QUESTION PAPER CODE 65/3/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \cdot 4 = -32$ $\frac{1}{2}$

2. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2}+\frac{1}{2}$

3. order 4, degree 2 $\frac{1}{2}+\frac{1}{2}$

4. $\sqrt{(-18)^2+(12)^2+(-4)^2} = 22$ $\frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let $e \in \mathbb{R}$ be the identity element.

then $a*e = e*a = a$ 1

$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1

$\therefore$ Identity element is $0 \in \mathbb{R}$

6. $kA = k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

$\therefore \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \Rightarrow 2k = 3a, 3k = 2b \text{ and } -4k = 24$ $\frac{1}{2}$

$\Rightarrow k = -6,\ a = \frac{-12}{3} = -4,\ b = \frac{-18}{2} = -9$ $1\frac{1}{2}$

7. $\int \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ 1

$= -\log|\sin x + \cos x| + c$ 1

8. $\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$$= \int\left[\frac{\sin(x+a)\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$$ $\frac{1}{2}$

$$= x\cdot\cos 2a - \sin 2a\cdot\log|\sin(x+a)| + c$$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int(\log x)^2\cdot 1dx = (\log x)^2\cdot x - \int 2\cdot \log x\cdot\frac{1}{x}\cdot x\, dx$ 1

$$= x\cdot(\log x)^2 - \left\{\log x\cdot 2x - \int\frac{1}{x}\cdot 2xdx\right\}$$ $\frac{1}{2}$

$$= x(\log x)^2 - 2x\log x + 2x + c$$ $\frac{1}{2}$

9. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

10. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a}\times\vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j}+\hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a}\cdot\vec{b}\times\vec{c} = 0$ $\frac{1}{2}$

$$\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3)$$

$$= 3 - 12 + 9 = 0$$ $1+\frac{1}{2}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

11. A = {(S, F, M), (S, M, F), (M, F, S), (F, M, S)}
B = {(S, F, M), (M, F, S)} 1

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$ 1

12. Given $2\,P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}$ and $P(X = x_4) = \frac{k}{5}$ $\frac{1}{2}$

$$\therefore\ k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$ 1

SECTION C

13. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let (a, b) $\in$ R for a, b $\in$ Z, then a – b is divisible by 2

$\Rightarrow$ (b – a) is also divisible by 2

$\therefore$ (b, a) $\in$ R $\Rightarrow$ R is symmetric ...(ii) 1

For a, b, c, $\in$ Z, Let (a, b) $\in$ R and (b, c) $\in$ R

$\therefore$ a – b = 2p, p $\in$ Z, and b – c = 2q, q $\in$ Z,

adding, a – c = 2 (p + q) $\Rightarrow$ (a – c) is divisible by 2

$\Rightarrow$ (a, c) $\in$ R, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$$\text{fof}(x) = f\left(\frac{4x+3}{6x-4}\right)$$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow \text{fof}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x $\Rightarrow$ fof = I $\Rightarrow$ $f^{-1} = f$ 1

14. Given $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$

$$\Rightarrow \tan^{-1}x - \left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{\pi}{6}$$ 1

$$\Rightarrow 2\tan^{-1}x = \frac{2\pi}{3} \Rightarrow \tan^{-1}x = \frac{\pi}{3}$$ 1

$$\Rightarrow x = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \qquad \therefore \sec^{-1}\frac{2}{x} = \sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$$ 1+1

15. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$R_1 \to R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ 1

$\therefore \Delta = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b}\begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix}$ $1+\frac{1}{2}$

$C_2 \to C_2 - cC_3$

$\Rightarrow \Delta = -\frac{2}{b}\begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix}$ 1

$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc.$ $\frac{1}{2}$

16. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log \sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1 - y(x+y)\cot x}{(x+y)\log \sin x - 1}$$ $\frac{1}{2}$

17. $y = (\sec^{-1} x)^2, x > 0$

$$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2 - 1}}$$ 1

$$\Rightarrow x\sqrt{x^2 - 1}\frac{dy}{dx} = 2\sqrt{y}$$ $\frac{1}{2}$

squaring both sides, we get

$$x^2(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y \quad \text{or} \quad (x^4 - x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$ $\frac{1}{2}$

differentiating w.r.t. x.

