)} \times \text{ar (CPD)} = \text{ar (APD)} \times \text{ar (BPC)}.$$

[*Hint* : From A and C, draw perpendiculars to BD.]

7. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) ar (PRQ) = $\frac{1}{2}$ ar (ARC) (ii) ar (RQC) = $\frac{3}{8}$ ar (ABC)

(iii) ar (PBQ) = ar (ARC)

8. In Fig. 11.34, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX $\perp$ DE meets BC at Y. Show that:

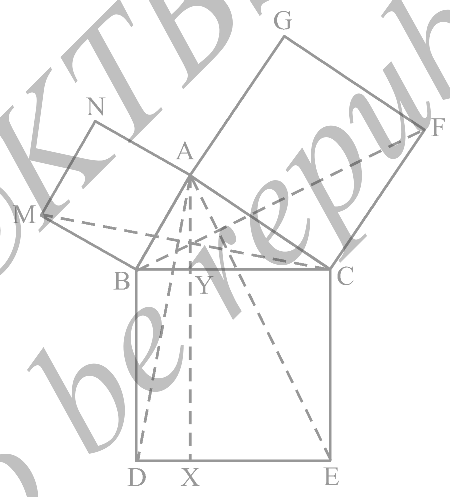


Fig. 11.34

(i) Δ MBC $\cong$ Δ ABD (ii) ar (BYXD) = 2 ar (MBC)

(iii) ar (BYXD) = ar (ABMN) (iv) Δ FCB $\cong$ Δ ACE

(v) ar (CYXE) = 2 ar (FCB) (vi) ar (CYXE) = ar (ACFG)

(vii) ar (BCED) = ar (ABMN) + ar (ACFG)

Note : Result (vii) is the famous *Theorem of Pythagoras*. You shall learn a simpler proof of this theorem in Class X.

11.5 Summary

In this chapter, you have studied the following points :

1. Area of a figure is a number (in some unit) associated with the part of the plane enclosed by that figure.

2. Two congruent figures have equal areas but the converse need not be true.

3. If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then ar (T) = ar (P) + ar (Q), where ar (X) denotes the area of figure X.

4. Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices, (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

5. Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.

6. Area of a parallelogram is the product of its base and the corresponding altitude.

7. Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.

8. If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.

9. Triangles on the same base (or equal bases) and between the same parallels are equal in area.

10. Area of a triangle is half the product of its base and the corresponding altitude.

11. Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.

12. A median of a triangle divides it into two triangles of equal areas.

CHAPTER 12

CIRCLES

12.1 Introduction

You may have come across many objects in daily life, which are round in shape, such as wheels of a vehicle, bangles, dials of many clocks, coins of denominations 50 p, Re 1 and Rs 5, key rings, buttons of shirts, etc. (see Fig.12.1). In a clock, you might have observed that the second's hand goes round the dial of the clock rapidly and its tip moves in a round path. This path traced by the tip of the second's hand is called a *circle*. In this chapter, you will study about circles, other related terms and some properties of a circle.

Fig. 12.1

12.2 Circles and Its Related Terms: A Review

Take a compass and fix a pencil in it. Put its pointed leg on a point on a sheet of a paper. Open the other leg to some distance. Keeping the pointed leg on the same point, rotate the other leg through one revolution. What is the closed figure traced by the pencil on paper? As you know, it is a circle (see Fig.12.2). How did you get a circle? You kept one point fixed (A in Fig.12.2) and drew all the points that were at a fixed distance from A. This gives us the following definition:

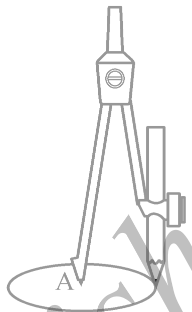


Fig. 12.2

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the *centre* of the circle and the fixed distance is called the *radius* of the circle. In Fig.12.3, O is the centre and the length OP is the radius of the circle.

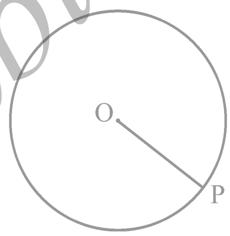


Fig. 12.3

Remark : Note that the line segment joining the centre and any point on the circle is also called a *radius* of the circle. That is, 'radius' is used in two senses-in the sense of a line segment and also in the sense of its length.

You are already familiar with some of the following concepts from Class VI. We are just recalling them.

A circle divides the plane on which it lies into three parts. They are: (i) inside the circle, which is also called the *interior* of the circle; (ii) the *circle* and (iii) outside the circle, which is also called the *exterior* of the circle (see Fig.12.4). The circle and its interior make up the *circular region*.

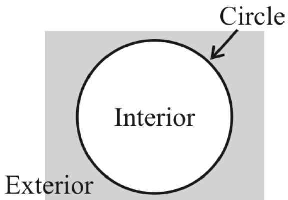


Fig. 12.4

If you take two points P and Q on a circle, then the line segment PQ is called a *chord* of the circle (see Fig. 12.5). The chord, which passes through the centre of the circle, is called a *diameter* of the circle. As in the case of radius, the word 'diameter' is also used in two senses, that is, as a line segment and also as its length. Do you find any other chord of the circle longer than a diameter? No, you see that a *diameter is the longest chord and all diameters have the same length, which is equal to two* *times the radius*. In Fig.12.5, AOB is a diameter of the circle. How many diameters does a circle have? Draw a circle and see how many diameters you can find.

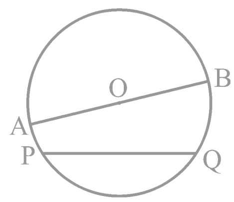


Fig. 12.5

A piece of a circle between two points is called an *arc*. Look at the pieces of the circle between two points P and Q in Fig.12.6. You find that there are two pieces, one longer and the other smaller (see Fig.12.7). The longer one is called the *major arc* PQ and the shorter one is called the *minor arc* PQ. The minor arc PQ is also denoted by $\widehat{PQ}$ and the major arc PQ by $\widehat{PRQ}$, where R is some point on the arc between P and Q. Unless otherwise stated, arc PQ or $\widehat{PQ}$ stands for minor arc PQ. When P and Q are ends of a diameter, then both arcs are equal and each is called a *semicircle*.

Fig. 12.6

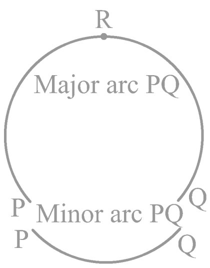


Fig. 12.7

The length of the complete circle is called its *circumference*. The region between a chord and either of its arcs is called a *segment* of the circular region or simply a *segment* of the circle. You will find that there are two types of segments also, which are the *major segment* and the *minor segment* (see Fig. 12.8). The region between an arc and the two radii, joining the centre to the end points of the arc is called a *sector*. Like segments, you find that the minor arc corresponds to the *minor sector* and the major arc corresponds to the *major sector*. In Fig. 12.9, the region OPQ is the minor sector and remaining part of the circular region is the major sector. When two arcs are equal, that is, each is a semicircle, then both segments and both sectors become the same and each is known as a *semicircular region*.

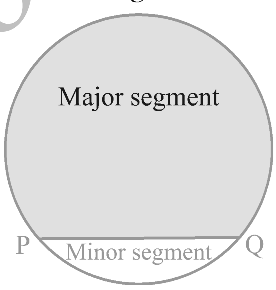


Fig. 12.8

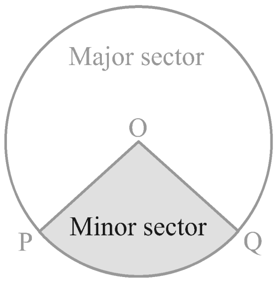


Fig. 12.9

EXERCISE 12.1

1. Fill in the blanks:
   (i) The centre of a circle lies in ______________ of the circle. (exterior/ interior)
   (ii) A point, whose distance from the centre of a circle is greater than its radius lies in ______________ of the circle. (exterior/ interior)
   (iii) The longest chord of a circle is a ______________ of the circle.
   (iv) An arc is a ______________ when its ends are the ends of a diameter.
   (v) Segment of a circle is the region between an arc and ___________ of the circle.
   (vi) A circle divides the plane, on which it lies, in ______________ parts.
2. Write True or False: Give reasons for your answers.
   (i) Line segment joining the centre to any point on the circle is a radius of the circle.
   (ii) A circle has only finite number of equal chords.
   (iii) If a circle is divided into three equal arcs, each is a major arc.
   (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
   (v) Sector is the region between the chord and its corresponding arc.
   (vi) A circle is a plane figure.

12.3 Angle Subtended by a Chord at a Point

Take a line segment PQ and a point R not on the line containing PQ. Join PR and QR (see Fig. 12.10). Then $\angle$ PRQ is called the angle subtended by the line segment PQ at the point R. What are angles POQ, PRQ and PSQ called in Fig. 12.11? $\angle$ POQ is the angle subtended by the chord PQ at the centre O, $\angle$ PRQ and $\angle$ PSQ are respectively the angles subtended by PQ at points R and S on the major and minor arcs PQ.

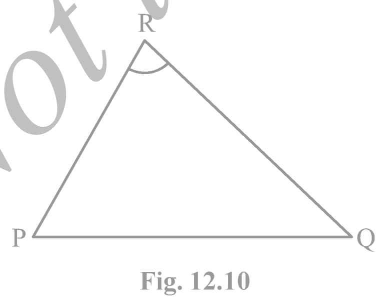


Fig. 12.10

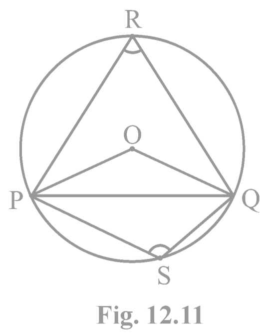


Fig. 12.11

Let us examine the relationship between the size of the chord and the angle subtended by it at the centre. You may see by drawing different chords of a circle and angles subtended by them at the centre that the longer is the chord, the bigger will be the angle subtended by it at the centre. What will happen if you take two equal chords of a circle? Will the angles subtended at the centre be the same or not?

Draw two or more equal chords of a circle and measure the angles subtended by them at the centre (see Fig.12.12). You will find that the angles subtended by them at the centre are equal. Let us give a proof of this fact.

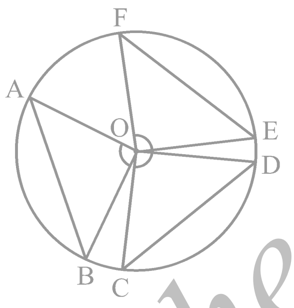


Fig. 12.12

Theorem 12.1 : *Equal chords of a circle subtend equal angles at the centre.*

Proof : You are given two equal chords AB and CD of a circle with centre O (see Fig.12.13). You want to prove that $\angle$ AOB = $\angle$ COD.

In triangles AOB and COD,

OA = OC (Radii of a circle)

OB = OD (Radii of a circle)

AB = CD (Given)

Therefore, Δ AOB $\cong$ Δ COD (SSS rule)

This gives $\angle$ AOB = $\angle$ COD

(Corresponding parts of congruent triangles)

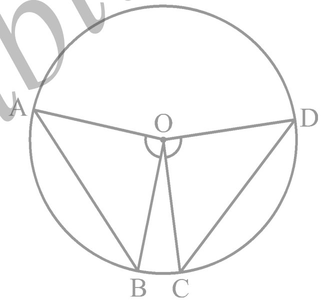


Fig. 12.13

Remark : For convenience, the abbreviation CPCT will be used in place of 'Corresponding parts of congruent triangles', because we use this very frequently as you will see.

Now if two chords of a circle subtend equal angles at the centre, what can you say about the chords? Are they equal or not? Let us examine this by the following activity:

Take a tracing paper and trace a circle on it. Cut it along the circle to get a disc. At its centre O, draw an angle AOB where A, B are points on the circle. Make another angle POQ at the centre equal to $\angle$AOB. Cut the disc along AB and PQ (see Fig. 12.14). You will get two segments ACB and PRQ of the circle. If you put one on the other, what do you observe? They cover each other, i.e., they are congruent. So AB = PQ.

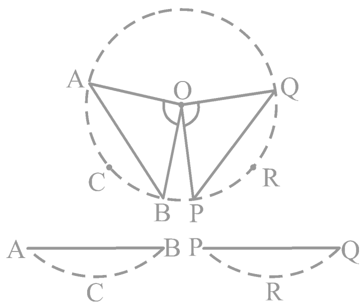


Fig. 12.14

Though you have seen it for this particular case, try it out for other equal angles too. The chords will all turn out to be equal because of the following theorem:

Theorem 12.2 : *If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.*

The above theorem is the converse of the Theorem 12.1. Note that in Fig. 12.13, if you take $\angle$ AOB = $\angle$ COD, then

$$\Delta \text{ AOB} \cong \Delta \text{ COD (Why?)}$$

Can you now see that AB = CD?

EXERCISE 12.2

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.
2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

12.4 Perpendicular from the Centre to a Chord

Activity : Draw a circle on a tracing paper. Let O be its centre. Draw a chord AB. Fold the paper along a line through O so that a portion of the chord falls on the other. Let the crease cut AB at the point M. Then, $\angle$ OMA = $\angle$ OMB = 90° or OM is perpendicular to AB. Does the point B coincide with A (see Fig.12.15)?

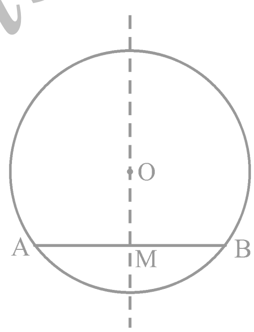


Fig. 12.15

Yes it will. So MA = MB.

Give a proof yourself by joining OA and OB and proving the right triangles OMA and OMB to be congruent. This example is a particular instance of the following result:

Theorem 12.3 : *The perpendicular from the centre of a circle to a chord bisects the chord.*

What is the converse of this theorem? To write this, first let us be clear what is assumed in Theorem 12.3 and what is proved. Given that the perpendicular from the centre of a circle to a chord is drawn and to prove that it bisects the chord. Thus in the converse, what the hypothesis is 'if a line from the centre bisects a chord of a circle' and what is to be proved is 'the line is perpendicular to the chord'. So the converse is:

Theorem 12.4 : *The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.*

Is this true? Try it for few cases and see. You will see that it is true for these cases. See if it is true, in general, by doing the following exercise. We will write the stages and you give the reasons.

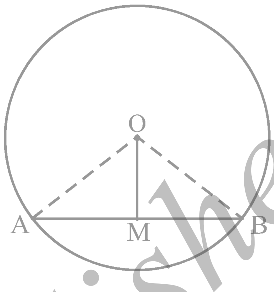


Fig. 12.16

Let AB be a chord of a circle with centre O and O is joined to the mid-point M of AB. You have to prove that OM $\perp$ AB. Join OA and OB (see Fig. 12.16). In triangles OAM and OBM,

$$OA = OB \quad \text{(Why ?)}$$

$$AM = BM \quad \text{(Why ?)}$$

$$OM = OM \quad \text{(Common)}$$

Therefore, $\Delta OAM \cong \Delta OBM$ (How ?)

This gives $\angle OMA = \angle OMB = 90°$ (Why ?)

12.5 Circle through Three Points

You have learnt in Chapter 6, that two points are sufficient to determine a line. That is, there is one and only one line passing through two points. A natural question arises. How many points are sufficient to determine a circle?

Take a point P. How many circles can be drawn through this point? You see that there may be as many circles as you like passing through this point [see Fig. 12.17(i)]. Now take two points P and Q. You again see that there may be an infinite number of circles passing through P and Q [see Fig.12.17(ii)]. What will happen when you take three points A, B and C? Can you draw a circle passing through three collinear points?

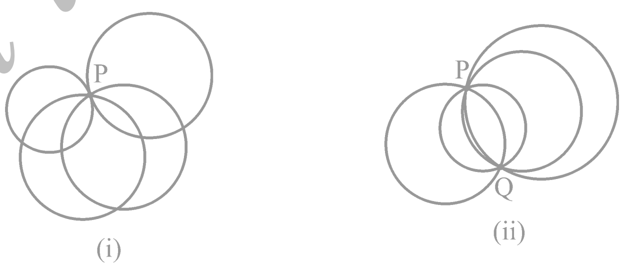


Fig. 12. 17

No. If the points lie on a line, then the third point will lie inside or outside the circle passing through two points (see Fig 12.18).

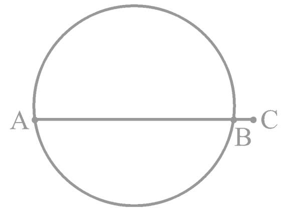
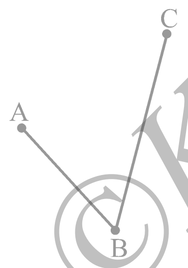
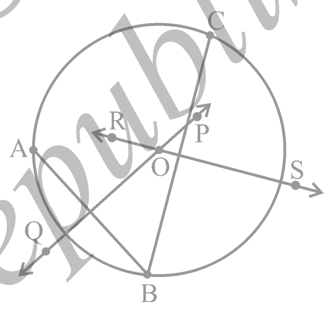


Fig. 12.18

So, let us take three points A, B and C, which are not on the same line, or in other words, they are not collinear [see Fig. 12.19(i)]. Draw perpendicular bisectors of AB and BC say, PQ and RS respectively. Let these perpendicular bisectors intersect at one point O. (Note that PQ and RS will intersect because they are not parallel) [see Fig. 12.19(ii)].



(i)



(ii)

Fig. 12.19

Now O lies on the perpendicular bisector PQ of AB, you have OA = OB, as every point on the perpendicular bisector of a line segment is equidistant from its end points, proved in Chapter 7.

Similarly, as O lies on the perpendicular bisector RS of BC, you get

$$OB = OC$$

So OA = OB = OC, which means that the points A, B and C are at equal distances from the point O. So if you draw a circle with centre O and radius OA, it will also pass through B and C. This shows that there is a circle passing through the three points A, B and C. You know that two lines (perpendicular bisectors) can intersect at only one point, so you can draw only one circle with radius OA. In other words, there is a unique circle passing through A, B and C. You have now proved the following theorem:

Theorem 12.5 : *There is one and only one circle passing through three given non-collinear points.*

Remark : If ABC is a triangle, then by Theorem 12.5, there is a unique circle passing through the three vertices A, B and C of the triangle. This circle is called the *circumcircle* of the Δ ABC. Its centre and radius are called respectively the *circumcentre* and the *circumradius* of the triangle.

Example 1 : Given an arc of a circle, complete the circle.

Solution : Let arc PQ of a circle be given. We have to complete the circle, which means that we have to find its centre and radius. Take a point R on the arc. Join PR and RQ. Use the construction that has been used in proving Theorem 12.5, to find the centre and radius.

Taking the centre and the radius so obtained, we can complete the circle (see Fig. 12.20).

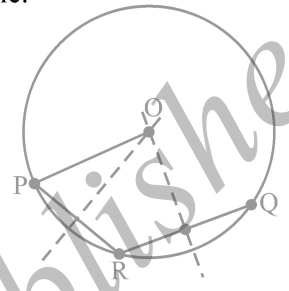


Fig. 12.20

EXERCISE 12.3

1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?
2. Suppose you are given a circle. Give a construction to find its centre.
3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

12.6 Equal Chords and Their Distances from the Centre

Let AB be a line and P be a point. Since there are infinite numbers of points on a line, if you join these points to P, you will get infinitely many line segments PL_1, PL_2, PM, PL_3, PL_4, etc. Which of these is the distance of AB from P? You may think a while and get the answer. Out of these line segments, the perpendicular from P to AB, namely PM in Fig. 12.21, will be the least. In Mathematics, we define this least length PM to be **the distance of AB from P**. So you may say that:

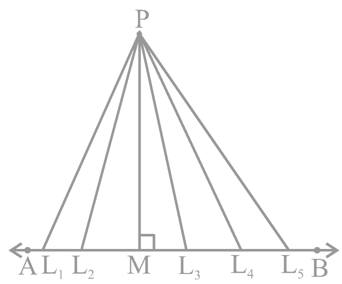


Fig. 12.21

The length of the perpendicular from a point to a line is the distance of the line from the point.

Note that if the point lies on the line, the distance of the line from the point is zero.

A circle can have infinitely many chords. You may observe by drawing chords of a circle that longer chord is nearer to the centre than the smaller chord. You may observe it by drawing several chords of a circle of different lengths and measuring their distances from the centre. What is the distance of the diameter, which is the longest chord from the centre? Since the centre lies on it, the distance is zero. Do you think that there is some relationship between the length of chords and their distances from the centre? Let us see if this is so.

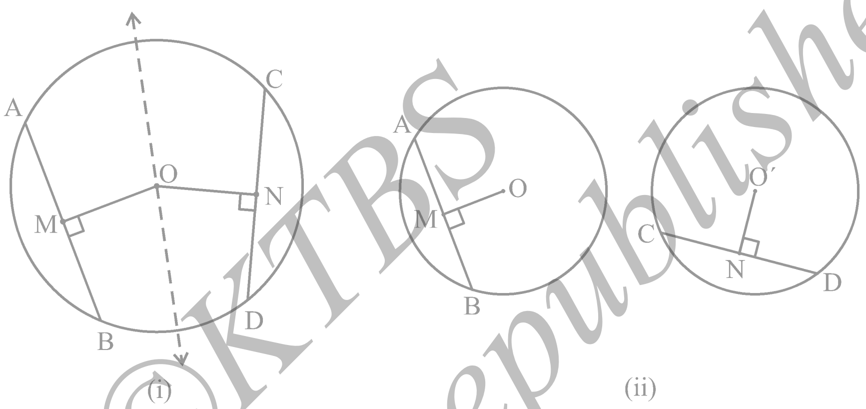


Fig. 12.22

Activity : Draw a circle of any radius on a tracing paper. Draw two equal chords AB and CD of it and also the perpendiculars OM and ON on them from the centre O. Fold the figure so that D falls on B and C falls on A [see Fig.12.22 (i)]. You may observe that O lies on the crease and N falls on M. Therefore, OM = ON. Repeat the activity by drawing congruent circles with centres O and O′ and taking equal chords AB and CD one on each. Draw perpendiculars OM and O′N on them [see Fig. 12.22(ii)]. Cut one circular disc and put it on the other so that AB coincides with CD. Then you will find that O coincides with O′ and M coincides with N. In this way you verified the following:

Theorem 12.6 : *Equal chords of a circle* (*or of congruent circles*) *are equidistant from the centre* (*or centres*).

Next, it will be seen whether the converse of this theorem is true or not. For this, draw a circle with centre O. From the centre O, draw two line segments OL and OM of equal length and lying inside the circle [see Fig. 12.23(i)]. Then draw chords PQ and RS of the circle perpendicular to OL and OM respectively [see Fig 12.23(ii)]. Measure the lengths of PQ and RS. Are these different? No, both are equal. Repeat the activity for more equal line segments and drawing the chords perpendicular to

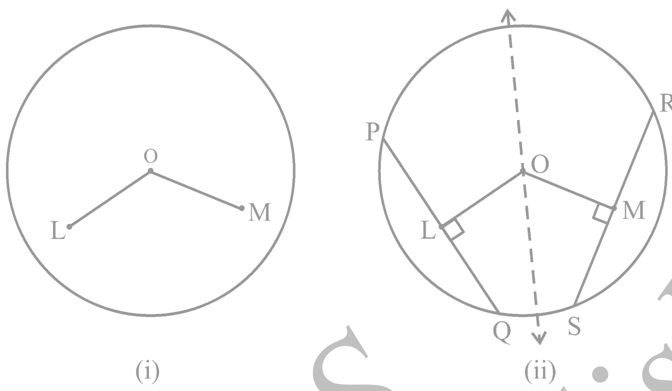


Fig. 12.23

them. This verifies the converse of the Theorem 12.6 which is stated as follows:

Theorem 12.7 : *Chords equidistant from the centre of a circle are equal in length.*

We now take an example to illustrate the use of the above results:

Example 2 : If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Solution : Given that AB and CD are two chords of a circle, with centre O intersecting at a point E. PQ is a diameter through E, such that $\angle$ AEQ = $\angle$ DEQ (see Fig.12.24). You have to prove that AB = CD. Draw perpendiculars OL and OM on chords AB and CD, respectively. Now

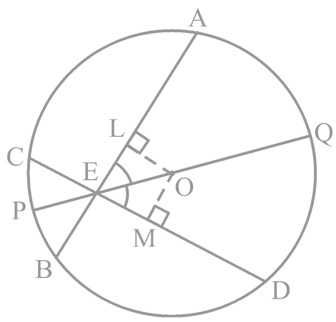


Fig. 12.24

$\angle$ LOE = 180° – 90° – $\angle$ LEO = 90° – $\angle$ LEO
(Angle sum property of a triangle)
= 90° – $\angle$ AEQ = 90° – $\angle$ DEQ
= 90° – $\angle$ MEO = $\angle$ MOE

In triangles OLE and OME,

	$\angle$ LEO = $\angle$ MEO	(Why ?)
	$\angle$ LOE = $\angle$ MOE	(Proved above)
	EO = EO	(Common)
Therefore,	Δ OLE $\cong$ Δ OME	(Why ?)
This gives	OL = OM	(CPCT)
So,	AB = CD	(Why ?)

EXERCISE 12.4

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.
2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.
3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.
4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 12.25).
5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Fig. 12.25

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

12.7 Angle Subtended by an Arc of a Circle

You have seen that the end points of a chord other than diameter of a circle cuts it into two arcs – one major and other minor. If you take two equal chords, what can you say about the size of arcs? Is one arc made by first chord equal to the corresponding arc made by another chord? In fact, they are more than just equal in length. They are congruent in the sense that if one arc is put on the other, without bending or twisting, one superimposes the other completely.

You can verify this fact by cutting the arc, corresponding to the chord CD from the circle along CD and put it on the corresponding arc made by equal chord AB. You will find that the arc CD superimpose the arc AB completely (see Fig. 12.26). This shows that equal chords make congruent arcs and conversely congruent arcs make equal chords of a circle. You can state it as follows:

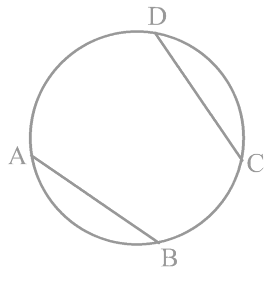


Fig. 12.26

If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.

Also the angle subtended by an arc at the centre is defined to be angle subtended by the corresponding chord at the centre in the sense that the minor arc subtends the angle and the major arc subtends the reflex angle. Therefore, in Fig 12.27, the angle subtended by the minor arc PQ at O is $\angle$POQ and the angle subtended by the major arc PQ at O is reflex angle POQ.

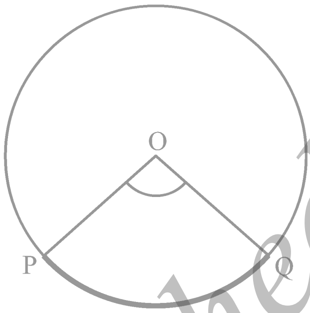


Fig. 12.27

In view of the property above and Theorem 12.1, the following result is true:

Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

Therefore, the angle subtended by a chord of a circle at its centre is equal to the angle subtended by the corresponding (minor) arc at the centre. The following theorem gives the relationship between the angles subtended by an arc at the centre and at a point on the circle.

