QUESTION PAPER CODE 65/1/N

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Finding $A^T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ $\frac{1}{2}$

Getting $\alpha = \frac{\pi}{4}$ or 45° $\frac{1}{2}$

2. k = 27 1

3. For a unique solution

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$ $\frac{1}{2}$

$\Rightarrow k \neq 0$ $\frac{1}{2}$

4. Getting equation as $\frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$ $\frac{1}{2}$

Sum of intercepts $\frac{5}{2} + 5 - 5 = \frac{5}{2}$ $\frac{1}{2}$

5. Getting $\lambda = -9$ and $\mu = 27$ $\frac{1}{2}$ each

6. $\vec{a} + \vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ $\frac{1}{2}$

Unit vector parallel to $\vec{a} + \vec{b}$ is $\frac{1}{7}(6\hat{i} - 3\hat{j} + 2\hat{k})$ $\frac{1}{2}$

SECTION B

7. $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1} 3x - \tan^{-1} x$ $\frac{1}{2}$

$\Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{2x}{1+3x^2}\right)$ $1\frac{1}{2}$

$\frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$ $\frac{1}{2}$

$2x(1 + 3x^2 - 2 + x^2) = 0$ $\frac{1}{2}$

$x = 0, \frac{1}{2}, -\frac{1}{2}$ 1

OR

Let $2x = \tan\theta$ 1

L. H. S $= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) - \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$ 1

$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$ 1

$= 3\theta - 2\theta$

$= \theta$ or $\tan^{-1} 2x$

$\therefore$ L. H. S = R. H. S 1

8. Getting matrix equation as $\begin{pmatrix}10 & 3\\ 3 & 10\end{pmatrix}\begin{pmatrix}E\\ H\end{pmatrix} = \begin{pmatrix}145\\ 180\end{pmatrix}$ $1\frac{1}{2}$

$\Rightarrow \begin{pmatrix}E\\ H\end{pmatrix} = \begin{pmatrix}10 & 3\\ 3 & 10\end{pmatrix}^{-1}\begin{pmatrix}145\\ 180\end{pmatrix}$

$\Rightarrow \begin{pmatrix}E\\ H\end{pmatrix} = \begin{pmatrix}10\\ 15\end{pmatrix}$

$\Rightarrow$ E = 10, H = 15 $1\frac{1}{2}$

The poor boy was charged ₹ 65 less

Value: Helping the poor 1

9. L.H.L = a + 3 $1\frac{1}{2}$

R.H.L = b/2 $1\frac{1}{2}$

f(x) is continuous at x = 0. So, a + 3 = 2 = b/2 $\frac{1}{2}$

$\Rightarrow$ a = –1 and b = 4 $\frac{1}{2}$

10. $\frac{dx}{dy} = \frac{\sin a}{\cos^2(a+y)}$ $1\frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{-2\cos(a+y)\sin(a+y)}{\sin a}\frac{dy}{dx}$

$= \frac{-\sin 2(a+y)}{\sin a}\frac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow \sin a\frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$ $\frac{1}{2}$

OR

Let 2x = sin θ 1

$\therefore\ y = \sin^{-1}\left(\frac{6x - \sqrt{1-4x^2}}{5}\right)$

$= \sin^{-1}\left(\frac{3}{5}\sin\theta - \frac{4}{5}\cos\theta\right)$

$= \sin^{-1}(\cos\alpha \sin\theta - \sin\alpha \cos\theta)$ $\quad [\cos\alpha = \frac{3}{5};\ \sin\alpha = \frac{4}{5}]$ 1

$= \sin^{-1}(\sin(\theta - \alpha))$

$= \theta - \alpha$ 1

$= \sin^{-1}(2x) - \alpha$

$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$ 1

11. Slope of the tangent = $3x^2 + 2 = 14$ 1

Points of contact (2, 8) and (–2, –16) 1

Equations of tangent

$14x - y - 20 = 0$ 1

and $14x - y + 12 = 0$ 1

12. Let 2x = t

$I = \frac{1}{2}\int \frac{(t-5)}{(t-3)^3} e^t\ dt$ 1

$= \frac{1}{2}\int \left[\frac{1}{(t-3)^2} - \frac{2}{(t-3)^3}\right] e^t\ dt$ 2

$= \frac{1}{2}\frac{1}{(t-3)^2} e^t + C = \frac{1}{2}\frac{1}{(2x-3)^2} e^{2x} + C$ 1

