

QUESTION PAPER CODE 65/1/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A^{-1}| = \frac{1}{|A|} \Rightarrow k = -1$ 1

2. $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{kx}{|x|} = -k$ $\frac{1}{2}$

$k = -3$ $\frac{1}{2}$

3. $\int_2^3 3^x dx = \left[\frac{3^x}{\log 3} \right]_2^3 = \frac{18}{\log 3}$ $\frac{1}{2} + \frac{1}{2}$

4. $\cos^2 90^\circ + \cos^2 60^\circ + \cos^2 \gamma = 1$ $\frac{1}{2}$

$\cos \gamma = \pm \frac{\sqrt{3}}{2}, \gamma = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$ $\frac{1}{2}$

SECTION B

5. Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix

A is skew symmetric

$\therefore A = -A'$ 1

$\Rightarrow a_{ij} = -a_{ji} \forall i, j$

For diagonal elements $i = j$,

$\Rightarrow 2a_{ii} = 0$

$\Rightarrow a_{ii} = 0 \Rightarrow$ diagonal elements are zero. 1

6. From the given equation

$2 \sin y \cos y \cdot \frac{dy}{dx} - \sin xy \cdot \left[x \cdot \frac{dy}{dx} + y \cdot 1 \right] = 0$ 1

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x \sin(xy)}$$

$$\therefore \left. \frac{dy}{dx} \right|_{x=1, y=\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)} \quad 1$$

7. $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{3}{4\pi r^2} \quad 1$$

$$S = 4\pi r^2$$

$$\Rightarrow \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} \quad \frac{1}{2}$$

$$\Rightarrow \left. \frac{dS}{dt} \right|_{r=2} = 3\text{cm}^2/\text{s} \quad \frac{1}{2}$$

8. $f(x) = 4x^3 - 18x^2 + 27x - 7$

$$f'(x) = 12x^2 - 36x + 27 \quad \frac{1}{2}$$

$$= 3(2x - 3)^2 \geq 0 \quad \forall x \in \mathbb{R} \quad 1$$

$$\Rightarrow f(x) \text{ is increasing on } \mathbb{R} \quad \frac{1}{2}$$

9. Equation of given line is $\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35} \quad \frac{1}{2}$

Its DR's $\left\langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35} \right\rangle$ or $\langle 7, -5, 1 \rangle \quad \frac{1}{2}$

Equation of required line is

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k}) \quad 1$$

$$\begin{aligned}
 10. \quad P(E \cap F') &= P(E) - P(E \cap F) && 1 \\
 &= P(E) - P(E) \cdot P(F) && \frac{1}{2} \\
 &= P(E)[1 - P(F)] \\
 &= P(E)P(F') && \frac{1}{2}
 \end{aligned}$$

\Rightarrow E and F' are independent events.

11. Let x necklaces and y bracelets are manufactured

\therefore L.P.P. is

Maximize profit, $P = 100x + 300y$

subject to constraints

$$x + y \leq 24$$

$$\frac{1}{2}x + y \leq 16 \text{ or } x + 2y \leq 32$$

$$x, y, \geq 1$$

$$\frac{1}{2} \times 3 = 1 \frac{1}{2}$$

$$\begin{aligned}
 12. \quad \int \frac{dx}{x^2 + 4x + 8} &= \int \frac{dx}{(x+2)^2 + (2)^2} && 1 \\
 &= \frac{1}{2} \tan^{-1} \frac{x+2}{2} + C && 1
 \end{aligned}$$

