

# Class X Math Paper Summative Assessment I

**Total marks of the paper:** 90

**Total time of the paper:** 3.5 hrs

**General Instructions:**

1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections A, B, C, and D. Section – A comprises of 8 questions of 1 mark each, Section – B comprises of 6 questions of 2 marks each, Section – C comprises of 10 questions of 3 marks each and Section – D comprises of 10 questions of 4 marks each.
3. Question numbers 1 to 8 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is not permitted.
6. An additional 15 minutes has been allotted to read this question paper only.

## Questions

1] Triangle ABC is similar to triangle DEF and their areas are

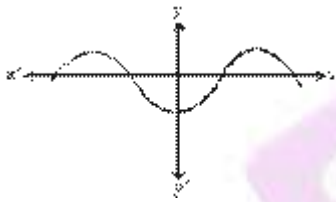
[Marks:1]

$64 \text{ cm}^2$  and  $121 \text{ cm}^2$  respectively. If  $EF = 15.4 \text{ cm}$ , then  $BC = ?$

- A. 13
- B. 8
- C. 11
- D. 11.2

2] The graph  $y = p(x)$  is shown below. How many zeroes does the polynomial  $p(x)$  have?

[Marks:1]



- A. 1
- B. 2
- C. 3
- D. 4

3] Which of the following is not a measure of central tendency :

[Marks:1]

- A. Mode
- B. Median

C. Mean

D. Range

4] The number  $7 \times 11 \times 13 + 13 + 13 \times 2$  is [Marks:1]

A. multiple of 7

B. Neither prime nor composite

C. Prime

D. Composite

5] HCF of the smallest composite number and the smallest prime number is [Marks:1]

A. 4

B. 0

C. 1

D. 2

6] For what value of 'K' will the system of equations  $3x + y = 1$ ,  
 $(2K - 1)x + (K - 1)y = 2K + 1$  have no solution. [Marks:1]

A. -2

B. 1

C. 3

D. 2

7]

If  $3\cot A = 4$ , then  $\frac{1 - \tan^2 A}{1 + \tan^2 A} =$

[Marks:1]

A.  $\frac{7}{16}$

B.  $\frac{16}{25}$

C.  $\frac{9}{25}$

D.  $\frac{7}{25}$

8]

If  $\tan A = \cot B$ , then  $A + B = ?$

[Marks:1]

A.  $60^\circ$

B.  $45^\circ$

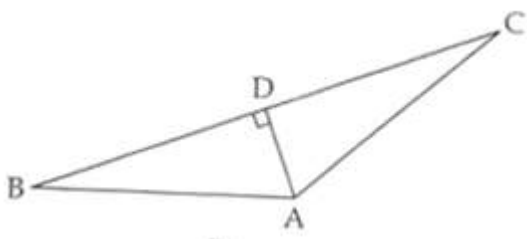
C.  $30^\circ$

D.  $90^\circ$

9]

In figure, If  $AD \perp BC$ , then prove that  $AB^2 + CD^2 = AC^2 + BD^2$

[Marks:2]



10]

Find LCM and HCF of 120 and 144 by fundamental theorem of Arithmetic.

[Marks:2]

11]

Solve the following pair of linear equations.

[Marks:2]

$3x + 4y = 10$  and  $2x - 2y = 2$

12] Prove that  $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$ . [Marks:2]

13] Write a frequency distribution table for the following data:

Marks	Above 0	Above 10	Above 20	Above 30	Above 40	Above 50
No. of students	30	28	21	15	10	0

[Marks:2]

14] If  $x - 2$  is a factor of  $x^3 + ax^2 + bx + 16$  and  $a - b = 16$ , find the value of  $a$  and  $b$ .

