EXERCISE 10(A)

1. In the figure alongside, 
   \( \text{AB} = \text{AC} \)
   \( \angle A = 48^0 \) and
   \( \angle ACD = 18^0 \)
   Show that: \( \text{BC} = \text{CD} \).

Solution:

In \( \triangle ABC \),

\[ \angle BAC + \angle ACB + \angle ABC = 180^0 \]
\[ 48^0 + \angle ACB + \angle ABC = 180^0 \]
But \( \angle ACB = \angle ABC \) [\( \text{AB} = \text{AC} \)]
\[ 2 \angle ABC = 180^0 - 48^0 \]
\[ 2 \angle ABC = 132^0 \]
\[ \angle ABC = 66^0 = \angle ACB \ldots \ldots \ (i) \]

\[ \angle ACB = 66^0 \]
\[ \angle ACD + \angle DCB = 66^0 \]
\[ 18^0 + \angle DCB = 66^0 \]
\[ \angle DCB = 48^0 \ldots \ldots \ (ii) \]

Now, In \( \triangle DCB \),

\[ \angle DBC = 66^0 \] [From (i)], Since \( \angle ABC = \angle DBC \)
\[ \angle DCB = 48^0 \] [From (ii)]
\[ \angle BDC = 180^0 - 48^0 - 66^0 \]
\[ \angle BDC = 66^0 \]
Since \( \angle BDC = \angle DBC \)
Hence, \( \text{BC} = \text{CD} \)
Equal angles have equal sides opposite to them.
2. Calculate:
   (i) \( \angle ADC \)
   (ii) \( \angle ABC \)
   (iii) \( \angle BAC \)

Solution:
Given: \( \angle ACE = 130^\circ \); \( AD = BD = CD \)

Proof:
(i)
\[
\angle ACD + \angle ACE = 130^\circ \quad \text{[DCE is a straight line]}
\]
\[
\Rightarrow \angle ACD = 180^\circ - 130^\circ
\]
\[
\Rightarrow \angle ACD = 50^\circ
\]
Now, \( CD = AD \)
\[
\Rightarrow \angle ACD = \angle DAC = 50^\circ \quad \text{(i)}
\]

[Since angles opposite to equal sides are equal]

In \( \triangle ADC \),
\[
\angle ACD = \angle DAC = 50^\circ
\]
\[
\angle ACD + \angle DAC + \angle ADC = 180^\circ
\]
\[
50^\circ + 50^\circ + \angle ADC = 180^\circ
\]
\[
\angle ADC = 180^\circ - 100^\circ
\]
\[
\angle ADC = 80^\circ
\]

(ii)
\[
\angle ADC = \angle ABD + \angle DAB \quad \text{[Exterior angle is equal to sum of opp. interior angles]}
\]

But \( AD = BD \)
\[
\therefore \angle DAB = \angle ABD
\]
\[
\Rightarrow 80^\circ = \angle ABD + \angle ABD
\]
\[
\Rightarrow 2\angle BD = 80^\circ
\]
\[
\Rightarrow \angle ABD = 40^\circ = \angle DAB \quad \text{(ii)}
\]

(iii)
\[
\angle BAC = \angle DAB - \angle DAC
\]

Substituting the values from (i) and (ii)
\[
\angle BAC = 40^\circ + 50^\circ
\]
\[
\Rightarrow \angle BAC = 90^\circ
\]
3. In the following figure, AB = AC; BC = CD and DE is parallel to BC. Calculate:
   (i) \( \angle CDE \)
   (ii) \( \angle DCE \)

Solution:

\[ \angle FAB = 128^\circ \quad [\text{Given}] \]
\[ \angle BAC + \angle FAB = 180^\circ \quad [\text{FAC is a st. line}] \]
\[ \Rightarrow \angle BAC = 180^\circ - 128^\circ \]
\[ \Rightarrow \angle BAC = 52^\circ \]

In \( \triangle ABC \),
\[ \angle A = 52^\circ \]
\[ \angle B = \angle C \quad [\text{Given } AB = AC \text{ and angles opposite to equal sides are equal}] \]
\[ \angle A + \angle B + \angle C = 180^\circ \]
\[ \Rightarrow \angle A + \angle B + \angle B = 180^\circ \]
\[ \Rightarrow 52^\circ + 2\angle B = 180^\circ \]
\[ \Rightarrow 2\angle B = 128^\circ \]
\[ \Rightarrow \angle B = 64^\circ = \angle C \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \:\ (i) \]
\[ \angle B = \angle ADE \quad [\text{Given } DE \parallel EC] \]

(i)
Now,
\[ \angle ADE + \angle CDE + \angle B = 180^\circ \quad [\text{ADB is a st. line}] \]
\[ \Rightarrow 64^\circ + \angle CDE + 64^\circ = 180^\circ \]
\[ \Rightarrow \angle CDE = 180^\circ - 128^\circ \]
\[ \Rightarrow \angle CDE = 52^\circ \]
(ii)
Given $DE \parallel BC$ and $DC$ is the transversal.

