QUESTION PAPER CODE 65/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A| = 8.$ 1

2. $k = 12.$ 1

3. $-\log|\sin 2x| + c$ OR $\log|\sec x| - \log|\sin x| + c.$ 1

4. Writing the equations as $\left.\begin{array}{l} 2x - y + 2z = 5 \\ 2x - y + 2z = 8 \end{array}\right\}$ $\frac{1}{2}$

$\Rightarrow$ Distance = 1 unit $\frac{1}{2}$

SECTION B

5. Any skew symmetric matrix of order 3 is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ 1

$\Rightarrow \quad |A| = -a(bc) + a(bc) = 0$ 1

OR

Since A is a skew-symmetric matrix $\quad \therefore \quad A^T = -A$ $\frac{1}{2}$

$\therefore \quad |A^T| = |-A| = (-1)^3.|A|$ $\frac{1}{2}$

$\Rightarrow \quad |A| = -|A|$ $\frac{1}{2}$

$\Rightarrow \quad 2|A| = 0$ or $|A| = 0.$ $\frac{1}{2}$

6. $f(x) = x^3 - 3x$

$\therefore \quad f'(c) = 3c^2 - 3 = 0$ $\frac{1}{2}$

$\therefore \quad c^2 = 1 \quad \Rightarrow \quad c = \pm 1.$ $\frac{1}{2}$

Rejecting c = 1 as it does not belong to $(-\sqrt{3}, 0)$, $\frac{1}{2}$

we get $c = -1.$ $\frac{1}{2}$

7. Let V be the volume of cube, then $\frac{dV}{dt} = 9 \text{ cm}^3/\text{s}$.

Surface area (S) of cube = $6x^2$, where x is the side.

then $V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3x^2} \cdot \frac{dV}{dt}$ 1

$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \frac{dx}{dt} = 12x \cdot \frac{1}{3x^2} \frac{dV}{dt}$ $\frac{1}{2}$

$= 4 \cdot \frac{1}{10} \cdot 9 = 3.6 \text{ cm}^2/\text{s}$ $\frac{1}{2}$

8. $f(x) = x^3 - 3x^2 + 6x - 100$

$f'(x) = 3x^2 - 6x + 6$ $\frac{1}{2}$

$= 3[x^2 - 2x + 2] = 3[(x-1)^2 + 1]$ 1

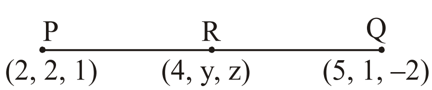
since $f'(x) > 0 \ \forall \ x \in \mathbb{R} \ \therefore f(x)$ is increasing on $\mathbb{R}$ $\frac{1}{2}$

9. Equation of line PQ is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$ $\frac{1}{2}$

Any point on the line is $(3\lambda + 2, -\lambda + 2, -3\lambda + 1)$ $\frac{1}{2}$

$3\lambda + 2 = 4 \Rightarrow \lambda = \frac{2}{3} \ \therefore$ z coord. $= -3\left(\frac{2}{3}\right) + 1 = -1.$ $\frac{1}{2} + \frac{1}{2}$

OR



Let R(4, y, z) lying on PQ divides PQ in the ratio k : 1

$\Rightarrow 4 = \frac{5k+2}{k+1} \Rightarrow k = 2.$ 1

$\therefore z = \frac{2(-2) + 1(1)}{2+1} = \frac{-3}{3} = -1.$ 1

10. Event A: Number obtained is even

B: Number obtained is red.

$P(A) = \frac{3}{6} = \frac{1}{2}, P(B) = \frac{3}{6} = \frac{1}{2}$ $\quad \frac{1}{2} + \frac{1}{2}$

$P(A \cap B) = P$ (getting an even red number) $= \frac{1}{6}$ $\quad \frac{1}{2}$

Since $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(P \cap B)$ which is $\frac{1}{6}$ $\quad \frac{1}{2}$

$\therefore$ A and B are not independent events.