OR

[Marks:2]

If  $m, n$  are the zeroes of polynomial  $ax^2 - 5x + c$ , find the value of  $a$  and  $c$  if  $m + n = m.n$

15] Find the mode of the following data:

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	3	12	32	20	6

[Marks:3]

16] In  $\triangle ABC$ , if  $AD$  is the median, then show that  $AB^2 + AC^2 = 2[AD^2 + BD^2]$ . [Marks:3]

17] Prove that:

[Marks:3]

$$\frac{1 + \cos A}{\sin A} + \frac{\sin A}{1 + \cos A} = 2 \operatorname{cosec} A$$

18] The sum of the numerator and denominator of a fraction is 8. If 3 is added to both the numerator and the denominator the fraction becomes  $\frac{3}{4}$ . Find the fraction. [Marks:3]

OR

Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference between the digit is 3, find the number.

19] Prove that  $\sqrt{11}$  is an irrational.

OR

[Marks:3]

Prove that  $2\sqrt{3} - 7$  is an irrational.

20] Find the median of the following data.

Class Interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	7	8	12	10	8	5

[Marks:3]

OR

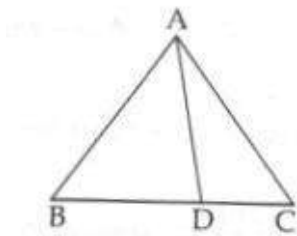
The mean of the following data is 53, find the missing frequencies.

Age in years	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	Total
Number of people	15	$f_1$	21	$f_2$	17	100

21] In figure,

$\triangle ABC$  is such that  $\angle ADC = \angle BAC$ . Prove that  $CA^2 = CB \times CD$ .

[Marks:3]



22] Prove that:

[Marks:3]

$$\frac{1 + \sec A}{\sec A} = \frac{\sin^2}{1 - \cos A}$$

OR

Without using trigonometric tables evaluate:

$$\frac{3 \tan 35^\circ \tan 40^\circ \tan 50^\circ \tan 55^\circ - \frac{1}{2} \tan^2 60^\circ}{4 (\cos^2 39^\circ + \cos^2 51^\circ)}$$

**23]** In dividing  $x^3 - 3x^2 + x + 2$  by a polynomial  $g(x)$  the quotient  $q(x)$  and the remainder  $r(x)$  were  $x - 2$  and  $-2x + 4$  respectively. Find  $g(x)$ . [Marks:3]

**24]** For what values of  $a$  and  $b$  does the following pairs of linear equations have an infinite number of solutions: [Marks:3]

$$2x + 3y = 7; a(x + y) - b(x - y) = 3a + b - 2$$

**25]** Use Euclid's division lemma to show that the square of any positive integer is either of the form  $3m$  or  $3m + 1$  for some integer  $m$ . [Marks:4]

**26]** 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

No. of letters	1-4	4-7	7-10	10-13	13-16	16-19
No. of surnames	6	30	40	16	4	4

[Marks:4]

What is the average length of a surname? Which average you will use? Justify.

**27]** Prove that: [Marks:4]

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$$

OR

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$$

**28]** State and prove Basic proportionality theorem.

OR

[Marks:4]

State and prove Pythagoras theorem.

**29]** If  $\tan \theta + \sin \theta = m$  and  $\tan \theta - \sin \theta = n$ . Show that  $m^2 - n^2 = 4\sqrt{mn}$ . [Marks:4]

**30]** What must be subtracted from  $x^3 - 6x^2 - 15x + 80$  so that the result is exactly divisible by  $x^2 + x - 12$ ? [Marks:4]

**31]** Solve graphically the pair of equations  $2x + 3y = 11$  and  $2x - 4y = -24$ . Hence find the value of coordinate of the vertices of the triangle so formed. [Marks:4]

**32]** The mode of the following frequency distribution is 55. Find the value of  $f_1$  and  $f_2$ .

Class Interval	0 - 10	15 - 30	30 - 45	45 - 60	60 - 75	75 - 90	Total
Frequency	6	7	$f_1$	15	10	$f_2$	51

[Marks:4]

**33]** Prove that ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides. [Marks:4]

**34]** Prove that :

[Marks:4]

$$\frac{1}{\sec \theta - \tan \theta} - \frac{1}{\cos \theta} = \frac{1}{\cos \theta} - \frac{1}{\sec \theta + \tan \theta}$$



## Class X Math Paper Summative Assessment I Solution

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paper: 90

Total time of the  
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## Solutions

1] 
$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow BC^2 = \frac{64}{121} \times (15.4)^2$$

$$\Rightarrow BC = \frac{8}{11} \times 15.4 = 11.2$$

2] Zeroes of a polynomial are the x- coordinates of the points where its graph crosses or touches the X- axis. Graph of  $y = p(x)$  intersects the X-axis at 4 points. Therefore, the polynomial  $p(x)$  has 4 zeroes

3] Range is not a measure of central tendency.