$\Rightarrow \angle CDE = \angle DCB = 52^\circ$ \ldots \ldots \text{(i)}

Also, $\angle ECB = 64^\circ$ \ldots \ldots \text{[From (i)]}

But,

$\angle ECB = \angle DCE + \angle DCB$

$\Rightarrow 64^\circ = \angle DCE + 52^\circ$

$\Rightarrow \angle DCE = 64^\circ - 52^\circ$

$\Rightarrow \angle DCE = 12^\circ$

4. Calculate $x$:

(i) \hspace{1cm}

(ii)

Solution:

(i) Let the triangle be $ABC$ and the altitude be $AD$. 

(ii) 

https://byjus.com
In \( \triangle ABD \),
\[
\angle DBA = \angle DAE = 37^\circ \quad \text{[Given \( BD = AD \) and angles opposite to equal sides are equal]}
\]

Now,
\[
\angle CDA = \angle DBA + \angle DAB \quad \text{[Exterior angle is equal to the sum of opp. interior angles]}
\]
\[
\therefore \angle CDA = 37^\circ + 37^\circ \\
\Rightarrow \angle CDA = 74^\circ
\]
Now in \( \triangle ADC \),
\[
\angle CDA = \angle CAD = 74^\circ \quad \text{[Given \( CD = AC \) and angles opposite to equal sides are equal]}
\]

Now,
\[
\angle CAD + \angle CDA + \angle ACD = 180^\circ \\
\Rightarrow 74^\circ + 74^\circ + x = 180^\circ \\
\Rightarrow x = 180^\circ - 148^\circ \\
\Rightarrow x = 32^\circ
\]

(ii) Let triangle be \( ABC \) and altitude be \( AD \).

![Diagram of triangle ABC with AD as altitude]

In \( \triangle ABD \),
\[
\angle DBA = \angle DAE = 50^\circ \quad \text{[Given \( BD = AD \) and angles opposite to equal sides are equal]}
\]

Now,
\[
\angle CDA = \angle DBA + \angle DAB \quad \text{[Exterior angle is equal to the sum of opp. interior angles]}
\]
\[
\therefore \angle CDA = 50^\circ + 50^\circ \\
\Rightarrow \angle CDA = 100^\circ
\]
5. In the figure, given below, AB = AC.
Prove that: $\angle BOC = \angle ACD$

Solution:

Let $\angle ABO = \angle OBC = x$ and $\angle ACO = \angle OCB = y$

In $\triangle ABC$,
$\angle BAC = 180^\circ - 2x - 2y \ldots \ldots (i)$

Since $\angle B = \angle C \quad [AB = AC]$

$\frac{1}{2}B = \frac{1}{2}C$

$\Rightarrow x = y$

Now,
$\angle ACD = 2x + \angle BAC \quad [\text{Exterior angle is equal to sum of opp. interior angles}]$

$\quad = 2x + 180^\circ - 2x - 2y \quad [\text{From (i)}]$

$\angle ACD = 180^\circ - 2y \ldots \ldots (ii)$
In \( \triangle OBC \),
\[ \angle BOC = 180^\circ - x - y \]
\[ \Rightarrow \angle BOC = 180^\circ - y - y \quad \text{[Already proved]} \]
\[ \Rightarrow \angle BOC = 180^\circ - 2y \quad \text{(iii)} \]

From (i) and (ii)
\[ \angle BOC = \angle ACD \]

6. In the figure given below, \( LM = LN \); angle \( PLN = 110^\circ \). Calculate:
   (i) \( \angle LMN \)
   (ii) \( \angle MLN \)

   \[ \text{Solution:} \]
   
   Given: \( \angle PLN = 110^\circ \)
   (i) We know that the sum of the measure of all the angles of a quadrilateral is \( 360^\circ \).
   In quad. PQNL,
   \[ \angle QPL + \angle PLN + \angle LNQ + \angle NQP = 360^\circ \]
   \[ \Rightarrow 90^\circ + 110^\circ + \angle LNQ + 90^\circ = 360^\circ \]
   \[ \Rightarrow \angle LNQ = 360^\circ - 290^\circ \]
   \[ \Rightarrow \angle LNQ = 70^\circ \]
   \[ \Rightarrow \angle LNM = 70^\circ \ldots \ldots \text{(i)} \]