SECTION C

$$\begin{aligned}
 13. \quad \text{Let } \frac{1}{2} \cos^{-1} \frac{a}{b} &= x && \frac{1}{2} \\
 \text{LHS} &= \tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = \frac{1 + \tan x}{1 - \tan x} + \frac{1 - \tan x}{1 + \tan x} && 1 \frac{1}{2} \\
 &= \frac{2(1 + \tan^2 x)}{1 - \tan^2 x} = \frac{2}{\cos 2x} && 1 \\
 &= \frac{2b}{a} = \text{RHS} && 1
 \end{aligned}$$

$$14. \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$= 3(x+y) \begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix} \quad 1$$

$$R_1 \rightarrow R_1 - R_2, R_3 \rightarrow R_3 - R_2$$

$$= 3(x+y) \begin{vmatrix} 0 & y & y \\ 1 & x & x+y \\ 0 & 2y & -y \end{vmatrix} \quad 1+1$$

$$= -3(x+y)(-y^2 - 2y^2) = 9y^2(x+y) \quad 1$$

OR

$$\text{Let } D = \begin{bmatrix} x & y \\ z & w \end{bmatrix} \quad \frac{1}{2}$$

$$CD = AB \Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} \quad 1+1$$

$$2x + 5z = 3, 3x + 8z = 43; 2y + 5w = 0, 3y + 8w = 22.$$

$$\text{Solving, we get } x = -191, y = -110, z = 77, w = 44 \quad 1$$

$$\therefore D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix} \quad \frac{1}{2}$$

$$15. y = (\sin x)^x + \sin^{-1} \sqrt{x}$$

$$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad 1$$

$$u = (\sin x)^x$$

$$\Rightarrow \log u = x \log \sin x \quad \frac{1}{2}$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x [x \cot x + \log \sin x] \quad 1$$

$$v = \sin^{-1} \sqrt{x}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}} \quad 1$$

$$\therefore \frac{dy}{dx} = (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{2\sqrt{x-x^2}} \quad \frac{1}{2}$$

OR

$$x^m \cdot y^n = (x+y)^{m+n}$$

$$\Rightarrow m \log x + n \log y = (m+n) \log (x+y) \quad 1$$

$$\Rightarrow \frac{m}{x} + \frac{n}{y} \cdot \frac{dy}{dx} = \frac{m+n}{x+y} \left(1 + \frac{dy}{dx}\right) \quad 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \quad \dots(i) \quad 1$$

$$\frac{d^2y}{dx^2} = \frac{x \frac{dy}{dx} - y}{x^2} = 0 \quad \dots(ii) \text{ (using (i))} \quad 1$$

$$16. \int \frac{2x}{(x^2+1)(x^2+2)^2} = \int \frac{dy}{(y+1)(y+2)^2} \quad [\text{by substituting } x^2 = y] \quad 1$$

$$= \int \frac{dy}{y+1} - \int \frac{dy}{y+2} - \int \frac{dy}{(y+2)^2} \quad (\text{using partial fraction}) \quad 1 \frac{1}{2}$$

$$= \log(y+1) - \log(y+2) + \frac{1}{y+2} + C \quad 1$$

$$= \log(x^2+1) - \log(x^2+2) + \frac{1}{x^2+2} + C \quad \frac{1}{2}$$

$$17. I = \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$$

$$= \int_0^\pi \frac{(\pi-x) \sin x}{1 + \cos^2 x} dx \quad 1$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x \, dx}{1 + \cos^2 x}$$

Put $\cos x = t$ and $-\sin x \, dx = dt$

$$= -\pi \int_1^{-1} \frac{dt}{1+t^2}$$

$$= \pi [\tan^{-1} t]_{-1}^1 = \frac{\pi^2}{2}$$

$$\Rightarrow I = \frac{\pi^2}{4}$$

1

 $\frac{1}{2}$ $\frac{1}{2}$

OR

$$I = \int_0^{3/2} |x \sin \pi x| \, dx$$

$$= \int_0^1 x \sin \pi x \cdot dx - \int_1^{3/2} x \sin \pi x \, dx$$

$$= \left[-x \frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_0^1 - \left[-x \frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_1^{3/2}$$

$$= \frac{2}{\pi} + \frac{1}{\pi^2}$$

 $\frac{1}{2}$ $\frac{1}{2}$

1

$$18. \quad x^2 - y^2 = C(x^2 + y^2)^2 \Rightarrow 2x - 2yy' = 2C(x^2 + y^2)(2x + 2yy')$$