Theorem 12.8 : *The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

Proof : Given an arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle. We need to prove that $\angle$ POQ = 2 $\angle$ PAQ.

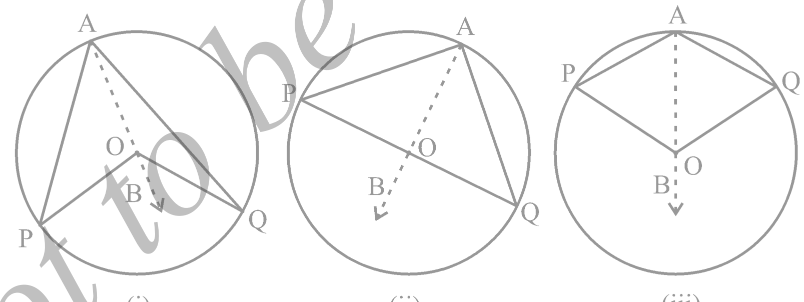


Fig. 12.28

Consider the three different cases as given in Fig. 12.28. In (i), arc PQ is minor; in (ii), arc PQ is a semicircle and in (iii), arc PQ is major.

Let us begin by joining AO and extending it to a point B.

In all the cases,

$$\angle \text{ BOQ} = \angle \text{ OAQ} + \angle \text{ AQO}$$

because an exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Also in Δ OAQ,

$$OA = OQ \qquad \text{(Radii of a circle)}$$

Therefore, $\angle OAQ = \angle OQA$ (Theorem 5.5)

This gives $\angle BOQ = 2 \angle OAQ$ (1)

Similarly, $\angle BOP = 2 \angle OAP$ (2)

From (1) and (2), $\angle BOP + \angle BOQ = 2(\angle OAP + \angle OAQ)$

This is the same as $\angle POQ = 2 \angle PAQ$ (3)

For the case (iii), where PQ is the major arc, (3) is replaced by

$$\text{reflex angle } POQ = 2 \angle PAQ$$

Remark : Suppose we join points P and Q and form a chord PQ in the above figures. Then $\angle PAQ$ is also called the angle formed in the segment PAQP.

In Theorem 12.8, A can be any point on the remaining part of the circle. So if you take any other point C on the remaining part of the circle (see Fig. 12.29), you have

$$\angle POQ = 2 \angle PCQ = 2 \angle PAQ$$

Therefore, $\angle PCQ = \angle PAQ$.

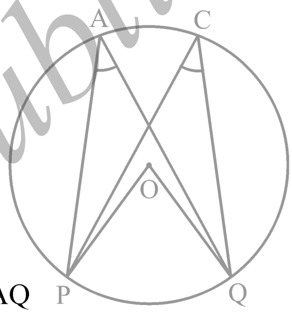


Fig. 12.29

This proves the following:

Theorem 12.9 : *Angles in the same segment of a circle are equal.*

Again let us discuss the case (ii) of Theorem 12.8 separately. Here $\angle PAQ$ is an angle in the segment, which is a semicircle. Also, $\angle PAQ = \frac{1}{2} \angle POQ = \frac{1}{2} \times 180° = 90°$. If you take any other point C on the semicircle, again you get that

$$\angle PCQ = 90°$$

Therefore, you find another property of the circle as:

Angle in a semicircle is a right angle.

The converse of Theorem 12.9 is also true. It can be stated as:

Theorem 12.10 : *If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).*

You can see the truth of this result as follows:

In Fig. 12.30, AB is a line segment, which subtends equal angles at two points C and D. That is

$$\angle ACB = \angle ADB$$

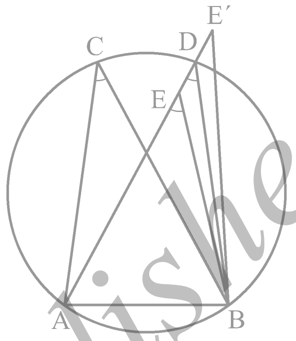


Fig. 12.30

To show that the points A, B, C and D lie on a circle let us draw a circle through the points A, C and B. Suppose it does not pass through the point D. Then it will intersect AD (or extended AD) at a point, say E (or E′).

If points A, C, E and B lie on a circle,

$\angle ACB = \angle AEB$ (Why?)

But it is given that $\angle ACB = \angle ADB$.

Therefore, $\angle AEB = \angle ADB$.

This is not possible unless E coincides with D. (Why?)

Similarly, E′ should also coincide with D.

12.8 Cyclic Quadrilaterals

A quadrilateral ABCD is called *cyclic* if all the four vertices of it lie on a circle (see Fig 12.31). You will find a peculiar property in such quadrilaterals. Draw several cyclic quadrilaterals of different sides and name each of these as ABCD. (This can be done by drawing several circles of different radii and taking four points on each of them.) Measure the opposite angles and write your observations in the following table.

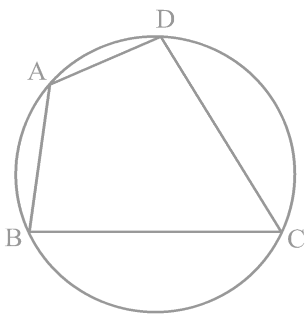


Fig. 12.31

S.No. of Quadrilateral	$\angle A$	$\angle B$	$\angle C$	$\angle D$	$\angle A + \angle C$	$\angle B + \angle D$
1.						
2.						
3.						
4.						
5.						
6.						

What do you infer from the table?

You find that $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$, neglecting the error in measurements. This verifies the following:

Theorem 12.11 : *The sum of either pair of opposite angles of a cyclic quadrilateral is* 180°.

In fact, the converse of this theorem, which is stated below is also true.

Theorem 12.12 : *If the sum of a pair of opposite angles of a quadrilateral is* 180°, *the quadrilateral is cyclic.*

You can see the truth of this theorem by following a method similar to the method adopted for Theorem 12.10.

Example 3 : In Fig. 12.32, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that $\angle AEB = 60°$.

Solution : Join OC, OD and BC.

Triangle ODC is equilateral (Why?)

Therefore, $\angle COD = 60°$

Now, $\angle CBD = \frac{1}{2} \angle COD$ (Theorem 10.8)

This gives $\angle CBD = 30°$

Again, $\angle ACB = 90°$ (Why ?)

So, $\angle BCE = 180° - \angle ACB = 90°$

Which gives $\angle CEB = 90° - 30° = 60°$, i.e., $\angle AEB = 60°$

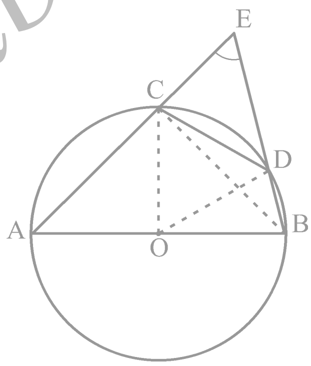


Fig. 12.32

Example 4 : In Fig 12.33, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.

Solution : $\angle CAD = \angle DBC = 55°$
(Angles in the same segment)

Therefore, $\angle DAB = \angle CAD + \angle BAC$
$= 55° + 45° = 100°$

But $\angle DAB + \angle BCD = 180°$
(Opposite angles of a cyclic quadrilateral)

So, $\angle BCD = 180° - 100° = 80°$

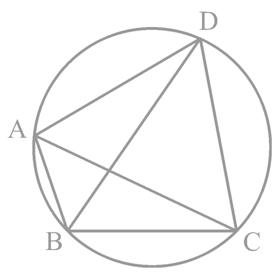


Fig. 12.33

Example 5 : Two circles intersect at two points A and B. AD and AC are diameters to the two circles (see Fig.12.34). Prove that B lies on the line segment DC.

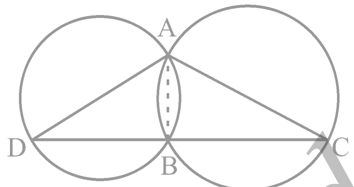


Fig. 12.34

Solution : Join AB.

$$\angle \text{ ABD} = 90° \quad \text{(Angle in a semicircle)}$$

$$\angle \text{ ABC} = 90° \quad \text{(Angle in a semicircle)}$$

So, $\angle \text{ ABD} + \angle \text{ ABC} = 90° + 90° = 180°$

Therefore, DBC is a line. That is B lies on the line segment DC.

Example 6 : Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.

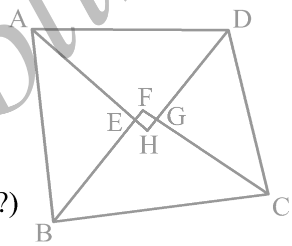


Fig. 12.35

Solution : In Fig. 12.35, ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral EFGH.

Now, $\angle \text{ FEH} = \angle \text{ AEB} = 180° - \angle \text{ EAB} - \angle \text{ EBA}$ (Why ?)

$$= 180° - \frac{1}{2}(\angle \text{ A} + \angle \text{ B})$$

and $\angle \text{ FGH} = \angle \text{ CGD} = 180° - \angle \text{ GCD} - \angle \text{ GDC}$ (Why ?)

$$= 180° - \frac{1}{2}(\angle \text{ C} + \angle \text{ D})$$

Therefore, $\angle \text{ FEH} + \angle \text{ FGH} = 180° - \frac{1}{2}(\angle \text{ A} + \angle \text{ B}) + 180° - \frac{1}{2}(\angle \text{ C} + \angle \text{ D})$

$$= 360° - \frac{1}{2}(\angle \text{ A} + \angle \text{ B} + \angle \text{ C} + \angle \text{ D}) = 360° - \frac{1}{2} \times 360°$$

$$= 360° - 180° = 180°$$

Therefore, by Theorem 12.12, the quadrilateral EFGH is cyclic.

EXERCISE 12.5

1. In Fig. 12.36, A,B and C are three points on a circle with centre O such that $\angle$ BOC = 30° and $\angle$ AOB = 60°. If D is a point on the circle other than the arc ABC, find $\angle$ADC.

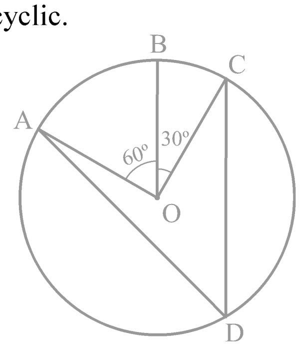


Fig. 12.36

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.
3. In Fig. 12.37, $\angle$ PQR = 100°, where P, Q and R are points on a circle with centre O. Find $\angle$ OPR.

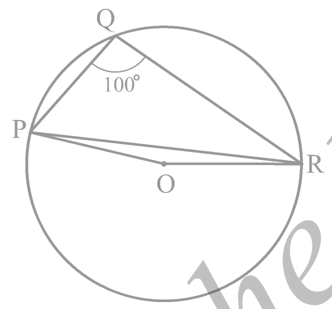


Fig. 12.37

4. In Fig. 12.38, $\angle$ ABC = 69°, $\angle$ ACB = 31°, find $\angle$ BDC.

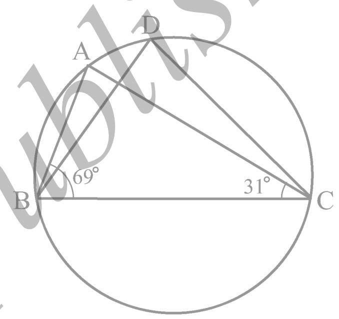


Fig. 12.38

5. In Fig. 12.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle$ BEC = 130° and $\angle$ ECD = 20°. Find $\angle$ BAC.

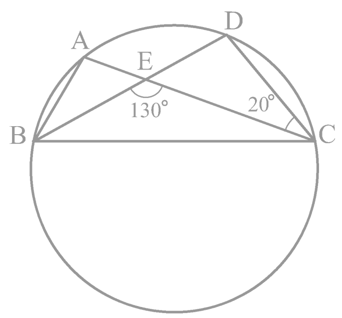


Fig. 12.39

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle$ DBC = 70°, $\angle$ BAC is 30°, find $\angle$ BCD. Further, if AB = BC, find $\angle$ ECD.
7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.
8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 12.40). Prove that $\angle ACP = \angle QCD$.

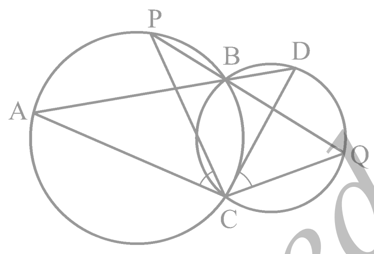


Fig. 12.40

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

12. Prove that a cyclic parallelogram is a rectangle.

EXERCISE 12.6 (Optional)*

1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.

8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}A$, $90° - \frac{1}{2}B$ and $90° - \frac{1}{2}C$.

*These exercises are not from examination point of view.

9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

10. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

12.9 Summary

In this chapter, you have studied the following points:

1. A circle is the collection of all points in a plane, which are equidistant from a fixed point in the plane.
2. Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
3. If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centres) are equal, the chords are equal.
4. The perpendicular from the centre of a circle to a chord bisects the chord.
5. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
6. There is one and only one circle passing through three non-collinear points.
7. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).
8. Chords equidistant from the centre (or corresponding centres) of a circle (or of congruent circles) are equal.
9. If two arcs of a circle are congruent, then their corresponding chords are equal and conversely if two chords of a circle are equal, then their corresponding arcs (minor, major) are congruent.
10. Congruent arcs of a circle subtend equal angles at the centre.
11. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
12. Angles in the same segment of a circle are equal.
13. Angle in a semicircle is a right angle.
14. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.
15. The sum of either pair of opposite angles of a cyclic quadrilateral is 180^0.
16. If sum of a pair of opposite angles of a quadrilateral is 180^0, the quadrilateral is cyclic.

CHAPTER 13

SURFACE AREAS AND VOLUMES

13.1 Introduction

Wherever we look, usually we see solids. So far, in all our study, we have been dealing with figures that can be easily drawn on our notebooks or blackboards. These are called *plane figures*. We have understood what rectangles, squares and circles are, what we mean by their perimeters and areas, and how we can find them. We have learnt these in earlier classes. It would be interesting to see what happens if we cut out many of these plane figures of the same shape and size from cardboard sheet and stack them up in a vertical pile. By this process, we shall obtain some *solid figures* (briefly called *solids*) such as a cuboid, a cylinder, etc. In the earlier classes, you have also learnt to find the surface areas and volumes of cuboids, cubes and cylinders. We shall now learn to find the surface areas and volumes of cuboids and cylinders in details and extend this study to some other solids such as cones and spheres.

13.2 Surface Area of a Cuboid and a Cube

Have you looked at a bundle of many sheets of paper? How does it look? Does it look like what you see in Fig. 13.1?

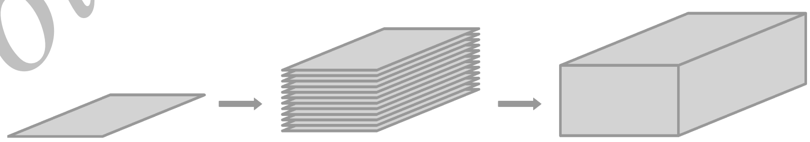
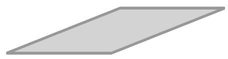


Fig. 13.1

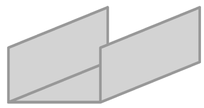
That makes up a cuboid. How much of brown paper would you need, if you want to cover this cuboid? Let us see:

First we would need a rectangular piece to cover the bottom of the bundle. That would be as shown in Fig. 13.2 (a)



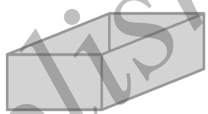
(a)

Then we would need two long rectangular pieces to cover the two side ends. Now, it would look like Fig. 13.2 (b).



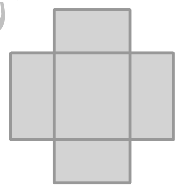
(b)

Now to cover the front and back ends, we would need two more rectangular pieces of a different size. With them, we would now have a figure as shown in Fig. 13.2(c).



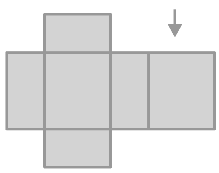
(c)

This figure, when opened out, would look like Fig. 13.2 (d).



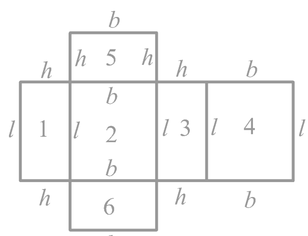
(d)

Finally, to cover the top of the bundle, we would require another rectangular piece exactly like the one at the bottom, which if we attach on the right side, it would look like Fig. 13.2(e).



(e)

So we have used six rectangular pieces to cover the complete outer surface of the cuboid.



(f)

Fig. 13.2

This shows us that the outer surface of a cuboid is made up of six rectangles (in fact, rectangular regions, called the faces of the cuboid), whose areas can be found by multiplying length by breadth for each of them separately and then adding the six areas together.

Now, if we take the length of the cuboid as l, breadth as b and the height as h, then the figure with these dimensions would be like the shape you see in Fig. 13.2(f).

So, the sum of the areas of the six rectangles is:

Area of rectangle 1 $(= l \times h)$

$+$

Area of rectangle 2 $(= l \times b)$

$+$

Area of rectangle 3 $(= l \times h)$

$+$

Area of rectangle 4 $(= l \times b)$

$+$

Area of rectangle 5 $(= b \times h)$

$+$

Area of rectangle 6 $(= b \times h)$

$= 2(l \times b) + 2(b \times h) + 2(l \times h)$

$= 2(lb + bh + hl)$

This gives us:

$$\boxed{\textbf{Surface Area of a Cuboid} = 2(lb + bh + hl)}$$

where l, b and h are respectively the three edges of the cuboid.

Note : The unit of area is taken as the square unit, because we measure the magnitude of a region by filling it with squares of side of unit length.

For example, if we have a cuboid whose length, breadth and height are 15 cm, 10 cm and 20 cm respectively, then its surface area would be:

$$2[(15 \times 10) + (10 \times 20) + (20 \times 15)] \text{ cm}^2$$
$$= 2(150 + 200 + 300) \text{ cm}^2$$
$$= 2 \times 650 \text{ cm}^2$$
$$= 1300 \text{ cm}^2$$

Recall that a cuboid, whose length, breadth and height are all equal, is called a *cube*. If each edge of the cube is a, then the surface area of this cube would be

$$2(a \times a + a \times a + a \times a), \text{ i.e., } 6a^2 \text{ (see Fig. 13.3), giving us}$$

$$\boxed{\textbf{Surface Area of a Cube} = 6a^2}$$

where a is the edge of the cube.

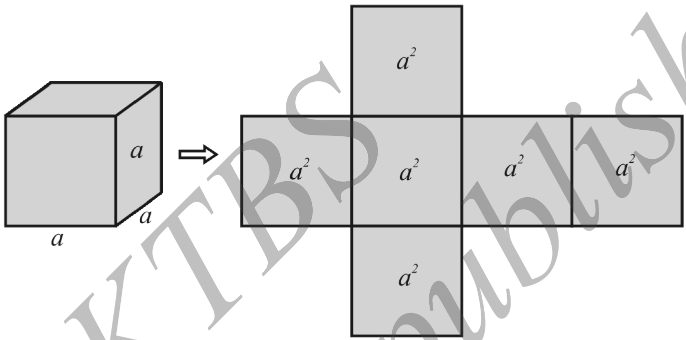


Fig. 13.3

Suppose, out of the six faces of a cuboid, we only find the area of the four faces, leaving the bottom and top faces. In such a case, the area of these four faces is called the **lateral surface area** of the cuboid. So, *lateral surface area of a cuboid of length l, breadth b and height h is equal to 2lh + 2bh or 2(l + b)h.* Similarly, *lateral surface area of a cube of side a is equal to $4a^2$.*

Keeping in view of the above, the surface area of a cuboid (or a cube) is sometimes also referred to as the **total surface area**. Let us now solve some examples.

Example 1 : Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden box covered with coloured paper with picture of Santa Claus on it (see Fig. 13.4). She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth and height as 80 cm, 40 cm and 20 cm respectively how many square sheets of paper of side 40 cm would she require?

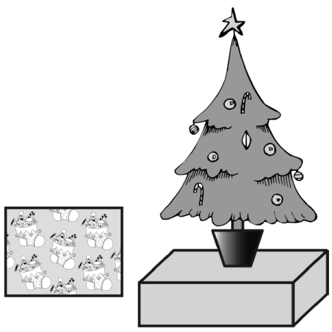


Fig. 13.4

Solution : Since Mary wants to paste the paper on the outer surface of the box; the quantity of paper required would be equal to the surface area of the box which is of the shape of a cuboid. The dimensions of the box are:

Length =80 cm, Breadth = 40 cm, Height = 20 cm.

The surface area of the box = $2(lb + bh + hl)$

$$= 2[(80 \times 40) + (40 \times 20) + (20 \times 80)] \text{ cm}^2$$

$$= 2[3200 + 800 + 1600] \text{ cm}^2$$

$$= 2 \times 5600 \text{ cm}^2 = 11200 \text{ cm}^2$$

The area of each sheet of the paper = 40×40 cm^2

$$= 1600 \text{ cm}^2$$

Therefore, number of sheets required $= \dfrac{\text{surface area of box}}{\text{area of one sheet of paper}}$

$$= \frac{11200}{1600} = 7$$

So, she would require 7 sheets.

Example 2 : Hameed has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm (see Fig. 13.5). Find how much he would spend for the tiles, if the cost of the tiles is ₹ 360 per dozen.

Solution : Since Hameed is getting the five outer faces of the tank covered with tiles, he would need to know the surface area of the tank, to decide on the number of tiles required.

Edge of the cubical tank = 1.5 m = 150 cm (= a)

So, surface area of the tank = $5 \times 150 \times 150$ cm^2

Area of each square tile = side × side = 25×25 cm^2

So, the number of tiles required $= \dfrac{\text{surface area of the tank}}{\text{area of each tile}}$

$$= \frac{5 \times 150 \times 150}{25 \times 25} = 180$$

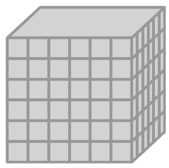


Fig. 13.5

Cost of 1 dozen tiles, i.e., cost of 12 tiles = ₹ 360

Therefore, cost of one tile = ₹ $\dfrac{360}{12}$ = ₹ 30

So, the cost of 180 tiles = 180 × ₹ 30 = ₹ 5400

EXERCISE 13.1

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:
   (i) The area of the sheet required for making the box.
   (ii) The cost of sheet for it, if a sheet measuring $1 m^2$ costs Rs 20.
2. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 7.50 per m^2.
3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹ 10 per m^2 is ₹ 15000, find the height of the hall.
   [**Hint :** Area of the four walls = Lateral surface area.]
4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?
5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.
   (i) Which box has the greater lateral surface area and by how much?
   (ii) Which box has the smaller total surface area and by how much?
6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.
   (i) What is the area of the glass?
   (ii) How much of tape is needed for all the 12 edges?
7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is ₹ 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.
8. Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

13.3 Surface Area of a Right Circular Cylinder

If we take a number of circular sheets of paper and stack them up as we stacked up rectangular sheets earlier, what would we get (see Fig. 13.6)?

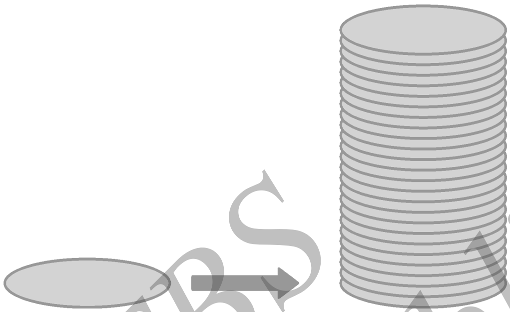


Fig. 13.6

Here, if the stack is kept vertically up, we get what is called a *right circular cylinder*, since it has been kept at right angles to the base, and the base is circular. Let us see what kind of cylinder is *not* a right circular cylinder.

In Fig 13.7 (a), you see a cylinder, which is certainly circular, but it is not at right angles to the base. So, we can *not* say this a *right* circular cylinder.

Of course, if we have a cylinder with a non circular base, as you see in Fig. 13.7 (b), then we also cannot call it a right circular cylinder.

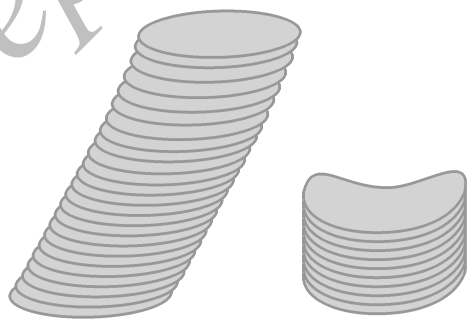


Fig. 13.7

Remark : Here, we will be dealing with only right circular cylinders. So, unless stated otherwise, the word cylinder would mean a right circular cylinder.

Now, if a cylinder is to be covered with coloured paper, how will we do it with the minimum amount of paper? First take a rectangular sheet of paper, whose length is just enough to go round the cylinder and whose breadth is equal to the height of the cylinder as shown in Fig. 13.8.

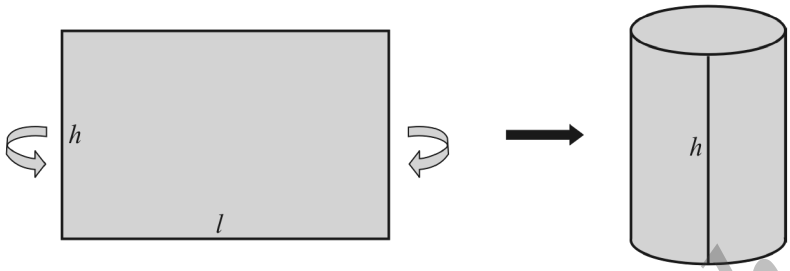


Fig. 13.8

The area of the sheet gives us the curved surface area of the cylinder. Note that the length of the sheet is equal to the circumference of the circular base which is equal to $2\pi r$.

So, curved surface area of the cylinder

$$= \text{area of the rectangular sheet} = \text{length} \times \text{breadth}$$
$$= \text{perimeter of the base of the cylinder} \times h$$
$$= 2\pi r \times h$$

Therefore, $\boxed{\textbf{Curved Surface Area of a Cylinder} = 2\pi rh}$

where r is the radius of the base of the cylinder and h is the height of the cylinder.

Remark : In the case of a cylinder, unless stated otherwise, 'radius of a cylinder' shall mean' base radius of the cylinder'.

If the top and the bottom of the cylinder are also to be covered, then we need two circles (infact, circular regions) to do that, each of radius r, and thus having an area of πr^2 each (see Fig. 13.9), giving us the total surface area as $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$.

