

QUESTION PAPER CODE 65/2/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A|^{-1} |A| = |I| \Rightarrow |A|^2 = 1$

 $\frac{1}{2}$

$\therefore |A| = 1 \text{ or } |A| = -1$

 $\frac{1}{2}$

2. For $x < 0$, $y = x |x| = -x^2$

 $\frac{1}{2}$

$\therefore \frac{dy}{dx} = -2x$

 $\frac{1}{2}$

3. Order = 2, Degree not defined

 $\frac{1}{2} + \frac{1}{2}$

4. D. Rs are 1, 1, 1

$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$

1

SECTION B

5. $\left. \begin{array}{l} \forall a, b \in \mathbb{R}, \sqrt{a^2 + b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R} \end{array} \right\}$

1

Also,

$\left. \begin{array}{l} a * (b * c) = a * \sqrt{b^2 + c^2} = \sqrt{a^2 + b^2 + c^2} \\ (a * b) * c = \sqrt{a^2 + b^2} * c = \sqrt{a^2 + b^2 + c^2} \end{array} \right\} \Rightarrow a * (b * c) = (a * b) * c \left. \vphantom{\begin{array}{l} a * (b * c) = a * \sqrt{b^2 + c^2} = \sqrt{a^2 + b^2 + c^2} \\ (a * b) * c = \sqrt{a^2 + b^2} * c = \sqrt{a^2 + b^2 + c^2} \end{array}} \right\} \therefore * \text{ is Associative}$

1

6. $(A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

1

$$7. \int \sqrt{3-2x-x^2} dx = \int \sqrt{2^2-(x+1)^2} dx \quad 1$$

$$= \frac{x+1}{2} \sqrt{3-2x-x^2} + 2 \sin^{-1} \left(\frac{x+1}{2} \right) + c \quad 1$$

$$8. \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \cdot \tan x + \operatorname{cosec} x \cdot \cot x) dx \quad 1$$

$$= \sec x - \operatorname{cosec} x + c \quad 1$$

OR

$$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{ (x-1)^{-2} - 2(x-1)^{-3} \} dx \quad 1$$

$$\left. \begin{aligned} &= e^x (x-1)^{-2} + c \\ &\text{or} \\ &\frac{e^x}{(x-1)^2} + c \end{aligned} \right\} \quad 1$$

9. Differentiating $y = Ae^{2x} + Be^{-2x}$, we get

$$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}, \text{ differentiate again to get,} \quad 1$$

$$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y \text{ or } \frac{d^2y}{dx^2} - 4y = 0 \quad 1$$

10. Let θ be the angle between $\vec{a}$ & $\vec{b}$, then

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} = \frac{7}{2.7} = \frac{1}{2} \quad 1$$

$$\Rightarrow \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \quad \frac{1}{2}$$

OR

$$(-3\hat{i} + 7\hat{j} + 5\hat{k}) \cdot \{ (-5\hat{i} + 7\hat{j} - 3\hat{k}) \times (7\hat{i} - 5\hat{j} - 3\hat{k}) \} = \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix} \quad 1$$

$$= -264 \quad \frac{1}{2}$$

$$\therefore \text{ Volume of cuboid} = 264 \text{ cubic units} \quad \frac{1}{2}$$

(2)

$$11. \quad P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$$

 $\frac{1}{2}$

$$P(A \cap B) = P(A) \cdot P(B|A) = 0.3 \times 0.5 = 0.15$$

 $\frac{1}{2}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70} \text{ or } \frac{3}{14}$$

1

$$12. \quad (i) \quad P(3 \text{ heads}) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{5}{16}$$

1

$$(ii) \quad P(\text{At most 3 heads}) = P(r \leq 3)$$

$$\left. \begin{aligned} &= 1 - P(4 \text{ heads or } 5 \text{ heads}) \\ &= 1 - \left\{ {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5 \left(\frac{1}{2}\right)^5 \right\} \\ &= \frac{26}{32} \text{ or } \frac{13}{16} \end{aligned} \right\}$$

1

OR

X = No. of heads in simultaneous toss of two coins.