OR

Writing $\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$\Rightarrow\ A = \frac{2}{5}, B = \frac{1}{5}, C = \frac{3}{5}$ 1

$\therefore\ I = \frac{1}{5}\int \frac{2x}{x^2+1} dx + \frac{1}{5}\int \frac{dx}{x^2+1} + \frac{3}{5}\int \frac{dx}{x+2}$ 2

$\Rightarrow\ I = \frac{1}{5}\log\left|x^2+1\right| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log\left|x+2\right| + C$ 1

13. Using property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ 1

$I = \int_{-2}^{2}\left(\frac{x^2}{1+5^x}\right)dx = \int_{-2}^{2}\left(\frac{x^2}{1+5^{-x}}\right)dx$ 1

$2I = \int_{-2}^{2} x^2\,dx$ 1

$2I = \frac{16}{3}$ or $I = \frac{8}{3}$ 1

14. Writing $x + 3 = A(-4 - 2x) + B$

$\Rightarrow \quad A = -\frac{1}{2},\ B = 1$ 1

$\therefore \quad I = -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx + \int\sqrt{\left(\sqrt{7}\right)^2 - (x+2)^2}\,dx$ 1

$I = -\frac{1}{3}(3-4-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}} + C$ 2

15. Writing linear equation $\frac{dy}{dx} + \frac{\cos x}{1+\sin x}y = -\frac{x}{1+\sin x}$ 1

$I.F = e^{\int\frac{\cos x}{1+\sin x}dx} = 1 + \sin x$ 1

General solution is: $y(1+\sin x) = -\frac{x^2}{2} + C$ 1

Particular solution is: $y(1+\sin x) = 1 - \frac{x^2}{2}$ 1

16. $\frac{dx}{dy} = \frac{2xe^v - y}{2ye^v}$

$\frac{x}{y} = v$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$ 1

$v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v}$

$2\int e^v dv = -\int\frac{dy}{y}$ 1

General solution is: $2e^v = -\log|y| + C$ or $2e^{x/y} = -\log|y| + C$ 1

Particular solution is: $2e^{x/y} + \log|y| = 2$ 1

17. $\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k},\ \overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k},\ \overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$ $1\frac{1}{2}$

For 4 points to be coplanar, $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

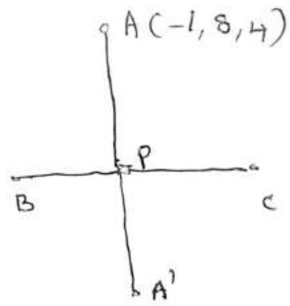
i.e., $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$ $1\frac{1}{2}$

$= -4(12 + 3) + 6(-3 + 24) - 2(1 + 32)$

$= -60 + 126 - 66 = 0$ which is true

Hence, points are coplanar. 1

18.



Equation of line BC: $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = r$ 1

General point on BC: $(2r, -2r - 1, -4r + 3)$

$\Rightarrow$ d.r.'s of AP: $(2r + 1, -2r - 9, -4r - 1)$ 1

As AP $\perp$ BC $\Rightarrow r = -1$

$\Rightarrow$ Co-ordinates of P: $(-2, 1, 7)$ 1

Hence, coordinates of Image of A: $(-3, -6, 10)$ 1

19. Let E_1 and E_2 be the events of drawing bag X and bag Y respectively.

Then, $P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

Let A be the event of drawing one white and one black ball from any one of the bag without replacement. Then,

$\Rightarrow$ $P(A/E_1) = \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} = \frac{16}{30}$

$P(A/E_2) = \frac{3}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{3}{5} = \frac{18}{30}$ $1\frac{1}{2}$