So, $\boxed{\textbf{Total Surface Area of a Cylinder} = 2\pi r(r + h)}$

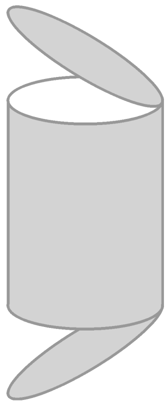


Fig. 13.9

where h is the height of the cylinder and r its radius.

Remark : You may recall from Chapter 1 that π is an irrational number. So, the value of π is a non-terminating, non-repeating decimal. But when we use its value in our calculations, we usually take its value as approximately equal to $\frac{22}{7}$ or 3.14.

Example 3 : Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. (see Fig 13.10). What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius? You may take $\pi = \frac{22}{7}$.

Solution : Radius of the base of the cylindrical kaleidoscope (r) = 3.5 cm.

Height (length) of kaleidoscope (h) = 25 cm.

Area of chart paper required = curved surface area of the kaleidoscope

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 3.5 \times 25 \text{ cm}^2$$

$$= 550 \text{ cm}^2$$

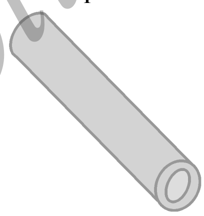


Fig. 13.10

EXERCISE 13.2

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.
2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?
3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see Fig. 13.11). Find its
   (i) inner curved surface area,
   (ii) outer curved surface area,
   (iii) total surface area.

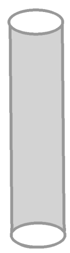


Fig. 13.11

4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.
5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹ 12.50 per m^2.
6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.
7. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find
   (i) its inner curved surface area,
   (ii) the cost of plastering this curved surface at the rate of ₹ 40 per m^2.
8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.
9. Find
   (i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.
   (ii) how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank.
10. In Fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.

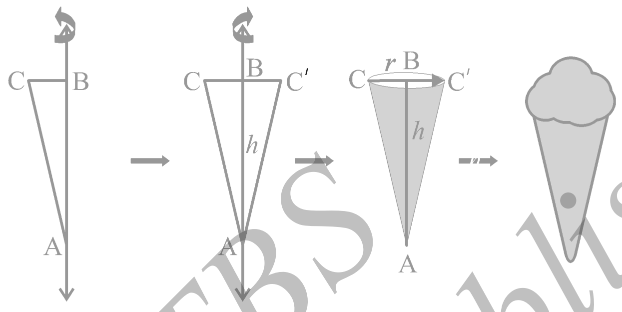
Fig. 13.12

11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

13.4 Surface Area of a Right Circular Cone

So far, we have been generating solids by stacking up congruent figures. Incidentally, such figures are called *prisms*. Now let us look at another kind of solid which is not a prism (These kinds of solids are called *pyramids.*). Let us see how we can generate them.

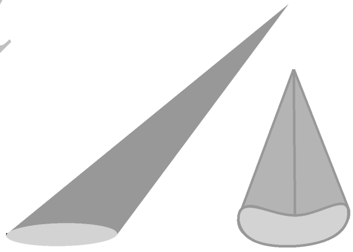
Activity : Cut out a right-angled triangle ABC right angled at B. Paste a long thick string along one of the perpendicular sides say AB of the triangle [see Fig. 13.13(a)]. Hold the string with your hands on either sides of the triangle and rotate the triangle about the string a number of times. What happens? Do you recognize the shape that the triangle is forming as it rotates around the string [see Fig. 13.13(b)]? Does it remind you of the time you had eaten an ice-cream heaped into a container of that shape [see Fig. 13.13 (c) and (d)]?



(a) (b) (c) (d)

Fig. 13.13

This is called a *right circular cone*. In Fig. 13.13(c) of the right circular cone, the point A is called the vertex, AB is called the height, BC is called the *radius* and AC is called the slant height of the cone. Here B will be the centre of circular base of the cone. The height, radius and slant height of the cone are usually denoted by h, r and l respectively. Once again, let us see what kind of cone we can *not* call a right circular cone. Here, you are (see Fig. 13.14)! What you see in these figures are not right circular cones; because in (a), the line joining its vertex to the centre of its base is not at right angle to the base, and in (b) the base is not circular.



(a) (b)

Fig. 13.14

As in the case of cylinder, since we will be studying only about right circular cones, remember that by 'cone' in this chapter, we shall mean a 'right circular cone.'

Activity : (i) Cut out a neatly made paper cone that does not have any overlapped paper, straight along its side, and opening it out, to see the shape of paper that forms the surface of the cone. (The line along which you cut the cone is the *slant height* of the cone which is represented by l). It looks like a part of a round cake.

(ii) If you now bring the sides marked A and B at the tips together, you can see that the curved portion of Fig. 13.15 (c) will form the circular base of the cone.

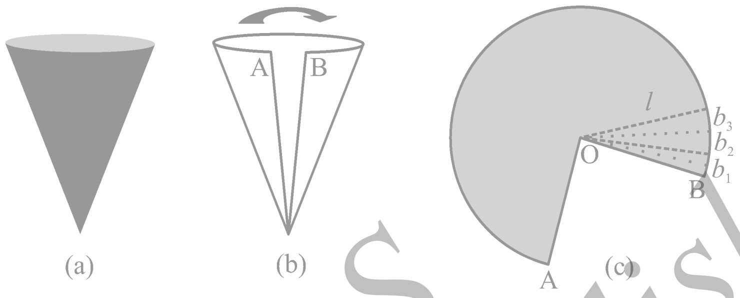


Fig. 13.15

(iii) If the paper like the one in Fig. 13.15 (c) is now cut into hundreds of little pieces, along the lines drawn from the point O, each cut portion is almost a small triangle, whose height is the slant height l of the cone.

(iv) Now the area of each triangle = $\frac{1}{2}$ × base of each triangle × l.

So, area of the entire piece of paper

$$= \text{sum of the areas of all the triangles}$$

$$= \frac{1}{2}b_1 l + \frac{1}{2}b_2 l + \frac{1}{2}b_3 l + \cdots = \frac{1}{2}l\left(b_1 + b_2 + b_3 + \cdots\right)$$

$$= \frac{1}{2} \times l \times \text{length of entire curved boundary of Fig. 13.15(c)}$$

(as $b_1 + b_2 + b_3 + \ldots$ makes up the curved portion of the figure)

But the curved portion of the figure makes up the perimeter of the base of the cone and the circumference of the base of the cone = $2\pi r$, where r is the base radius of the cone.

So, $\boxed{\textbf{Curved Surface Area of a Cone} = \frac{1}{2} \times l \times 2\pi r = \pi r l}$

where r is its base radius and l its slant height.

Note that $l^2 = r^2 + h^2$ (as can be seen from Fig. 13.16), by applying Pythagoras Theorem. Here h is the *height* of the cone.

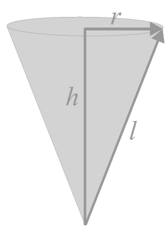


Fig. 13.16

Therefore, $l = \sqrt{r^2 + h^2}$

Now if the base of the cone is to be closed, then a circular piece of paper of radius r is also required whose area is πr^2.

So, $\boxed{\textbf{Total Surface Area of a Cone} = \pi rl + \pi r^2 = \pi r(l + r)}$

Example 4 : Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.

Solution : Curved surface area $= \pi rl$

$$= \frac{22}{7} \times 7 \times 10 \text{ cm}^2$$

$$= 220 \text{ cm}^2$$

Example 5 : The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone (Use $\pi = 3.14$).

Solution : Here, $h = 16$ cm and $r = 12$ cm.

So, from $l^2 = h^2 + r^2$, we have

$$l = \sqrt{16^2 + 12^2} \text{ cm} = 20 \text{ cm}$$

So, curved surface area $= \pi rl$

$$= 3.14 \times 12 \times 20 \text{ cm}^2$$

$$= 753.6 \text{ cm}^2$$

Further, total surface area $= \pi rl + \pi r^2$

$$= (753.6 + 3.14 \times 12 \times 12) \text{ cm}^2$$

$$= (753.6 + 452.16) \text{ cm}^2$$

$$= 1205.76 \text{ cm}^2$$

Example 6 : A corn cob (see Fig. 13.17), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

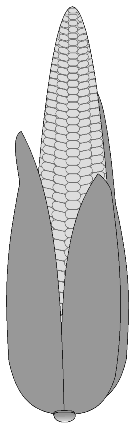


Fig. 13.17

Solution : Since the grains of corn are found only on the curved surface of the corn cob, we would need to know the curved surface area of the corn cob to find the total number of grains on it. In this question, we are given the height of the cone, so we need to find its slant height.

Here, $l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + 20^2}$ cm

$= \sqrt{404.41}$ cm = 20.11 cm

Therefore, the curved surface area of the corn cob = πrl

$= \frac{22}{7} \times 2.1 \times 20.11 \text{ cm}^2 = 132.726 \text{ cm}^2 = 132.73 \text{ cm}^2$ (approx.)

Number of grains of corn on 1 cm^2 of the surface of the corn cob = 4

Therefore, number of grains on the entire curved surface of the cob

= 132.73 × 4 = 530.92 = 531 (approx.)

So, there would be approximately 531 grains of corn on the cob.

EXERCISE 13.3

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.
2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.
3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.
4. A conical tent is 10 m high and the radius of its base is 24 m. Find

   (i) slant height of the tent.

   (ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹ 70.
5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3.14$).
6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m^2.
7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.
8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

13.5 Surface Area of a Sphere

What is a sphere? Is it the same as a circle? Can you draw a circle on a paper? Yes, you can, because a circle is a plane closed figure whose every point lies at a constant distance (called **radius**) from a fixed point, which is called the **centre** of the circle. Now if you paste a string along a diameter of a circular disc and rotate it as you had rotated the triangle in the previous section, you see a new solid (see Fig 13.18). What does it resemble? A ball? Yes. It is called a **sphere**.

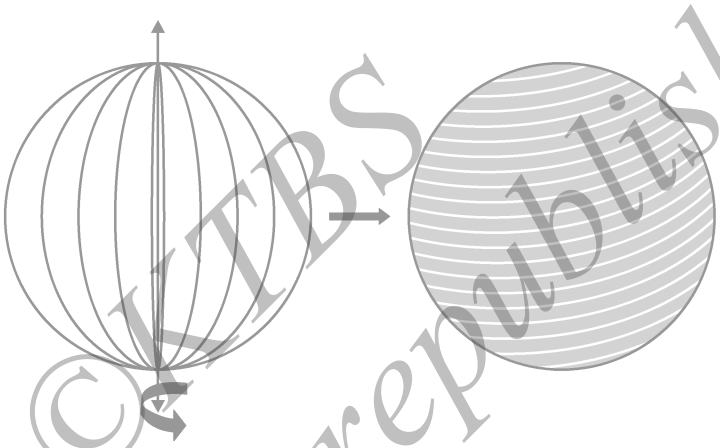


Fig. 13.18

Can you guess what happens to the centre of the circle, when it forms a sphere on rotation? Of course, it becomes the centre of the sphere. So, *a sphere is a three dimensional figure* (*solid figure*), *which is made up of all points in the space, which lie at a constant distance called the radius, from a fixed point called the centre of the sphere.*

Note : A sphere is like the surface of a ball. The word *solid sphere* is used for the solid whose surface is a sphere.

Activity : Have you ever played with a top or have you at least watched someone play with one? You must be aware of how a string is wound around it. Now, let us take a rubber ball and drive a nail into it. Taking support of the nail, let us wind a string around the ball. When you have reached the 'fullest' part of the ball, use pins to keep the string in place, and continue to wind the string around the remaining part of the ball, till you have completely covered the ball [see Fig. 13.19(a)]. Mark the starting and finishing points on the string, and slowly unwind the string from the surface of the ball.

Now, ask your teacher to help you in measuring the diameter of the ball, from which you easily get its radius. Then on a sheet of paper, draw four circles with radius equal to the radius of the ball. Start filling the circles one by one, with the string you had wound around the ball [see Fig. 13.19(b)].

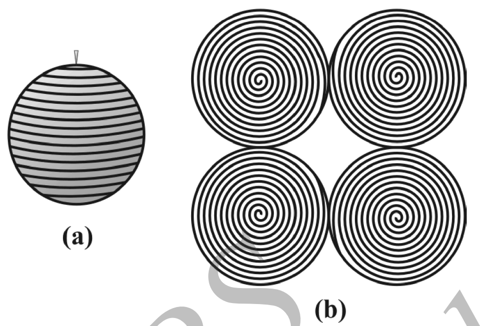


Fig. 13.19

What have you achieved in all this?

The string, which had completely covered the surface area of the sphere, has been used to completely fill the regions of four circles, all of the same radius as of the sphere.

So, what does that mean? This suggests that the surface area of a sphere of radius r

$= 4$ times the area of a circle of radius $r = 4 \times (\pi r^2)$

So, $\boxed{\textbf{Surface Area of a Sphere} = 4 \pi r^2}$

where r is the radius of the sphere.

How many faces do you see in the surface of a sphere? There is only one, which is curved.

Now, let us take a solid sphere, and slice it exactly 'through the middle' with a plane that passes through its centre. What happens to the sphere?

Yes, it gets divided into two equal parts (see Fig. 13.20)! What will each half be called? It is called a **hemisphere**. (Because 'hemi' also means 'half')

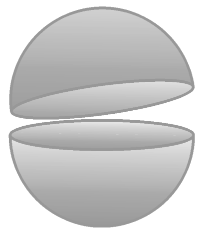


Fig. 13.20

And what about the surface of a hemisphere? How many faces does it have?

Two! There is a curved face and a flat face (base).

The curved surface area of a hemisphere is half the surface area of the sphere, which is $\frac{1}{2}$ of $4\pi r^2$.

Therefore, **Curved Surface Area of a Hemisphere = $2\pi r^2$**

where r is the radius of the sphere of which the hemisphere is a part.

Now taking the two faces of a hemisphere, its surface area $2\pi r^2 + \pi r^2$

So, **Total Surface Area of a Hemisphere = $3\pi r^2$**

Example 7 : Find the surface area of a sphere of radius 7 cm.

Solution : The surface area of a sphere of radius 7 cm would be

$$4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 616 \text{ cm}^2$$

Example 8 : Find (i) the curved surface area and (ii) the total surface area of a hemisphere of radius 21 cm.

Solution : The curved surface area of a hemisphere of radius 21 cm would be

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 2772 \text{ cm}^2$$

(ii) the total surface area of the hemisphere would be

$$3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 4158 \text{ cm}^2$$

Example 9 : The hollow sphere, in which the circus motorcyclist performs his stunts, has a diameter of 7 m. Find the area available to the motorcyclist for riding.

Solution : Diameter of the sphere = 7 m. Therefore, radius is 3.5 m. So, the riding space available for the motorcyclist is the surface area of the 'sphere' which is given by

$$4\pi r^2 = 4 \times \frac{22}{7} \times 3.5 \times 3.5 \text{ m}^2$$

$$= 154 \text{ m}^2$$

Example 10 : A hemispherical dome of a building needs to be painted (see Fig. 13.21). If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹ 5 per 100 cm^2.

Solution : Since only the rounded surface of the dome is to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done. Now, circumference of the dome = 17.6 m. Therefore, $17.6 = 2\pi r$.

So, the radius of the dome = $17.6 \times \frac{7}{2 \times 22}$ m = 2.8 m

The curved surface area of the dome = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 2.8 \times 2.8 \text{ m}^2$$

$$= 49.28 \text{ m}^2$$

Now, cost of painting 100 cm^2 is ₹ 5.

So, cost of painting 1 m^2 = ₹ 500

Therefore, cost of painting the whole dome

= ₹ 500 × 49.28

= ₹ 24640

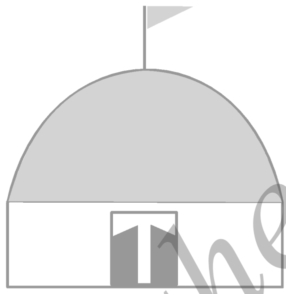


Fig. 13.21

EXERCISE 13.4

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Find the surface area of a sphere of radius:
   (i) 10.5 cm (ii) 5.6 cm (iii) 14 cm
2. Find the surface area of a sphere of diameter:
   (i) 14 cm (ii) 21 cm (iii) 3.5 m
3. Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)
4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.
5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹ 16 per 100 cm^2.
6. Find the radius of a sphere whose surface area is 154 cm^2.
7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.
8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.
9. A right circular cylinder just encloses a sphere of radius r (see Fig. 13.22). Find
   (i) surface area of the sphere,
   (ii) curved surface area of the cylinder,
   (iii) ratio of the areas obtained in (i) and (ii).

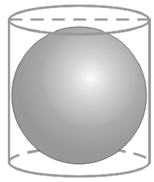


Fig. 13.22

13.6 Volume of a Cuboid

You have already learnt about volumes of certain figures (objects) in earlier classes. Recall that solid objects occupy space. The measure of this occupied space is called the **Volume** of the object.

Note : If an object is solid, then the space occupied by such an object is measured, and is termed the **Volume** of the object. On the other hand, if the object is hollow, then interior is empty, and can be filled with air, or some liquid that will take the shape of its container. In this case, the volume of the substance that can fill the interior is called the **capacity of the container**. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. Hence, the unit of measurement of either of the two is cubic unit.

So, if we were to talk of the volume of a cuboid, we would be considering the measure of the space occupied by the cuboid.

Further, the area or the volume is measured as the magnitude of a region. So, correctly speaking, we should be finding the area of a circular region, or volume of a cuboidal region, or volume of a spherical region, etc. But for the sake of simplicity, we say, find the area of a circle, volume of a cuboid or a sphere even though these mean only their boundaries.

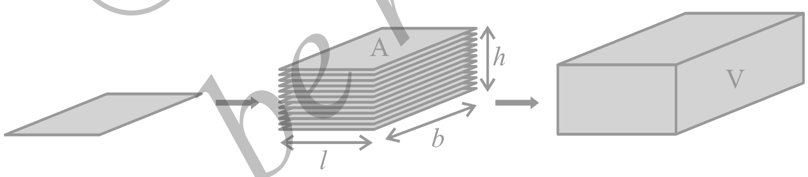


Fig. 13.23

Observe Fig. 13.23. Suppose we say that the area of each rectangle is A, the height up to which the rectangles are stacked is h and the volume of the cuboid is V. Can you tell what would be the relationship between V, A and h?

The area of the plane region occupied by each rectangle × height

= Measure of the space occupied by the cuboid

So, we get $\quad A \times h = V$

That is, **Volume of a Cuboid = base area × height = length × breadth × height**

or $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid.

Note : When we measure the magnitude of the region of a space, that is, the space occupied by a solid, we do so by counting the number of cubes of edge of unit length that can fit into it exactly. Therefore, the unit of measurement of volume is cubic unit.

Again $\boxed{\textbf{Volume of a Cube = edge} \times \textbf{edge} \times \textbf{edge} = a^3}$

where a is the edge of the cube (see Fig. 13.24).

So, if a cube has edge of 12 cm,

then volume of the cube = $12 \times 12 \times 12 \text{ cm}^3$

$= 1728 \text{ cm}^3$.

Recall that you have learnt these formulae in earlier classes. Now let us take some examples to illustrate the use of these formulae:

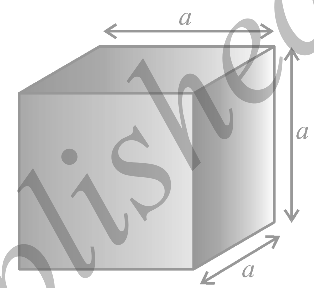


Fig. 13.24

Example11 : A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, how many bricks would be required?

Solution : Since the wall with all its bricks makes up the space occupied by it, we need to find the volume of the wall, which is nothing but a cuboid.

Here, Length = 10 m = 1000 cm

Thickness = 24 cm

Height = 4 m = 400 cm

Therefore, Volume of the wall = length × thickness × height

$= 1000 \times 24 \times 400 \text{ cm}^3$

Now, each brick is a cuboid with length = 24 cm, breadth = 12 cm and height = 8 cm

So, volume of each brick = length × breadth × height

$= 24 \times 12 \times 8 \text{ cm}^3$

So, number of bricks required $= \dfrac{\text{volume of the wall}}{\text{volume of each brick}}$

$$= \frac{1000 \times 24 \times 400}{24 \times 12 \times 8}$$

$= 4166.6$

So, the wall requires 4167 bricks.

Example 12 : A child playing with building blocks, which are of the shape of cubes, has built a structure as shown in Fig. 13.25. If the edge of each cube is 3 cm, find the volume of the structure built by the child.

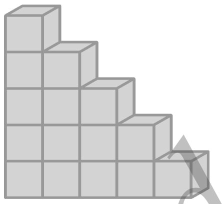


Fig. 13.25

Solution : Volume of each cube = edge × edge × edge

$= 3 \times 3 \times 3 \text{ cm}^3 = 27 \text{ cm}^3$

Number of cubes in the structure = 15

Therefore, volume of the structure = $27 \times 15 \text{ cm}^3$

$= 405 \text{ cm}^3$

EXERCISE 13.5

1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?
2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? ($1 \text{ m}^3 = 1000\ l$)
3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?
4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹ 30 per m^3.
5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.
6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?
7. A godown measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.
8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.
9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

13.7 Volume of a Cylinder

Just as a cuboid is built up with rectangles of the same size, we have seen that a right circular cylinder can be built up using circles of the same size. So, using the same argument as for a cuboid, we can see that the volume of a cylinder can be obtained as : base area × height

$$= \text{area of circular base} \times \text{height} = \pi r^2 h$$

So, $\boxed{\textbf{Volume of a Cylinder} = \pi r^2 h}$

where r is the base radius and h is the height of the cylinder.

Example 13 : The pillars of a temple are cylindrically shaped (see Fig. 13.26). If each pillar has a circular base of radius 20 cm and height 10 m, how much concrete mixture would be required to build 14 such pillars?

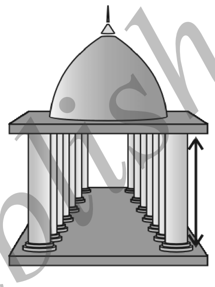


Fig. 13.26

Solution : Since the concrete mixture that is to be used to build up the pillars is going to occupy the entire space of the pillar, what we need to find here is the volume of the cylinders.

Radius of base of a cylinder = 20 cm

Height of the cylindrical pillar = 10 m = 1000 cm

So, volume of each cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 20 \times 20 \times 1000 \text{ cm}^3$$

$$= \frac{8800000}{7} \text{ cm}^3$$

$$= \frac{8.8}{7} \text{ m}^3 \text{ (Since 1000000 cm}^3 = 1\text{m}^3)$$

Therefore, volume of 14 pillars = volume of each cylinder × 14

$$= \frac{8.8}{7} \times 14 \text{ m}^3$$

$$= 17.6 \text{ m}^3$$

So, 14 pillars would need 17.6 m^3 of concrete mixture.

Example 14 : At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses (see Fig. 13.27) of radius 3 cm up to a height of 8 cm, and sold for ₹ 3 each. How much money does the stall keeper receive by selling the juice completely?

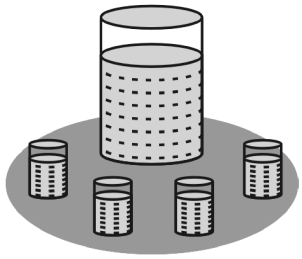


Fig. 13.27

Solution : The volume of juice in the vessel

= volume of the cylindrical vessel

= $\pi R^2 H$

(where R and H are taken as the radius and height respectively of the vessel)

= $\pi \times 15 \times 15 \times 32$ cm^3

Similarly, the volume of juice each glass can hold = $\pi r^2 h$

(where r and h are taken as the radius and height respectively of each glass)

= $\pi \times 3 \times 3 \times 8$ cm^3

So, number of glasses of juice that are sold

$$= \frac{\text{volume of the vessel}}{\text{volume of each glass}}$$

$$= \frac{\pi \times 15 \times 15 \times 32}{\pi \times 3 \times 3 \times 8}$$

= 100

Therefore, amount received by the stall keeper = ₹ 3 × 100

= ₹ 300

EXERCISE 13.6

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? (1000 cm^3 = 1l)
2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.
3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?
4. If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. (Use π = 3.14)

5. It costs ₹ 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹ 20 per m^2, find

   (i) inner curved surface area of the vessel,

   (ii) radius of the base,

   (iii) capacity of the vessel.

6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

13.8 Volume of a Right Circular Cone

In Fig 13.28, can you see that there is a right circular cylinder and a right circular cone of the same base radius and the same height?

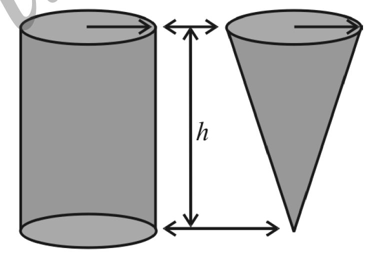


Fig. 13.28

Activity : Try to make a hollow cylinder and a hollow cone like this with the same base radius and the same height (see Fig. 13.28). Then, we can try out an experiment that will help us, to see practically what the volume of a right circular cone would be!

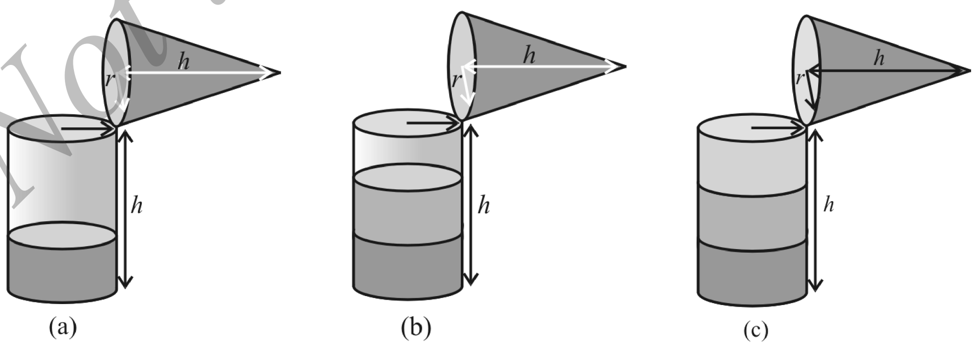


Fig. 13.29

So, let us start like this.

Fill the cone up to the brim with sand once, and empty it into the cylinder. We find that it fills up only a part of the cylinder [see Fig. 13.29(a)].

When we fill up the cone again to the brim, and empty it into the cylinder, we see that the cylinder is still not full [see Fig. 13.29(b)].

When the cone is filled up for the third time, and emptied into the cylinder, it can be seen that the cylinder is also full to the brim [see Fig. 13.29(c)].

With this, we can safely come to the conclusion that three times the volume of a cone, makes up the volume of a cylinder, which has the same base radius and the same height as the cone, which means that the volume of the cone is one-third the volume of the cylinder.

So, $\boxed{\textbf{Volume of a Cone} = \frac{1}{3}\pi r^2 h}$

where r is the base radius and h is the height of the cone.

Example 15 : The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.