$$X: \quad 0 \quad 1 \quad 2$$

 $\frac{1}{2}$

$$P(x): \quad 1/4 \quad 1/2 \quad 1/4$$

 $1 \frac{1}{2}$

SECTION C

$$13. \quad R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$$

For $1 \in A$, $(1, 1) \notin R \Rightarrow R$ is not reflexive

1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric

 $1 \frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow R$ is not transitive

 $1 \frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

$\therefore$ 'f' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f' is onto

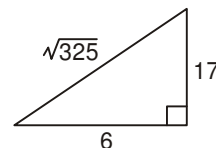
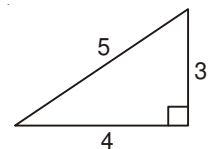
$\therefore$ f is invertible with, $f^{-1}: Y \rightarrow N$ and $f^{-1}(y) = \frac{y-3}{4}$ or $f^{-1}(x) = \frac{x-3}{4}$, $x \in Y$ 1

14. $\sin \left[\cos^{-1} \left(\frac{4}{5} \right) + \tan^{-1} \left(\frac{2}{3} \right) \right]$

$$= \sin \left[\tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{2}{3} \right) \right]$$

$$= \sin \left[\tan^{-1} \left(\frac{3/4 + 2/3}{1 - 3/4 \cdot 2/3} \right) \right] = \sin \left[\tan^{-1} \left(\frac{17}{6} \right) \right]$$

$$= \sin \left[\sin^{-1} \left(\frac{17}{\sqrt{325}} \right) \right] = \frac{17}{\sqrt{325}}$$



1

1 $\frac{1}{2}$ 1 $\frac{1}{2}$

15.
$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \left(\begin{array}{l} \text{By applying,} \\ C_1 \rightarrow C_1 + C_2 + C_3 \end{array} \right)$$

1

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \left\{ \begin{array}{l} \text{By applying,} \\ R_2 \rightarrow R_2 - R_1; \\ R_3 \rightarrow R_3 - R_1 \end{array} \right.$$

2

$$= (a+b+c) \{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$$

$$= 3(a+b+c) (ab + bc + ca)$$

1

$$16. \quad x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x\sqrt{1+y} = -y\sqrt{1+x} \quad \frac{1}{2}$$

$$\text{Squaring to get: } x^2(1+y) = y^2(1+x) \quad \frac{1}{2}$$

$$\text{Simplifying to get: } (x-y)(x+y+xy) = 0 \quad 1$$

$$\text{As, } x \neq y \quad \therefore y = -\frac{x}{1+x} \quad 1$$

Differentiating w.r.t. 'x', we get:

$$\frac{dy}{dx} = \frac{-1(1+x) - (-x) \cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2} \quad 1$$

OR

$$(\cos x)^y = (\sin y)^x \Rightarrow y \cdot \log(\cos x) = x \cdot \log(\sin y) \quad 1$$

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} \cdot \log(\cos x) + y(-\tan x) = \log(\sin y) + x \cdot \cot y \cdot \frac{dy}{dx} \quad 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \cdot \tan x + \log(\sin y)}{\log(\cos x) - x \cot y} \quad 1$$

$$17. \quad (x-a)^2 + (y-b)^2 = c^2, \quad c > 0$$

Differentiating both sides with respect to 'x', we get

$$2(x-a) + 2(y-b) \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x-a}{y-b} \quad 1 \frac{1}{2}$$

Differentiating again with respect to 'x', we get;

$$\frac{d^2y}{dx^2} = -\frac{(y-b) - (x-a) \cdot \frac{dy}{dx}}{(y-b)^2} = \frac{-c^2}{(y-b)^3} \quad (\text{By substituting } \frac{dy}{dx}) \quad 1 + \frac{1}{2}$$

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c \quad 1$$

Which is a constant independent of 'a' & 'b'.