11. Let A works for x day and B for y days.

$\therefore$ L.P.P. is Minimize C = 300x + 400y $\quad \frac{1}{2}$

Subject to: $\begin{cases} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{cases}$ $\quad 1\frac{1}{2}$

12. $\int \frac{dx}{5 - 8x - x^2} = \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$ $\quad 1$

$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21} + (x+4)}{\sqrt{21} - (x+4)} \right| + c$ $\quad 1$

SECTION C

13. $\tan^{-1} \frac{x-3}{x-4} + \tan^{-1} \frac{x+3}{x+4} = \frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1} \left(\frac{\frac{x-3}{x-4} + \frac{x+3}{x+4}}{1 - \frac{x-3}{x-4} \cdot \frac{x+3}{x+4}} \right) = \frac{\pi}{4}$ $\quad 1\frac{1}{2}$

$\Rightarrow \quad \frac{2x^2 - 24}{-7} = 1 \Rightarrow x^2 = \frac{17}{2}$ $\quad 1\frac{1}{2}$

$\Rightarrow \quad x = \pm\sqrt{\frac{17}{2}}$ $\quad 1$

14. $\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1+1

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding

$(a - 1)^2 \cdot (a - 1) = (a - 1)^3$. 1

OR

Let $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow \begin{pmatrix} 2a-c & 2b-d \\ a & b \\ -3a+4c & -3b+4d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow \quad 2a - c = -1, \quad 2b - d = -8$

$a = 1, \quad b = -2$ 1

$-3a + 4c = 9, \quad -3b + 4d = 22$

Solving to get $a = 1, b = -2, c = 3, d = 4$

$\therefore \quad A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ 1

15. $x^y + y^x = a^b$

Let $u + v = a^b$, where $x^y = u$ and $y^x = v$.

$\therefore \quad \frac{du}{dx} + \frac{dv}{dx} = 0 \qquad ...(i)$ $\frac{1}{2}$

$$y \log x = \log u \Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right]$$ 1

$$x \log y = \log v \Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$$ 1

Putting in (i) $x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = 0$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^x \log y + y \cdot x^{y-1}}{x^y \cdot \log x + x \cdot y^{x-1}}$$ 1

OR

$$e^y \cdot (x+1) = 1 \Rightarrow e^y \cdot 1 + (x+1) \cdot e^y \cdot \frac{dy}{d} = 0$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{(x+1)}$$ 1

$$\frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2} = \left(\frac{dy}{dx}\right)^2$$ $1\frac{1}{2}$

16. $I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta = \int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)} d\theta$ $\frac{1}{2}$

$$= \int \frac{dt}{(4+t^2)(1+4t^2)}, \text{ where } \sin\theta = t$$ 1

$$= \int \frac{-\frac{1}{15}}{4+t^2} dt + \int \frac{\frac{4}{15}}{1+4t^2} dt$$ 1

$$= -\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right) + \frac{4}{30}\tan^{-1}(2t) + c$$ 1

$$= -\frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right) + \frac{2}{15}\tan^{-1}(2\sin\theta) + c$$ $\frac{1}{2}$

17. $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx = \int_0^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x} dx$ 1

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx = \pi \int_0^{\pi} \tan x (\sec x - \tan x) dx$

$I = \frac{\pi}{2} \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1) dx$ 1

$= \frac{\pi}{2} [\sec x - \tan x + x]_0^{\pi}$ 1

$= \frac{\pi(\pi - 2)}{2}$ 1

OR

$I = \int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$

$= \int_1^4 (x-1) dx - \int_1^2 (x-2) dx + \int_2^4 (x-2) dx - \int_1^4 (x-4) dx$ 2

$= \left.\frac{(x-1)^2}{2}\right]_1^4 - \left.\frac{(x-2)^2}{2}\right]_1^2 + \left.\frac{(x-2)^2}{2}\right]_2^4 - \left.\frac{(x-4)^2}{2}\right]_1^4$ 1

$= \frac{9}{2} + \frac{1}{2} + 2 + \frac{9}{2} = 11\frac{1}{2}$ or $\frac{23}{2}$ 1

18. Given differential equation can be written as

$(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x \Rightarrow \frac{dy}{dx} + \frac{1}{1 + x^2} y = \frac{\tan^{-1} x}{1 + x^2}$ 1

Integrating factor = $e^{\tan^{-1} x}$. 1

$\therefore$ Solution is $y \cdot e^{\tan^{-1} x} = \int \tan^{-1} x \cdot e^{\tan^{-1} x} \frac{1}{1 + x^2} dx$ 1

$\Rightarrow y \cdot e^{\tan^{-1} x} = e^{\tan^{-1} x} \cdot (\tan^{-1} x - 1) + c$ 1

or $y = (\tan^{-1} x - 1) + c \cdot e^{-\tan^{-1} x}$

19. $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ 1

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$, are not parallel vectors, and $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ $\therefore$ A, B, C form a triangle 1

Also $\overrightarrow{BC} \cdot \overrightarrow{CA} = 0$ $\therefore$ A, B, C form a right triangle 1

Area of $\Delta = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{BC}| = \frac{1}{2}\sqrt{210}$ 1

20. Given points, A, B, C, D are coplanar, if the

vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, i.e.