4]  $7 \times 11 \times 13 + 13 + 13 \times 2 = 13(7 \times 11 + 1 + 2) = 13(80)$

5] Smallest composite number = 4 and the smallest prime number = 2. HCF of 2 and 4 = 2.

6] For the system of linear equations:  $a_1x + b_1y + c_1 = 0$ , and

$$a_2x + b_2y + c_2 = 0 \text{ to have no solution } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

For the system of equations  $3x + y = 1$ ,

$(2K - 1)x + (K - 1)y = 2K + 1$   $a_1 = 3$ ,  $b_1 = 1$ ,  $c_1 = 1$ , and

$a_2 = 2K - 1$   $b_2 = K - 1$   $c_2 = 2K + 1$

$$\frac{3}{2K - 1} = \frac{1}{K - 1} \neq \frac{1}{2K + 1}$$

This gives  $K=2$ .

7] Given that  $3\cot A=4$ , we have,  $\cot A=\frac{4}{3}$

Then,  $\tan A=\frac{3}{4}$

Now,  $(1-\tan^2 A)=1-\frac{9}{16}=\frac{7}{16}$

And,  $(1+\tan^2 A)=1+\frac{9}{16}=\frac{25}{16}$

Therefore,  $\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{7}{25}$

8]  $\tan A=\cot B$

$\tan A=\tan(90^\circ-B)$

$A=90^\circ-B$  or  $A+B=90^\circ$

9] In  $\triangle ADC$ ,  $AD^2 = AC^2 - CD^2$

In  $\triangle ABD$ ,  $AD^2 = AB^2 - DB^2$

$AB^2 - DB^2 = AC^2 - CD^2$

$\Rightarrow AB^2 + CD^2 = AC^2 + BD^2$

10]  $120 = 2^3 \times 3 \times 5$

$144 = 2^4 \times 3^2$

$LCM = 2^4 \times 3^2 \times 5 = 720$

$HCF = 2^3 \times 3 = 24$

11]  $3x + 4y = 10$

$$4x - 4y = 4$$

$$\hline 7x = 14$$

$$x = 2$$

$$x - y = 1 \Rightarrow y = x - 1 = 2 - 1 = 1$$

The solution  $x = 2, y = 1$

12] We know that  $\sin^2\theta + \cos^2\theta = 1$  Taking cube of both sides  $(\sin^2\theta + \cos^2\theta)^3 = 1^3$

$$(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3 \sin^2\theta \cdot \cos^2\theta (\sin^2\theta + \cos^2\theta) = 1$$

$$\text{Therefore, } \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$$

13]

Marks	No. of students
0 - 10	2
10 - 20	7
20 - 30	6
30 - 40	5
40 - 50	10
Total	30

14]

$$\text{Let } p(x) = x^3 + ax^2 + bx + 16$$

$$g(x) = x - 2 \text{ is a factor of } p(x)$$

$$\therefore p(2) = 0$$

$$\text{so, } p(2) = 8 + 4a + 2b + 16 = 0$$

$$= 4a + 2b = -24$$

$$= 2a + b = -12 \quad \text{(i)}$$

Also  $a - b = 16 \quad \text{(ii)}$

Solving (i) and (ii)

