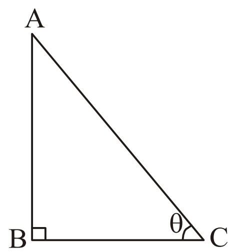
QUESTION PAPER CODE 30/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. 

$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$ $\frac{1}{2}$

$\Rightarrow \quad \theta = 60°$ $\frac{1}{2}$

2. $\frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units $\frac{1}{2}$

$\therefore \quad d = 9$ units $\frac{1}{2}$

3. Favourable outcomes are $-1, 0, 1$ $\frac{1}{2}$

$\therefore$ Required Probability $= \frac{3}{7}$ $\frac{1}{2}$

4. $\sqrt{(4-1)^2 + (k-0)^2} = 5$ $\frac{1}{2}$

$\Rightarrow \quad k = \pm 4$ $\frac{1}{2}$

SECTION B

5. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$ 1

$\Rightarrow \quad (\sqrt{2}\,x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow \quad x = \frac{-5}{\sqrt{2}}, -\sqrt{2}$

or $\quad \frac{-5\sqrt{2}}{2}, -\sqrt{2}$ 1

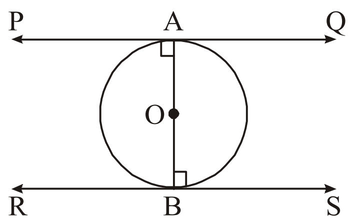
6. A.P. formed is 208, 216, 224, ..., 496 — 1

$a_n = 496$

$\Rightarrow \quad 208 + (n-1) \times 8 = 496$ — $\frac{1}{2}$

$\Rightarrow \quad n = 37$ — $\frac{1}{2}$

7.



$\angle PAO = \angle OBS = 90°$ — 1

But these are alternate interior angles

$\therefore \quad PQ \parallel RS$ — 1

8. $x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow \quad x^2 + 2kx + (k^2 - k + 2) = 0$ — $\frac{1}{2}$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow \quad 4k^2 - 4k^2 + 4k - 8 = 0$ — 1

$\Rightarrow \quad k = 2$ — $\frac{1}{2}$

9. Correct construction — 2

10. $PA = PC + CA = PC + CQ$

$\Rightarrow \quad 12 = PC + 3 \Rightarrow PC = 9$ cm — 1

$PD = 9$ cm

$\therefore \quad PC + PD = 18$ cm — 1

SECTION C

11. $a_m = \frac{1}{n} \quad \Rightarrow \quad a + (m-1)d = \frac{1}{n} \qquad ...(1)$ — $\frac{1}{2}$

$a_n = \frac{1}{m} \quad \Rightarrow \quad a + (n-1)d = \frac{1}{m} \qquad ...(2)$ — $\frac{1}{2}$

Solving (1) and (2), $a = \frac{1}{mn}$ and $d = \frac{1}{mn}$ 1

$$S_{mn} = \frac{mn}{2}\left[2 \times \frac{1}{mn} + (mn-1) \times \frac{1}{mn}\right]$$

$$= \frac{1}{2}(mn+1)$$ 1

12. $S_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots$ upto n terms

$$= \underbrace{(4 + 4 + \ldots + 4)}_{\text{n times}} - \frac{1}{n}(1 + 2 + 3 + \ldots + n)$$ 1

$$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$$ $\frac{1}{2}+1$

$$= \frac{7n-1}{2}$$ $\frac{1}{2}$

13. $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

For equal roots, $B^2 - 4AC = 0$ $\frac{1}{2}$

$\Rightarrow \quad 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$ 1

$\Rightarrow \quad m^2c^2 - c^2 - m^2c^2 + a^2 + m^2a^2 = 0$ 1

$\Rightarrow \quad c^2 = a^2(1 + m^2)$ $\frac{1}{2}$

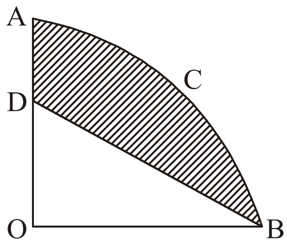
14. $\frac{3}{4} \times$ Volume of conical vessel = Volume of cylindrical vessel 1

Let the height of cylindrical vessel be h

$\Rightarrow \quad \frac{3}{\cancel{4}} \times \frac{1}{3} \times \pi \times 5 \times 5 \times \cancel{24}^{6} = \pi \times 10 \times 10 \times h$ 1

$\Rightarrow \quad h = \frac{3}{2}\text{cm}$ or 1.5 cm 1

15.