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k}, \overrightarrow{AC} = -\hat{i} - 3\hat{j} - 8\hat{k}, \overrightarrow{AD} = \hat{i} + (\lambda - 9)\hat{k}$ $1\frac{1}{2}$

are coplanar

i.e., $\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & \lambda - 9 \end{vmatrix} = 0$ 1

$-2[-3\lambda + 27] + 4[-\lambda + 17] - 6(3) = 0$ 1

$\Rightarrow \lambda = 2.$ $\frac{1}{2}$

21. Writing

+	1	3	5	7
1	×	4	6	8
3	4	×	8	10
5	6	8	×	12
7	8	10	12	×

$\therefore$ X :	4	6	8	10	12	1
P(X) :	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1
xP(X) :	$\frac{4}{6}$	$\frac{6}{6}$	$\frac{16}{6}$	$\frac{10}{6}$	$\frac{12}{6}$	
x^2P(X) :	$\frac{16}{6}$	$\frac{36}{6}$	$\frac{128}{6}$	$\frac{100}{6}$	$\frac{144}{6}$	

$\Sigma x P(x) = \frac{48}{6} = 8 \quad \therefore \text{Mean} = 8$ 1

Variance $= \Sigma x^2 P(x) - [\Sigma x P(x)]^2 = \frac{424}{6} - 64 = \frac{20}{3}$ 1

22. Let E_1: Selecting a student with 100% attendance
E_2: Selecting a student who is not regular 1

A: selected student attains A grade.

$P(E_1) = \frac{30}{100}$ and $P(E_2) = \frac{70}{100}$ $\frac{1}{2}$

$P(A/E_1) = \frac{70}{100}$ and $P(A/E_2) = \frac{10}{100}$ $\frac{1}{2}$

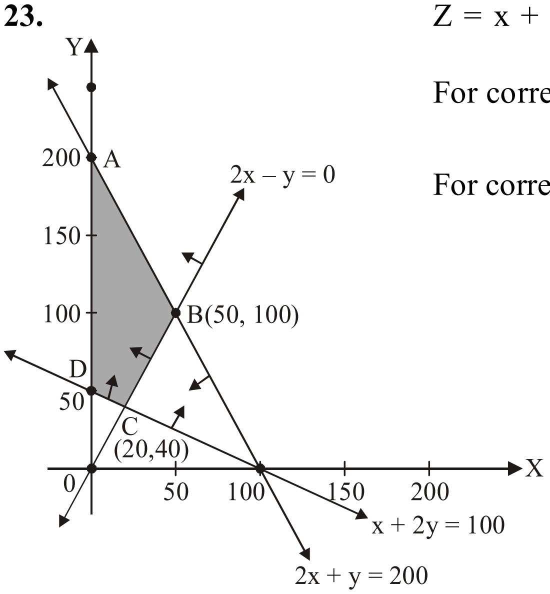
$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{30}{100}\times\frac{70}{100}}{\frac{30}{100}\times\frac{70}{100}+\frac{70}{100}\times\frac{10}{100}}$$

$$= \frac{3}{4}$$ 1

Regularity is required everywhere or any relevant value 1

23. $Z = x + 2y$ s.t $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0$



For correct graph of three lines $1\frac{1}{2}$

For correct shading $1\frac{1}{2}$

$Z(A) = 0 + 400 = 400$

$Z(B) = 50 + 200 = 250$

$Z(C) = 20 + 80 = 100$

$Z(D) = 0 + 100 = 100$

$\therefore$ Max (= 400) at $x = 0,\ y = 200$ 1

SECTION D

24. Getting $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ ...(i) $1\frac{1}{2}$

Given equations can be written as $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$ 1

$\Rightarrow \quad AX = B$

From (i) $A^{-1} = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ 1

$\therefore \quad X = A^{-1}B = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$ 1