Solution : From $l^2 = r^2 + h^2$, we have

$$r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2}\text{ cm} = 7\sqrt{7}\text{ cm}$$

So, volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{7} \times 7\sqrt{7} \times 21\ \text{cm}^3$

$= 7546\ \text{cm}^3$

Example 16 : Monica has a piece of canvas whose area is 551 m^2. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m^2, find the volume of the tent that can be made with it.

Solution : Since the area of the canvas = 551 m^2 and area of the canvas lost in wastage is 1 m^2, therefore the area of canvas available for making the tent is $(551 - 1)\ m^2 = 550\ m^2$.

Now, the surface area of the tent = 550 m^2 and the required base radius of the conical tent = 7 m

Note that a tent has only a curved surface (the floor of a tent is not covered by canvas!!).

Therefore, curved surface area of tent = 550 m^2.

That is, $\pi r l = 550$

or, $\frac{22}{7} \times 7 \times l = 550$

or, $l = 3\frac{550}{22}$ m = 25 m

Now, $l^2 = r^2 + h^2$

Therefore, $h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2}$ m $= \sqrt{625 - 49}$ m $= \sqrt{576}$ m
$= 24$ m

So, the volume of the conical tent $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$ m^3 = 1232 m^3.

EXERCISE 13.7

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Find the volume of the right circular cone with
(i) radius 6 cm, height 7 cm (ii) radius 3.5 cm, height 12 cm

2. Find the capacity in litres of a conical vessel with
(i) radius 7 cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm

3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$)

4. If the volume of a right circular cone of height 9 cm is 48 π cm^3, find the diameter of its base.

5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find
(i) height of the cone (ii) slant height of the cone
(iii) curved surface area of the cone

7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

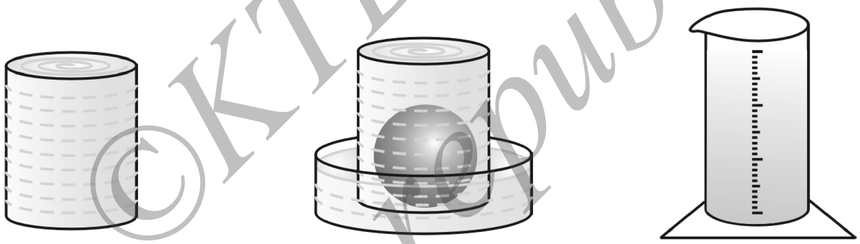
8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

13.9 Volume of a Sphere

Now, let us see how to go about measuring the volume of a sphere. First, take two or three spheres of different radii, and a container big enough to be able to put each of the spheres into it, one at a time. Also, take a large trough in which you can place the container. Then, fill the container up to the brim with water [see Fig. 13.30(a)].

Now, carefully place one of the spheres in the container. Some of the water from the container will over flow into the trough in which it is kept [see Fig. 13.30(b)]. Carefully pour out the water from the trough into a measuring cylinder (i.e., a graduated cylindrical jar) and measure the water over flowed [see Fig. 13.30(c)]. Suppose the radius of the immersed sphere is r (you can find the radius by measuring the diameter of the sphere). Then evaluate $\frac{4}{3}\pi r^3$. Do you find this value almost equal to the measure of the volume over flowed?



(a) (b) (c)

Fig. 13.30

Once again repeat the procedure done just now, with a different size of sphere. Find the radius R of this sphere and then calculate the value of $\frac{4}{3}\pi R^3$. Once again this value is nearly equal to the measure of the volume of the water displaced (over flowed) by the sphere. What does this tell us? We know that the volume of the sphere is the same as the measure of the volume of the water displaced by it. By doing this experiment repeatedly with spheres of varying radii, we are getting the same result, namely, the volume of a sphere is equal to $\frac{4}{3}\pi$ times the cube of its radius. This gives us the idea that

$$\boxed{\textbf{Volume of a Sphere} = \frac{4}{3}\pi r^3}$$

where r is the radius of the sphere.

Later, in higher classes it can be proved also. But at this stage, we will just take it as true.

Since a hemisphere is half of a sphere, can you guess what the volume of a hemisphere will be? Yes, it is $\frac{1}{2}$ of $\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$.

So, $\boxed{\textbf{Volume of a Hemisphere} = \frac{2}{3}\pi r^3}$

where r is the radius of the hemisphere.

Let us take some examples to illustrate the use of these formulae.

Example 17 : Find the volume of a sphere of radius 11.2 cm.

Solution : Required volume $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 11.2 \times 11.2 \times 11.2 \text{ cm}^3 = 5887.32 \text{ cm}^3$$

Example 18 : A shot-putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm^3, find the mass of the shot-putt.

Solution : Since the shot-putt is a solid sphere made of metal and its mass is equal to the product of its volume and density, we need to find the volume of the sphere.

Now, volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.9 \times 4.9 \times 4.9 \text{ cm}^3$$

$$= 493 \text{ cm}^3 \text{ (nearly)}$$

Further, mass of 1 cm^3 of metal is 7.8 g.

Therefore, mass of the shot-putt = 7.8 × 493 g

= 3845.44 g = 3.85 kg (nearly)

Example 19 : A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

Solution : The volume of water the bowl can contain

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \text{ cm}^3 = 89.8 \text{ cm}^3$$

EXERCISE 13.8

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. Find the volume of a sphere whose radius is
   (i) 7 cm (ii) 0.63 m
2. Find the amount of water displaced by a solid spherical ball of diameter
   (i) 28 cm (ii) 0.21 m
3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?
4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?
5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?
6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.
7. Find the volume of a sphere whose surface area is 154 cm^2.
8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 498.96. If the cost of white-washing is ₹ 2.00 per square metre, find the
   (i) inside surface area of the dome, (ii) volume of the air inside the dome.
9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the
   (i) radius r' of the new sphere, (ii) ratio of S and S′.
10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

EXERCISE 13.9 (Optional)*

1. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see Fig. 13.31). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.

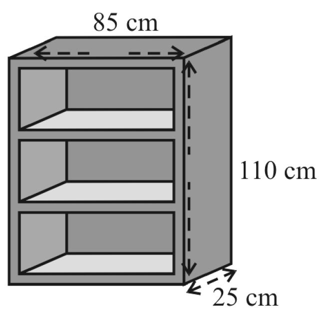


Fig. 13.31

*These exercises are not from examination point of view.

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in Fig 13.32. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.

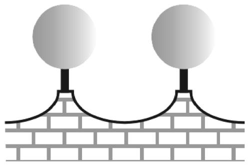


Fig. 13.32

3. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

13.10 Summary

In this chapter, you have studied the following points:

1. **Surface area of a cuboid = $2\,(lb + bh + hl)$**
2. **Surface area of a cube = $6a^2$**
3. **Curved surface area of a cylinder = $2\pi rh$**
4. **Total surface area of a cylinder = $2\pi r(r + h)$**
5. **Curved surface area of a cone = πrl**
6. **Total surface area of a right circular cone = $\pi rl + \pi r^2$, i.e., $\pi r(l + r)$**
7. **Surface area of a sphere of radius $r = 4\,\pi\, r^2$**
8. **Curved surface area of a hemisphere = $2\pi r^2$**
9. **Total surface area of a hemisphere = $3\pi r^2$**
10. **Volume of a cuboid = $l \times b \times h$**
11. **Volume of a cube = a^3**
12. **Volume of a cylinder = $\pi r^2 h$**
13. **Volume of a cone = $\frac{1}{3}\pi r^2 h$**
14. **Volume of a sphere of radius $r = \frac{4}{3}\pi\, r^3$**
15. **Volume of a hemisphere = $\frac{2}{3}\pi\, r^3$**

[Here, letters l, b, h, a, r, etc. have been used in their usual meaning, depending on the context.]

CHAPTER 14

STATISTICS

14.1 Introduction

Everyday we come across a lot of information in the form of facts, numerical figures, tables, graphs, etc. These are provided by newspapers, televisions, magazines and other means of communication. These may relate to cricket batting or bowling averages, profits of a company, temperatures of cities, expenditures in various sectors of a five year plan, polling results, and so on. These facts or figures, which are numerical or otherwise, collected with a definite purpose are called *data*. Data is the plural form of the Latin word *datum*. Of course, the word 'data' is not new for you. You have studied about data and data handling in earlier classes.

Our world is becoming more and more information oriented. Every part of our lives utilises data in one form or the other. So, it becomes essential for us to know how to extract meaningful information from such data. This extraction of meaningful information is studied in a branch of mathematics called *Statistics*.

The word 'statistics' appears to have been derived from the Latin word 'status' meaning 'a (political) state'. In its origin, statistics was simply the collection of data on different aspects of the life of people, useful to the State. Over the period of time, however, its scope broadened and statistics began to concern itself not only with the collection and presentation of data but also with the interpretation and drawing of inferences from the data. Statistics deals with collection, organisation, analysis and interpretation of data. The word 'statistics' has different meanings in different contexts. Let us observe the following sentences:

1. May I have the latest copy of 'Educational Statistics of India'.

2. I like to study 'Statistics' because it is used in day-to-day life.

In the first sentence, statistics is used in a plural sense, meaning numerical data. These may include a number of educational institutions of India, literacy rates of various states, etc. In the second sentence, the word 'statistics' is used as a singular noun, meaning the subject which deals with the collection, presentation, analysis of data as well as drawing of meaningful conclusions from the data.

In this chapter, we shall briefly discuss all these aspects regarding data.

14.2 Collection of Data

Let us begin with an exercise on gathering data by performing the following activity.

Activity 1 : Divide the students of your class into four groups. Allot each group the work of collecting one of the following kinds of data:

(i) Heights of 20 students of your class.

(ii) Number of absentees in each day in your class for a month.

(iii) Number of members in the families of your classmates.

(iv) Heights of 15 plants in or around your school.

Let us move to the results students have gathered. How did they collect their data in each group?

(i) Did they collect the information from each and every student, house or person concerned for obtaining the information?

(ii) Did they get the information from some source like available school records?

In the first case, when the information was collected by the investigator herself or himself with a definite objective in her or his mind, the data obtained is called *primary data*.

In the second case, when the information was gathered from a source which already had the information stored, the data obtained is called *secondary data*. Such data, which has been collected by someone else in another context, needs to be used with great care ensuring that the source is reliable.

By now, you must have understood how to collect data and distinguish between primary and secondary data.

EXERCISE 14.1

1. Give five examples of data that you can collect from your day-to-day life.

2. Classify the data in Q.1 above as primary or secondary data.

14.3 Presentation of Data

As soon as the work related to collection of data is over, the investigator has to find out ways to present them in a form which is meaningful, easily understood and gives its main features at a glance. Let us now recall the various ways of presenting the data through some examples.

Example 1 : Consider the marks obtained by 10 students in a mathematics test as given below:

55	36	95	73	60	42	25	78	75	62

The data in this form is called *raw data*.

By looking at it in this form, can you find the highest and the lowest marks?

Did it take you some time to search for the maximum and minimum scores? Wouldn't it be less time consuming if these scores were arranged in ascending or descending order? So let us arrange the marks in ascending order as

25	36	42	55	60	62	73	75	78	95

Now, we can clearly see that the lowest marks are 25 and the highest marks are 95.

The difference of the highest and the lowest values in the data is called the *range* of the data. So, the range in this case is 95 – 25 = 70.

Presentation of data in ascending or descending order can be quite time consuming, particularly when the number of observations in an experiment is large, as in the case of the next example.

Example 2 : Consider the marks obtained (out of 100 marks) by 30 students of Class IX of a school:

10	20	36	92	95	40	50	56	60	70
92	88	80	70	72	70	36	40	36	40
92	40	50	50	56	60	70	60	60	88

Recall that the number of students who have obtained a certain number of marks is called the *frequency* of those marks. For instance, 4 students got 70 marks. So the frequency of 70 marks is 4. To make the data more easily understandable, we write it in a table, as given below:

Table 14.1

Marks	Number of students (i.e., the frequency)
10	1
20	1
36	3
40	4
50	3
56	2
60	4
70	4
72	1
80	1
88	2
92	3
95	1
Total	30

Table 14.1 is called *an ungrouped frequency distribution table,* or simply a *frequency distribution table*. Note that you can use also *tally marks* in preparing these tables, as in the next example.

Example 3 : 100 plants each were planted in 100 schools during Van Mahotsava. After one month, the number of plants that survived were recorded as :

95	67	28	32	65	65	69	33	98	96
76	42	32	38	42	40	40	69	95	92
75	83	76	83	85	62	37	65	63	42
89	65	73	81	49	52	64	76	83	92
93	68	52	79	81	83	59	82	75	82
86	90	44	62	31	36	38	42	39	83
87	56	58	23	35	76	83	85	30	68
69	83	86	43	45	39	83	75	66	83
92	75	89	66	91	27	88	89	93	42
53	69	90	55	66	49	52	83	34	36

To present such a large amount of data so that a reader can make sense of it easily, we condense it into groups like 20-29, 30-39, . . ., 90-99 (since our data is from 23 to 98). These groupings are called 'classes' or 'class-intervals', and their size is called the *class-size* or *class width*, which is 10 in this case. In each of these classes, the least number is called the *lower class limit* and the greatest number is called the *upper class limit,* e.g., in 20-29, 20 is the 'lower class limit' and 29 is the 'upper class limit'.

Also, recall that using tally marks, the data above can be condensed in tabular form as follows:

Table 14.2

Number of plants survived	**Tally Marks**	**Number of schools (frequency)**
20 - 29	𝍷𝍷𝍷	3
30 - 39	𝍸 𝍸 𝍷𝍷𝍷𝍷	14
40 - 49	𝍸 𝍸 𝍷𝍷	12
50 - 59	𝍸 𝍷𝍷𝍷	8
60 - 69	𝍸 𝍸 𝍸 𝍷𝍷𝍷	18
70 - 79	𝍸 𝍸	10
80 - 89	𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷	23
90 - 99	𝍸 𝍸 𝍷𝍷	12
Total		100

Presenting data in this form simplifies and condenses data and enables us to observe certain important features at a glance. This is called *a grouped frequency distribution table*. Here we can easily observe that 50% or more plants survived in 8 + 18 + 10 + 23 + 12 = 71 schools.

We observe that the classes in the table above are non-overlapping. Note that we could have made more classes of shorter size, or fewer classes of larger size also. For instance, the intervals could have been 22-26, 27-31, and so on. So, there is no hard and fast rule about this except that the classes should not overlap.

Example 4 : Let us now consider the following frequency distribution table which gives the weights of 38 students of a class:

Table 14.3

Weights (in kg)	Number of students
31 - 35	9
36 - 40	5
41 - 45	14
46 - 50	3
51 - 55	1
56 - 60	2
61 - 65	2
66 - 70	1
71 - 75	1
Total	38

Now, if two new students of weights 35.5 kg and 40.5 kg are admitted in this class, then in which interval will we include them? We cannot add them in the ones ending with 35 or 40, nor to the following ones. This is because there are gaps in between the upper and lower limits of two consecutive classes. So, we need to divide the intervals so that the upper and lower limits of consecutive intervals are the same. For this, we find the difference between the upper limit of a class and the lower limit of its succeeding class. We then add half of this difference to each of the upper limits and subtract the same from each of the lower limits.

For example, consider the classes 31 - 35 and 36 - 40.

The lower limit of 36 - 40 = 36

The upper limit of 31 - 35 = 35

The difference = 36 – 35 = 1

So, half the difference = $\frac{1}{2}$ = 0.5

So the new class interval formed from 31 - 35 is (31 – 0.5) - (35 + 0.5), i.e., 30.5 - 35.5.

Similarly, the new class formed from the class 36 - 40 is (36 – 0.5) - (40 + 0.5), i.e., 35.5 - 40.5.

Continuing in the same manner, the continuous classes formed are:

30.5-35.5, 35.5-40.5, 40.5-45.5, 45.5-50.5, 50.5-55.5, 55.5-60.5, 60.5 - 65.5, 65.5 - 70.5, 70.5 - 75.5.

Now it is possible for us to include the weights of the new students in these classes. But, another problem crops up because 35.5 appears in both the classes 30.5 - 35.5 and 35.5 - 40.5. In which class do you think this weight should be considered?

If it is considered in both classes, it will be counted twice.

By convention, we consider 35.5 in the class 35.5 - 40.5 and not in 30.5 - 35.5. Similarly, 40.5 is considered in 40.5 - 45.5 and not in 35.5 - 40.5.

So, the new weights 35.5 kg and 40.5 kg would be included in 35.5 - 40.5 and 40.5 - 45.5, respectively. Now, with these assumptions, the new frequency distribution table will be as shown below:

Table 14.4

Weights (in kg)	Number of students
30.5-35.5	9
35.5-40.5	6
40.5-45.5	15
45.5-50.5	3
50.5-55.5	1
55.5-60.5	2
60.5-65.5	2
65.5-70.5	1
70.5-75.5	1
Total	40

Now, let us move to the data collected by you in Activity 1. This time we ask you to present these as frequency distribution tables.

Activity 2 : Continuing with the same four groups, change your data to frequency distribution tables.Choose convenient classes with suitable class-sizes, keeping in mind the range of the data and the type of data.

EXERCISE 14.2

1. The blood groups of 30 students of Class VIII are recorded as follows:

   A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,

   A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.

   Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?

2. The distance (in km) of 40 engineers from their residence to their place of work were found as follows:

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

   Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?

3. The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89

   (i) Construct a grouped frequency distribution table with classes 84 - 86, 86 - 88, etc.

   (ii) Which month or season do you think this data is about?

   (iii) What is the range of this data?

4. The heights of 50 students, measured to the nearest centimetres, have been found to be as follows:

161	150	154	165	168	161	154	162	150	151
162	164	171	165	158	154	156	172	160	170
153	159	161	170	162	165	166	168	165	164
154	152	153	156	158	162	160	161	173	166
161	159	162	167	168	159	158	153	154	159

   (i) Represent the data given above by a grouped frequency distribution table, taking the class intervals as 160 - 165, 165 - 170, etc.

   (ii) What can you conclude about their heights from the table?

5. A study was conducted to find out the concentration of sulphur dioxide in the air in

parts per million (ppm) of a certain city. The data obtained for 30 days is as follows:

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	0.04

(i) Make a grouped frequency distribution table for this data with class intervals as 0.00 - 0.04, 0.04 - 0.08, and so on.

(ii) For how many days, was the concentration of sulphur dioxide more than 0.11 parts per million?

6. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Prepare a frequency distribution table for the data given above.

7. The value of π upto 50 decimal places is given below:

3.14159265358979323846264338327950288419716939937510

(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

(ii) What are the most and the least frequently occurring digits?

8. Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:

1	6	2	3	5	12	5	8	4	8
10	3	4	12	2	8	15	1	17	6
3	2	8	5	9	6	8	7	14	12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5 - 10.

(ii) How many children watched television for 15 or more hours a week?

9. A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:

2.6	3.0	3.7	3.2	2.2	4.1	3.5	4.5
3.5	2.3	3.2	3.4	3.8	3.2	4.6	3.7
2.5	4.4	3.4	3.3	2.9	3.0	4.3	2.8
3.5	3.2	3.9	3.2	3.2	3.1	3.7	3.4
4.6	3.8	3.2	2.6	3.5	4.2	2.9	3.6

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 - 2.5.

14.4 Graphical Representation of Data

The representation of data by tables has already been discussed. Now let us turn our attention to another representation of data, i.e., the graphical representation. It is well said that one picture is better than a thousand words. Usually comparisons among the individual items are best shown by means of graphs. The representation then becomes easier to understand than the actual data. We shall study the following graphical representations in this section.

(A) Bar graphs

(B) Histograms of uniform width, and of varying widths

(C) Frequency polygons

(A) Bar Graphs

In earlier classes, you have already studied and constructed bar graphs. Here we shall discuss them through a more formal approach. Recall that a bar graph is a pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis (say, the x-axis), depicting the variable. The values of the variable are shown on the other axis (say, the y-axis) and the heights of the bars depend on the values of the variable.

Example 5 : In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:

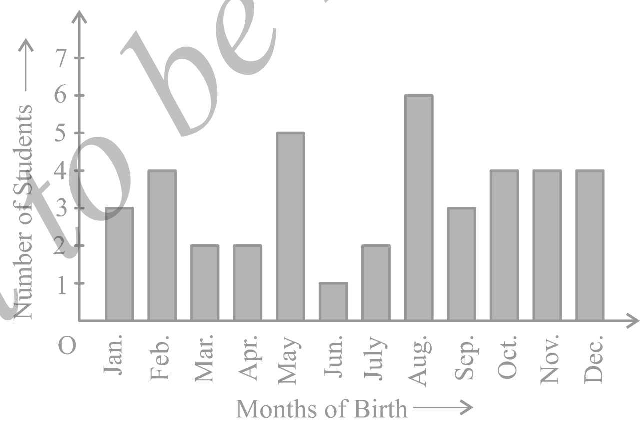


Fig. 14.1

Observe the bar graph given above and answer the following questions:

(i) How many students were born in the month of November?

(ii) In which month were the maximum number of students born?

Solution : Note that the variable here is the 'month of birth', and the value of the variable is the 'Number of students born'.

(i) 4 students were born in the month of November.

(ii) The Maximum number of students were born in the month of August.

Let us now recall how a bar graph is constructed by considering the following example.

Example 6 : A family with a monthly income of Rs 20,000 had planned the following expenditures per month under various heads:

Table 14.5

Heads	**Expenditure (in thousand rupees)**
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Draw a bar graph for the data above.

Solution : We draw the bar graph of this data in the following steps. Note that the unit in the second column is thousand rupees. So, '4' against 'grocery' means Rs 4000.

1. We represent the Heads (variable) on the horizontal axis choosing any scale, since the width of the bar is not important. But for clarity, we take equal widths for all bars and maintain equal gaps in between. Let one Head be represented by one unit.
2. We represent the expenditure (value) on the vertical axis. Since the maximum expenditure is Rs 5000, we can choose the scale as 1 unit = Rs 1000.
3. To represent our first Head, i.e., grocery, we draw a rectangular bar with width 1 unit and height 4 units.
4. Similarly, other Heads are represented leaving a gap of 1 unit in between two consecutive bars.

The bar graph is drawn in Fig. 14.2.

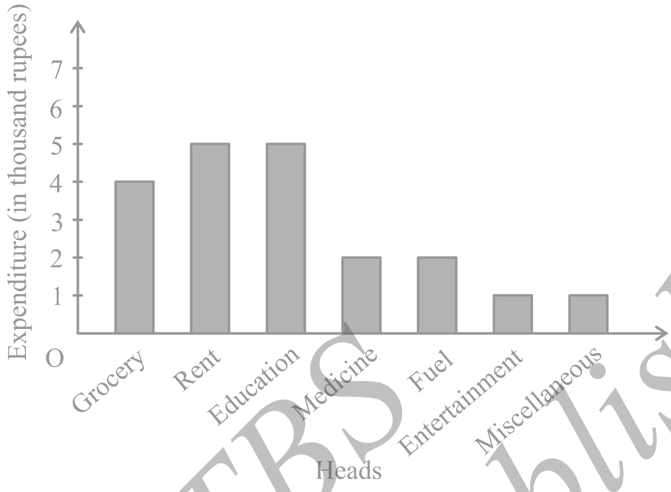


Fig. 14.2

Here, you can easily visualise the relative characteristics of the data at a glance, e.g., the expenditure on education is more than double that of medical expenses. Therefore, in some ways it serves as a better representation of data than the tabular form.

Activity 3 : Continuing with the same four groups of Activity 1, represent the data by suitable bar graphs.

Let us now see how a frequency distribution table for *continuous* class intervals can be represented graphically.

(B) Histogram

This is a form of representation like the bar graph, but it is used for continuous class intervals. For instance, consider the frequency distribution Table 14.6, representing the weights of 36 students of a class:

Table 14.6

Weights (in kg)	Number of students
30.5 - 35.5	9
35.5 - 40.5	6
40.5 - 45.5	15
45.5 - 50.5	3
50.5 - 55.5	1
55.5 - 60.5	2
Total	36

Let us represent the data given above graphically as follows:

(i) We represent the weights on the horizontal axis on a suitable scale. We can choose the scale as 1 cm = 5 kg. Also, since the first class interval is starting from 30.5 and not zero, we show it on the graph by marking a *kink* or a break on the axis.

(ii) We represent the number of students (frequency) on the vertical axis on a suitable scale. Since the maximum frequency is 15, we need to choose the scale to accomodate this maximum frequency.

(iii) We now draw rectangles (or rectangular bars) of width equal to the class-size and lengths according to the frequencies of the corresponding class intervals. For example, the rectangle for the class interval 30.5 - 35.5 will be of width 1 cm and length 4.5 cm.

(iv) In this way, we obtain the graph as shown in Fig. 14.3:

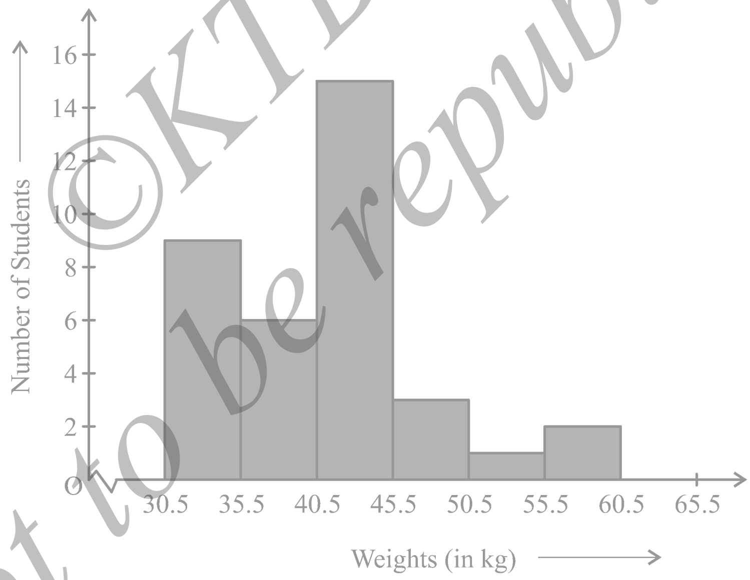


Fig. 14.3

Observe that since there are no gaps in between consecutive rectangles, the resultant graph appears like a solid figure. This is called a *histogram*, which is a graphical representation of a grouped frequency distribution with continuous classes. Also, unlike a bar graph, the width of the bar plays a significant role in its construction.

Here, in fact, areas of the rectangles erected are proportional to the corresponding frequencies. However, since the widths of the rectangles are all equal, the lengths of the rectangles are proportional to the frequencies. That is why, we draw the lengths according to (iii) above.

Now, consider a situation different from the one above.

Example 7 : A teacher wanted to analyse the performance of two sections of students in a mathematics test of 100 marks. Looking at their performances, she found that a few students got under 20 marks and a few got 70 marks or above. So she decided to group them into intervals of varying sizes as follows: 0 - 20, 20 - 30, . . ., 60 - 70, 70 - 100. Then she formed the following table:

Table 14.7

Marks	Number of students
0 - 20	7
20 - 30	10
30 - 40	10
40 - 50	20
50 - 60	20
60 - 70	15
70 - above	8
Total	90

A histogram for this table was prepared by a student as shown in Fig. 14.4.