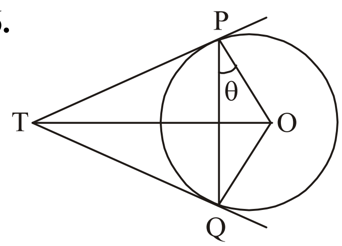


Area of shaded region = Area of quadrant OACB – Area of ΔODB 1

$$= \left(\frac{22}{7} \times \frac{3.5 \times 3.5}{4} - \frac{1}{2} \times 3.5 \times 2\right) \text{cm}^2$$ 1

$$= \frac{49}{8} \text{ or } 6.125 \text{ cm}^2$$ 1

16.



Let $\angle OPQ = \theta$

$\Rightarrow \quad \angle TPQ = 90° - \theta = \angle TQP$ 1

$\angle TPQ + \angle TQP + \angle PTQ = 180°$

$\Rightarrow \quad 90° - \theta + 90° - \theta + \angle PTQ = 180°$ $1\frac{1}{2}$

$\Rightarrow \quad \angle PTQ = 2\theta$

$= 2\angle OPQ$ $\frac{1}{2}$

17. A(–2, 0), B(2, 0), C(0, 2)

AB = 4 units, BC = $2\sqrt{2}$ units, AC = $2\sqrt{2}$ units 1

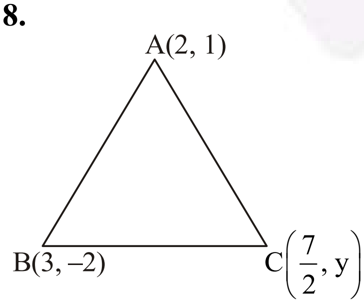
P(–4, 0), Q(4, 0), R(0, 4)

PQ = 8 units, QR = $4\sqrt{2}$ units, PR = $4\sqrt{2}$ units 1

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{2}$$ 1

$\therefore \quad \Delta ABC \sim \Delta PQR$

18.



$ar(\Delta ABC) = 5$ sq.units

$$\Rightarrow \quad \frac{1}{2}\left[2(-2-y) + 3(y-1) + \frac{7}{2}(1+2)\right] = 5$$ $1\frac{1}{2}$

$$\Rightarrow \quad y + \frac{7}{2} = 10$$ 1

$$\Rightarrow \quad y = \frac{13}{2}$$ $\frac{1}{2}$

19. Total number of outcomes = 36

(i) Favourable outcomes are

(1, 1,) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1) (2, 2) (2, 3)

(2, 4) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1) i.e., 15

$\therefore$ P(sum less than 7) = $\frac{15}{36}$ or $\frac{5}{12}$ 1

(ii) Favourable outcomes are

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3)

(2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (4, 1)

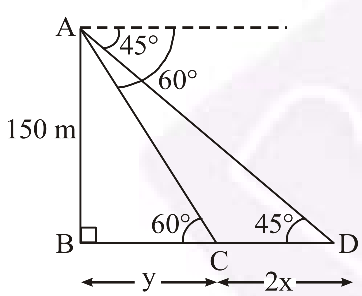
(4, 2) (4, 3) (5, 1) (5, 2) (5, 3) (6, 1) (6, 2) i.e., 25

P(product less than 16) = $\frac{25}{36}$ 1

(iii) Favourable outcomes are

$\therefore$ P(doublet of odd number) = $\frac{3}{36}$ or $\frac{1}{12}$ 1

20.



Correct Figure $\frac{1}{2}$

Let the speed of boat be x m/min

$\therefore$ CD = 2x

$$\frac{150}{y} = \tan 60° \Rightarrow y = \frac{150}{\sqrt{3}} = 50\sqrt{3}$$ 1

$$\frac{150}{y+2x} = \tan 45° \Rightarrow 150 = 50\sqrt{3} + 2x$$

$\Rightarrow$ $x = 25(3-\sqrt{3})$ 1

$\therefore$ Speed = $25(3-\sqrt{3})$ m/min

= $1500(3-\sqrt{3})$ m/hr. $\frac{1}{2}$

SECTION D

21. Correct construction of given triangle 2

Correct construction of similar triangle 2

22. Correct figure, given, to prove and construction $\frac{1}{2}\times4=2$

Correct proof 2

23. $\frac{S_m}{S_n}=\frac{m^2}{n^2} \Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]}=\frac{m^2}{n^2}$ 1

$\Rightarrow \quad \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n}$ 1

Solving we get $d=2a$ 1

$\frac{a_m}{a_n}=\frac{a+(m-1)d}{a+(n-1)d}=\frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$

$=\frac{2m-1}{2n-1}$ 1

24. Let the speed of stream be x km/hr.

$\therefore$ Speed of boat upstream $=(15-x)$ km/hr. $\frac{1}{2}$

Speed of boat downstream $=(15+x)$ km/hr. $\frac{1}{2}$

$\frac{30}{15-x}+\frac{30}{15+x}=4\frac{1}{2}=\frac{9}{2}$ 1

$\Rightarrow \quad \frac{30(15+x+15-x)}{(15-x)(15+x)}=\frac{9}{2}$

$\Rightarrow \quad 200=225-x^2$ 1

$x=5$ (Rejecting -5)