Using Bayes' Theorem, we have

$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$ 1

$= \frac{\frac{1}{2} \times \frac{18}{30}}{\frac{1}{2} \times \frac{16}{30} + \frac{1}{2} \times \frac{18}{30}} = \frac{9}{17}$ 1

OR

Let A_i and B_i be the events of throwing 10 by A and B in the respective ith turn. Then,

$P(A_i) = P(B_i) = \frac{1}{12}$ and $P(\overline{A_i}) = P(\overline{B_i}) = \frac{11}{12}$ $1 + \frac{1}{2}$

Probability of winning A, when A starts first

$= \frac{1}{12} + \left(\frac{11}{12}\right)^2 \frac{1}{12} + \left(\frac{11}{12}\right)^4 \frac{1}{12} + \ldots$ 1

$= \frac{1/12}{1 - (11/12)^2}$

$= \frac{12}{23}$ 1

Probability of winning of B $= 1 - P(A) = 1 - \frac{12}{23} = \frac{11}{23}$ $\frac{1}{2}$

SECTION C

20. The variate X takes values 3, 4, 5, and 6 — 2

$P(X=3)=\frac{1}{20}; P(X=4)=\frac{3}{20}; P(X=5)=\frac{6}{20}; P(X=6)=\frac{10}{20};$ — 1

Probability distribution is:

X	3	4	5	6
P(X)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$

1

Mean = $\Sigma XP(X) = \frac{105}{20} = \frac{21}{4}$ — 1

Variance = $\Sigma X^2P(X) - (\Sigma X P(X))^2 = \frac{63}{80}$ — 1

21. Proving * is commutative — $1\frac{1}{2}$

Proving * is associative — $1\frac{1}{2}$

Getting identity element as (0, 0) — $1\frac{1}{2}$

Getting inverse of (a, b) as (–a, –b) — $1\frac{1}{2}$

22. Getting $\frac{dy}{d\theta} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$ — 2

Equating $\frac{dy}{d\theta}$ to 0 and getting critical point as $\cos\theta = 0$ i.e., $\theta = \frac{\pi}{2}$ — 1

For all $\theta, 0 \le \theta \le \frac{\pi}{2}, \frac{dy}{d\theta} \ge 0$ — 2

Hence, y is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$ — 1

OR

Correct Figure — 1

Writing $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{2} l^3 \sin^2\theta\cos\theta$ — 1

Getting $\frac{dV}{d\theta} = \frac{\pi}{2} l^3 [2\sin\theta\cos^2\theta - \sin^3\theta]$ — 1

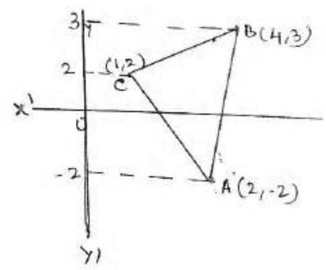
For maxima and minima, $\frac{dV}{d\theta} = 0$ — $\frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$ or $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ — $1\frac{1}{2}$

Getting $\frac{d^2V}{d\theta^2}$ negative — 1

Hence, volume of the cone is maximum when semi-vertical angle is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

23. Correct Figure 1



Writing equations of three sides in terms of y as

$$x_{AB} = \frac{2}{5}(y+2)+2;\ x_{BC} = 3(y-3)+4;\ x_{AC} = \frac{-1}{4}(y+2)+2 \quad 1$$

$$\text{Area} = \int_{-2}^{3}\left(\frac{2}{5}(y+2)+2\right)dy - \int_{-2}^{2}\left(-\frac{1}{4}(y+2)+2\right)dy - \int_{2}^{3}\left(3(y-3)+4\right)dy \quad 1$$

$$= \left.\frac{2}{10}(y+2)^2+2y\right]_{-2}^{3} - \left[-\frac{1}{8}(y+2)^2+2y\right]_{-2}^{2} - \left[\frac{3}{2}(y-3)^2+4y\right]_{2}^{3} \quad 2$$