$$3a = 4 \Rightarrow a = \frac{4}{3}$$

OR

Given polynomial  $p(x) = ax^2 - 5x + c$

$$\text{Sum of zeroes } m + n = \frac{+5}{a}$$

$$\text{Product of zeroes } mn = \frac{c}{a}$$

Given:  $m + n = mn = 10$

$$\frac{5}{a} = 10 \Rightarrow a = \frac{1}{2}$$

$$\frac{c}{a} = 10$$

$$\frac{c}{\frac{1}{2}} = 10$$

$$2c = 10$$

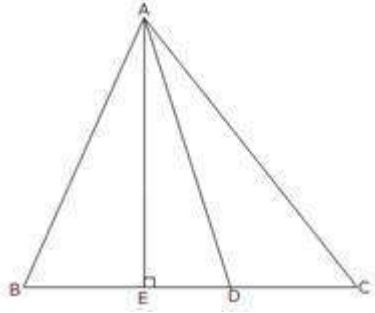
$$\Rightarrow c = 5$$

15] Modal Class = 20-30 ? = 20,  $f_0 = 12$ ,  $f_1 = 32$ ,  $f_2 = 20$ ,  $h = 10$  Mode =

$$L + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h = 20 + \left( \frac{32 - 12}{64 - 12 - 20} \right) 10$$

$$\text{Mode} = 20 + \frac{20 \times 10}{32} = 26.25$$

16]



Construction - draw  $AE \perp BC$

In right triangle AEB and AEC

$$AB^2 + AC^2 = BC^2 + AE^2 + EC^2 + AC^2$$

$$= 2AE^2 + (BD - ED)^2 + (ED + DC)^2$$

$$= 2AE^2 + 2ED^2 + BD^2 + DC^2$$

$$\therefore BD = DC$$

$$AB^2 + AC^2 = 2AE^2 + 2ED^2 + 2BD^2$$

$$= 2 [AE^2 + ED^2] + 2BD^2$$

$$= 2 (AD^2 + BD^2)$$

17]

$$\text{LHS} = \frac{(1 + \cos A)^2 + \sin^2 A}{\sin A (1 + \cos A)}$$

$$= \frac{1 + 2 \cos A + \cos^2 A + \sin^2 A}{\sin A (1 + \cos A)}$$

$$= \frac{2 (1 + \cos A)}{\sin A (1 + \cos A)}$$

$$= \frac{2 (1 + \cos A)}{\sin A (1 + \cos A)} = 2 \operatorname{cosec} A = \text{RHS}$$

18]

Let the fraction be:  $\frac{x}{y}$

According to the question

$$x + y = 8$$

$$\frac{x + 3}{y + 3} = \frac{3}{4} \Rightarrow 4x - 3y = 3$$

On solving we get  $x = 3, y = 5$

The fraction is  $\frac{3}{5}$

OR

Let the tens and units digits of the number be  $x$  and  $y$  respectively then

$$7(10x + y) = 4(10y + x)$$

$$\Rightarrow 70x + 7y = 40y + 4x$$

$$\Rightarrow 66x = 33y$$

$$\Rightarrow y = 2x$$

Also,  $y - x = 3$

On solving,  $x = 3, y = 6$

Number = 36.

19]

Let  $\sqrt{11}$  be a rational number

$$\therefore \text{Let } \sqrt{11} = \frac{p}{q}$$

$$\sqrt{11}q = p$$

$$11q^2 = p^2$$

11 divides  $p^2$  hence 11 divides  $P$ .

Let  $p = 11c$

$$\sqrt{11}q \cdot 11q^2 = 121 c^2$$

Or  $q^2 = 11c^2$

11 divides  $q^2$  Hence 11 divides  $q$

From (1) and (2)  $p$  and  $q$  have a common factor 11 which contradicts our assumption.  $\sqrt{11}$  is irrational

OR

Let  $2\sqrt{3} - 7$  is rational

$$\therefore 2\sqrt{3} - 7 = \frac{p}{q}$$

$$\sqrt{3} = \frac{p + 7q}{2q}$$

Since  $p$  and  $q$  are integer  $\frac{p + 7q}{2q}$  is rational

$\therefore \sqrt{3}$  is rational

But we have that  $\sqrt{3}$  is irrational

$\therefore$  our assumption is wrong. Hence  $2\sqrt{3} - 7$  is irrational.