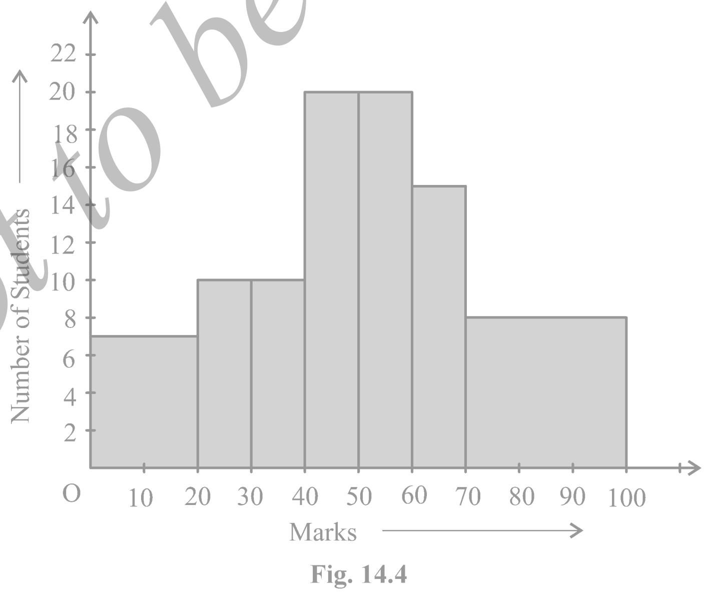


Fig. 14.4

Carefully examine this graphical representation. Do you think that it correctly represents the data? No, the graph is giving us a misleading picture. As we have mentioned earlier, the areas of the rectangles are proportional to the frequencies in a histogram. Earlier this problem did not arise, because the widths of all the rectangles were equal. But here, since the widths of the rectangles are varying, the histogram above does not give a correct picture. For example, it shows a greater frequency in the interval 70 - 100, than in 60 - 70, which is not the case.

So, we need to make certain modifications in the lengths of the rectangles so that the areas are again proportional to the frequencies.

The steps to be followed are as given below:

1. Select a class interval with the minimum class size. In the example above, the minimum class-size is 10.
2. The lengths of the rectangles are then modified to be proportionate to the class-size 10.

For instance, when the class-size is 20, the length of the rectangle is 7. So when the class-size is 10, the length of the rectangle will be $\frac{7}{20} \times 10 = 3.5$.

Similarly, proceeding in this manner, we get the following table:

Table 14.8

Marks	**Frequency**	**Width of the class**	**Length of the rectangle**
0 - 20	7	20	$\frac{7}{20} \times 10 = 3.5$
20 - 30	10	10	$\frac{10}{10} \times 10 = 10$
30 - 40	10	10	$\frac{10}{10} \times 10 = 10$
40 - 50	20	10	$\frac{20}{10} \times 10 = 20$
50 - 60	20	10	$\frac{20}{10} \times 10 = 20$
60 - 70	15	10	$\frac{15}{10} \times 10 = 15$
70 - 100	8	30	$\frac{8}{30} \times 10 = 2.67$

Since we have calculated these lengths for an interval of 10 marks in each case, we may call these lengths as "proportion of students per 10 marks interval".

So, the correct histogram with varying width is given in Fig. 14.5.

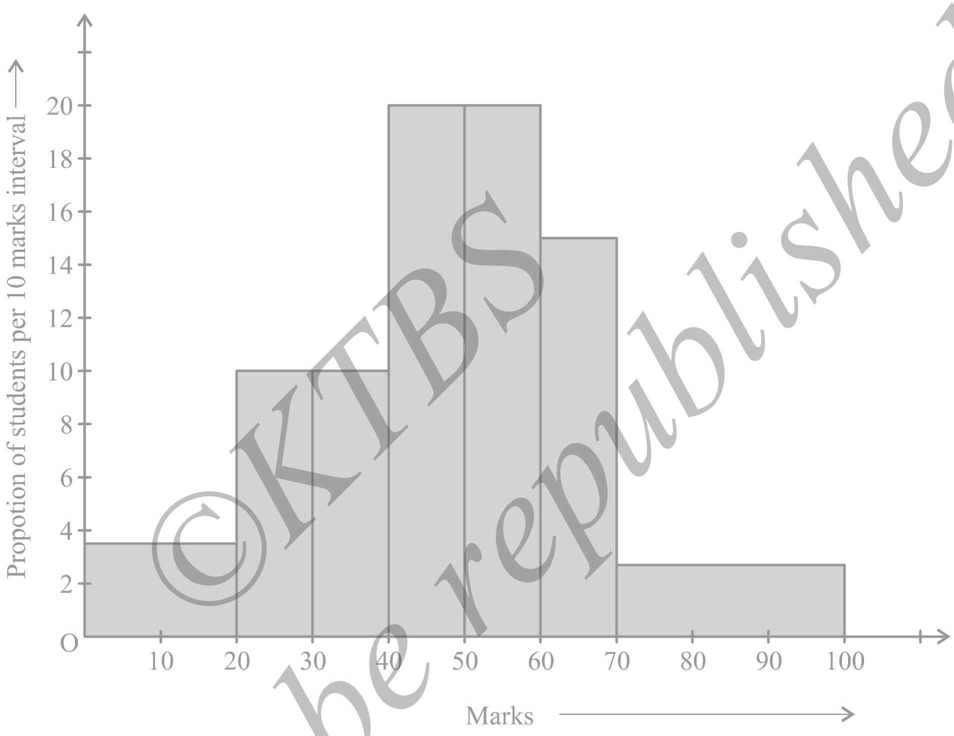


Fig. 14.5

(C) Frequency Polygon

There is yet another visual way of representing quantitative data and its frequencies. This is a polygon. To see what we mean, consider the histogram represented by Fig. 14.3. Let us join the mid-points of the upper sides of the adjacent rectangles of this histogram by means of line segments. Let us call these mid-points B, C, D, E, F and G. When joined by line segments, we obtain the figure BCDEFG (see Fig. 14.6). To complete the polygon, we assume that there is a class interval with frequency zero before 30.5 - 35.5, and one after 55.5 - 60.5, and their mid-points are A and H, respectively. ABCDEFGH is the frequency polygon corresponding to the data shown in Fig. 14.3. We have shown this in Fig. 14.6.

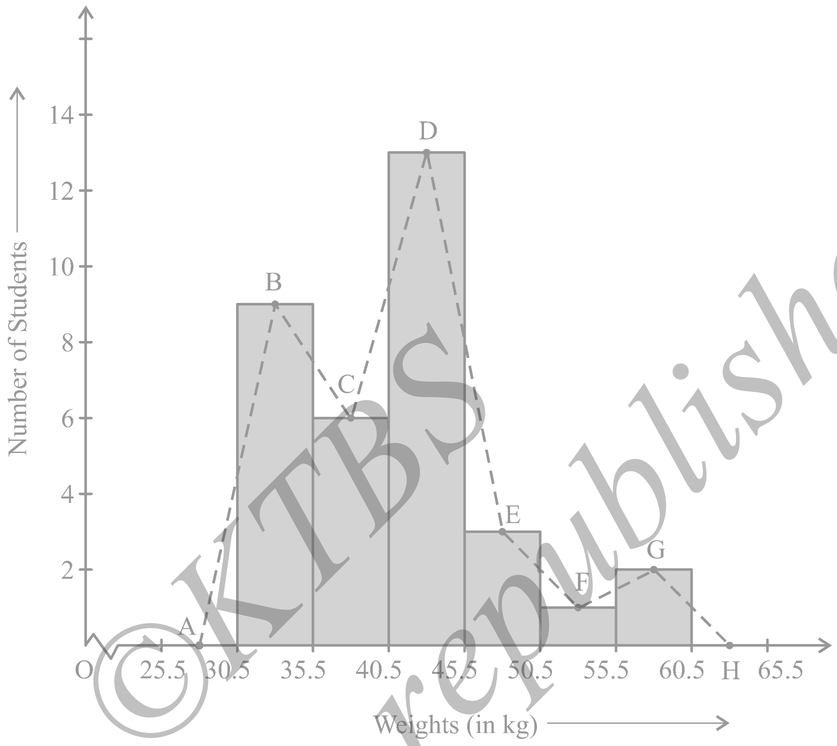


Fig. 14.6

Although, there exists no class preceding the lowest class and no class succeeding the highest class, addition of the two class intervals with zero frequency enables us to make the area of the frequency polygon the same as the area of the histogram. Why is this so? (**Hint :** Use the properties of congruent triangles.)

Now, the question arises: how do we complete the polygon when there is no class preceding the first class? Let us consider such a situation.

Example 8 : Consider the marks, out of 100, obtained by 51 students of a class in a test, given in Table 14.9.

Table 14.9

Marks	Number of students
0 - 10	5
10 - 20	10
20 - 30	4
30 - 40	6
40 - 50	7
50 - 60	3
60 - 70	2
70 - 80	2
80 - 90	3
90 - 100	9
Total	51

Draw a frequency polygon corresponding to this frequency distribution table.

Solution : Let us first draw a histogram for this data and mark the mid-points of the tops of the rectangles as B, C, D, E, F, G, H, I, J, K, respectively. Here, the first class is 0-10. So, to find the class preceeding 0-10, we extend the horizontal axis in the negative direction and find the mid-point of the imaginary class-interval (–10) - 0. The first end point, i.e., B is joined to this mid-point with zero frequency on the negative direction of the horizontal axis. The point where this line segment meets the vertical axis is marked as A. Let L be the mid-point of the class succeeding the last class of the given data. Then OABCDEFGHIJKL is the frequency polygon, which is shown in Fig. 14.7.

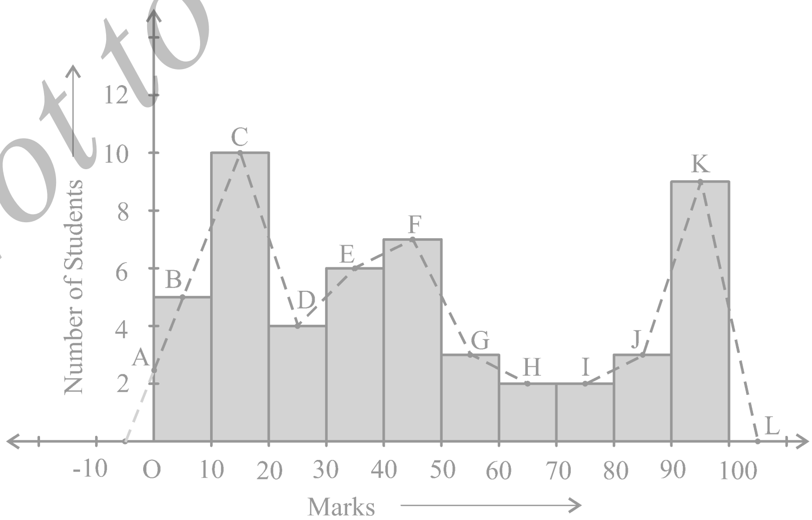


Fig. 14.7

Frequency polygons can also be drawn independently without drawing histograms. For this, we require the mid-points of the class-intervals used in the data. These mid-points of the class-intervals are called **class-marks**.

To find the class-mark of a class interval, we find the sum of the upper limit and lower limit of a class and divide it by 2. Thus,

$$\textbf{Class-mark} = \frac{\textbf{Upper limit + Lower limit}}{\textbf{2}}$$

Let us consider an example.

Example 9 : In a city, the weekly observations made in a study on the cost of living index are given in the following table:

Table 14.10

Cost of living index	Number of weeks
140 - 150	5
150 - 160	10
160 - 170	20
170 - 180	9
180 - 190	6
190 - 200	2
Total	52

Draw a frequency polygon for the data above (without constructing a histogram).

Solution : Since we want to draw a frequency polygon without a histogram, let us find the class-marks of the classes given above, that is of 140 - 150, 150 - 160,....

For 140 - 150, the upper limit = 150, and the lower limit = 140

So, the class-mark = $\frac{150+140}{2} = \frac{290}{2} = 145.$

Continuing in the same manner, we find the class-marks of the other classes as well.

So, the new table obtained is as shown in the following table:

Table 14.11

Classes	Class-marks	Frequency
140 - 150	145	5
150 - 160	155	10
160 - 170	165	20
170 - 180	175	9
180 - 190	185	6
190 - 200	195	2
Total		52

We can now draw a frequency polygon by plotting the class-marks along the horizontal axis, the frequencies along the vertical-axis, and then plotting and joining the points B(145, 5), C(155, 10), D(165, 20), E(175, 9), F(185, 6) and G(195, 2) by line segments. We should not forget to plot the point corresponding to the class-mark of the class 130 - 140 (just before the lowest class 140 - 150) with zero frequency, that is, A(135, 0), and the point H (205, 0) occurs immediately after G(195, 2). So, the resultant frequency polygon will be ABCDEFGH (see Fig. 14.8).

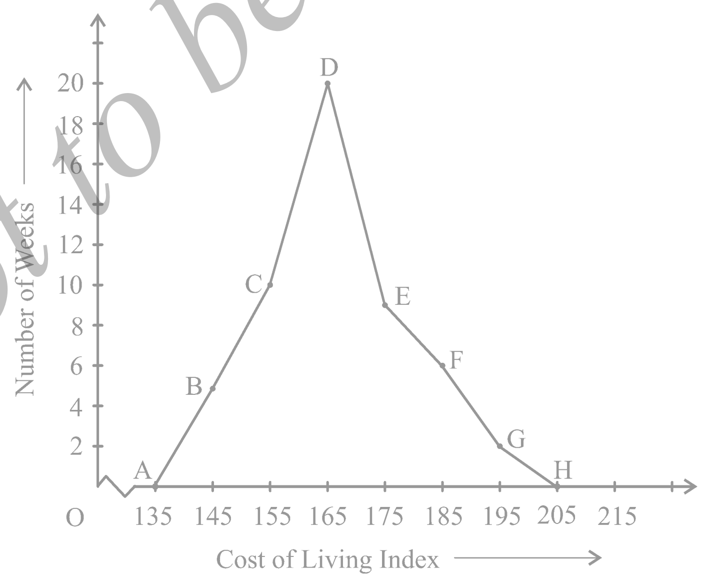


Fig. 14.8

Frequency polygons are used when the data is continuous and very large. It is very useful for comparing two different sets of data of the same nature, for example, comparing the performance of two different sections of the same class.

EXERCISE 14.3

1. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 - 44 (in years) worldwide, found the following figures (in %):

S.No.	Causes	Female fatality rate (%)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the information given above graphically.

(ii) Which condition is the major cause of women's ill health and death worldwide?

(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

2. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below.

Section	Number of girls per thousand boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph.

3. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.

(ii) Which political party won the maximum number of seats?

4. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:

Length (in mm)	**Number of leaves**
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

(i) Draw a histogram to represent the given data. [Hint: First make the class intervals continuous]

(ii) Is there any other suitable graphical representation for the same data?

(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

5. The following table gives the life times of 400 neon lamps:

Life time (in hours)	**Number of lamps**
300 - 400	14
400 - 500	56
500 - 600	60
600 - 700	86
700 - 800	74
800 - 900	62
900 - 1000	48

(i) Represent the given information with the help of a histogram.

(ii) How many lamps have a life time of more than 700 hours?

6. The following table gives the distribution of students of two sections according to the marks obtained by them:

Section A		**Section B**	
Marks	**Frequency**	**Marks**	**Frequency**
0 - 10	3	0 - 10	5
10 - 20	9	10 - 20	19
20 - 30	17	20 - 30	15
30 - 40	12	30 - 40	10
40 - 50	9	40 - 50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

7. The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:

Number of balls	**Team A**	**Team B**
1 - 6	2	5
7 - 12	1	6
13 - 18	8	2
19 - 24	9	10
25 - 30	4	5
31 - 36	5	6
37 - 42	6	3
43 - 48	10	4
49 - 54	6	8
55 - 60	2	10

Represent the data of both the teams on the same graph by frequency polygons.

[**Hint :** First make the class intervals continuous.]

8. A random survey of the number of children of various age groups playing in a park was found as follows:

Age (in years)	Number of children
1 - 2	5
2 - 3	3
3 - 5	6
5 - 7	12
7 - 10	9
10 - 15	10
15 - 17	4

Draw a histogram to represent the data above.

9. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

Number of letters	Number of surnames
1 - 4	6
4 - 6	30
6 - 8	44
8 - 12	16
12 - 20	4

(i) Draw a histogram to depict the given information.

(ii) Write the class interval in which the maximum number of surnames lie.

14.5 Measures of Central Tendency

Earlier in this chapter, we represented the data in various forms through frequency distribution tables, bar graphs, histograms and frequency polygons. Now, the question arises if we always need to study all the data to 'make sense' of it, or if we can make out some important features of it by considering only certain representatives of the data. This is possible, by using measures of central tendency or averages.

Consider a situation when two students Mary and Hari received their test copies. The test had five questions, each carrying ten marks. Their scores were as follows:

Question Numbers	1	2	3	4	5
Mary's score	10	8	9	8	7
Hari's score	4	7	10	10	10

Upon getting the test copies, both of them found their average scores as follows:

$$\text{Mary's average score} = \frac{42}{5} = 8.4$$

$$\text{Hari's average score} = \frac{41}{5} = 8.2$$

Since Mary's average score was more than Hari's, Mary claimed to have performed better than Hari, but Hari did not agree. He arranged both their scores in ascending order and found out the middle score as given below:

Mary's Score	7	8	8	9	10
Hari's Score	4	7	10	10	10

Hari said that since his middle-most score was 10, which was higher than Mary's middle-most score, that is 8, his performance should be rated better.

But Mary was not convinced. To convince Mary, Hari tried out another strategy. He said he had scored 10 marks more often (3 times) as compared to Mary who scored 10 marks only once. So, his performance was better.

Now, to settle the dispute between Hari and Mary, let us see the three measures they adopted to make their point.

The average score that Mary found in the first case is the *mean*. The 'middle' score that Hari was using for his argument is the *median*. The most often scored mark that Hari used in his second strategy is the *mode*.

Now, let us first look at the mean in detail.

The **mean** (or **average**) of a number of observations is the sum of the values of all the observations divided by the total number of observations.

It is denoted by the symbol $\bar{x}$, read as 'x bar'.

Let us consider an example.

Example 10 : 5 people were asked about the time in a week they spend in doing social work in their community. They said 10, 7, 13, 20 and 15 hours, respectively.

Find the mean (or average) time in a week devoted by them for social work.

Solution : We have already studied in our earlier classes that the mean of a certain number of observations is equal to $\frac{\text{Sum of all the observations}}{\text{Total number of observations}}$. To simplify our working of finding the mean, let us use a variable x_i to denote the ith observation. In this case, i can take the values from 1 to 5. So our first observation is x_1, second observation is x_2, and so on till x_5.

Also $x_1 = 10$ means that the value of the first observation, denoted by x_1, is 10. Similarly, $x_2 = 7$, $x_3 = 13$, $x_4 = 20$ and $x_5 = 15$.

$$\text{Therefore, the mean } \bar{x} = \frac{\text{Sum of all the observations}}{\text{Total number of observations}}$$

$$= \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$= \frac{10 + 7 + 13 + 20 + 15}{5} = \frac{65}{5} = 13$$

So, the mean time spent by these 5 people in doing social work is 13 hours in a week.

Now, in case we are finding the mean time spent by 30 people in doing social work, writing $x_1 + x_2 + x_3 + \ldots + x_{30}$ would be a tedious job. We use the Greek symbol Σ (for the letter Sigma) for *summation*. Instead of writing $x_1 + x_2 + x_3 + \ldots + x_{30}$, we write $\sum_{i=1}^{30} x_i$, which is read as 'the sum of x_i as i varies from 1 to 30'.

So, $$\bar{x} = \frac{\sum_{i=1}^{30} x_i}{30}$$

Similarly, for n observations $$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Example 11 : Find the mean of the marks obtained by 30 students of Class IX of a school, given in Example 2.

Solution : Now, $$\bar{x} = \frac{x_1 + x_2 + \cdots + x_{30}}{30}$$

$$\sum_{i=1}^{30} x_i = 10 + 20 + 36 + 92 + 95 + 40 + 50 + 56 + 60 + 70 + 92 + 88$$
$$80 + 70 + 72 + 70 + 36 + 40 + 36 + 40 + 92 + 40 + 50 + 50$$
$$56 + 60 + 70 + 60 + 60 + 88 = 1779$$

So, $$\bar{x} = \frac{1779}{30} = 59.3$$

Is the process not time consuming? Can we simplify it? Note that we have formed a frequency table for this data (see Table 14.1).

The table shows that 1 student obtained 10 marks, 1 student obtained 20 marks, 3 students obtained 36 marks, 4 students obtained 40 marks, 3 students obtained 50 marks, 2 students obtained 56 marks, 4 students obtained 60 marks, 4 students obtained 70 marks, 1 student obtained 72 marks, 1 student obtained 80 marks, 2 students obtained 88 marks, 3 students obtained 92 marks and 1 student obtained 95 marks.

So, the total marks obtained = $(1 \times 10) + (1 \times 20) + (3 \times 36) + (4 \times 40) + (3 \times 50) + (2 \times 56) + (4 \times 60) + (4 \times 70) + (1 \times 72) + (1 \times 80) + (2 \times 88) + (3 \times 92) + (1 \times 95)$

$= f_1x_1 + \ldots + f_{13}x_{13}$, where f_i is the frequency of the ith entry inTable 14.1.

In brief, we write this as $\sum_{i=1}^{13} f_i x_i$.

So, the total marks obtained $= \sum_{i=1}^{13} f_i x_i$

$= 10 + 20 + 108 + 160 + 150 + 112 + 240 + 280 + 72 + 80 + 176 + 276 + 95$

$= 1779$

Now, the total number of observations

$= \sum_{i=1}^{13} f_i$

$= f_1 + f_2 + \ldots + f_{13}$

$= 1 + 1 + 3 + 4 + 3 + 2 + 4 + 4 + 1 + 1 + 2 + 3 + 1$

$= 30$

So, the mean $\bar{x} = \dfrac{\text{Sum of all the observations}}{\text{Total number of observations}} = \left(\dfrac{\sum_{i=1}^{13} f_i x_i}{\sum_{i=1}^{13} f_i} \right)$

$= \dfrac{1779}{30} = 59.3$

This process can be displayed in the following table, which is a modified form of Table 14.1.

Table 14.12

Marks (x_i)	Number of students (f_i)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
	$\sum_{i=1}^{13} f_i = 30$	$\sum_{i=1}^{13} f_i x_i = 1779$

Thus, in the case of an ungrouped frequency distribution, you can use the formula

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

for calculating the mean.

Let us now move back to the situation of the argument between Hari and Mary, and consider the second case where Hari found his performance better by finding the middle-most score. As already stated, this measure of central tendency is called the *median*.

The **median** is that value of the given number of observations, which divides it into exactly two parts. So, when the data is arranged in ascending (or descending) order the median of ungrouped data is calculated as follows:

(i) When the number of observations (n) is odd, the median is the value of the $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation. For example, if $n = 13$, the value of the $\left(\frac{13+1}{2}\right)^{\text{th}}$, i.e., the 7th observation will be the median [see Fig. 14.9 (i)].

(ii) When the number of observations (n) is even, the median is the mean of the $\left(\frac{n}{2}\right)^{\text{th}}$ and the $\left(\frac{n}{2}+1\right)^{\text{th}}$ observations. For example, if $n = 16$, the mean of the values of the $\left(\frac{16}{2}\right)^{\text{th}}$ and the $\left(\frac{16}{2}+1\right)^{\text{th}}$ observations, i.e., the mean of the values of the 8th and 9th observations will be the median [see Fig. 14.9 (ii)].

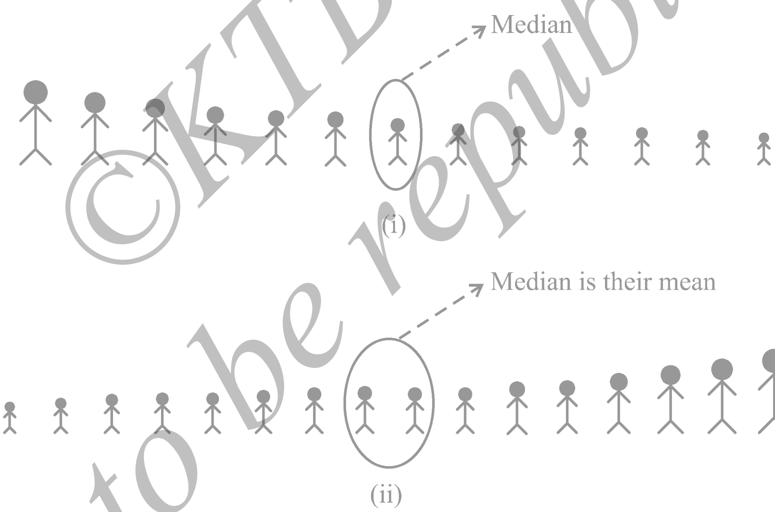


Fig. 14.9

Let us illustrate this with the help of some examples.

Example 12 : The heights (in cm) of 9 students of a class are as follows:

155 160 145 149 150 147 152 144 148

Find the median of this data.

Solution : First of all we arrange the data in ascending order, as follows:

144 145 147 148 149 150 152 155 160

Since the number of students is 9, an odd number, we find out the median by finding the height of the $\left(\frac{n+1}{2}\right)$th $= \left(\frac{9+1}{2}\right)$th = the 5th student, which is 149 cm.

So, the median, i.e., the medial height is 149 cm.

Example 13 : The points scored by a Kabaddi team in a series of matches are as follows:

17, 2, 7, 27, 15, 5, 14, 8, 10, 24, 48, 10, 8, 7, 18, 28

Find the median of the points scored by the team.

Solution : Arranging the points scored by the team in ascending order, we get

2, 5, 7, 7, 8, 8, 10, 10, 14, 15, 17, 18, 24, 27, 28, 48.

There are 16 terms. So there are two middle terms, i.e. the $\frac{16}{2}$th and $\left(\frac{16}{2}+1\right)$th, i.e., the 8th and 9th terms.

So, the median is the mean of the values of the 8th and 9th terms.

i.e, the median $= \frac{10+14}{2} = 12$

So, the medial point scored by the Kabaddi team is 12.

Let us again go back to the unsorted dispute of Hari and Mary.

The third measure used by Hari to find the average was the *mode*.

The **mode** is that value of the observation which occurs most frequently, i.e., an observation with the maximum frequency is called the mode.

The readymade garment and shoe industries make great use of this measure of central tendency. Using the knowledge of mode, these industries decide which size of the product should be produced in large numbers.

Let us illustrate this with the help of an example.

Example 14 : Find the mode of the following marks (out of 10) obtained by 20 students:

4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9

Solution : We arrange this data in the following form :

2, 3, 3, 4, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 9, 9, 9, 9, 10, 10

Here 9 occurs most frequently, i.e., four times. So, the mode is 9.