$$= 15-6-\frac{5}{2} \text{ or } \frac{13}{2} \quad 1$$

24. Required equation of the plane is

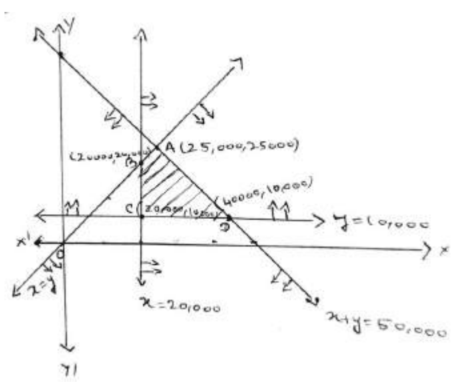
$$\vec{r}.[(1-2\lambda)\hat{i}+(-2+\lambda)\hat{j}+(3+\lambda)\hat{k}] = 4-5\lambda \quad 2$$

Intercept of the plane on x-axis = Intercept of the plane on y-axis

$$\Rightarrow \frac{4-5\lambda}{1-2\lambda} = \frac{4-5\lambda}{\lambda-2} \text{ i.e., } \lambda = 1, \frac{4}{5} \left(\text{rejecting } \lambda = \frac{4}{5}\right) \quad 2$$

Required equation of the plane is $\vec{r}.(-\hat{i}-\hat{j}+4\hat{k})+1=0$ 2

25. Let the investment in bond A be ₹ x and in bond B be ₹ y



Objective function is: $Z = \frac{x}{10}+\frac{9}{100}y$ 1

Subject to constraints

$x+y \geq 50000;\ x \geq 20,000;\ y \geq 10,000;\ x \geq y$ (*) 2

Correct Figure 2

Vertices of feasible region are A, B, C, and D

Point	$Z = \frac{x}{10}+\frac{9}{100}y$	Value
A(25,000, 25000)	2500 + 2250	4750
B(20,000, 20,000)	2000 + 1800	3800
C(20,000, 10,000)	2000 + 900	2900
D(40,000, 10,000)	4000 + 900	4900

Return is maximum when ₹ 40000 are invested in Bond A and ₹ 10000 in Bond B

Maximum return is ₹ 4900 1

Since there are more than 3 constraints, student may be given full 6 marks even if reaches upto (*).

26. $\Delta = \begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix}$

$R_1 \to zR_1,\ R_2 \to xR_2,\ R_3 \to yR_3$

$\Delta = \dfrac{1}{xyz}\begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ x^2z & x(z+y)^2 & x^2y \\ y^2z & y^2x & y(z+x)^2 \end{vmatrix}$ 1

$= \dfrac{xyz}{xyz}\begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix}$ 1

$C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$\Delta = \begin{vmatrix} (x+y)^2 - z^2 & 0 & z^2 \\ 0 & (z+y)^2 - x^2 & x^2 \\ y^2 - (z+x)^2 & y^2 - (z+x)^2 & (z+x)^2 \end{vmatrix}$

$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ y-z-x & y-z-x & (z+x)^2 \end{vmatrix}$ 1

$R_3 \to R_3 - R_1 - R_2$ we get

$= (x+y+z)^2\begin{vmatrix} x+y-z & 0 & z^2 \\ 0 & z+y-x & x^2 \\ -2x & -2z & 2xz \end{vmatrix}$ 1

$C_1 \to C_1 + \dfrac{C_3}{z},\ C_2 \to C_2 + \dfrac{C_3}{x}$ we get

$\Delta = (x+y+z)^2\begin{vmatrix} x+y & \dfrac{z^2}{x} & z^2 \\ \dfrac{x^2}{z} & z+y & x^2 \\ 0 & 0 & 2xz \end{vmatrix}$ 1

$= 2xyz\ (x + y + z)^3$ 1

OR

For getting $A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$ $1\frac{1}{2}$

For getting $A^3 = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$ $1\frac{1}{2}$

Simplifying $A^3 - 6A^2 + 7A + kI_3$ as $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix}$ 2

Equating $\begin{pmatrix} k-2 & 0 & 0 \\ 0 & k-2 & 0 \\ 0 & 0 & k-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow \quad k - 2 = 0$

$k = 2$ 1