20]

Class	Frequency	



Interval		
0 - 20	7	7
20 - 40	8	15
40 - 60	12	27
60 - 80	10	37
80 - 100	8	45
100 - 120	5	50
Total	50	

$$\frac{N}{2} = \frac{50}{2} = 25$$

Median class 40 - 60

$$\ell = 40, f = 12 \text{ CF} = 15 \text{ h} = 20$$

$$\begin{aligned} \text{Median} &= \ell + \left( \frac{\frac{N}{2} - \text{CF}}{f} \right) h \\ &= 40 + \left( \frac{25 - 15}{12} \right) 20 \\ &= 40 + \left( \frac{25 - 15}{12} \right) 20 \\ &= 40 + \frac{200}{12} = 56.7 = 56.7 \end{aligned}$$

OR

Age (in	No. of	Value	fixi

Years)	people $f_i$	$x_i$	
0 - 20	15	10	150
20 - 40	$f_1$	30	$30f_1$
40 - 60	21	50	1050
60 - 80	$f_2$	70	$70f_2$
80 - 100	17	90	1530
Total	100		$2730 + 30f_1 + 70f_2$

$$53 + f_1 + f_2 = 100$$

$$f_1 + f_2 = 47 \dots(1)$$

$$53 = \frac{2730 + 30f_1 + 70f_2}{100}$$

$$5300 - 2730 = 30f_1 + 70f_2$$

$$30f_1 + 70f_2 = 2570$$

$$\text{Or } 3f_1 + 7f_2 = 257 \dots(2)$$

$$\begin{array}{r} \underline{4f_2 = 116} \\ \Rightarrow f_2 = \frac{116}{4} = 29 \end{array}$$

$$f_1 + f_2 = 47$$

$$f_2 = 47 - 29 = 18$$

21]

In  $\triangle ABC$  and  $\triangle DAC$

$$\angle BAC = \angle ADC$$

$$\angle C = \angle C$$

$$\triangle ABC \sim \triangle DAC \text{ (AA)}$$

$$\frac{CB}{CA} = \frac{CA}{CD}$$

$$\Rightarrow CA^2 = CB \times CD$$

22]

$$\text{LHS} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{(\cos A + 1) \cos A}{\cos A}$$

$$= 1 + \cos A$$

$$= \frac{(1 + \cos A)(1 - \cos A)}{1 - \cos A}$$

$$= \frac{1 - \cos^2 A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\frac{3 \tan 35^\circ \cot(90^\circ - 55^\circ) \tan 40^\circ \cot(90^\circ - 50^\circ) - \frac{1}{2}(\sqrt{3})^2}{4(\cos^2 39^\circ)}$$

OR

$$= \frac{3 - \frac{3}{2}}{4} = \frac{6 - 3}{8} = \frac{3}{8}$$

23]

$$p(x) = x^3 - 3x^2 + x + 2$$

$$g(x) = ?$$

$$q(x) = x - 2 \quad r(x) = -2x + 4$$

$$p(x) = g(x) + r(x)$$

$$x^3 - 3x^2 + x + 2 = g(x)(x - 2) - 2x + 4$$

$$g(x) = \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$= x^2 - x + 1$$

24] The system has infinitely many solutions:

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

(1)                      (2)                      (3)

Equating (1) and (2), we get  $a = 5b$

Equating (2) and (3), we get  $2a - 4b = 6$

On solving, we get  $b = 1$  and  $a = 5$ .

25] If  $a$  and  $b$  are one two positive integers. Then  $a = bq + r$ ,  $0 \leq r < b$  Let  $b = 3$

Therefore,  $r = 0, 1, 2$  Therefore,  $a = 3q$  or  $a = 3q + 1$  or  $a = 3q + 2$

If  $a = 3q$   $a^2 = 9q^2 = 3(3q^2) = 3m$  or where  $m = 3q^2$   $a = 3q + 1$   $a^2 = 9q^2 +$

$6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$  where  $m = 3q^2 + 2q$  or  $a = 3q + 2$   $a^2 = 9q^2 +$

$12q + 4 = 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$ , where  $m = 3q^2 + 4q + 1$

Therefore, the squares of any positive integer is either of the form  $3m$  or  $3m + 1$ .