Example 15 : Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The labourers draw a salary of ₹ 5,000 per month each while the supervisor gets ₹ 15,000 per month. Calculate the mean, median and mode of the salaries of this unit of the factory.

Solution : Mean = $\dfrac{5000+5000+5000+5000+15000}{5} = \dfrac{35000}{5} = 7000$

So, the mean salary is ₹ 7000 per month.

To obtain the median, we arrange the salaries in ascending order:

5000, 5000, 5000, 5000, 15000

Since the number of employees in the factory is 5, the median is given by the $\left(\dfrac{5+1}{2}\right)$th $= \dfrac{6}{2}$th = 3rd observation. Therefore, the median is ₹ 5000 per month.

To find the mode of the salaries, i.e., the modal salary, we see that 5000 occurs the maximum number of times in the data 5000, 5000, 5000, 5000, 15000. So, the modal salary is ₹ 5000 per month.

Now compare the three measures of central tendency for the given data in the example above. You can see that the mean salary of ₹ 7000 does not give even an approximate estimate of any one of their wages, while the medial and modal salaries of ₹ 5000 represents the data more effectively.

Extreme values in the data affect the mean. This is one of the weaknesses of the mean. So, if the data has a few points which are very far from most of the other points, (like 1,7,8,9,9) then the mean is not a good representative of this data. Since the median and mode are not affected by extreme values present in the data, they give a better estimate of the average in such a situation.

Again let us go back to the situation of Hari and Mary, and compare the three measures of central tendency.

Measures of central tendency	**Hari**	**Mary**
Mean	8.2	8.4
Median	10	8
Mode	10	8

This comparison helps us in stating that these measures of central tendency are not sufficient for concluding which student is better. We require some more information to conclude this, which you will study about in the higher classes.

EXERCISE 14.4

1. The following number of goals were scored by a team in a series of 10 matches:

   2, 3, 4, 5, 0, 1, 3, 3, 4, 3

   Find the mean, median and mode of these scores.

2. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:

   41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60

   Find the mean, median and mode of this data.

3. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x.

   29, 32, 48, 50, x, $x+2$, 72, 78, 84, 95

4. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

5. Find the mean salary of 60 workers of a factory from the following table:

Salary (in ₹)	**Number of workers**
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1
Total	60

6. Give one example of a situation in which
   (i) the mean is an appropriate measure of central tendency.
   (ii) the mean is not an appropriate measure of central tendency but the median is an appropriate measure of central tendency.

14.6 Summary

In this chapter, you have studied the following points:

1. Facts or figures, collected with a definite purpose, are called data.
2. Statistics is the area of study dealing with the presentation, analysis and interpretation of data.
3. How data can be presented graphically in the form of bar graphs, histograms and frequency polygons.
4. The three measures of central tendency for **ungrouped data** are:

   (i) Mean : It is found by adding all the values of the observations and dividing it by the total number of observations. It is denoted by $\bar{x}$.

   So, $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$. For an ungrouped frequency distribution, it is $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$.

   (ii) Median : It is the value of the middle-most observation (s).

   If n is an odd number, the median = value of the $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ observation.

   If n is an even number, median = Mean of the values of the $\left(\dfrac{n}{2}\right)^{\text{th}}$ and $\left(\dfrac{n}{2}+1\right)^{\text{th}}$ observations.

   (iii) Mode : The mode is the most frequently occurring observation.

CHAPTER 15

PROBABILITY

It is remarkable that a science, which began with the consideration of games of chance, should be elevated to the rank of the most important subject of human knowledge. — ***Pierre Simon Laplace***

15.1 Introduction

In everyday life, we come across statements such as

(1) It will **probably** rain today.

(2) I **doubt** that he will pass the test.

(3) **Most probably**, Kavita will stand first in the annual examination.

(4) **Chances** are high that the prices of diesel will go up.

(5) There is a 50-50 **chance** of India winning a toss in today's match.

The words 'probably', 'doubt', 'most probably', 'chances', etc., used in the statements above involve an element of uncertainty. For example, in (1), 'probably rain' will mean it may rain or may not rain today. We are predicting rain today based on our past experience when it rained under similar conditions. Similar predictions are also made in other cases listed in (2) to (5).

The uncertainty of 'probably' etc can be measured numerically by means of 'probability' in many cases.

Though probability started with gambling, it has been used extensively in the fields of Physical Sciences, Commerce, Biological Sciences, Medical Sciences, Weather Forecasting, etc.

15.2 Probability – an Experimental Approach

In earlier classes, you have had a glimpse of probability when you performed experiments like tossing of coins, throwing of dice, etc., and observed their *outcomes*. You will now learn to measure the chance of occurrence of a particular *outcome* in an experiment.

Blaise Pascal (1623–1662)
Fig. 15.1

The concept of probability developed in a very strange manner. In 1654, a gambler Chevalier de Mere, approached the well-known 17th century French philosopher and mathematician Blaise Pascal regarding certain dice problems. Pascal became interested in these problems, studied them and discussed them with another French mathematician, Pierre de Fermat. Both Pascal and Fermat solved the problems independently. This work was the beginning of Probability Theory.

Pierre de Fermat (1601–1665)
Fig. 15.2

The first book on the subject was written by the Italian mathematician, J.Cardan (1501–1576). The title of the book was 'Book on Games of Chance' (Liber de Ludo Aleae), published in 1663. Notable contributions were also made by mathematicians J. Bernoulli (1654–1705), P. Laplace (1749–1827), A.A. Markov (1856–1922) and A.N. Kolmogorov (born 1903).

Activity 1 : (i) Take any coin, toss it ten times and note down the number of times a head and a tail come up. Record your observations in the form of the following table

Table 15.1

Number of times the coin is tossed	Number of times head comes up	Number of times tail comes up
10	—	—

Write down the values of the following fractions:

$$\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$$

and

$$\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$$

(ii) Toss the coin twenty times and in the same way record your observations as above. Again find the values of the fractions given above for this collection of observations.

(iii) Repeat the same experiment by increasing the number of tosses and record the number of heads and tails. Then find the values of the corresponding fractions.

You will find that as the number of tosses gets larger, the values of the fractions come closer to 0.5. To record what happens in more and more tosses, the following group activity can also be performed:

Acitivity 2 : Divide the class into groups of 2 or 3 students. Let a student in each group toss a coin 15 times. Another student in each group should record the observations regarding heads and tails. [Note that coins of the same denomination should be used in all the groups. It will be treated as if only one coin has been tossed by all the groups.]

Now, on the blackboard, make a table like Table 15.2. First, Group 1 can write down its observations and calculate the resulting fractions. Then Group 2 can write down its observations, but will calculate the fractions for the combined data of Groups 1 and 2, and so on. (We may call these fractions as *cumulative fractions*.) We have noted the first three rows based on the observations given by one class of students.

Table 15.2

Group (1)	Number of heads (2)	Number of tails (3)	$\dfrac{\text{Cumulative number of heads}}{\text{Total number of times the coin is tossed}}$ (4)	$\dfrac{\text{Cumulative number of tails}}{\text{Total number of times the coin is tossed}}$ (5)
1	3	12	$\dfrac{3}{15}$	$\dfrac{12}{15}$
2	7	8	$\dfrac{7+3}{15+15}=\dfrac{10}{30}$	$\dfrac{8+12}{15+15}=\dfrac{20}{30}$
3	7	8	$\dfrac{7+10}{15+30}=\dfrac{17}{45}$	$\dfrac{8+20}{15+30}=\dfrac{28}{45}$
4	⋮	⋮	⋮	⋮

What do you observe in the table? You will find that as the total number of tosses of the coin increases, the values of the fractions in Columns (4) and (5) come nearer and nearer to 0.5.

Activity 3 : (i) Throw a die* 20 times and note down the number of times the numbers

*A die is a well balanced cube with its six faces marked with numbers from 1 to 6, one number on one face. Sometimes dots appear in place of numbers.

1, 2, 3, 4, 5, 6 come up. Record your observations in the form of a table, as in Table 15.3:

Table 15.3

Number of times a die is thrown	**Number of times these scores turn up**					
	1	**2**	**3**	**4**	**5**	**6**
20						

Find the values of the following fractions:

$$\frac{\text{Number of times 1 turned up}}{\text{Total number of times the die is thrown}}$$

$$\frac{\text{Number of times 2 turned up}}{\text{Total number of times the die is thrown}}$$

$$\vdots$$

$$\frac{\text{Number of times 6 turned up}}{\text{Total number of times the die is thrown}}$$

(ii) Now throw the die 40 times, record the observations and calculate the fractions as done in (i).

As the number of throws of the die increases, you will find that the value of each fraction calculated in (i) and (ii) comes closer and closer to $\frac{1}{6}$.

To see this, you could perform a group activity, as done in Activity 2. Divide the students in your class, into small groups. One student in each group should throw a die ten times. Observations should be noted and cumulative fractions should be calculated.

The values of the fractions for the number 1 can be recorded in Table 15.4. This table can be extended to write down fractions for the other numbers also or other tables of the same kind can be created for the other numbers.

Table 15.4

Group (1)	Total number of times a die is thrown in a group (2)	$\frac{\text{Cumulative number of times 1 turned up}}{\text{Total number of times the die is thrown}}$ (3)
1	—	—
2	—	—
3	—	—
4	—	—

The dice used in all the groups should be almost the same in size and appearence. Then all the throws will be treated as throws of the same die.

What do you observe in these tables?

You will find that as the total number of throws gets larger, the fractions in Column (3) move closer and closer to $\frac{1}{6}$.

Activity 4 : (i) Toss two coins simultaneously ten times and record your observations in the form of a table as given below:

Table 15.5

Number of times the two coins are tossed	Number of times no head comes up	Number of times one head comes up	Number of times two heads come up
10	—	—	—

Write down the fractions:

$$A = \frac{\text{Number of times no head comes up}}{\text{Total number of times two coins are tossed}}$$

$$B = \frac{\text{Number of times one head comes up}}{\text{Total number of times two coins are tossed}}$$

$$C = \frac{\text{Number of times two heads come up}}{\text{Total number of times two coins are tossed}}$$

Calculate the values of these fractions.

Now increase the number of tosses (as in Activitiy 2). You will find that the more the number of tosses, the closer are the values of A, B and C to 0.25, 0.5 and 0.25, respectively.

In Activity 1, each toss of a coin is called a *trial*. Similarly in Activity 3, each throw of a die is a *trial*, and each simultaneous toss of two coins in Activity 4 is also a *trial*.

So, *a trial is an action which results in one or several outcomes*. The possible outcomes in Activity 1 were Head and Tail; whereas in Activity 3, the possible outcomes were 1, 2, 3, 4, 5 and 6.

In Activity 1, the getting of a head in a particular throw is an *event with outcome 'head'*. Similarly, *getting a tail is an event with outcome 'tail'*. In Activity 2, the getting of a particular number, say 1, is an *event* with outcome 1.

If our experiment was to throw the die for getting an even number, then the event would consist of three outcomes, namely, 2, 4 and 6.

So, an *event* for an experiment is the collection of some outcomes of the experiment. In Class X, you will study a more formal definition of an event.

So, can you now tell what the events are in Activity 4?

With this background, let us now see what probability is. Based on what we directly observe as the outcomes of our trials, we find the *experimental* or *empirical* probability.

Let n be the total number of trials. The *empirical probability* P(E) of an event E happening, is given by

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{The total number of trials}}$$

In this chapter, we shall be finding the empirical probability, though we will write 'probability' for convenience.

Let us consider some examples.

To start with let us go back to Activity 2, and Table 15.2. In Column (4) of this table, what is the fraction that you calculated? Nothing, but it is the empirical probability of getting a head. Note that this probability kept changing depending on the number of trials and the number of heads obtained in these trials. Similarly, the empirical probability of getting a tail is obtained in Column (5) of Table 15.2. This is $\frac{12}{15}$ to start with, then it is $\frac{2}{3}$, then $\frac{28}{45}$, and so on.

So, the empirical probability depends on the number of trials undertaken, and the number of times the outcomes you are looking for coming up in these trials.

Activity 5 : Before going further, look at the tables you drew up while doing Activity 3. Find the probabilities of getting a 3 when throwing a die a certain number of times. Also, show how it changes as the number of trials increases.

Now let us consider some other examples.

Example 1 : A coin is tossed 1000 times with the following frequencies:

Head : 455, Tail : 545

Compute the probability for each event.

Solution : Since the coin is tossed 1000 times, the total number of trials is 1000. Let us call the events of getting a head and of getting a tail as E and F, respectively. Then, the number of times E happens, i.e., the number of times a head come up, is 455.

So, the probability of E = $\dfrac{\text{Number of heads}}{\text{Total number of trials}}$

i.e., $$P(E) = \frac{455}{1000} = 0.455$$

Similarly, the probability of the event of getting a tail = $\dfrac{\text{Number of tails}}{\text{Total number of trials}}$

i.e., $$P(F) = \frac{545}{1000} = 0.545$$

Note that in the example above, P(E) + P(F) = 0.455 + 0.545 = 1, and E and F are the only two possible outcomes of each trial.

Example 2 : Two coins are tossed simultaneously 500 times, and we get

Two heads : 105 times

One head : 275 times

No head : 120 times

Find the probability of occurrence of each of these events.

Solution : Let us denote the events of getting two heads, one head and no head by E_1, E_2 and E_3, respectively. So,

$$P(E_1) = \frac{105}{500} = 0.21$$

$$P(E_2) = \frac{275}{500} = 0.55$$

$$P(E_3) = \frac{120}{500} = 0.24$$

Observe that $P(E_1) + P(E_2) + P(E_3) = 1$. Also E_1, E_2 and E_3 cover all the outcomes of a trial.

Example 3 : A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table :

Table 15.6

Outcome	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

Find the probability of getting each outcome.

Solution : Let E_i denote the event of getting the outcome i, where $i = 1, 2, 3, 4, 5, 6$. Then

$$\text{Probability of the outcome } 1 = P(E_1) = \frac{\text{Frequency of 1}}{\text{Total number of times the die is thrown}}$$

$$= \frac{179}{1000} = 0.179$$

Similarly, $P(E_2) = \dfrac{150}{1000} = 0.15$, $P(E_3) = \dfrac{157}{1000} = 0.157$,

$P(E_4) = \dfrac{149}{1000} = 0.149$, $P(E_5) = \dfrac{175}{1000} = 0.175$

and $P(E_6) = \dfrac{190}{1000} = 0.19$.

Note that $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$

Also note that:

(i) The probability of each event lies between 0 and 1.

(ii) The sum of all the probabilities is 1.

(iii) $E_1, E_2, \ldots, E_6$ cover all the possible outcomes of a trial.

Example 4 : On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in Table 15.7 :

Table 15.7

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

Without looking at the page, the pencil is placed on one of these numbers, i.e., the number is chosen at *random*. What is the probability that the digit in its unit place is 6?

Solution : The probability of digit 6 being in the unit place

$$= \frac{\text{Frequency of 6}}{\text{Total number of selected telephone numbers}}$$

$$= \frac{14}{200} = 0.07$$

You can similarly obtain the empirical probabilities of the occurrence of the numbers having the other digits in the unit place.

Example 5 : The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

(i) What is the probability that on a given day it was correct?

(ii) What is the probability that it was not correct on a given day?

Solution : The total number of days for which the record is available = 250

(i) P(the forecast was correct on a given day)

$$= \frac{\text{Number of days when the forecast was correct}}{\text{Total number of days for which the record is available}}$$

$$= \frac{175}{250} = 0.7$$

(ii) The number of days when the forecast was not correct = 250 – 175 = 75

So, P(the forecast was not correct on a given day) = $\frac{75}{250} = 0.3$

Notice that:

P(forecast was correct on a given day) + P(forecast was not correct on a given day)

$= 0.7 + 0.3 = 1$

Example 6 : A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Table 15.8

Distance (in km)	less than 4000	4000 to 9000	9001 to 14000	more than 14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that :

(i) it will need to be replaced before it has covered 4000 km?

(ii) it will last more than 9000 km?

(iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

Solution : (i) The total number of trials = 1000.

The frequency of a tyre that needs to be replaced before it covers 4000 km is 20.

So, P(tyre to be replaced before it covers 4000 km) = $\frac{20}{1000} = 0.02$

(ii) The frequency of a tyre that will last more than 9000 km is 325 + 445 = 770

So, P(tyre will last more than 9000 km) = $\frac{770}{1000} = 0.77$

(iii) The frequency of a tyre that requires replacement between 4000 km and 14000 km is 210 + 325 = 535.

So, P(tyre requiring replacement between 4000 km and 14000 km) = $\frac{535}{1000} = 0.535$

Example 7 : The percentage of marks obtained by a student in the monthly unit tests are given below:

Table 15.9

Unit test	I	II	III	IV	V
Percentage of marks obtained	69	71	73	68	74

Based on this data, find the probability that the student gets more than 70% marks in a unit test.

Solution : The total number of unit tests held is 5.
The number of unit tests in which the student obtained more than 70% marks is 3.

So, P(scoring more than 70% marks) = $\frac{3}{5}$ = 0.6

Example 8 : An insurance company selected 2000 drivers at random (i.e., without any preference of one driver over another) in a particular city to find a relationship between age and accidents. The data obtained are given in the following table:

Table 15.10

Age of drivers (in years)	Accidents in one year				
	0	**1**	**2**	**3**	**over 3**
18 - 29	440	160	110	61	35
30 - 50	505	125	60	22	18
Above 50	360	45	35	15	9

Find the probabilities of the following events for a driver chosen at random from the city:

(i) being 18-29 years of age *and* having exactly 3 accidents in one year.

(ii) being 30-50 years of age *and* having one or more accidents in a year.

(iii) having no accidents in one year.

Solution : Total number of drivers = 2000.

(i) The number of drivers who are 18-29 years old and have exactly 3 accidents in one year is 61.

So, P (driver is 18-29 years old with exactly 3 accidents) = $\frac{61}{2000}$

$= 0.0305 \approx 0.031$

(ii) The number of drivers 30-50 years of age and having one or more accidents in one year = 125 + 60 + 22 + 18 = 225

So, P(driver is 30-50 years of age and having one or more accidents)

$= \frac{225}{2000} = 0.1125 \approx 0.113$

(iii) The number of drivers having no accidents in one year = 440 + 505 + 360

= 1305

Therefore, P(drivers with no accident) = $\frac{1305}{2000}$ = 0.653

Example 9 : Consider the frequency distribution table (Table 14.3, Example 4, Chapter 14), which gives the weights of 38 students of a class.

(i) Find the probability that the weight of a student in the class lies in the interval 46-50 kg.

(ii) Give two events in this context, one having probability 0 and the other having probability 1.

Solution : (i) The total number of students is 38, and the number of students with weight in the interval 46 - 50 kg is 3.

So, P(weight of a student is in the interval 46 - 50 kg) = $\frac{3}{38}$ = 0.079

(ii) For instance, consider the event that a student weighs 30 kg. Since no student has this weight, the probability of occurrence of this event is 0. Similarly, the probability of a student weighing more than 30 kg is $\frac{38}{38}$ = 1.

Example 10 : Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Table 15.11

Bag	1	2	3	4	5
Number of seeds germinated	40	48	42	39	41

What is the probability of germination of

(i) more than 40 seeds in a bag?

(ii) 49 seeds in a bag?

(iii) more that 35 seeds in a bag?

Solution : Total number of bags is 5.

(i) Number of bags in which more than 40 seeds germinated out of 50 seeds is 3.

P(germination of more than 40 seeds in a bag) = $\frac{3}{5}$ = 0.6

(ii) Number of bags in which 49 seeds germinated = 0.

P(germination of 49 seeds in a bag) = $\frac{0}{5}$ = 0.

(iii) Number of bags in which more than 35 seeds germinated = 5.

So, the required probability = $\frac{5}{5}$ = 1.

Remark : In all the examples above, you would have noted that the probability of an event can be any fraction from 0 to 1.

EXERCISE 15.1

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

2. 1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

(i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

3. Refer to Example 5, Section 14.4, Chapter 14. Find the probability that a student of the class was born in August.

4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

5. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in ₹)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is

(i) earning ₹ 10000 – 13000 per month and owning exactly 2 vehicles.

(ii) earning ₹ 16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than ₹ 7000 per month and does not own any vehicle.

(iv) earning ₹ 13000 – 16000 per month and owning more than 2 vehicles.

(v) owning not more than 1 vehicle.

6. Refer to Table 14.7, Chapter 14.

(i) Find the probability that a student obtained less than 20% in the mathematics test.

(ii) Find the probability that a student obtained marks 60 or above.

7. To know the opinion of the students about the subject *statistics*, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random

(i) likes statistics, (ii) does not like it.

8. Refer to Q.2, Exercise 14.2. What is the empirical probability that an engineer lives:

(i) less than 7 km from her place of work?

(ii) more than or equal to 7 km from her place of work?

(iii) within $\frac{1}{2}$ km from her place of work?

9. **Activity :** Note the frequency of two-wheelers, three-wheelers and four-wheelers going past during a time interval, in front of your school gate. Find the probability that any one vehicle out of the total vehicles you have observed is a two-wheeler.

10. **Activity :** Ask all the students in your class to write a 3-digit number. Choose any student from the room at random. What is the probability that the number written by her/him is divisible by 3? Remember that a number is divisible by 3, if the sum of its digits is divisible by 3.

11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

    4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

    Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

12. In Q.5, Exercise 14.2, you were asked to prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days.

13. In Q.1, Exercise 14.2, you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

15.3 Summary

In this chapter, you have studied the following points:

1. An event for an experiment is the collection of some outcomes of the experiment.
2. The empirical (or experimental) probability P(E) of an event E is given by

$$P(E) = \frac{\text{Number of trials in which E has happened}}{\text{Total number of trials}}$$

3. The Probability of an event lies between 0 and 1 (0 and 1 inclusive).

APPENDIX 2

INTRODUCTION TO MATHEMATICAL MODELLING

A2.1 Introduction

Right from your earlier classes, you have been solving problems related to the real-world around you. For example, you have solved problems in simple interest using the formula for finding it. The formula (or equation) is a relation between the interest and the other three quantities that are related to it, the principal, the rate of interest and the period. This formula is an example of a **mathematical model**. A **mathematical model** is a mathematical relation that describes some real-life situation.

Mathematical models are used to solve many real-life situations like:

- launching a satellite.
- predicting the arrival of the monsoon.
- controlling pollution due to vehicles.
- reducing traffic jams in big cities.

In this chapter, we will introduce you to the process of constructing mathematical models, which is called **mathematical modelling**. In mathematical modelling, we take a real-world problem and write it as an equivalent mathematical problem. We then solve the mathematical problem, and interpret its solution in terms of the real-world problem. After this we see to what extent the solution is valid in the context of the real-world problem. So, the *stages* involved in mathematical modelling are formulation, solution, interpretation and validation.

We will start by looking at the process you undertake when solving word problems, in Section A2.2. Here, we will discuss some word problems that are similar to the ones you have solved in your earlier classes. We will see later that the steps that are used for solving word problems are some of those used in mathematical modelling also.

In the next section, that is Section A2.3, we will discuss some simple models.

In Section A2.4, we will discuss the overall process of modelling, its advantages and some of its limitations.

A2.2 Review of Word Problems

In this section, we will discuss some word problems that are similar to the ones that you have solved in your earlier classes. Let us start with a problem on direct variation.

Example 1 : I travelled 432 kilometres on 48 litres of petrol in my car. I have to go by my car to a place which is 180 km away. How much petrol do I need?

Solution : We will list the steps involved in solving the problem.

Step 1 : Formulation : You know that farther we travel, the more petrol we require, that is, the amount of petrol we need varies directly with the distance we travel.

Petrol needed for travelling 432 km = 48 litres

Petrol needed for travelling 180 km = ?

Mathematical Description : Let

x = distance I travel

y = petrol I need

y varies directly with x.

So, $y = kx$, where k is a constant.

I can travel 432 kilometres with 48 litres of petrol.

So, $y = 48, x = 432$.

Therefore, $k = \frac{y}{x} = \frac{48}{432} = \frac{1}{9}$.

Since $y = kx$,

therefore, $$y = \frac{1}{9}x \qquad (1)$$

Equation or Formula (1) describes the relationship between the petrol needed and distance travelled.

Step 2 : Solution : We want to find the petrol we need to travel 180 kilometres; so, we have to find the value of y when $x = 180$. Putting $x = 180$ in (1), we have

$$y = \frac{180}{9} = 20.$$

Step 3 : **Interpretation :** Since $y = 20$, we need 20 litres of petrol to travel 180 kilometres.

Did it occur to you that you may not be able to use the formula (1) in all situations? For example, suppose the 432 kilometres route is through mountains and the 180 kilometres route is through flat plains. The car will use up petrol at a faster rate in the first route, so we cannot use the same rate for the 180 kilometres route, where the petrol will be used up at a slower rate. So the formula works if all such conditions that affect the rate at which petrol is used are the same in both the trips. Or, if there is a difference in conditions, the effect of the difference on the amount of petrol needed for the car should be very small. The petrol used will vary directly with the distance travelled only in such a situation. We assumed this while solving the problem.

Example 2 : Suppose Sudhir has invested ₹ 15,000 at 8% simple interest per year. With the return from the investment, he wants to buy a washing machine that costs ₹ 19,000. For what period should he invest ₹ 15,000 so that he has enough money to buy a washing machine?

Solution : Step 1 : **Formulation of the problem :** Here, we know the principal and the rate of interest. The interest is the amount Sudhir needs in addition to 15,000 to buy the washing machine. We have to find the number of years.

Mathematical Description : The formula for simple interest is $I = \frac{Pnr}{100}$,

where
P = Principal,
n = Number of years,
r % = Rate of interest
I = Interest earned

Here, the principal = ₹ 15,000

The money required by Sudhir for buying a washing machine = ₹ 19,000

So, the interest to be earned = ₹ (19,000 – 15,000)

= ₹ 4,000

The number of years for which ₹ 15,000 is deposited = n

The interest on ₹ 15,000 for n years at the rate of 8% = I

Then,
$$I = \frac{15000 \times n \times 8}{100}$$

So, $$I = 1200n \qquad (1)$$

gives the relationship between the number of years and interest, if ₹ 15000 is invested at an annual interest rate of 8%.

We have to find the period in which the interest earned is ₹ 4000. Putting I = 4000 in (1), we have

$$4000 = 1200n \qquad (2)$$

Step 2 : Solution of the problem : Solving Equation (2), we get

$$n = \frac{4000}{1200} = 3\frac{1}{3}.$$

Step 3 : Interpretation : Since $n = 3\frac{1}{3}$ and one third of a year is 4 months, Sudhir can buy a washing machine after 3 years and 4 months.

Can you guess the assumptions that you have to make in the example above? We have to assume that the interest rate remains the same for the period for which we calculate the interest. Otherwise, the formula $I = \frac{Pnr}{100}$ will not be valid. We have also assumed that the price of the washing machine does not increase by the time Sudhir has gathered the money.