   In \( \triangle LMN \),
   \( LM = LN \) \quad \text{[Given]}
   \[ \therefore \angle LNM = \angle LMN \quad \text{[angles opp. to equal sides are equal]} \]
   \[ \Rightarrow \angle LMN = 70^\circ \ldots \ldots \text{(i)} \quad \text{[From (i)]} \]

(ii)
In $\triangle LMN$,
$\angle LMN + \angle LNM + \angle MLN = 180^\circ$

But, $\angle LNM = \angle LMN = 70^\circ$  \[\text{[From (i) and (ii)]}\]
$\therefore 70^\circ + 70^\circ + \angle MLN = 180^\circ$
$\Rightarrow \angle MLN = 180^\circ - 140^\circ$
$\Rightarrow \angle MLN = 40^\circ$

7. An isosceles triangle $ABC$ has $AC=BC$. CD bisects $AB$ at $D$ and $\angle CAB=55^\circ$.
Find:
(i) $\angle DCB$
(ii) $\angle CBD$.

Solution:

In $\triangle ABC$,
$AC = BC$  \[\text{[Given]}\]
$\therefore \angle CAB = \angle CBD$  \[\text{[angles opp. to equal sides are equal]}\]
$\Rightarrow \angle CBD = 55^\circ$

In $\triangle ABC$,
$\angle CBA + \angle CAB + \angle ACB = 180^\circ$
but, $\angle CAB = \angle CBA = 55^\circ$
$\Rightarrow 55^\circ + 55^\circ + \angle ACE = 180^\circ$
$\Rightarrow \angle ACB = 180^\circ - 110^\circ$
$\Rightarrow \angle ACB = 70^\circ$
Now,
In \( \triangle ACD \) and \( \triangle BCD \),
\[ AC = BC \quad [\text{Given}] \]
\[ CD = CD \quad [\text{Common}] \]
\[ AD = BD \quad [\text{Given: CD bisects AB}] \]

\[ \therefore \triangle ACD \cong \triangle BCD \]
\[ \Rightarrow \angle DCA = \angle DCB \]
\[ \Rightarrow \angle DCB = \frac{\angle ACB}{2} - \frac{70^\circ}{2} \]
\[ \Rightarrow \angle DCB = 35^\circ \]

8. Find \( x \):

Solution:
Let us name the figure as following:

In \( \triangle ABC \),
\[ AD = AC \quad [\text{Given}] \]

\[ \therefore \angle ADC = \angle ACD \quad [\text{angles opposite to equal sides are equal}] \]
\[ \Rightarrow \angle ADC = 42^\circ \]
9. In the triangle ABC, BD bisects angle B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.

Solution:
In \(\triangle ABD\) and \(\triangle DBC\),

\[
\begin{align*}
BD &= BD & \text{[Common]} \\
\angle DBA &= \angle DBC & \text{[each equal to 90°]} \\
\angle ABD &= \angle DBC & \text{[BD bisects \(\angle ABC\)]} \\
\therefore \triangle ABD &\cong \triangle DBC & \text{[ASA criterion]} \\
\end{align*}
\]

Hence, 

We know that,
\[
\begin{align*}
\angle CBA &= 21° \\
\angle BCA &= 42° \\
\therefore x &= 21° + 42° \\
\Rightarrow x &= 63°
\end{align*}
\]
AD = DC
\(x + 1 = y + 2\)
\(\Rightarrow x = y + 1 \ldots (i)\)
and AB = BC
\(3x + 1 = 5y - 2\)
Substituting the value of x from (i)
\(3(y + 1) + 1 = 5y - 2\)
\(\Rightarrow 3y + 3 + 1 = 5y - 2\)
\(\Rightarrow 3y + 4 = 5y - 2\)
\(\Rightarrow 2y = 6\)
\(\Rightarrow y = 3\)
Putting \(y = 3\) in (i)
\(x = 3 + 1\)
\(\therefore x = 4\)

10. In the given figure; AE || BD, AC || ED and AB = AC. Find \(\angle a, \angle b\) and \(\angle c\).

Solution:
Let P and Q be the points as shown below:

Given: \(\angle PDQ = 58^\circ\)
\(\angle PDQ = \angle EDC = 58^\circ\) [Vertically opp. angles]
\(\angle EDC = \angle ACB = 58^\circ\) [Corresponding angles \(\because\) AC || ED]
11. In the following figure; AC=CD, AD=BD and $\angle C=58^\circ$.