1

$$\Rightarrow (x - yy') = \frac{x^2 - y^2}{y^2 + x^2} (2x + 2yy') \Rightarrow (y^2 + x^2)(x - yy') = (x^2 - y^2)(2x + 2yy')$$

1

$$\Rightarrow [-2y(x^2 - y^2) - y(y^2 + x^2)] \frac{dy}{dx} = 2x(x^2 - y^2) - x(y^2 + x^2)$$

1

$$\Rightarrow (y^3 - 3x^2y) \frac{dy}{dx} = (x^3 - 3xy^2)$$

$$\Rightarrow (y^3 - 3x^2y)dy = (x^3 - 3xy^2)dx$$

1

Hence $x^2 - y^2 = C(x^2 + y^2)^2$ is the solution of given differential equation.

$$19. [\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = c_2 - c_3 \quad 1$$

$$(a) \quad c_1 = 1, \quad c_2 = 2$$

$$[\vec{a} \vec{b} \vec{c}] = 2 - c_3 \quad 1$$

$$\because \vec{a}, \vec{b}, \vec{c} \text{ are coplanar } [\vec{a} \vec{b} \vec{c}] = 0 \Rightarrow c_3 = 2 \quad 1$$

$$(b) \quad c_2 = -1, \quad c_3 = 1$$

$$[\vec{a} \vec{b} \vec{c}] = c_2 - c_3 = -2 \neq 0$$

\Rightarrow No value of c_1 can make $\vec{a}, \vec{b}, \vec{c}$ coplanar 1

$$20. |\vec{a}| = |\vec{b}| = |\vec{c}| \text{ and } \vec{a} \cdot \vec{b} = 0 = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} \quad \dots(i) \quad 1$$

Let α, β and γ be the angles made by $(\vec{a} + \vec{b} + \vec{c})$ with \vec{a}, \vec{b} and \vec{c} respectively

$$(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a} = |\vec{a} + \vec{b} + \vec{c}| |\vec{a}| \cos \alpha$$

$$\Rightarrow \alpha = \cos^{-1} \left(\frac{|\vec{a}|}{|\vec{a} + \vec{b} + \vec{c}|} \right)$$

$$\text{Similarly, } \beta = \cos^{-1} \left(\frac{|\vec{b}|}{|\vec{a} + \vec{b} + \vec{c}|} \right) \text{ and } \gamma = \cos^{-1} \left(\frac{|\vec{c}|}{|\vec{a} + \vec{b} + \vec{c}|} \right) \quad 1$$

using (i), we get $\alpha = \beta = \gamma$ $\frac{1}{2}$

$$\text{Now } |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \quad 1$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 3|\vec{a}|^2 \text{ (using (i))}$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3} |\vec{a}|$$

$$\therefore \alpha = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right) = \beta = \gamma \quad \frac{1}{2}$$

21.

x	P(x)
0	p
1	p
2	k
3	k

$$\sum p(x) = 1 \Rightarrow 2p + 2k = 1 \Rightarrow k = \frac{1}{2} - p$$

1

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	p	0	0
1	p	p	p
2	$\frac{1}{2} - p$	$1 - 2p$	$2 - 4p$
3	$\frac{1}{2} - p$	$\frac{3}{2} - 3p$	$\frac{9}{2} - 9p$
		$\frac{5}{2} - 4p$	$\frac{13}{2} - 12p$

2

As per problem, $\sum p_i x_i^2 = 2 \sum p_i x_i$

$$\Rightarrow p = \frac{3}{8}$$

1

22. Let H_1 be the event that 6 appears on throwing a die
 H_2 be the event that 6 does not appear on throwing a die

E be the event that he reports it is six

1

$$P(H_1) = \frac{1}{6}, P(H_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$$