26]

CI	1-4	4-7	7-10	10-13	13-16	16-19
fi	6	30	40	16	4	4

xi	2.5	5.5	8.5	11.5	14.5	17.5
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Average most suitable here is the Mode because we are interested in knowing the length of surname for maximum no. of people

Since the maximum frequency is 40 and it lies in the class interval 7-10.

Therefore, modal class = 7-10

$f_0 = 30, h = 3, f_1 = 40, f_2 = 16$

$$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 7 + \left( \frac{40 - 30}{2 \times 40 - 30 - 16} \right) \times 3$$

$$= 7 + .88 = 7.88 \text{ years (approx.)}$$

27]

$$\begin{aligned} \text{LHS} &= \frac{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}{1 - \frac{\cos \theta}{\sin \theta}} = \frac{\frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)}}{1 - \frac{\cos \theta}{\sin \theta}} \\ &= \frac{\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta (\sin \theta - \cos \theta)}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} \\ &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} \\ &= \frac{\sin^2 \theta}{\sin \theta \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} = \tan \theta + \cot \theta + 1 = \text{RHS} \end{aligned}$$

OR

$$\text{LHS} = (\csc A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$$

$$= \left( \frac{1}{\sin A} - \sin A \right) \left( \frac{1}{\cos A} - \cos A \right) \left( \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)$$

$$= \left( \frac{1 - \sin^2 A}{\sin A} \right) \times \left( \frac{1 - \cos^2 A}{\cos A} \right) \times \left( \frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A} \right)$$

$$= \frac{\cos^2 A}{\cos A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos A \cdot \sin A}$$

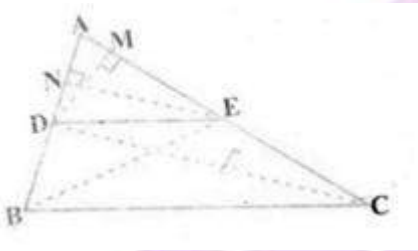
$$= 1$$

$$= \text{RHS}$$

28] Basic proportionality theorem

**Statement** If a line is drawn parallel of one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

**Given:** A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively (see fig.)



To prove that  $\frac{AD}{BD} = \frac{AE}{EC}$ .

**Construction:** Let us join BE and CD and then draw  $DM \perp AC$  and  $EN \perp AB$ .

**Proof:**

Now, area of  $\triangle ADE \left( = \frac{1}{2} \text{ base} \times \text{height} \right) = \frac{1}{2} AD \times EN$ .

Note that  $\triangle BDE$  and  $\triangle DEC$  are on the same base DE and between the same parallels BC and DE.

So,  $\text{ar}(\triangle BDE) = \text{ar}(\triangle DEC)$

Therefore, from (1), (2) and (3), we have :

$$\frac{AD}{DB} = \frac{AE}{EC} \text{ Hence proved.}$$

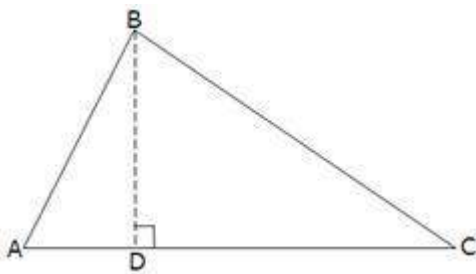
OR

**Pythagoras Theorem : Statement:**In a right angled triangle,the square of the hypotenuse is equal to the sum of squares of the other two sides.