In $\triangle ABC$,
$AB = AC$ [Given]
$\therefore \angle ACB = \angle ABC = 58^\circ$ [angles opp. to equal sides are equal]

Now,
$\angle ACB + \angle ABC + \angle BAC = 180^\circ$
$\Rightarrow 58^\circ + 58^\circ + b = 180^\circ$
$\Rightarrow a = 180^\circ - 116^\circ$
$\Rightarrow a = 64^\circ$

Since AE $||$ BD and AC is the transversal
$\angle ABC = b$ [Corresponding angles]
$\therefore b = 58^\circ$

Also since AE $||$ BD and ED is the transversal
$\angle EDC = c$ [Corresponding angles]
$\therefore c = 58^\circ$

Find $\angle CAB$.

**Solution:**

In $\triangle ACD$,
$AC = CD$ [Given]
$\therefore \angle CAD = \angle CDA$
$\angle ACD = 58^\circ$ [Given]

$\angle ACD + \angle CDA + \angle CAD = 180^\circ$
$\Rightarrow 58^\circ + 2\angle CAD = 180^\circ$
$\Rightarrow 2\angle CAD = 122^\circ$
$\Rightarrow \angle CAD = \angle CDA = 61^\circ$ [i]

Now,
\[ \angle CDA = \angle DAB + \angle DBA \quad \text{[Ext. angle is equal to sum of opp. int. angles]} \]

But,
\[ \angle DAB = \angle DBA \quad \text{[Given: } AD = DB \] \]
\[ \therefore \angle DAB + \angle DAB = \angle CDA \]
\[ \Rightarrow 2\angle DAB = 61^\circ \]
\[ \Rightarrow \angle DAB = 30.5^\circ \quad \ldots \ldots \quad \text{(i)} \]

In \( \triangle ABC \),
\[ \angle CAB = \angle CAD + \angle DAB \]
\[ \therefore \angle CAB = 61^\circ + 30.5^\circ \]
\[ \Rightarrow \angle AB = 91.5^\circ \]

12. In the figure of Q.no.11, given above, if \( AC = AD = CD = BD \); find \( \angle ABC \).

**Solution:**

In \( \triangle ACD \),
\[ AC = AD = CD \quad \text{[Given]} \]
Hence, \( ACD \) is an equilateral triangle
\[ \therefore \angle ACD = \angle CAD = \angle DAB = 60^\circ \]
\[ \angle CDA = \angle DAB + \angle ABD \quad \text{[Ext. angle is equal to sum of opp. int. angles]} \]

But,
\[ \angle DAB = \angle ABD \quad \text{[Given: } AD = DB \] \]
\[ \therefore \angle ABD + \angle ABD = \angle CDA \]
\[ \Rightarrow 2\angle ABD = 60^\circ \]
\[ \Rightarrow \angle ABD = \angle ABC = 30^\circ \]

13. In triangle \( ABC \); \( AB = AC \) and \( \angle A = \angle B = 85^\circ \); find \( \angle A \).

**Solution:**
Let $\angle A = 8x$ and $\angle E = 5x$

Given: $AB = AC$

$\Rightarrow \angle B = \angle C = 5x$ [Angles opp. to equal sides are equal]

Now,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 8x + 5x + 5x = 180^\circ$

$\Rightarrow 18x = 180^\circ$

$\Rightarrow x = 10^\circ$

Given that:

$\angle A = 8x$

$\Rightarrow \angle A = 8 \times 10^\circ$

$\Rightarrow \angle A = 80^\circ$

14. In triangle $ABC$; $\angle A=60^\circ$, $\angle C=40^\circ$ and bisector of $\angle ABC$ meets side $AC$ at point $P$. Show that $BP=CP$.

Solution:

In $\triangle ABC$,

$\angle A = 60^\circ$

$\angle C = 40^\circ$

$\therefore \angle B = 180^\circ - 60^\circ - 40^\circ$

$\Rightarrow \angle B = 80^\circ$

Now,

$BP$ is the bisector of $\angle ABC$

$\therefore \angle PBC = \frac{\angle ABC}{2}$

$\Rightarrow \angle PBC = 40^\circ$

In $\triangle PBC$

$\angle PBC = \angle PCB = 40^\circ$

$\therefore BP = CP$ [Sides opp. to equal angles are equal]
15. In triangle ABC; \( \angle ABC = 90^\circ \), and P is a point on AC such that \( \angle PBC = \angle PCB \). Show that PA = PB.