18. Let (α, β) be the point on the curve where normal

passes through $(-1, 4) \therefore \alpha^2 = 4\beta$, also $\frac{dy}{dx} = \frac{x}{2}$

$$\text{Slope of normal at } (\alpha, \beta) = \frac{-1}{\left[\frac{dy}{dx}\right]_{(\alpha, \beta)}} = \frac{-1}{\frac{\alpha}{2}} = \frac{-2}{\alpha} \quad 1$$

$$\text{Equation of normal: } y - 4 = \frac{-2}{\alpha}(x + 1) \quad \frac{1}{2}$$

$$(\alpha, \beta) \text{ lies on the normal} \Rightarrow \beta - 4 = \frac{-2}{\alpha}(\alpha + 1)$$

$$\text{Putting } \beta = \frac{\alpha^2}{4}, \text{ we get; } \alpha^3 - 8\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha^2 + 2\alpha - 4) = 0 \quad 1$$

$$\text{For } \alpha = 2 \text{ Equation of normal is: } x + y - 3 = 0 \quad 1$$

$$\text{For } \alpha = \pm\sqrt{5} - 1; \text{ Equation of normal is: } y - 4 = \frac{-2}{\pm\sqrt{5} - 1}(x + 1) \quad \frac{1}{2}$$

$$19. \int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} dx = \frac{3}{5} \int \frac{1}{x + 2} dx + \frac{1}{5} \int \frac{2x}{x^2 + 1} dx + \frac{1}{5} \int \frac{1}{x^2 + 1} dx \quad 2$$

$$= \frac{3}{5} \log |x + 2| + \frac{1}{5} \log |x^2 + 1| + \frac{1}{5} \tan^{-1} x + c \quad 2$$

$$20. \left. \begin{aligned} \int_0^a f(x) dx &= - \int_a^0 f(a - t) dx \quad \text{Put } x = a - t, dx = -dt \\ \text{Upper limit} &= t = a - x = a - a = 0 \\ \text{Lower limit} &= t = a - x = a - 0 = a \\ &= \int_0^a f(a - t) dt = \int_0^a f(a - x) dx \end{aligned} \right\} \quad 1$$

$$\text{Let } I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx \quad \therefore I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x} dx \quad \dots(ii) \quad 1$$

Adding, (i) and (ii) we get

$$2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx = \frac{\pi}{2\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$$

$$= \frac{\pi}{2\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx \quad \frac{1}{2}$$

$$\Rightarrow 2I = \frac{\pi}{2\sqrt{2}} \left\{ \log \left| \sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right) \right| \right\}_0^{\pi/2} \quad \frac{1}{2}$$

$$= \frac{\pi}{2\sqrt{2}} \{ \log(\sqrt{2} + 1) - \log(\sqrt{2} - 1) \} \quad \frac{1}{2}$$

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}} \{ \log(\sqrt{2} + 1) - \log(\sqrt{2} - 1) \} \text{ or } \frac{\pi}{4\sqrt{2}} \log\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right) \quad \frac{1}{2}$$

21. The given differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right) \quad \frac{1}{2}$$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$, to get 1

$$v + x \frac{dv}{dx} = v - \tan v \Rightarrow x \frac{dv}{dx} = -\tan v$$

$$\Rightarrow \cot v \, dv = -\frac{1}{x} dx, \quad 1$$

Integrating both sides we get,

$$\left. \begin{aligned} \log |\sin v| &= -\log |x| + \log c \\ \Rightarrow \log |\sin v| &= \log \left| \frac{c}{x} \right| \end{aligned} \right\} \quad 1$$

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x \cdot \sin\left(\frac{y}{x}\right) = c \quad \frac{1}{2}$$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1 + \sin x} \cdot y = \frac{-x}{1 + \sin x}; \quad 1 \frac{1}{2}$$

$$\text{I.F.} = e^{\int \frac{\cos x}{1 + \sin x} dx} = e^{\log(1 + \sin x)} = 1 + \sin x \quad 1$$

$\therefore$ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1 + \sin x} \times (1 + \sin x) dx + c \quad 1$$

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1 + \sin x)} + \frac{c}{1 + \sin x} \quad \frac{1}{2}$$

22. $\vec{a} \cdot \frac{(\vec{b} + \vec{c})}{|\vec{b} + \vec{c}|} = 1 \quad 1$

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2 + \lambda)^2 + 36 + 4} \quad 1 \frac{1}{2}$$

$$\Rightarrow \lambda + 6 = \sqrt{(2 + \lambda)^2 + 40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1 \quad \frac{1}{2}$$