**Given:** A right triangle ABC right angled at B.

**To prove:** that  $AC^2 = AB^2 + BC^2$

**Construction:**Let us draw  $BD \perp AC$  (See fig.)



**Proof :**

Now,  $\triangle ADB \sim \triangle ABC$  (Using Theorem: If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse ,then triangles on both sides of the perpendicular are similar to the whole triangle and to each other)

$$\text{So, } \frac{AD}{AB} = \frac{AB}{AC} \quad (\text{Sides are proportional})$$

$$\text{Or, } AD \cdot AC = AB^2 \quad (1)$$

$$\text{Also, } \triangle BDC \sim \triangle ABC \quad (\text{Theorem})$$

$$\text{So, } \frac{CD}{BC} = \frac{BC}{AC}$$

$$\text{Or, } CD \cdot AC = BC^2$$

Adding (1) and (2),

$$AD \cdot AC + CD \cdot AC = AB^2 + BC^2$$

$$\text{OR, } AC(AD + CD) = AB^2 + BC^2$$

$$\text{OR, } AC \cdot AC = AB^2 + BC^2$$

$$\text{OR, } AC^2 = AB^2 + BC^2$$

Hence proved.

29]

$$m^2 - n^2 = (m + n)(m - n)$$

$$= [(\tan \theta + \sin \theta) + \tan \theta - \sin \theta]$$

$$= 2 \tan \theta \cdot 2 \sin \theta$$

$$= 4 \tan \theta \cdot \sin \theta$$

$$= 4 \sqrt{\tan^2 \theta \sin^2 \theta}$$

$$= 4 \sqrt{(\sec^2 \theta - 1) \sin^2 \theta}$$

$$= 4 \sqrt{\sec^2 \theta - \sin^2 \theta}$$

$$= 4 \sqrt{\tan^2 \theta - \sin^2 \theta}$$

$$= 4 \sqrt{mn}$$

30]

$$\text{Let } p(x) = x^3 - 6x^2 - 15x + 80$$

Let say that we subtracted  $ax + b$  so that it is exactly divisible by

$$x^2 + x - 12$$



$$s(x) = x^3 - 6x^2 - 15x + 80 - (ax + b)$$

$$= x^3 - 6x^2 - (15 + a)x + (80 - b)$$

Dividend = Divisor x Quotient + Remainder

But remainder = 0

$\therefore$  Dividend = Divisor x Quotient

$$s(x) = (x^2 + x - 12) \times \text{quotient}$$

$$s(x) = x^3 - 6x^2 - (15 + a)x + (80 - b)$$

$$x(x^2 + x - 12) - 7(x^2 + x - 12)$$

$$= x^3 + x^2 - 7x^2 - 12x - 7x + 84$$

$$= x^3 - 6x^2 - 19x + 84$$

$$\text{Hence, } x^3 - 6x^2 - 19x + 84 = x^3 - 6x^2 - (15 + a)x + (80 - b)$$

$$-15 - a = -19 \quad \Rightarrow a = +4$$

$$\text{and } 80 - b = 84 \quad \Rightarrow b = -4$$

Hence if in  $p(x)$  we subtracted  $4x - 4$  then it is exactly divisible by

$x^2 + x - 12$ .

31] We have to solve the pair of equations graphically

$$2x + 3y = 11 \dots (1)$$

$$2x - 4y = -24 \dots (2)$$

For (1)

X	1	4	-2
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y	3	1	5
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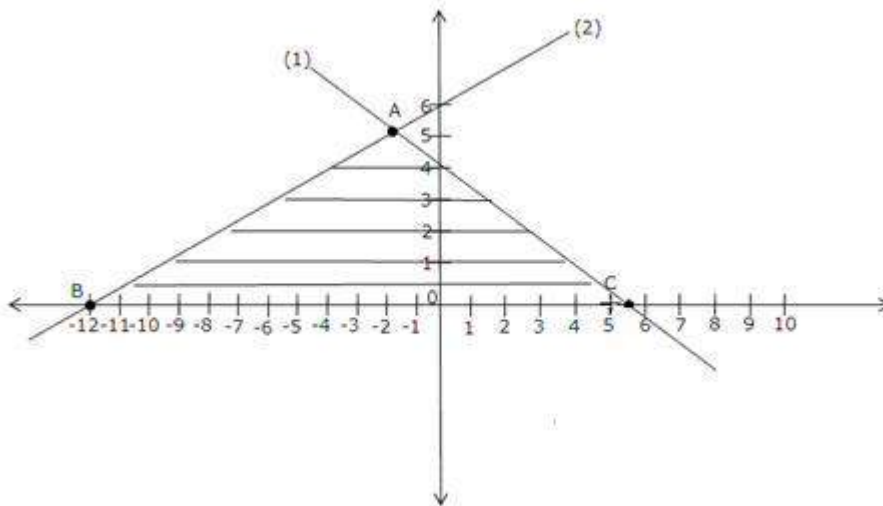
For (2)

X	-12	0	-10
y	0	6	1

point of intersection  $x = -2, y = 5$

The triangle formed is shaded as  $\triangle ABC$  coordinates are

A (-2,5) B (-12,0) C(5.5,0).