1

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)}$$

 $\frac{1}{2}$

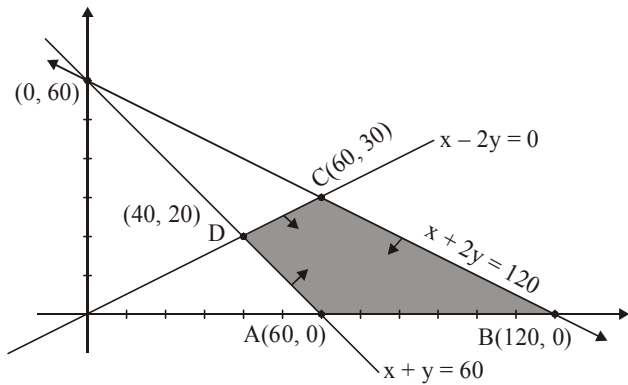
$$= \frac{4}{9}$$

 $\frac{1}{2}$

Relevant value: Yes, Truthness leads to more respect in society.

1

23.



Correct graph of 3 lines

 $1\frac{1}{2}$

Correct shade of 3 lines

 $1\frac{1}{2}$

$$Z = 5x + 10y$$

$$Z|_{A(60, 0)} = 300$$

$$Z|_{B(120, 0)} = 600$$

$$Z|_{C(60, 30)} = 600$$

$$Z|_{D(40, 20)} = 400$$

Minimum value of $Z = 300$ at $x = 60, y = 0$

1

SECTION D

$$24. \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1

$$AB = I \Rightarrow A^{-1} = B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

1

Given equations in matrix form are:

$$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

1

$$A/X = C$$

 $\frac{1}{2}$

$$\Rightarrow X = (A')^{-1} C = (A^{-1})'C$$

1

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

1

$$\Rightarrow x = 0, y = 5, z = 3$$

 $\frac{1}{2}$

25. Clearly $f^{-1}(y) = g(y): [-5, \infty) \rightarrow \mathbb{R}_+$ and,

$$f \circ g(y) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = 9\left(\frac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\frac{\sqrt{y+6}-1}{3}\right) - 5 = y \quad 2$$

$$\text{and } (g \circ f)(x) = g(9x^2 + 6x - 5) = \frac{\sqrt{9x^2 + 6x + 1} - 1}{3} = x \quad 2$$

$$\therefore g = f^{-1} \quad 1$$

$$(i) \quad f^{-1}(10) = \frac{\sqrt{16}-1}{3} = 1 \quad \frac{1}{2}$$

$$(ii) \quad f^{-1}(y) = \frac{4}{3} \Rightarrow y = 19 \quad \frac{1}{2}$$

OR

Note: Some short comings have been observed in this question which makes the question unsolvable.

So, 6 marks may be given for a genuine attempt.

$$a * b = a - b + ab \quad \forall a, b \in A = \mathbb{Q} - [1]$$

$$b * a = b - a + ba$$

$$(a * b) \neq b * a \Rightarrow * \text{ is not commutative.} \quad 1 \frac{1}{2}$$

$$(a * b) * c = (a - b + ab) * c$$

$$= a - b - c + ab + ac - bc + abc$$

$$a * (b * c) = a * (b - c + bc)$$

$$= a - b + c + ab - ac - bc + abc$$

$$(a * b) * c \neq a * (b * c) \quad 1 \frac{1}{2}$$

$\Rightarrow *$ is not associative.

Existence of identity

$$a * e = a - e + ae = a$$

$$\Rightarrow e(a - 1) = 0$$

$$\Rightarrow e = 0$$

$\therefore e$ is not unique

\therefore No identity element exists.

$$a * b = e = b * a$$

\therefore No identity element exists.