$\therefore$ Unit vector along $(\vec{b} + \vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k} \quad 1$

23. Lines are perpendicular

$$\therefore -3(3\lambda) + 2\lambda(2) + 2(-5) = 0 \Rightarrow \lambda = -2 \quad 2$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix} = -63 \neq 0 \quad 1 \frac{1}{2}$$

$\therefore$ Lines are not intersecting $\frac{1}{2}$

SECTION D

$$24. \quad |A| = 11; \text{Adj } (A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix} \quad 1+2$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A = \frac{1}{11} \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix} \quad \frac{1}{2}$$

$$\text{Taking; } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

The system of equations in matrix form is

$$A \cdot X = B \quad \therefore X = A^{-1} \cdot B \quad 1$$

$\therefore$ Solution is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad 1$$

$$\therefore x = 1, y = 1, z = 1 \quad \frac{1}{2}$$

OR

We know: $A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad 1$$

$$\begin{aligned}
& R_2 \rightarrow R_2 - 2R_1 \\
\Rightarrow & \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \\
& R_1 \leftrightarrow R_2 \\
\Rightarrow & \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \\
& R_2 \rightarrow R_2 - 2R_1 \\
\Rightarrow & \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \\
& R_2 \rightarrow R_2 + 2R_3 \\
\Rightarrow & \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} A \\
& R_2 \leftrightarrow R_3 \\
\Rightarrow & \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A \\
& R_2 \rightarrow R_2 - 3R_3 \\
\Rightarrow & \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \\
& R_1 \rightarrow R_1 - R_2 - 2R_3 \\
\Rightarrow & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A
\end{aligned}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

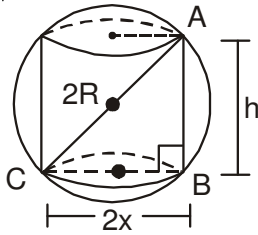
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25.

Correct Figure

1



$$\text{In rt. } \triangle ABC; 4x^2 + h^2 = 4R^2, x^2 = \frac{4R^2 - h^2}{4}$$

1

$$V(\text{Volume of cylinder}) = \pi x^2 h = \frac{\pi}{4}(4R^2 h - h^3)$$

1

$$V'(h) = \frac{\pi}{4}(4R^2 - 3h^2); V''(h) = \frac{\pi}{4}(-6h)$$

 $\frac{1}{2} + \frac{1}{2}$

$$V'(h) = 0 \Rightarrow h = \frac{2R}{\sqrt{3}}$$

1

$$V''\left(\frac{2R}{\sqrt{3}}\right) = \frac{-6\pi}{4}\left(\frac{2R}{\sqrt{3}}\right) < 0 \Rightarrow \text{Volume 'V' is max.}$$

 $\frac{1}{2}$

$$\text{for } h = \frac{2R}{\sqrt{3}}$$

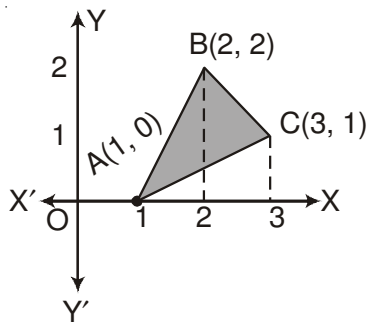
$$\text{Max. Volume: } V = \frac{4}{3\sqrt{3}}\pi R^3$$

 $\frac{1}{2}$

26.

Correct Figure

1



$$\left. \begin{array}{l} \text{Equation of line AB: } y = 2(x - 1) \\ \text{Equation of line BC: } y = 4 - x \\ \text{Equation of line AC: } y = \frac{1}{2}(x - 1) \end{array} \right\}$$

 $1 \frac{1}{2}$

$$\text{ar}(\triangle ABC) = 2 \int_1^2 (x - 1) dx + \int_2^3 (4 - x) dx - \frac{1}{2} \int_1^3 (x - 1) dx$$