$$(x^4 - x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3 - 2x)\left(\frac{dy}{dx}\right)^2 = 4 \cdot \frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$$ $\frac{1}{2}$

18. Curve $y = \frac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{(x^2 - 5x + 6) \cdot 1 - (x-7)(2x-5)}{(x^2 - 5x + 6)^2}$$ $\frac{1}{2}$

at (7, 0) $\frac{dy}{dx} = \frac{20}{(20)^2} = \frac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\frac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \frac{1}{20}(x - 7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x - 7)$

or $20x + y = 140$. $\frac{1}{2}$

19. $I = \int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\, dx = dt$ $\frac{1}{2}$

$\therefore I = \int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3} \right) dt$ $1\frac{1}{2}$

$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \frac{1}{2}\log(\sin^2 x + 1) - \frac{1}{2}\log(\sin^2 + 3) + c.$ $\frac{1}{2}$

20. RHS $= \int_a^b f(a+b-x)\, dx = -\int_b^a f(t)\, dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \int_a^b f(t)\, dt = \int_a^b f(x)\, dx = \text{LHS}$ $\frac{1}{2}$

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ ...(i) $\frac{1}{2}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

21. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\ dx = \cot x.dx.$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$ 1

I.F. = $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ 1

$\therefore$ Solution is, $y\cdot(1+x^2) = \int \cot x\ dx = \log|\sin x| + c$ 1+1

or $y = \frac{1}{1+x^2}\cdot\log|\sin x| + \frac{c}{1+x^2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$ $\therefore\ \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c} = 0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\,\vec{a}\cdot\vec{b} - 8\,\vec{b}\cdot\vec{c} + 12\,\vec{a}\cdot\vec{c}$ 1

$= 9(1)^2 + 4(2)^2 + 4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a}-2\vec{b}+2\vec{c}| = \sqrt{61}$ 1

23. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$\Rightarrow (5\lambda+2)\cdot 1 + (-5)(2\lambda) + 1(3) = 0$ $\frac{1}{2}$

$-5\lambda + 5 = 0 \Rightarrow \lambda = 1$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7} = \frac{y-2}{-5} - \frac{z-1}{1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $= \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|} = \frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$= \frac{135}{|\vec{b}_1\times\vec{b}_2|} \neq 0$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 1(7) - 1(-3) + 1(-1) = 9$ 1

$$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ 2

$$\Rightarrow A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ $\frac{1}{2}$

Given equations can be written as $\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\11\\0\end{bmatrix}$

or $AX = B \Rightarrow X = A^{-1}B$ $\frac{1}{2}$

$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}\begin{bmatrix}6\\11\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$ $1\frac{1}{2}$

$\therefore\ x = 1,\ y = 2,\ z = 3$ $\frac{1}{2}$

OR

Let: $\begin{bmatrix}2 & 3 & 1\\2 & 4 & 1\\3 & 7 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A$ 1

$R_1 \leftrightarrow R_3 \begin{bmatrix}3 & 7 & 2\\2 & 4 & 1\\2 & 3 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix}A$

$\begin{matrix}R_1 \to R_1 - R_3\\R_2 \to R_2 - R_3\end{matrix} \begin{bmatrix}1 & 4 & 1\\0 & 1 & 0\\2 & 3 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\1 & 0 & 0\end{bmatrix}A$

$R_3 \to R_3 - 2R_1 \begin{bmatrix}1 & 4 & 1\\0 & 1 & 0\\0 & -5 & -1\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\3 & 0 & -2\end{bmatrix}A$

$R_3 \to R_3 + 5R_2 \begin{bmatrix}1 & 4 & 1\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\-2 & 5 & -2\end{bmatrix}A$ 4

$\begin{matrix}R_1 \to R_1 - 4R_2\\R_3 \to -R_3\end{matrix} \begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & -4 & 1\\-1 & 1 & 0\\2 & -5 & 2\end{bmatrix}A$

$R_1 \to R_1 - R_3 \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & -1\\-1 & 1 & 0\\2 & -5 & 2\end{bmatrix}A$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$ 1