Example 3 : A motorboat goes upstream on a river and covers the distance between two towns on the riverbank in six hours. It covers this distance downstream in five hours. If the speed of the stream is 2 km/h, find the speed of the boat in still water.

Solution : Step 1 : Formulation : We know the speed of the river and the time taken to cover the distance between two places. We have to find the speed of the boat in still water.

Mathematical Description : Let us write x for the speed of the boat, t for the time taken and y for the distance travelled. Then

$$y = tx \qquad (1)$$

Let d be the distance between the two places.

While going upstream, the actual speed of the boat

= speed of the boat – speed of the river,

because the boat is travelling against the flow of the river.

So, the speed of the boat upstream = $(x - 2)$ km/h

It takes 6 hours to cover the distance between the towns upstream. So, from (1), we get $$d = 6(x - 2) \qquad (2)$$

When going downstream, the speed of the river has to be *added* to the speed of the boat.

So, the speed of the boat downstream = $(x + 2)$ km/h

The boat takes 5 hours to cover the same distance downstream. So,

$$d = 5(x + 2) \tag{3}$$

From (2) and (3), we have

$$5(x + 2) = 6(x - 2) \tag{4}$$

Step 2 : Finding the Solution

Solving for x in Equation (4), we get $x = 22$.

Step 3 : Interpretation

Since $x = 22$, therefore the speed of the motorboat in still water is 22 km/h.

In the example above, we know that the speed of the river is not the same everywhere. It flows slowly near the shore and faster at the middle. The boat starts at the shore and moves to the middle of the river. When it is close to the destination, it will slow down and move closer to the shore. So, there is a small difference between the speed of the boat at the middle and the speed at the shore. Since it will be close to the shore for a small amount of time, this difference in speed of the river will affect the speed only for a small period. So, we can ignore this difference in the speed of the river. We can also ignore the small variations in speed of the boat. Also, apart from the speed of the river, the friction between the water and surface of the boat will also affect the actual speed of the boat. We also assume that this effect is very small.

So, we have assumed that

1. The speed of the river and the boat remains constant all the time.
2. The effect of friction between the boat and water and the friction due to air is negligible.

We have found the speed of the boat in still water with the *assumptions* (*hypotheses*) above.

As we have seen in the word problems above, there are 3 *steps in solving a word problem. These are*

1. **Formulation :** We analyse the problem and see which factors have a major influence on the solution to the problem. These are the **relevant factors**. In our first example, the relevant factors are the distance travelled and petrol consumed. We ignored the other factors like the nature of the route, driving speed, etc. Otherwise, the problem would have been more difficult to solve. The factors that we ignore are the **irrelevant factors**.

We then describe the problem mathematically, in the form of one or more mathematical equations.

2. **Solution :** We find the solution of the problem by solving the mathematical equations obtained in Step 1 using some suitable method.
3. **Interpretation :** We see what the solution obtained in Step 2 means in the context of the original word problem.

Here are some exercises for you. You may like to check your understanding of the steps involved in solving word problems by carrying out the three steps above for the following problems.

EXERCISE A 2.1

In each of the following problems, clearly state what the relevant and irrelevant factors are while going through Steps 1, 2 and 3 given above.

1. Suppose a company needs a computer for some period of time. The company can either hire a computer for ₹ 2,000 per month or buy one for ₹ 25,000. If the company has to use the computer for a long period, the company will pay such a high rent, that buying a computer will be cheaper. On the other hand, if the company has to use the computer for say, just one month, then hiring a computer will be cheaper. Find the number of months beyond which it will be cheaper to buy a computer.
2. Suppose a car starts from a place A and travels at a speed of 40 km/h towards another place B. At the same instance, another car starts from B and travels towards A at a speed of 30 km/h. If the distance between A and B is 100 km, after how much time will the cars meet?
3. The moon is about 3,84,000 km from the earth, and its path around the earth is nearly circular. Find the speed at which it orbits the earth, assuming that it orbits the earth in 24 hours. (Use $\pi = 3.14$)
4. A family pays ₹ 1000 for electricity on an average in those months in which it does not use a water heater. In the months in which it uses a water heater, the average electricity bill is ₹ 1240. The cost of using the water heater is ₹ 8.00 per hour. Find the average number of hours the water heater is used in a day.

A2.3 Some Mathematical Models

So far, nothing was new in our discussion. In this section, we are going to add another step to the three steps that we have discussed earlier. This step is called *validation.* What does validation mean? Let us see. In a real-life situation, we cannot accept a model that gives us an answer that does not match the reality. This process of checking the answer against reality, and modifying the mathematical description if necessary, is called *validation*. This is a very important step in modelling. We will introduce you to this step in this section.

First, let us look at an example, where we do not have to modify our model after validation.

Example 4 : Suppose you have a room of length 6 m and breadth 5 m. You want to cover the floor of the room with square mosaic tiles of side 30 cm. How many tiles will you need? Solve this by constructing a mathematical model.

Solution : Formulation : We have to consider the area of the room and the area of a tile for solving the problem. The side of the tile is 0.3 m. Since the length is 6 m, we can fit in $\frac{6}{0.3} = 20$ tiles along the length of the room in one row (see Fig. A2.1.).

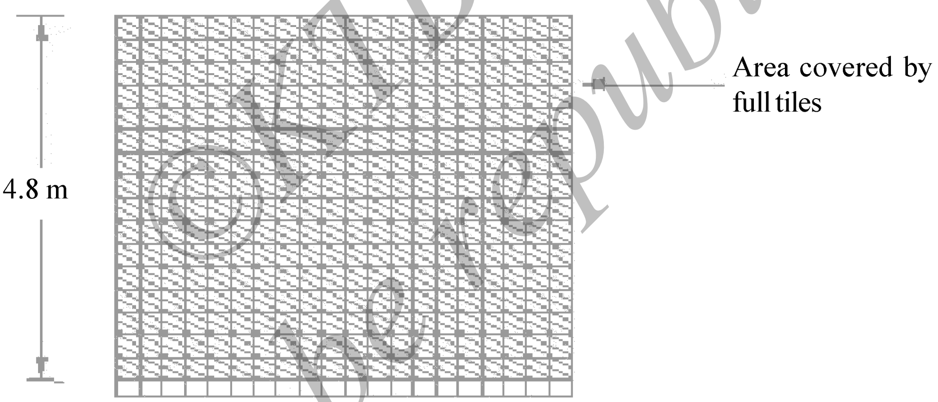


Fig. A2.1

Since the breadth of the room is 5 metres, we have $\frac{5}{0.3} = 16.67$. So, we can fit in 16 tiles in a column. Since $16 \times 0.3 = 4.8$, $5 - 4.8 = 0.2$ metres along the breadth will not be covered by tiles. This part will have to be covered by cutting the other tiles. The breadth of the floor left uncovered, 0.2 metres, is more than half the length of a tile, which is 0.3 m. So we cannot break a tile into two equal halves and use both the halves to cover the remaining portion.

Mathematical Description : We have:

Total number of tiles required = (Number of tiles along the length × Number of tiles along the breadth) + Number of tiles along the uncovered area (1)

Solution : As we said above, the number of tiles along the length is 20 and the number of tiles along the breadth is 16. We need 20 more tiles for the last row. Substituting these values in (1), we get $(20 \times 16) + 20 = 320 + 20 = 340$.

Interpretation : We need 340 tiles to cover the floor.

Validation : In real-life, your mason may ask you to buy some extra tiles to replace those that get damaged while cutting them to size. This number will of course depend upon the skill of your mason! But, we need not modify Equation (1) for this. This gives you a rough idea of the number of tiles required. So, we can stop here.

Let us now look at another situation now.

Example 5 : In the year 2000, 191 member countries of the U.N. signed a declaration. In this declaration, the countries agreed to achieve certain development goals by the year 2015. These are called the *millennium development goals*. One of these goals is to promote gender equality. One indicator for deciding whether this goal has been achieved is the ratio of girls to boys in primary, secondary and tertiary education. India, as a signatory to the declaration, is committed to improve this ratio. The data for the percentage of girls who are enrolled in primary schools is given in Table A2.1.

Table A2.1

Year	Enrolment (in %)
1991-92	41.9
1992-93	42.6
1993-94	42.7
1994-95	42.9
1995-96	43.1
1996-97	43.2
1997-98	43.5
1998-99	43.5
1999-2000	43.6*
2000-01	43.7*
2001-02	44.1*

Source : *Educational statistics, webpage of Department of Education, GOI.*

* *indicates that the data is provisional.*

Using this data, mathematically describe the rate at which the proportion of girls enrolled in primary schools grew. Also, estimate the year by which the enrolment of girls will reach 50%.

Solution : Let us first convert the problem into a mathematical problem.

Step 1 : Formulation : Table A2.1 gives the enrolment for the years 1991-92, 1992-93, etc. Since the students join at the beginning of an academic year, we can take the years as 1991, 1992, etc. Let us assume that the percentage of girls who join primary schools will continue to grow at the same rate as the rate in Table A2.1. So, the number of years is important, not the specific years. (To give a similar situation, when we find the simple interest for, say, ₹ 1500 at the rate of 8% for three years, it does not matter whether the three-year period is from 1999 to 2002 or from 2001 to 2004. What is important is the interest rate in the years being considered). Here also, we will see how the enrolment grows after 1991 by comparing the number of years that has passed after 1991 and the enrolment. Let us take 1991 as the 0th year, and write 1 for 1992 since 1 year has passed in 1992 after 1991. Similarly, we will write 2 for 1993, 3 for 1994, etc. So, Table A2.1 will now look like as Table A2.2.

Table A2.2

Year	Enrolment (in %)
0	41.9
1	42.6
2	42.7
3	42.9
4	43.1
5	43.2
6	43.5
7	43.5
8	43.6
9	43.7
10	44.1

The increase in enrolment is given in the following table :

Table A2.3

Year	Enrolment (in %)	Increase
0	41.9	0
1	42.6	0.7
2	42.7	0.1
3	42.9	0.2
4	43.1	0.2
5	43.2	0.1
6	43.5	0.3
7	43.5	0
8	43.6	0.1
9	43.7	0.1
10	44.1	0.4

At the end of the one-year period from 1991 to 1992, the enrolment has increased by 0.7% from 41.9% to 42.6%. At the end of the second year, this has increased by 0.1%, from 42.6% to 42.7%. From the table above, we cannot find a definite relationship between the number of years and percentage. But the increase is fairly steady. Only in the first year and in the 10th year there is a jump. The mean of the values is

$$\frac{0.7 + 0.1 + 0.2 + 0.2 + 0.1 + 0.3 + 0 + 0.1 + 0.1 + 0.4}{10} = 0.22$$

Let us assume that the enrolment steadily increases at the rate of 0.22 per cent.

Mathematical Description : We have assumed that the enrolment increases steadily at the rate of 0.22% per year.

So, the Enrolment Percentage (EP) in the first year = 41.9 + 0.22

EP in the second year = 41.9 + 0.22 + 0.22 = 41.9 + 2 × 0.22

EP in the third year = 41.9 + 0.22 + 0.22 + 0.22 = 41.9 + 3 × 0.22

So, the enrolment percentage in the nth year = $41.9 + 0.22n$, for $n \geq 1$. (1)

Now, we also have to find the number of years by which the enrolment will reach 50%. So, we have to find the value of n in the equation or formula

$$50 = 41.9 + 0.22n \quad (2)$$

Step 2 : Solution : Solving (2) for n, we get

$$n = \frac{50 - 41.9}{0.22} = \frac{8.1}{0.22} = 36.8$$

Step 3 : Interpretation : Since the number of years is an integral value, we will take the next higher integer, 37. So, the enrolment percentage will reach 50% in 1991 + 37 = 2028.

In a word problem, we generally stop here. But, since we are dealing with a real-life situation, we have to see to what extent this value matches the real situation.

Step 4 : Validation: Let us check if Formula (2) is in agreement with the reality. Let us find the values for the years we already know, using Formula (2), and compare it with the known values by finding the difference. The values are given in Table A2.4.

Table A2.4

Year	Enrolment (in %)	Values given by (2) (in %)	Difference (in %)
0	41.9	41.90	0
1	42.6	42.12	0.48
2	42.7	42.34	0.36
3	42.9	42.56	0.34
4	43.1	42.78	0.32
5	43.2	43.00	0.20
6	43.5	43.22	0.28
7	43.5	43.44	0.06
8	43.6	43.66	–0.06
9	43.7	43.88	–0.18
10	44.1	44.10	0.00

As you can see, some of the values given by Formula (2) are less than the actual values by about 0.3% or even by 0.5%. This can give rise to a difference of about 3 to 5 years since the increase per year is actually 1% to 2%. We may decide that this much of a difference is acceptable and stop here. In this case, (2) is our mathematical model.

Suppose we decide that this error is quite large, and we have to improve this model. Then we have to go back to Step 1, the formulation, and change Equation (2). Let us do so.

Step 1 : Reformulation : We still assume that the values increase steadily by 0.22%, but we will now introduce a correction factor to reduce the error. For this, we find the mean of all the errors. This is

$$\frac{0 + 0.48 + 0.36 + 0.34 + 0.32 + 0.2 + 0.28 + 0.06 - 0.06 - 0.18 + 0}{10} = 0.18$$

We take the mean of the errors, and correct our formula by this value.

Revised Mathematical Description : Let us now add the mean of the errors to our formula for enrolment percentage given in (2). So, our corrected formula is:

Enrolment percentage in the nth year $= 41.9 + 0.22n + 0.18 = 42.08 + 0.22n$, for $n \geq 1$ (3)

We will also modify Equation (2) appropriately. The new equation for n is:

$$50 = 42.08 + 0.22n \quad (4)$$

Step 2 : Altered Solution : Solving Equation (4) for n, we get

$$n = \frac{50 - 42.08}{0.22} = \frac{7.92}{0.22} = 36$$

Step 3 : Interpretation: Since $n = 36$, the enrolment of girls in primary schools will reach 50% in the year 1991 + 36 = 2027.

Step 4 : Validation: Once again, let us compare the values got by using Formula (4) with the actual values. Table A2.5 gives the comparison.

Table A2.5

Year	Enrolment (in %)	Values given by (2)	Difference between values	Values given by (4)	Difference between values
0	41.9	41.90	0	41.9	0
1	42.6	42.12	0.48	42.3	0.3
2	42.7	42.34	0.36	42.52	0.18
3	42.9	42.56	0.34	42.74	0.16
4	43.1	42.78	0.32	42.96	0.14
5	43.2	43.00	0.2	43.18	0.02
6	43.5	43.22	0.28	43.4	0.1
7	43.5	43.44	0.06	43.62	– 0.12
8	43.6	43.66	– 0.06	43.84	– 0.24
9	43.7	43.88	– 0.18	44.06	– 0.36
10	44.1	44.10	0	44.28	– 0.18

As you can see, many of the values that (4) gives are closer to the actual value than the values that (2) gives. The mean of the errors is 0 in this case.

We will stop our process here. So, Equation (4) is our mathematical description that gives a mathematical relationship between years and the percentage of enrolment of girls of the total enrolment. We have constructed a mathematical model that describes the growth.

The process that we have followed in the situation above is called mathematical modelling.

We have tried to construct a mathematical model with the mathematical tools that we already have. There are better mathematical tools for making predictions from the data we have. But, they are beyond the scope of this course. Our aim in constructing this model is to explain the process of modelling to you, not to make accurate predictions at this stage.

You may now like to model some real-life situations to check your understanding of our discussion so far. Here is an Exercise for you to try.

EXERCISE A2.2

1. We have given the timings of the gold medalists in the 400-metre race from the time the event was included in the Olympics, in the table below. Construct a mathematical model relating the years and timings. Use it to estimate the timing in the next Olympics.

Table A2.6

Year	Timing (in seconds)
1964	52.01
1968	52.03
1972	51.08
1976	49.28
1980	48.88
1984	48.83
1988	48.65
1992	48.83
1996	48.25
2000	49.11
2004	49.41

A2.4 The Process of Modelling, its Advantages and Limitations

Let us now conclude our discussion by drawing out aspects of mathematical modelling that show up in the examples we have discussed. With the background of the earlier sections, we are now in a position to give a brief overview of the steps involved in modelling.

Step 1 : Formulation : You would have noticed the difference between the formulation part of Example 1 in Section A2.2 and the formulation part of the model we discussed in A2.3. In Example 1, all the information is in a readily usable form. But, in the model given in A2.3 this is not so. Further, it took us some time to find a mathematical description. We tested our first formula, but found that it was not as good as the second one we got. This is usually true in general, i.e. when trying to model real-life situations; the first model usually needs to be revised. When we are solving a real-life problem, formulation can require a lot of time. For example, Newton's three laws of motion, which are mathematical descriptions of motion, are simple enough to state. But, Newton arrived at these laws after studying a large amount of data and the work the scientists before him had done.

Formulation involves the following three steps :

(i) **Stating the problem :** Often, the problem is stated vaguely. For example, the broad goal is to ensure that the enrolment of boys and girls are equal. This may mean that 50% of the total number of boys of the school-going age and 50% of the girls of the school-going age should be enrolled. The other way is to ensure that 50% of the school-going children are girls. In our problem, we have used the second approach.

(ii) **Identifying relevant factors :** Decide which quantities and relationships are important for our problem and which are unimportant and can be neglected. For example, in our problem regarding primary schools enrolment, the percentage of girls enrolled in the previous year can influence the number of girls enrolled this year. This is because, as more and more girls enrol in schools, many more parents will feel they also have to put their daughters in schools. But, we have ignored this factor because this may become important only after the enrolment crosses a certain percentage. Also, adding this factor may make our model more complicated.

(iii) **Mathematical Description :** Now suppose we are clear about what the problem is and what aspects of it are more relevant than the others. Then we have to find a relationship between the aspects involved in the form of an equation, a graph or any other suitable mathematical description. If it is an equation, then every important aspect should be represented by a variable in our mathematical equation.

Step 2 : **Finding the solution :** The mathematical formulation does not give the solution. We have to solve this mathematical equivalent of the problem. This is where your mathematical knowledge comes in useful.

Step 3 : **Interpretating the solution :** The mathematical solution is some value or values of the variables in the model. We have to go back to the real-life problem and see what these values mean in the problem.

Step 4 : **Validating the solution :** As we saw in A2.3, after finding the solution we will have to check whether the solution matches the reality. If it matches, then the mathematical model is acceptable. If the mathematical solution does not match, *we go back to the formulation step* again and try to improve our model.

This step in the process is one major difference between solving word problems and mathematical modelling. This is one of the most important step in modelling that is missing in word problems. Of course, it is possible that in some real-life situations, we do not need to validate our answer because the problem is simple and we get the correct solution right away. This was so in the first model we considered in A2.3.

We have given a summary of the order in which the steps in mathematical modelling are carried out in Fig. A2.2 below. Movement from the validation step to the formulation step is shown using a **dotted arrow**. This is because it may not be necessary to carry out this step again.

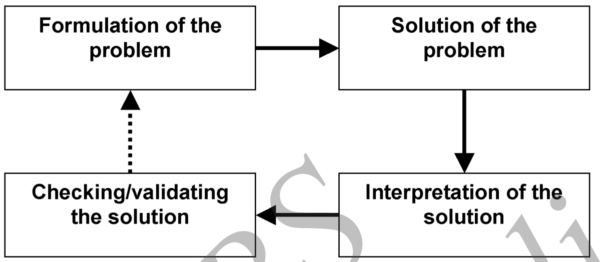


Fig.A2.2

Now that you have studied the stages involved in mathematical modelling, let us discuss some of its aspects.

The *aim* of mathematical modelling is to get some useful information about a real-world problem by converting it into a mathematical problem. This is especially useful when it is not possible or very expensive to get information by other means such as direct observation or by conducting experiments.

You may also wonder why we should undertake mathematical modelling? Let us look at some **advantages of modelling**. Suppose we want to study the corrosive effect of the discharge of the Mathura refinery on the Taj Mahal. We would not like to carry out experiments on the Taj Mahal directly since it may not be safe to do so. Of course, we can use a scaled down physical model, but we may need special facilities for this, which may be expensive. Here is where mathematical modelling can be of great use.

Again, suppose we want to know how many primary schools we will need after 5 years. Then, we can only solve this problem by using a mathematical model. Similarly, it is only through modelling that scientists have been able to explain the existence of so many phenomena.

You saw in Section A2.3, that we could have tried to improve the answer in the second example with better methods. But we stopped because we do not have the mathematical tools. This can happen in real-life also. Often, we have to be satisfied with very approximate answers, because mathematical tools are not available. For example, the model equations used in modelling weather are so complex that mathematical tools to find exact solutions are not available.

You may wonder to what extent we should try to improve our model. Usually, to improve it, we need to take into account more factors. When we do this, we add more variables to our mathematical equations. We may then have a very complicated model that is difficult to use. A model must be simple enough to use. A good model balances two factors:

1. Accuracy, i.e., how close it is to reality.
2. Ease of use.

For example, Newton's laws of motion are very simple, but powerful enough to model many physical situations.

So, is mathematical modelling the answer to all our problems? Not quite! It has its limitations.

Thus, we should keep in mind that a model is *only a simplification* of a real-world problem, and the two are not the same. It is something like the difference between a map that gives the physical features of a country, and the country itself. We can find the height of a place above the sea level from this map, but we cannot find the characteristics of the people from it. So, we should use a model only for the purpose it is supposed to serve, remembering all the factors we have neglected while constructing it. We should apply the model only within the limits where it is applicable. In the later classes, we shall discuss this aspect a little more.

EXERCISE A2.3

1. How are the solving of word problems that you come across in textbooks different from the process of mathematical modelling?

2. Suppose you want to minimise the waiting time of vehicles at a traffic junction of four roads. Which of these factors are important and which are not?

(i) Price of petrol.

(ii) The rate at which the vehicles arrive in the four different roads.

(iii) The proportion of slow-moving vehicles like cycles and rickshaws and fast moving vehicles like cars and motorcycles.

A2.5 Summary

In this Appendix, you have studied the following points :

1. The steps involved in solving word problems.

2. Construction of some mathematical models.

3. The steps involved in mathematical modelling given in the box below.

1. **Formulation :**
   (i) Stating the question
   (ii) Identifying the relevant factors
   (iii) Mathematical description
2. **Finding the solution.**
3. **Interpretation of the solution in the context of the real-world problem.**
4. **Checking/validating to what extent the model is a good representation of the problem being studied.**

4. The aims, advantages and limitations of mathematical modelling.

ANSWERS/HINTS

EXERCISE 8.1

1. $\frac{\sqrt{3}}{4}a^2$, $900\sqrt{3}\,\text{cm}^2$ **2.** ₹ 1650000 **3.** $20\sqrt{2}\ \text{m}^2$

4. $21\sqrt{11}\,\text{cm}^2$ **5.** $9000\ \text{cm}^2$ **6.** $9\sqrt{15}\ \text{cm}^2$

EXERCISE 8.2

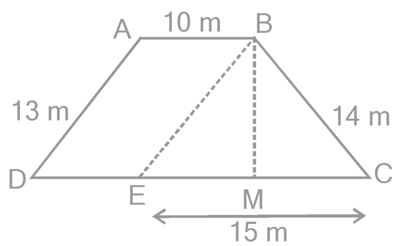
1. $65.5\ \text{m}^2$ (approx.) **2.** $15.2\ \text{cm}^2$ (approx.) **3.** $19.4\ \text{cm}^2$ (approx.)

4. 12 cm **5.** $48\ \text{m}^2$ **6.** $1000\sqrt{6}\ \text{cm}^2$, $1000\sqrt{6}\ \text{cm}^2$

7. Area of shade I = Area of shade II = $256\ \text{cm}^2$ and area of shade III = $17.92\ \text{cm}^2$

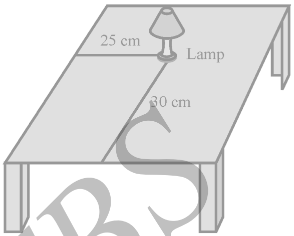
8. ₹ 705.60 **9.** $196\ \text{m}^2$

[See the figure. Find area of Δ BEC = $84\ \text{m}^2$, then find the height BM.]

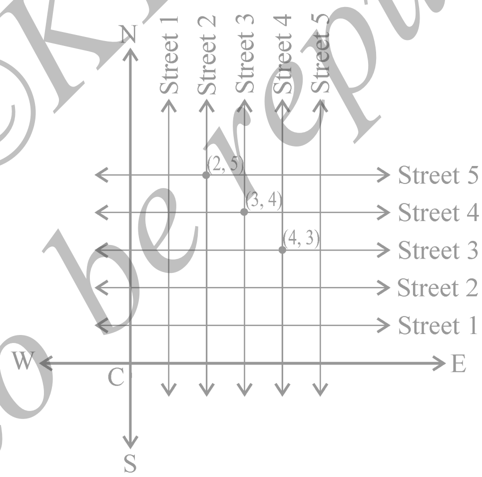


EXERCISE 9.1

1. Consider the lamp as a point and table as a plane. Choose any two perpendicular edges of the table. Measure the distance of the lamp from the longer edge, suppose it is 25 cm. Again, measure the distance of the lamp from the shorter edge, and suppose it is 30 cm. You can write the position of the lamp as (30, 25) or (25, 30), depending on the order you fix.



2. The Street plan is shown in figure given below.



Both the cross-streets are marked in the figure above. They are *uniquely* found because of the two reference lines we have used for locating them.

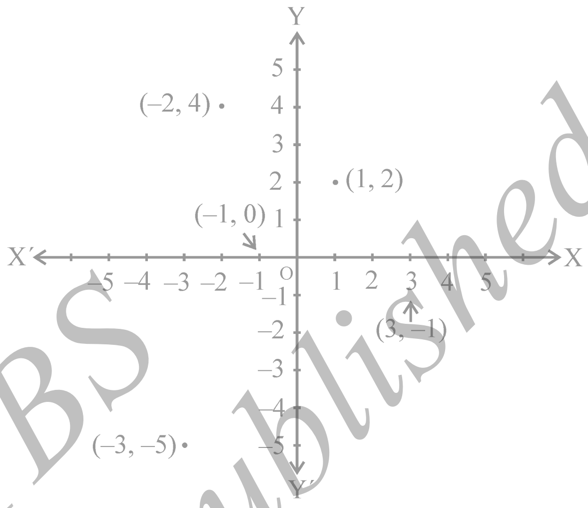
EXERCISE 9.2

1. (i) The x - axis and the y - axis (ii) Quadrants (iii) The origin

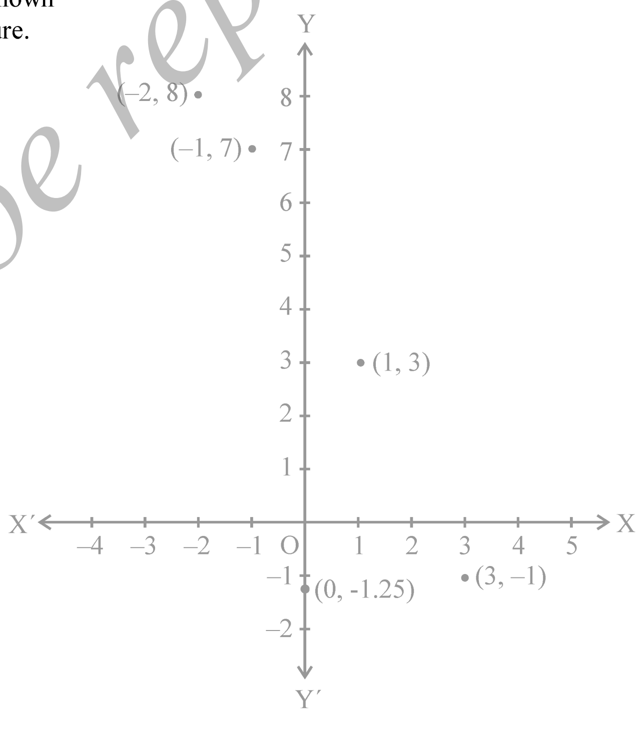
2. (i) $(-5, 2)$ (ii) $(5, -5)$ (iii) E (iv) G (v) 6 (vi) -3 (vii) $(0, 5)$ (viii) $(-3, 0)$

EXERCISE 9.3

1. The point (–2, 4) lies in quadrant II, the point (3, –1) lies in the quadrant IV, the point (– 1, 0) lies on the negative x - axis, the point (1, 2) lies in the quadrant I and the point (–3, –5) lies in the quadrant III. Locations of the points are shown in the adjoining figure.