 $1 \frac{1}{2}$

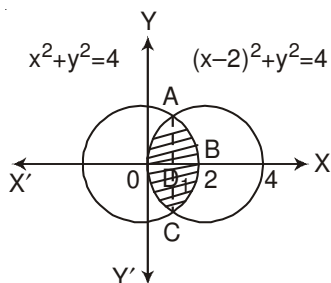
$$= (x - 1)^2 \Big|_1^2 - \frac{1}{2} (4 - x)^2 \Big|_2^3 - \frac{1}{4} (x - 1)^2 \Big|_1^3$$

 $1 \frac{1}{2}$

$$= 1 + \frac{3}{2} - 1 = \frac{3}{2}$$

 $\frac{1}{2}$

OR



Correct Figure

1

Getting the point of intersection as $x = 1$

1

Area (OABCO) = $4 \times \text{ar}(\text{ABD})$

$$= 4 \int_1^2 \sqrt{2^2 - x^2} dx$$

2

$$= 4 \left\{ \frac{x\sqrt{4-x^2}}{2} + 2 \sin^{-1} \left(\frac{x}{2} \right) \right\}_1^2$$

1

$$= \left(\frac{8\pi}{3} - 2\sqrt{3} \right)$$

1

27. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$$(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$$

2

Vector equation of plane is:

$$\{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$$

$$\Rightarrow \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$$

1

Cartesian Equation of plane is: $-9x - 3y + z + 14 = 0$

1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$$-9(x - 2) - 3(y - 3) + (z - 7) = 0$$

$$\Rightarrow -9x - 3y + z + 20 = 0$$

1

$$\text{Distance between parallel planes} = \left| \frac{-14 + 20}{\sqrt{91}} \right| = \frac{6}{\sqrt{91}}$$

1

OR

$$\text{Equation of line: } \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} \quad 1$$

$$\text{Equation of plane: } \begin{vmatrix} x-2 & y & z-3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0 \quad 1$$

$$\Rightarrow -3(x-2) + 3y - 3(z-3) = 0$$

$$\Rightarrow x - y + z - 5 = 0 \quad 1$$

$$\text{General point on line: } \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$$

$$\text{is: } P(3k+2, 4k-1, 2k+2); \text{ Putting in the equation of plane} \quad 1$$

$$\text{we get, } 3k+2-4k+1+2k+2=5 \Rightarrow k=0 \quad 1$$

$$\therefore \text{ Point of intersection is: } (2, -1, 2) \quad 1$$

$$\begin{array}{l} 28. \text{ Let } E_1 = \text{Event that two-headed coin is chosen} \\ \quad E_2 = \text{Event that biased coin is chosen} \\ \quad E_3 = \text{Event that unbiased coin is chosen} \\ \quad A = \text{Event that coin tossed shows head} \end{array} \quad 1$$

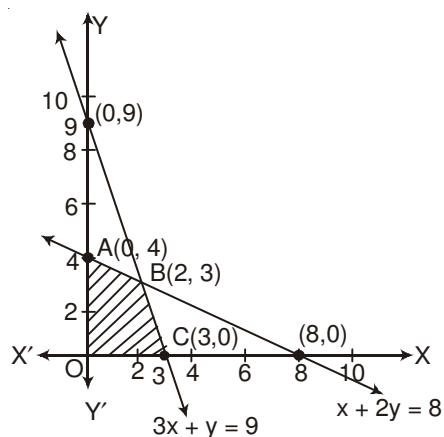
$$\text{Then, } P(E_1) = P(E_2) = P(E_3) = 1/3 \quad 1$$

$$P(A|E_1) = 1, P(A|E_2) = \frac{75}{100} = \frac{3}{4}, P(A|E_3) = \frac{1}{2} \quad 1 \frac{1}{2}$$

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{4}{9} \quad 2 + \frac{1}{2}$$

29.



let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:

Maximize profit: $z = 40x + 50y$ (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left. \begin{array}{l} 3x + y \leq 9 \\ x + 2y \leq 8 \\ x, y \geq 0 \end{array} \right\}$$

 $2\frac{1}{2}$

Correct graph:

2

Corner point

Value of z (₹)

A(0, 4)

200

B(2, 3)

230 (Max)

 $\frac{1}{2}$

C(3, 0)

120

 $\therefore$ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units

 $\frac{1}{2}$