25. Let Given surface area of open cylinder be S.

Then $S = 2\pi rh + \pi r^2$

$$\Rightarrow h = \frac{S - \pi r^2}{2\pi r}$$ 1

Volume $V = \pi r^2 h$ $\frac{1}{2}$

$$V = \pi r^2 \left[\frac{S - \pi r^2}{2\pi r}\right] = \frac{1}{2}[Sr - \pi r^3]$$ 1

$$\frac{dV}{dr} = \frac{1}{2}[S - 3\pi r^2]$$ $\frac{1}{2}$

$$\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2 \text{ or } 2\pi rh + \pi r^2 = 3\pi r^2$$ 1

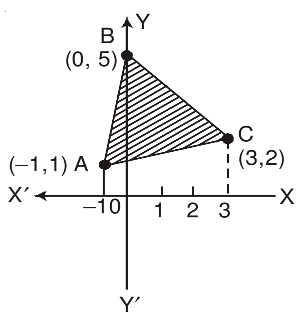
$\Rightarrow 2\pi rh = 2\pi r^2$ $\Rightarrow h = r$ 1

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$ 1

$\therefore$ For volume to be maximum, height = radius

26. Let the points be A (–1, 1), B (0, 5) and C (3, 2)

Correct Figure 1



Equation of $AB : y = 4x + 5$

$BC : y = 5 - x$

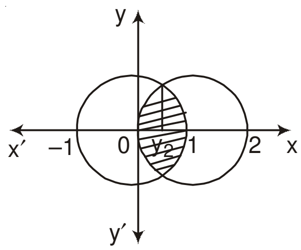
$AC : y = \frac{1}{4}(x + 5)$ $1\frac{1}{2}$

$$\text{Req. Area} = \int_{-1}^{0}(4x+5)\,dx + \int_{0}^{3}(5-x)\,dx - \int_{-1}^{3}\frac{1}{4}(x+5)\,dx$$ $1\frac{1}{2}$

$$\therefore A = \left[\frac{(4x+5)^2}{8}\right]_{-1}^{0} + \left[\frac{(5-x)^2}{-2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{(x+5)^2}{2}\right]_{-1}^{3}$$ 1

$$= 3+\frac{21}{2}-6=\frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2+y^2 = 1$$

and $x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$

$$\Rightarrow x = \frac{1}{2}$$ 1

$$\therefore \text{ Required area } = 2\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx\right]$$ 2

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+2\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$= 2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]+2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=0$$ 1

$$\Rightarrow 16(x-2)+24(y-5)+32(z+3)=0$$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$$\frac{x-3}{4}=\frac{y-1}{4}=\frac{z-5}{6} \text{ or } \frac{x-3}{2}=\frac{y-1}{2}=\frac{z-5}{3}$$ ...(ii) 1

Any point on (ii) is $(2\lambda+3, 2\lambda+1, 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$, is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is ∥ to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

28. Let the events be

E_1 : bag I is selected
E_2 : bag II is selected
A : getting a red ball
} 1

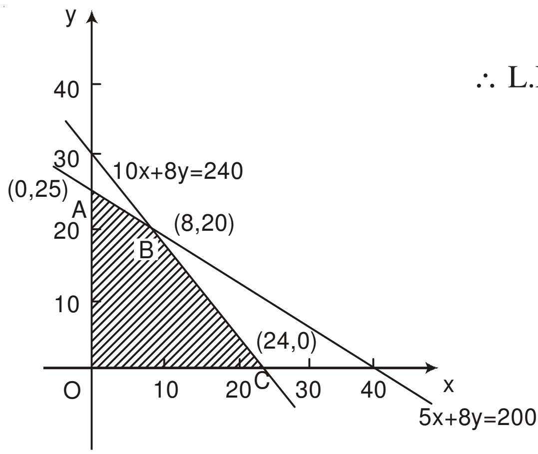
$P(E_1)=P(E_2)=\frac{1}{2}$ $\frac{1}{2}$

$P(A/E_1)=\frac{3}{9}=\frac{1}{3}$; $P(A/E_2)=\frac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$ 2

$$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that $5x + 8y \leq 200$
$10x + 8y \leq 240$
$x, y \geq 0$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20. 1