$= \frac{1}{8}\begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ 1

$\Rightarrow x = 3, \ y = -2, \ z = -1$ $\frac{1}{2}$

25. Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$ $\frac{1}{2}$

$\Rightarrow \quad \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4} \Rightarrow (4x_1+3)(3x_2+4) = (3x_1+4)(4x_2+3)$

$\Rightarrow \quad 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12_1x_2 + 16x_2 + 9x_1 + 12$

$\Rightarrow \quad 16(x_1 - x_2) - 9(x_1 - x_2) = 0 \Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence f is a 1–1 function 2

Let $y = \frac{4x+3}{3x+4}$, for $y \in R - \left\{\frac{4}{3}\right\}$

$3xy + 4y = 4x + 3 \Rightarrow 4x - 3xy = 4y - 3$

$\Rightarrow \quad x = \frac{4y-3}{4-3y} \quad \therefore \ \forall \, y \in R - \left\{\frac{4}{3}\right\}, x \in R - \left\{-\frac{4}{3}\right\}$

Hence f is ONTO and so bijective 2

and $f^{-1}(y) = \dfrac{4y-3}{4-3y}; y \in R - \left\{\dfrac{4}{3}\right\}$ 1

$f^{-1}(0) = -\dfrac{3}{4}$ $\frac{1}{2}$

and $f^{-1}(x) = 2 \Rightarrow \dfrac{4x-3}{4-3x} = 2$

$\Rightarrow \quad 4x - 3 = 8 - 6x$

$\Rightarrow \quad 10x = 11 \Rightarrow x = \dfrac{11}{10}$ $\frac{1}{2}$

OR

$(a, b) * (c, d) = (ac, b + ad); (a, b), (c, d) \in A$

$(c, d) * (a, b) = (ca, d + bc)$

Since $b + ad \neq d + bc \Rightarrow *$ is NOT comutative $1\frac{1}{2}$

for associativity, we have,

$[(a,b) * (c, d)] * (e, f) = (ac, b + ad) * (e, f) = (ace, b + ad + acf)$

$(a, b) * [(c, d) * (e, f)] = (a, b) * (ce, d + cf) = (ace, b + ad + acf)$ $1\frac{1}{2}$

$\Rightarrow *$ is associative

(i) Let (e, f) be the identity element in A

Then $(a, b) * (e, f) = (a, b) = (e, f) * (a, b)$

$\Rightarrow \quad (ae, b + af) = (a, b) = (ae, f + be)$

$\Rightarrow \quad e = 1, f = 0 \Rightarrow (1, 0)$ is the identity element $1\frac{1}{2}$

(ii) Let (c, d) be the inverse element for (a, b)

$\Rightarrow \quad (a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$

$\Rightarrow \quad (ac, b + ad) = (1, 0) = (ac, d + bc)$

$\Rightarrow ac = 1 \Rightarrow c = \dfrac{1}{a}$ and $b + ad = 0 \Rightarrow d = -\dfrac{b}{a}$ and $d + bc = 0 \Rightarrow d = -bc = -b\left(\dfrac{1}{a}\right)$

$\Rightarrow \left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$, $a \neq 0$ is the inverse of $(a, b) \in A$ $1\frac{1}{2}$

26. Let the sides of cuboid be x, x, y

$\Rightarrow$ $x^2y = k$ and $S = 2(x^2 + xy + xy) = 2(x^2 + 2xy)$ $\frac{1}{2}+1$

$\therefore$ $S = 2\left[x^2 + 2x\frac{k}{x^2}\right] = 2\left[x^2 + \frac{2k}{x}\right]$ 1

$\frac{ds}{dx} = 2\left[2x - \frac{2k}{x^2}\right]$ 1

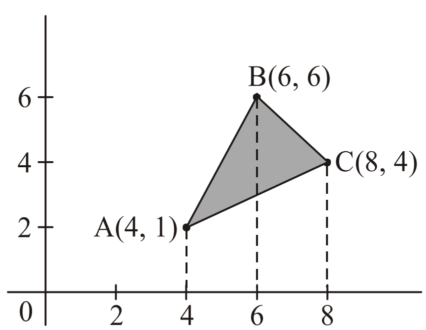
$\therefore$ $\frac{ds}{dx} = 0 \Rightarrow x^3 = k = x^2y \Rightarrow x = y$ 1