Solution:

Let \( \angle PBC = \angle PCB = x \)

In the right angled triangle ABC,
\( \angle ABC = 90^\circ \)
\( \angle ACB = x \)

\( \Rightarrow \angle EAC = 180^\circ - (90^\circ + x) \)
\( \Rightarrow \angle EAC = (90^\circ - x) \ldots \ldots \text{(i)} \)

And

\( \angle ABP = \angle ABC - \angle PBC \)
\( \Rightarrow \angle ABP = 90^\circ - x \ldots \ldots \text{(ii)} \)

Hence in the triangle ABP:
\( \angle BAP = \angle ABP \)
Hence,

\( PA = PB \) [sides opp. to equal angles are equal]

16. ABC is an equilateral triangle. Its side BC is produced upto point E such that C is midpoint of BE. Calculate the measure of angles ACE and AEC.

Solution:
17. In triangle ABC, D is a point in AB such that AC = CD = DB. If \( \angle B = 28^\circ \), find the angle ACD.

Solution:

\[
\Delta ABC \text{ is an equilateral triangle} \\
\Rightarrow \text{Side } AB = \text{Side } AC \\
\Rightarrow \angle ABC = \angle ACB \quad \text{[If two sides of a triangle are equal, then angles opposite to them are equal]} \\
\text{Similarly, Side } AC = \text{Side } BC \\
\Rightarrow \angle CAB = \angle ABC \quad \text{[If two sides of a triangle are equal, then angles opposite to them are equal]} \\
\text{Hence, } \angle ABC = \angle CAB = \angle ACB = \gamma \text{(say)} \\
\text{As the sum of all the angles of the triangle is } 180^\circ \\
\angle ABC + \angle CAB + \angle ACB = 180^\circ \\
\Rightarrow 3\gamma = 180^\circ \\
\Rightarrow \gamma = 60^\circ \\
\angle ABC = \angle CAB = \angle ACB = 60^\circ \\
\text{Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.} \\
\Rightarrow \angle CAB + \angle CBA = \angle ACE \\
\Rightarrow 60^\circ + 60^\circ = \angle ACE \\
\Rightarrow \angle ACE = 120^\circ \\
\text{Now } \Delta ACE \text{ is an isosceles triangle with } AC = CE \\
\Rightarrow \angle EAC = \angle AEC \\
\text{Sum of all the angles of a triangle is } 180^\circ \\
\angle EAC + \angle AEC + \angle ACE = 180^\circ \\
\Rightarrow 2\angle AEC + 120^\circ = 180^\circ \\
\Rightarrow 2\angle AEC = 180^\circ - 120^\circ \\
\Rightarrow \angle AEC = 30^\circ 
\]
18. In the given alongside figure, $AD=AB=AC$, $BD$ is parallel to $CA$ and $\angle ACB=65^\circ$. Find the $\angle DAC$. 

Solution:
Concise Selina Solutions for Class 9 Maths Chapter 10-Isosceles Triangle

We can see that the ΔABC is an isosceles triangle with Side AB = Side AC.
⇒ ∠ACB = ∠ABC
As ∠ACB = 65°
hence ∠ABC = 65°
Sum of all the angles of a triangle is 180°
∠ACB + ∠CAB + ∠ABC = 180°
65° + 65° + ∠CAB = 180°
∠CAB = 180° - 130°
∠CAB = 50°
As BD is parallel to CA
Therefore, ∠CAB = ∠DBA since they are alternate angles.
∠CAB = ∠DBA = 50°
We see that ΔADB is an isosceles triangle with Side AD = Side AB.
⇒ ∠ADB = ∠DBA = 50°
Sum of all the angles of a triangle is 180°
∠ADB + ∠DAB + ∠DBA = 180°
50° + ∠DAB + 50° = 180°
∠DAB = 180° - 100° = 80°
∠DAB = 80°
The angle DAC is sum of angle DAB and CAB.
∠DAC = ∠CAB + ∠DAB
∠DAC = 50° + 80°
∠DAC = 130°

19. Prove that a triangle ABC is isosceles, if:
   (i) Altitude AD bisects ∠BAC, or
   (ii) Bisector of ∠BAC is perpendicular to base BC.