\Rightarrow Inverse element does not exist.

$$e * a = e - a + ea = a$$

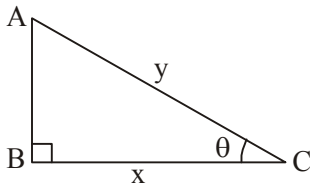
$$\Rightarrow e(1 + a) = 2a$$

$$\Rightarrow e = \frac{2a}{1+a}$$

$1 \frac{1}{2}$

$1 \frac{1}{2}$

26.



Given $x + y = k$

$$\text{Area of } \Delta = \frac{1}{2} x \sqrt{y^2 - x^2}$$

$$\text{Let } Z = \frac{1}{4} x^2 (y^2 - x^2)$$

$$= \frac{1}{4} x^2 [(k - x)^2 - x^2]$$

$$= \frac{1}{4} [k^2 x^2 - 2kx^3]$$

$$\frac{dz}{dx} = \frac{1}{4} [2k^2 x - 6kx^2] = 0 \Rightarrow k - 3x = 0 \Rightarrow x = \frac{k}{3}$$

$$\Rightarrow x + y - 3x = 0 \text{ or } y = 2x$$

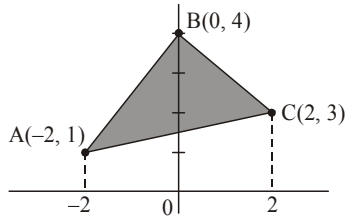
$$\frac{d^2z}{dx^2} = \frac{1}{4} [2k^2 - 12kx]$$

$$\left. \frac{d^2z}{dx^2} \right|_{x=\frac{k}{3}} = \frac{1}{4} [2k^2 - 4k^2] = -\frac{k^2}{2} < 0$$

\therefore Area will be maximum for $2x = y$

$$\text{but } \frac{x}{y} = \cos \theta \Rightarrow \cos \theta = \frac{x}{2x} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

27.



$$\text{Equation of AB: } y = \frac{3}{2}x + 4$$

Correct Figure: 1

$$\text{Equation of BC; } y = 4 - \frac{x}{2}$$

$$\text{Equation of AC; } y = \frac{1}{2}x + 2$$

 $1\frac{1}{2}$

$$\text{Required area} = \int_{-2}^0 \left(\frac{3}{2}x + 4 \right) dx + \int_0^2 \left(4 - \frac{x}{2} \right) dx - \int_{-2}^2 \left(\frac{1}{2}x + 2 \right) dx$$

1

$$= \left[\frac{3x^2}{4} + 4x \right]_{-2}^0 + \left[4x - \frac{x^2}{4} \right]_0^2 - \left[\frac{x^2}{4} + 2x \right]_{-2}^2$$

 $1\frac{1}{2}$

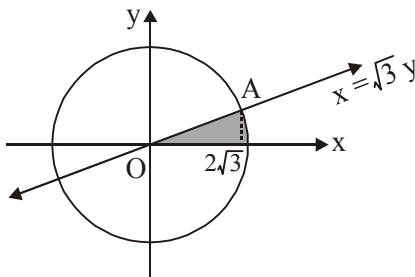
$$= 5 + 7 - 8$$

1

$$= 4 \text{ sq. units}$$

OR

Note: In this problem, two regions are possible instead of a unique one, so full 6 marks may be given for finding the area of either region correctly.