$\frac{d^2s}{dx^2} = 2\left[2 + \frac{4k}{x^3}\right] > 0$ $\therefore$ x = y will given minimum surface area 1

and x = y, means sides are equal

$\therefore$ Cube will have minimum surface area $\frac{1}{2}$

27. Figure 1



$$\left.\begin{aligned} &\text{Equation of AB}: y = \frac{5}{2}x - 9 \\ &\text{Equation of BC}: y = 12 - x \\ &\text{Equation of AC}: y = \frac{3}{4}x - 2 \end{aligned}\right\}$$ $1\frac{1}{2}$

$\therefore$ Area (A) $= \int_4^6\left(\frac{5}{2}x - 9\right)dx + \int_6^8(12 - x)\,dx - \int_4^8\left(\frac{3}{4}x - 2\right)dx$ 1

$= \left[\frac{5}{4}x^2 - 9x\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3}{8}x^2 - 2x\right]_4^8$ $1\frac{1}{2}$

$= 7 + 10 - 10 = 7$ sq.units 1

OR

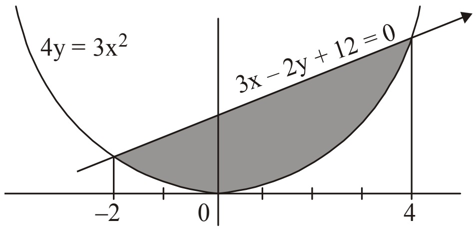


Figure 1

$$4y = 3x^2 \text{ and } 3x - 2y + 12 = 0 \Rightarrow 4\left(\frac{3x+12}{2}\right) = 3x^2$$

$$\Rightarrow 3x^2 - 6x - 24 = 0 \text{ or } x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$$

$\Rightarrow$ x-coordinates of points of intersection are $x = -2$, $x = 4$ 1

$$\therefore \text{ Area (A)} = \int_{-2}^{4}\left[\frac{1}{2}(3x+12) - \frac{3}{4}x^2\right]dx \qquad 1\frac{1}{2}$$

$$= \left[\frac{1}{2}\frac{(3x+12)^2}{6} - \frac{3}{4}\frac{x^3}{3}\right]_{-2}^{4} \qquad 1\frac{1}{2}$$

$$= 45 - 18 = 27 \text{ sq.units} \qquad 1$$

28. $$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+\frac{2y}{x}}{1-\frac{y}{x}} \qquad \frac{1}{2}$$

$$\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \therefore \; v + x\frac{dv}{dx} = \frac{1+2v}{1-v} \qquad \frac{1}{2}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1+2v-v+v^2}{v-1} \Rightarrow \int \frac{v-1}{v^2+v+1}dv = -\frac{dx}{x} \qquad 1$$

$$\Rightarrow \int \frac{2v+1-3}{v^2+v+1}dv = \int -\frac{2}{x}dx \Rightarrow \int \frac{2v+1}{v^2+v+1}dv - 3\int \frac{1}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dv = -\int \frac{2}{x}dx \qquad 1+1$$

$$\Rightarrow \log|v^2+v+1| - 3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log|x|^2 + c \qquad 1$$

$$\Rightarrow \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = c \qquad \frac{1}{2}$$

$$x = 1,\; y = 0 \Rightarrow c = -2\sqrt{3}\cdot\frac{\pi}{6} = -\frac{\sqrt{3}}{3}\pi \qquad \frac{1}{2}$$

$$\therefore \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) + \frac{\sqrt{3}}{3}\pi = 0$$

29. Equation of line through (3, – 4, – 5) and (2, –3, 1) is

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \qquad \text{...(i)}$$ 1

Eqn. of plane through the three given points is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0 \Rightarrow (x-1)(12) - (y-2)(-6) + (z-3)(6) = 0$$

or $2x + y + z - 7 = 0$ ...(ii) 2

Any point on line (i) is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 1$

$\Rightarrow \lambda = 2$ 1

Required point is (1, –2, 7) 1

OR

Equation of plane cutting intercepts (say, a, b, c) on the axes is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, with A(a, 0, 0), B(0, b, 0) and C(0, 0, c) 1

distance of this plane from orgin is $3p = \dfrac{|-1|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$ $1\frac{1}{2}$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2} \qquad \text{...(i)}$$ 1

Centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (x, y, z)$ 1

$\Rightarrow$ a = 3x, b = 3y, c = 3z, we get from (i) $\frac{1}{2}$

$$\frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9p^2} \text{ or } \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$$ 1