2. Positions of the points are shown by dots in the adjoining figure.



EXERCISE 10.1

1. $x - 2y = 0$

2. (i) $2x + 3y - 9.3\overline{5} = 0$; $a = 2$, $b = 3$, $c = -9.3\overline{5}$

(ii) $x - \frac{y}{5} - 10 = 0$; $a = 1$, $b = \frac{-1}{5}$, $c = -10$

(iii) $-2x + 3y - 6 = 0$; $a = -2$, $b = 3$, $c = -6$

(iv) $1.x - 3y + 0 = 0$; $a = 1$, $b = -3$, $c = 0$

(v) $2x + 5y + 0 = 0$; $a = 2$, $b = 5$, $c = 0$

(vi) $3x + 0.y + 2 = 0$; $a = 3$, $b = 0$, $c = 2$

(vii) $0.x + 1.y - 2 = 0$; $a = 0$, $b = 1$, $c = -2$

(viii) $-2x + 0.y + 5 = 0$; $a = -2$, $b = 0$, $c = 5$

EXERCISE 10.2

1. (iii), because for every value of x, there is a corresponding value of y and vice-versa.

2. (i) (0, 7), (1, 5), (2, 3), (4, –1)

(ii) $(1, 9 - \pi)$, $(0, 9)$, $(-1, 9 + \pi)$, $\left(\frac{9}{\pi}, 0\right)$

(iii) (0, 0), (4, 1), (– 4, 1), $\left(2, \frac{1}{2}\right)$

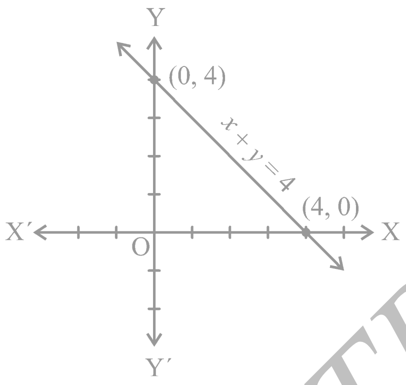
3. (i) No (ii) No

(iii) Yes (iv) No (v) No

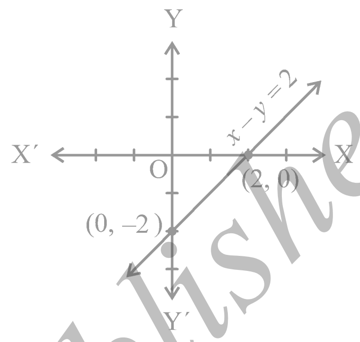
4. 7

EXERCISE 10.3

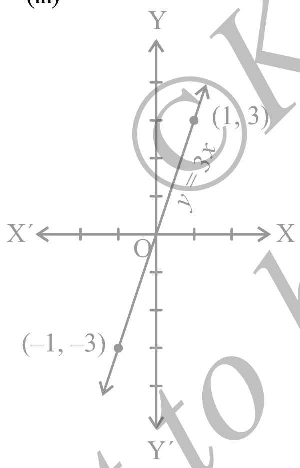
1. (i)



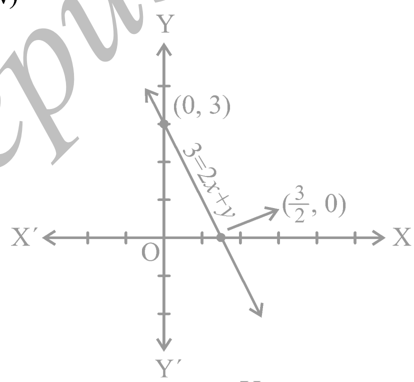
(ii)



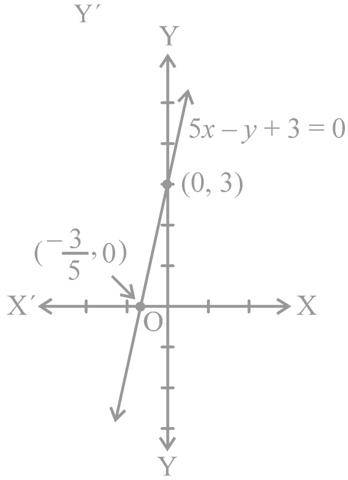
(iii)



(iv)



2. $7x - y = 0$ and $x + y = 16$; infintely many [Through a point infinitely many lines can be drawn]

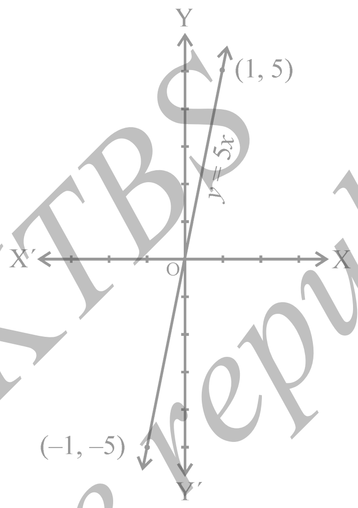


3. $\dfrac{5}{3}$ **4.** $5x - y + 3 = 0$

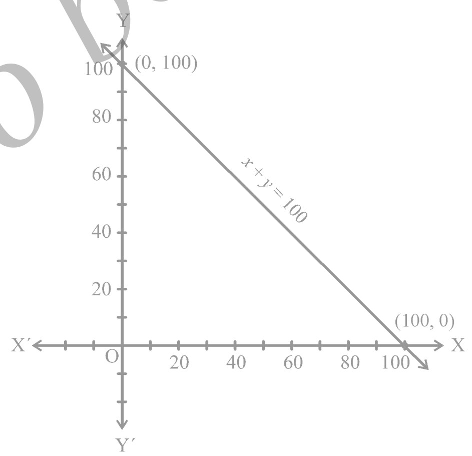
5. For Fig. 4.6, $x + y = 0$ and for Fig. 4.7, $y = -x + 2$.

6. Supposing x is the distance and y is the work done. Therefore according to the problem the equation will be $y = 5x$.

(i) 10 units (ii) 0 unit



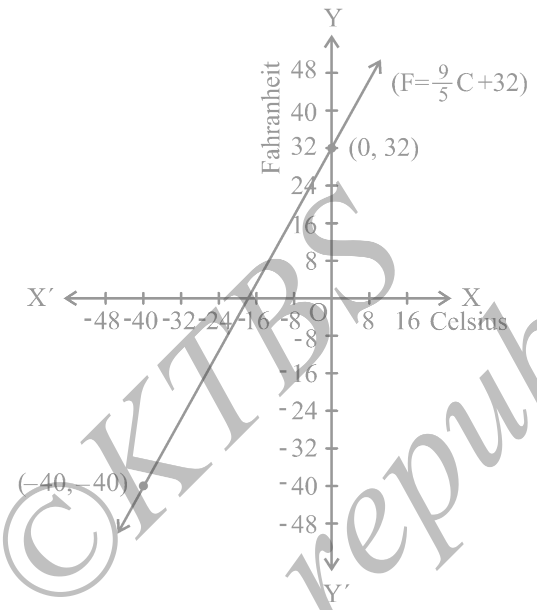
7. $x + y = 100$



8. (i) See adjacent figure. (ii) 86° F

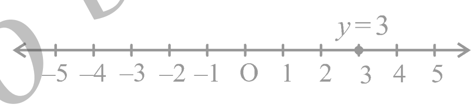
(iii) 35° C (iv) 32° F, –17.8° C (approximately)

(v) Yes, – 40° (both in F and C)

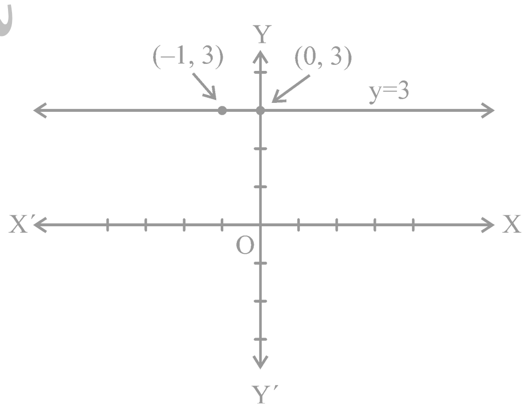


EXERCISE 10.4

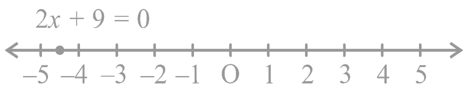
1. (i)



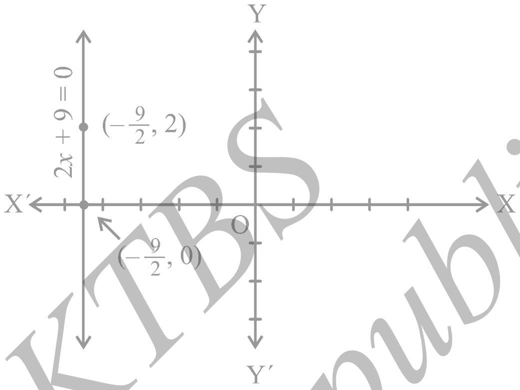
(ii)



2. (i)



(ii)



EXERCISE 11.1

1. (i) Base DC, parallels DC and AB; (iii) Base QR, parallels QR and PS;

(v) Base AD, parallels AD and BQ

EXERCISE 11.2

1. 12.8 cm. **2.** Join EG; Use result of Example 2.

6. Wheat in Δ APQ and pulses in other two triangles or pulses in Δ APQ and wheat in other two triangles.

EXERCISE 11.3

4. Draw CM $\perp$ AB and DN $\perp$ AB. Show CM = DN. **12.** See Example 4.

EXERCISE 11.4 (Optional)

7. Use result of Example 3 repeatedly.

EXERCISE 12.1

1. (i) Interior (ii) Exterior (iii) Diameter

(iv) Semicircle (v) The chord (vi) Three

2. (i) True (ii) False (iii) False

(iv) True (v) False (vi) True

EXERCISE 12.2

1. Prove exactly as Theorem 10.1 by considering chords of congruent circles.

2. Use SAS axiom of congruence to show the congruence of the two triangles.

EXERCISE 12.3

1. 0, 1, 2. Two   2. Proceed as in Example 1.

3. Join the centres O, O′ of the circles to the mid-point M of the common chord AB. Then, show $\angle$ OMA = 90° and $\angle$ O′MA = 90°.

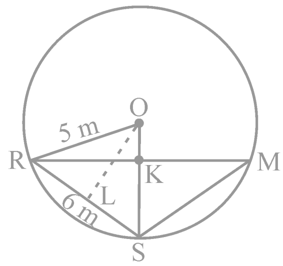
EXERCISE 12.4

1. 6 cm. First show that the line joining centres is perpendicular to the radius of the smaller circle and then that common chord is the diameter of the smaller circle.

2. If AB, CD are equal chords of a circle with centre O intersecting at E, draw perpendiculars OM on AB and ON on CD and join OE. Show that right triangles OME and ONE are congruent.

3. Proceed as in Example 2.

4. Draw perpendicular OM on AD.



5. Represent Reshma, Salma and Mandip by R, S and M respectively. Let KR = x m (see figure).

   Area of Δ ORS = $\frac{1}{2}x \times 5$. Also, area of Δ ORS = $\frac{1}{2}$ RS × OL = $\frac{1}{2} \times 6 \times 4$.

   Find x and hence RM.

6. Use the properties of an equilateral triangle and also Pythagoras Theorem.

EXERCISE 12.5

1. 45°   2. 150°, 30°   3. 10°

4. 80°   5. 110°   6. $\angle$BCD = 80° and $\angle$ECD = 50°

8. Draw perpendiculars AM and BN on CD (AB || CD and AB < CD). Show ΔAMD ≅ ΔBNC. This gives $\angle$C = $\angle$D and, therefore, $\angle$A + $\angle$C = 180°.

EXERCISE 12.6 (Optional)

2. Let O be the centre of the circle. Then perpendicular bisector of both the chords will be same and passes through O. Let r be the radius, then $r^2 = \left(\frac{11}{2}\right)^2 + x^2 = \left(\frac{5}{2}\right)^2 + (6-x)^2$, where x is length of the perpendicular from O on the chord of length 11 cm. This gives $x = 1$. So, $r = \frac{5\sqrt{5}}{2}$ cm. **3.** 3 cm.

4. Let $\angle$ AOC = x and $\angle$ DOE = y. Let $\angle$ AOD = z. Then $\angle$ EOC = z and $x + y + 2z = 360°$.

$\angle ODB = \angle OAD + \angle DOA = 90° - \frac{1}{2}z + z = 90° + \frac{1}{2}z$. Also $\angle OEB = 90° + \frac{1}{2}z$

8. $\angle ABE = \angle ADE$, $\angle ADF = \angle ACF = \frac{1}{2}\angle C$.

Therefore, $\angle EDF = \angle ABE + \angle ADF = \frac{1}{2}(\angle B + \angle C) = \frac{1}{2}(180° - \angle A) = 90° - \frac{1}{2}\angle A$.

9. Use Q. 1, Ex. 10.2 and Theorem 10.8.

10. Let angle-bisector of $\angle$ A intersect circumcircle of Δ ABC at D. Join DC and DB.

Then $\angle BCD = \angle BAD = \frac{1}{2}\angle A$ and $\angle DBC = \angle DAC = \frac{1}{2}\angle A$.

Therefore, $\angle BCD = \angle DBC$ or, DB = DC. So, D lies on the perpendicular bisector of BC.

EXERCISE 13.1

1. (i) 5.45 m^2 (ii) ₹ 109 **2.** ₹ 555 **3.** 6 m **4.** 100 bricks.

5. (i) Lateral surface area of cubical box is greater by 40 cm^2.

(ii) Total surface area of cuboidal box is greater by 10 cm^2.

6. (i) 4250 cm^2 of glass

(ii) 320 cm of tape. [Calculate the sum of all the edges (The 12 edges consist of 4 lengths, 4 breadths and 4 heights)].

7. ₹ 2184 **8.** 47 m^2

EXERCISE 13.2

1. 2 cm **2.** 7.48 m^2 **3.** (i) 968 cm^2 (ii) 1064.8 cm^2 (iii) 2038.08 cm^2

[Total surface area of a pipe is (inner curved surface area + outer curved surface area + areas of the two bases). Each base is a ring of area given by $\pi\ (R^2 - r^2)$, where R = outer radius and r = inner radius].

4. 1584 m^2 **5.** ₹ 68.75 **6.** 1 m

7. (i) 110 m^2 (ii) ₹ 4400 **8.** 4.4 m^2

9. (i) 59.4 m^2 (ii) 95.04 m^2

[Let the actual area of steel used be x m^2. Since $\frac{1}{12}$ of the actual steel used was wasted, the area of steel which has gone into the tank = $\frac{11}{12}$ of x. This means that the actual area of steel used = $\frac{12}{11} \times 87.12\,m^2$]

10. 2200 cm^2; Height of the cylinder should be treated as (30 + 2.5 + 2.5) cm

11. 7920 cm^2

EXERCISE 13.3

1. 165 cm^2 **2.** 1244.57 m^2 **3.** (i) 7 cm (ii) 462 cm^2

4. (i) 26 m (ii) ₹ 137280 **5.** 63 m **6.** ₹ 1155

7. 5500 cm^2 **8.** ₹ 384.34 (approx.)

EXERCISE 13.4

1. (i) 1386 cm^2 (ii) 394.24 cm^2 (iii) 2464 cm^2

2. (i) 616 cm^2 (ii) 1386 cm^2 (iii) 38.5 m^2

3. 942 cm^2 **4.** 1 : 4 **5.** ₹ 27.72

6. 3.5 cm **7.** 1 : 16 **8.** 173.25 cm^2

9. (i) $4\pi r^2$ (ii) $4\pi r^2$ (iii) 1 : 1

EXERCISE 13.5

1. 180 cm^3 **2.** 135000 litres **3.** 4.75 m **4.** ₹ 4320 **5.** 2 m

6. 3 days **7.** 16000 **8.** 6 cm, 4 : 1 **9.** 4000 m^3

EXERCISE 13.6

1. 34.65 litres

2. 3.432 kg [Volume of a pipe = $\pi h \times (R^2 - r^2)$, where R is the outer radius and r is the inner radius].

3. The cylinder has the greater capacity by 85 cm^3.

4. (i) 3 cm (ii) 141.3 cm^3

5. (i) 110 m^2 (ii) 1.75 m (iii) 96.25 *kl* **6.** 0.4708 m^2

7. Volume of wood = 5.28 cm^3, Volume of graphite = 0.11 cm^3.

8. 38500 cm^3 or 38.5*l* of soup

EXERCISE 13.7

1. (i) 264 cm^3 (ii) 154 cm^3 **2.** (i) 1.232 *l* (ii) $\frac{11}{35}\,l$

3. 10 cm **4.** 8 cm **5.** 38.5 *kl*

6. (i) 48 cm (ii) 50 cm (iii) 2200 cm^2

7. 100π cm^3 **8.** 240π cm^3; 5 : 12

9. 86.625x m^3, 99.825 m^2

EXERCISE 13.8

1. (i) $1437\frac{1}{3}$ cm^3 (ii) 1.05 m^3 (approx.)

2. (i) $11498\frac{2}{3}$ cm^3 (ii) 0.004851 m^3 **3.** 345.39 g (approx.) **4.** $\frac{1}{64}$

5. 0.303*l* (approx.) **6.** 0.06348 m^3 (approx.)

7. $179\frac{2}{3}$ cm^3

8. (i) 249.48 m^2 (ii) 523.9 m^3 (approx.)

9. (i) $3r$ (ii) 1 : 9

10. 22.46 mm^3 (approx.)

EXERCISE 13.9 (Optional)

1. ₹ 6275

2. ₹ 2784.32 (approx.) [Rememeber to subtract the part of the sphere that is resting on the support while calculating the cost of silver paint].

3. 43.75%

EXERCISE 14.1

1. Five examples of data that we can gather from our day-to-day life are :

(i) Number of students in our class.

(ii) Number of fans in our school.

(iii) Electricity bills of our house for last two years.

(iv) Election results obtained from television or newspapers.

(v) Literacy rate figures obtained from Educational Survey.

Of course, remember that there can be many more different answers.

2. Primary data; (i), (ii) and (iii)

Secondary data; (iv) and (v)

EXERCISE 14.2

1.

Blood group	Number of students
A	9
B	6
O	12
AB	3
Total	30

Most common – O , Rarest – AB

2.

Distances (in km)	**Tally Marks**	**Frequency**
0 - 5	𝍸	5
5 - 10	𝍸 𝍸 𝍷	11
10 - 15	𝍸 𝍸 𝍷	11
15 - 20	𝍸 𝍷𝍷𝍷𝍷	9
20 - 25	𝍷	1
25 - 30	𝍷	1
30 - 35	𝍷𝍷	2
Total		40

3. (i)

Relative humidity (in %)	**Frequency**
84 - 86	1
86 - 88	1
88 - 90	2
90 - 92	2
92 - 94	7
94 - 96	6
96 - 98	7
98 - 100	4
Total	30

(ii) The data appears to be taken in the rainy season as the relative humidity is high.

(iii) Range = 99.2 – 84.9 = 14.3

4. (i)

Heights (in cm)	Frequency
150 - 155	12
155 - 160	9
160 - 165	14
165 - 170	10
170 - 175	5
Total	50

(ii) One conclusion that we can draw from the above table is that more than 50% of students are shorter than 165 cm.

5. (i)

Concentration of Sulphur dioxide (in ppm)	Frequency
0.00 - 0.04	4
0.04 - 0.08	9
0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2
Total	30

(ii) The concentration of sulphur dioxide was more than 0.11 ppm for 8 days.

6.

Number of heads	Frequency
0	6
1	10
2	9
3	5
Total	30

7. (i)

Digits	Frequency
0	2
1	5
2	5
3	8
4	4
5	5
6	4
7	4
8	5
9	8
Total	50

(ii) The most frequently occurring digits are 3 and 9. The least occurring is 0.

8. (i)

Number of hours	Frequency
0 - 5	10
5 - 10	13
10 - 15	5
15 - 20	2
Total	30

(ii) 2 children.

9.

Life of batteries (in years)	Frequency
2.0 - 2.5	2
2.5 - 3.0	6
3.0 - 3.5	14
3.5 - 4.0	11
4.0 - 4.5	4
4.5 - 5.0	3
Total	40

EXERCISE 14.3

1. (ii) Reproductive health conditions.

3. (ii) Party A **4.** (ii) Frequency polygon (iii) No **5.** (ii) 184

8.

Age (in years)	Frequency	Width	Length of the rectangle
1 - 2	5	1	$\frac{5}{1} \times 1 = 5$
2 - 3	3	1	$\frac{3}{1} \times 1 = 3$
3 - 5	6	2	$\frac{6}{2} \times 1 = 3$
5 - 7	12	2	$\frac{12}{2} \times 1 = 6$
7 - 10	9	3	$\frac{9}{3} \times 1 = 3$
10 - 15	10	5	$\frac{10}{5} \times 1 = 2$
15 - 17	4	2	$\frac{4}{2} \times 1 = 2$

Now, you can draw the histogram, using these lengths.

9. (i)

Number of letters	Frequency	Width of interval	Length of rectangle
1 - 4	6	3	$\frac{6}{3} \times 2 = 4$
4 - 6	30	2	$\frac{30}{2} \times 2 = 30$
6 - 8	44	2	$\frac{44}{2} \times 2 = 44$
8 - 12	16	4	$\frac{16}{4} \times 2 = 8$
12 - 20	4	8	$\frac{4}{8} \times 2 = 1$

Now, draw the histogram.

(ii) 6 - 8

EXERCISE 14.4

1. Mean = 2.8; Median = 3; Mode = 3
2. Mean = 54.8; Median = 52; Mode = 52
3. $x = 62$ **4.** 14
5. Mean salary of 60 workers is Rs 5083.33.

EXERCISE 15.1

1. $\frac{24}{30}$, i.e., $\frac{4}{5}$ **2.** (i) $\frac{19}{60}$ (ii) $\frac{407}{750}$ (iii) $\frac{211}{1500}$ **3.** $\frac{3}{20}$ **4.** $\frac{9}{25}$

5. (i) $\frac{29}{2400}$ (ii) $\frac{579}{2400}$ (iii) $\frac{1}{240}$ (iv) $\frac{1}{96}$ (v) $\frac{1031}{1200}$ **6.** (i) $\frac{7}{90}$ (ii) $\frac{23}{90}$

7. (i) $\frac{27}{40}$ (ii) $\frac{13}{40}$ **8.** (i) $\frac{9}{40}$ (ii) $\frac{31}{40}$ (iii) 0 **11.** $\frac{7}{11}$ **12.** $\frac{1}{15}$ **13.** $\frac{1}{10}$

EXERCISE A2.1

1. Step 1: **Formulation :**

The relevant factors are the time period for hiring a computer, and the two costs given to us. We assume that there is no significant change in the cost of purchasing or hiring the computer. So, we treat any such change as irrelevant. We also treat all brands and generations of computers as the same, i.e. these differences are also irrelevant.

The expense of hiring the computer for x months is ₹ $2000x$. If this becomes more than the cost of purchasing a computer, we will be better off buying a computer. So, the equation is

$$2000x = 25000 \qquad (1)$$

Step 2 : **Solution :** Solving (1), $x = \frac{25000}{2000} = 12.5$

Step 3 : **Interpretation :** Since the cost of hiring a computer becomes more **after** 12.5 months, it is cheaper to buy a computer, if you have to use it for more than 12 months.

2. Step1 : **Formulation :** We will assume that cars travel at a constant speed. So, any change of speed will be treated as irrelevant. If the cars meet after x hours, the first car would have travelled a distance of $40x$ km from A and the second car would have travelled $30x$ km, so that it will be at a distance of $(100 – 30x)$ km from A. So the equation will be $40x = 100 – 30x$, i.e., $70x = 100$.

Step 2 : **Solution :** Solving the equation, we get $x = \frac{100}{70}$.

Step 3 : **Interpretation :** $\frac{100}{70}$ is approximately 1.4 hours. So, the cars will meet after 1.4 hours.

3. Step1: **Formulation :** The speed at which the moon orbits the earth is $\frac{\text{Length of the orbit}}{\text{Time taken}}$.

Step 2 : **Solution :** Since the orbit is nearly circular, the length is $2 \times \pi \times 384000$ km $= 2411520$ km

The moon takes 24 hours to complete one orbit.

So, speed $= \frac{2411520}{24} = 100480$ km/hour.

Step 3 : **Interpretation :** The speed is 100480 km/h.

4. **Formulation :** An assumption is that the difference in the bill is only because of using the water heater.

Let the average number of hours for which the water heater is used $= x$

Difference per month due to using water heater = ₹ 1240 – ₹ 1000 = ₹ 240

Cost of using water heater for one hour = ₹ 8

So, the cost of using the water heater for 30 days $= 8 \times 30 \times x$

Also, the cost of using the water heater for 30 days = Difference in bill due to using water heater

So, $240x = 240$

Solution : From this equation, we get $x = 1$.

Interpretation : Since $x = 1$, the water heater is used for an average of 1 hour in a day.

EXERCISE A2.2

1. We will not discuss any particular solution here. You can use the same method as we used in last example, or any other method you think is suitable.

EXERCISE A2.3

1. We have already mentioned that the formulation part could be very detailed in real-life situations. Also, we do not validate the answer in word problems. Apart from this word problem have a 'correct answer'. This need not be the case in real-life situations.

2. The important factors are (ii) and (iii). Here (i) is not an important factor although it can have an effect on the number of vehicles sold.