Correct Figure

1

x-coordinate of points of intersection is $x = \pm 2\sqrt{3}$

1

Required area

$$= \int_0^{2\sqrt{3}} \frac{x}{\sqrt{3}} \cdot dx + \int_{2\sqrt{3}}^4 \sqrt{4^2 - x^2} dx$$

 $1\frac{1}{2}$

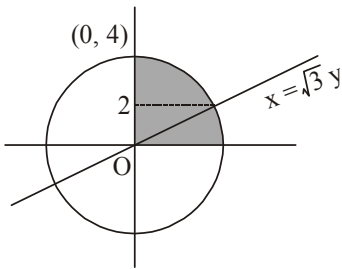
$$= \left[\frac{x^2}{2\sqrt{3}} \right]_0^{2\sqrt{3}} + \left[\frac{x\sqrt{16-x^2}}{2} + 8\sin^{-1} \frac{x}{4} \right]_{2\sqrt{3}}^4$$

 $1\frac{1}{2}$

$$= 2\sqrt{3} + 8 \left(\frac{\pi}{2} - \frac{\pi}{3} \right) - 2\sqrt{3}$$

$$= \frac{4\pi}{3} \text{ sq. units}$$

1

Alternate Solution

Correct figure

1

y-co-ordinate of point of intersection is $y = 2$

1

Required Area

$$= \sqrt{3} \int_0^2 y \, dx + \int_2^4 \sqrt{(4)^2 - y^2} \, dy$$

 $1 \frac{1}{2}$

$$= \sqrt{3} \left[\frac{y^2}{2} \right]_0^2 + \left[\frac{y\sqrt{16-y^2}}{2} + 8 \sin^{-1} \frac{y}{4} \right]_2^4$$

 $1 \frac{1}{2}$

$$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}$$

$$= \frac{8\pi}{3} \text{ sq.units}$$

1

28. The given equation can be written as

$$\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$$

1

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

1

 \therefore Solution is

$$y \cdot x = \int (x \cos x + \sin x) dx + c$$

1

$$\Rightarrow y \cdot x = x \sin x + c$$

1

$$\text{or } y = \sin x + \frac{c}{x}$$

$$\text{when } x = \frac{\pi}{2}, y = 1, \text{ we get } c = 0$$

1

Required solution is $y = \sin x$

1

29. Equation of family of planes

$$\vec{r} \cdot [(2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(\hat{i} - \hat{j})] = 1 - 4\lambda$$

1

$$\Rightarrow \vec{r} \cdot [(2 + \lambda)\hat{i} + (-3 - \lambda)\hat{j} + 4\hat{k}] = 1 - 4\lambda \quad \dots(i) \quad 1$$

plane (i) is perpendicular to

$$\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$$

$$2(2 + \lambda) - 1(-3 - \lambda) + 1(4) = 0 \Rightarrow \lambda = -\frac{11}{3} \quad 1+1$$

Substituting $\lambda = -\frac{11}{3}$ in equation (i), we get

$$\vec{r} \cdot \left(-\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + 4\hat{k} \right) = \frac{47}{3}$$

$$\Rightarrow \boxed{\vec{r} \cdot (-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47} \quad \text{(vector equation)} \quad \left. \vphantom{\vec{r} \cdot (-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47} \right\} \quad (ii) \quad 1$$

$$\text{or } \boxed{-5x + 2y + 12z - 47 = 0} \quad \text{(cartesian equation)}$$

Line $\frac{x-1}{1} = \frac{y-2}{1/2} = \frac{z-4}{1/3}$ lies on the plane

\therefore (i) Point P(1, 2, 4) satisfies equation (ii) $\frac{1}{2}$

and $a_1a_2 + b_1b_2 + c_1c_2 = -5 + 1 + 4 = 0$ $\frac{1}{2}$

\Rightarrow Line is perpendicular to the normal of plane \therefore Plane contains the given line

OR

Equation of line L_1 passing through (1, 2, -4) is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \quad 1$$

$$L_2: \frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

$$L_3: \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

$\therefore L_1 \perp L_2 \Rightarrow 3a - 16b + 7c = 0$ 1

$L_1 \perp L_3 \Rightarrow 3a + 8b - 5c = 0$ 1

Solving, we get

$$\frac{a}{24} = \frac{b}{36} = \frac{c}{72} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6} \quad 1$$

\therefore Required cartesian equation of line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \quad 1$$

Vector equation

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \quad 1$$