$\therefore$ Speed of stream $=5$ km/hr 1

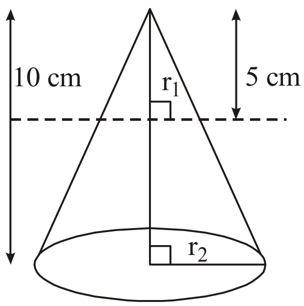
25. Area of traingle with vertices (a, a^2), (b, b^2) and $(0, 0)$ is

$$\frac{1}{2}|a(b^2) + b(-a^2) + 0|$$ 2

$$= \frac{1}{2}ab(b - a) \neq 0 \quad \text{as} \quad a \neq b \neq 0$$ 2

$\therefore$ Given points are not collinear

26.



$$\frac{5}{10} = \frac{r_1}{r_2}$$

$\Rightarrow \quad r_2 = 2r_1$ 1

Ratio of volumes of two parts

$$= \frac{\text{Volume of smaller cone}}{\text{Volume of frustum}}$$

$$= \frac{\frac{1}{3}\pi \times r_1^2 \times 5}{\frac{1}{3} \times \pi \times 5[r_1^2 + r_2^2 + r_1 r_2]} = \frac{r_1^2}{r_1^2 + 4r_1^2 + 2r_1^2}$$ $1\frac{1}{2}+1$

$$= \frac{1}{7}$$ $\frac{1}{2}$

27. For Peter,

Total number of outcomes = 36

Favourable outcome is (5, 5)

$\therefore$ P(Peter getting the number 25) $= \frac{1}{36}$ $1\frac{1}{2}$

For Rina, Total number of outcomes = 6

Favourable outcome is 5.

$\therefore$ P (Rina getting the number 25) $= \frac{1}{6}$ $1\frac{1}{2}$

$\therefore$ Rina has the better chance 1

28. Area of minor segment

$$= \frac{22}{7} \times 10 \times 10 \times \frac{\cancel{60}^{1}}{\cancel{360}^{6}} - \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= 10 \times 10 \left[\frac{22}{7} \times \frac{1}{6} - \frac{\sqrt{3}}{4} \right]$$

$$= \frac{100}{84}(44 - 21\sqrt{3})\ \text{cm}^2 \quad \text{or} \quad \frac{25}{21}(44 - 21\sqrt{3})\ \text{cm}^2$$ $2\frac{1}{2}$

Area of major segment

$$= \left[\frac{22}{7} \times 10 \times 10 - \frac{100}{84}(44 - 21\sqrt{3}) \right] \text{cm}^2$$

$$= \frac{100}{84}(220 + 21\sqrt{3})\ \text{cm}^2 \quad \text{or} \quad \frac{25}{21}(220 + 21\sqrt{3})\ \text{cm}^2$$ $1\frac{1}{2}$

29.

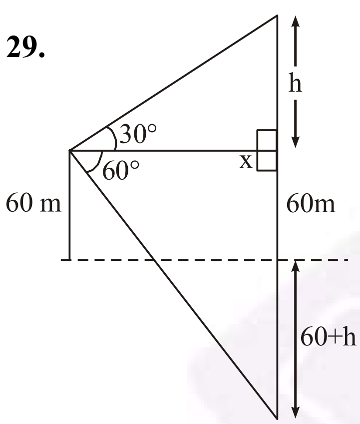


Figure 1

$$\frac{h}{x} = \tan 30° \Rightarrow x = h\sqrt{3}$$ 1

$$\frac{60 + 60 + h}{x} = \tan 60°$$

$$\Rightarrow \quad \frac{120 + h}{x} = \sqrt{3}$$ 1

$$\Rightarrow \quad 120 + h = h\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad h = 60$$ $\frac{1}{2}$

$\therefore$ height of cloud from surface of water = (60 + 60)m = 120 m $\frac{1}{2}$

30. Area of shaded region

= Area of square + Area of 2 major sectors. $1\frac{1}{2}$

$$= \left[28 \times 28 + 2 \times \frac{22}{7} \times 14 \times 14 \times \frac{270°}{360°}\right] \text{cm}^2$$ $1\frac{1}{2}$

$$= 28 \times 28\left(1 + \frac{33}{28}\right) = 1708 \text{ cm}^2$$ 1

31. Volume of water in cylindrical tank.

= Volume of water in park. 1

$\Rightarrow \quad \frac{22}{7} \times 1 \times 1 \times 5 = 25 \times 20 \times h$, where h is the height of standing water. $1\frac{1}{2}$

$\Rightarrow \quad h = \frac{11}{350}$ m or $\frac{22}{7}$ cm $\frac{1}{2}$

Conservation of water or any other relevant value. 1