32]

Class Interval	Frequency
0 - 15	6
15 - 30	7
30 - 45	f <sub>1</sub>

45 - 60	15
60 - 75	10
75 - 90	f <sub>2</sub>
Total	51

Mode = 55 (Given)

∴ Modal Class 45 - 60

$$l = 45, f_0 = f_1 \text{ and } f_1 = 15$$

$$f_2 = 10 \text{ h} = 15$$

$$38 + f_1 + f_2 = 51$$

$$f_1 + f_2 = 51 - 38$$

$$f_1 + f_2 = 13 \quad \dots(1)$$

$$55 = 45 + \left[ \frac{(15 - f_1)}{30 - f_1 - 10} \right] \times 15$$

$$10 = \frac{(15 - f_1) 15}{20 - f_1}$$

$$200 - f_1 = 225 - 15f_1$$

$$5f_1 = 25$$

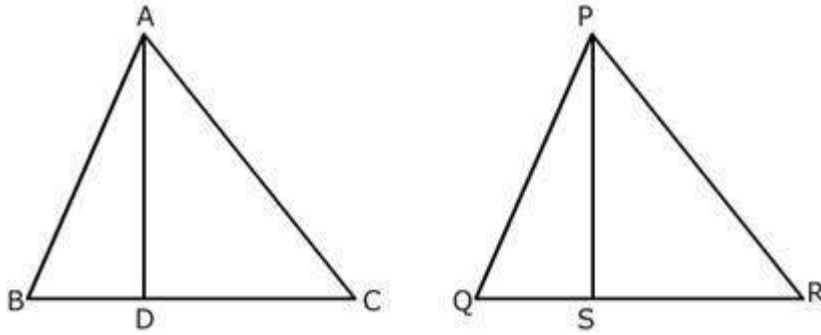
$$f_1 = 5$$

$$f_1 + f_2 = 13$$

$$\Rightarrow f_2 = 13 - 5 = 8$$

The missing frequencies are 5 and 8.

33]



Statement: The ratio

of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given:  $\triangle ABC \sim \triangle PQR$  To Prove:  $\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$  Construction: Draw  $AD \perp BC$  and  $PS \perp QR$

Proof:

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS} = \frac{BC}{QR} \times \frac{AD}{PS}$$

$\triangle ADB \sim \triangle PSQ$  (AA) Therefore,  $\frac{AD}{PS} = \frac{AB}{PQ}$  ... (iii)

But  $\triangle ABC \sim \triangle PQR$  Therefore,  $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$  ... (iv)

Therefore,  $\frac{AD}{PS} = \frac{BC}{QR}$  Therefore,  $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$

From (iii)  $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

34]

$$\text{LHS} = \frac{1}{\sec \theta - \tan \theta} - \frac{1}{\cos \theta}$$

$$= \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta - \tan \theta} - \sec \theta$$

$$= \frac{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta)} - \sec \theta$$

$$= (\sec \theta + \tan \theta) - \sec \theta = \tan \theta$$

$$\begin{aligned}\text{RHS} &= \frac{1}{\cos \theta} - \frac{1}{\sec \theta + \tan \theta} \\ &= \sec \theta - \frac{\sec^2 \theta - \tan^2 \theta}{\sec \theta + \tan \theta} \\ &= \sec \theta - \frac{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{(\sec \theta + \tan \theta)} \\ &= \sec \theta - (\sec \theta - \tan \theta) = \tan \theta\end{aligned}$$

Hence, LHS = RHS.