Solution:
(i) In ΔABC, let the altitude AD bisects ∠BAC. Then we have to prove that the ΔABC is isosceles.
In triangles ADB and ADC,
\( \angle BAD = \angle CAD \) (AD is bisector of \( \angle BAC \))
AD = AD (common)
\( \angle ADB = \angle ADC \) (Each equal to 90°)
\( \Rightarrow \triangle ADB \cong \triangle ADC \) (by ASA congruence criterion)
\( \Rightarrow AB = AC \) (cpct)
Hence, \( \triangle ABC \) is an isosceles.

(ii) In \( \triangle ABC \), the bisector of \( \angle BAC \) is perpendicular to the base BC. We have to prove that the \( \triangle ABC \) is isosceles.

In triangles ADB and ADC,
\( \angle BAD = \angle CAD \) (AD is bisector of \( \angle BAC \))
AD = AD (common)
\( \angle ADB = \angle ADC \) (Each equal to 90°)
\( \Rightarrow \triangle ADB \cong \triangle ADC \) (by ASA congruence criterion)
\( \Rightarrow AB = AC \) (cpct)
Hence, \( \triangle ABC \) is an isosceles.

20. In the given figure;
AB=BC and AD=EC.
Prove that: BD=BE.
In \( \triangle ABC \),
AB = BC (given)
\( \Rightarrow \) \( \angle BCA = \angle BAC \) (Angles opposite to equal sides are equal)
\( \Rightarrow \) \( \angle BCD = \angle BAE \) ....(i)

Given, AD = EC
\( \Rightarrow \) AD + DE = EC + DE (Adding DE on both sides)
\( \Rightarrow \) AE = CD ....(ii)

Now, in triangles ABE and CBD,
AB = BC (given)
\( \angle BAE = \angle BCD \) [From (i)]
AE = CD [From (ii)]
\( \Rightarrow \) \( \triangle ABE \cong \triangle CBD \)
\( \Rightarrow \) BE = BD (cpct)
1. If the equal sides of an isosceles triangle are produced, prove that the exterior angles so formed are obtuse and equal.

Solution:

Construction:
AB is produced to D and AC is produced to E so that exterior angles $\angle DBC$ and $\angle ECB$ is formed.

In $\triangle ABC$,
$AB = AC$ [Giver]

$\therefore \angle C = \angle B$ .... (i) [angles opp. to equal sides are equal]

Since angle B and angle C are acute they cannot be right angles or obtuse angles.

$\angle ABC + \angle DBC = 180^\circ$ [ABD is a st. line]

$\Rightarrow \angle DBC = 180^\circ - \angle ABC$ .... (ii)

Similarly,

$\angle ACB + \angle ECB = 180^\circ$ [ABD is a st. line]

$\Rightarrow \angle ECB = 180^\circ - \angle ACB$ .... (iii)

$\Rightarrow \angle ECB = 180^\circ - \angle C$ .... (iv) [from (i) and (iii)]

$\Rightarrow \angle DBC = \angle ECB$ .... (v) [from (ii) and (iv)]

Now,
\[ \angle DEC = 180^\circ - \angle B \]
But \( \angle B \) is an acute angle
\[ \therefore \angle DBC = 180^\circ - \text{Acute angle} = \text{obtuse angle} \]

Similarly,
\[ \angle ECB = 180^\circ - \angle C \]
But \( \angle C \) is an acute angle
\[ \therefore \angle EBC = 180^\circ - \text{Acute angle} = \text{obtuse angle} \]
Hence, exterior angles formed are obtuse and equal.

2. In the given figure, \( AB = AC \). Prove that:
   (i) \( DP = DQ \)
   (ii) \( AP = AQ \)
   (iii) \( AD \) bisects angle \( A \)

Solution:

\[ \text{Construction:} \]

Join \( AD \).
In $\triangle ABC$,
$AB = AC$ [Given]
\[\therefore \angle C = \angle B \ldots \text{(i)}\] [angles opp. to equal sides are equal]

(i)
In $\triangle BPD$ and $\triangle CQD$,
$\angle BPD = \angle CQD$ [Each = $90^\circ$]
$\angle B = \angle C$ [proved]
BD = DC [Given]
\[\therefore \triangle BPD \cong \triangle CQD \quad \text{(AAS criterion)}\]
\[\therefore BP = CQ \quad \text{[cpct]}\]

(ii)
We have already proved that $\triangle BPD \cong \triangle CQD$
Hence, $BP = CQ$ [cpct]
Now,
$AB = AC$ [Given]
$\Rightarrow AB - BP = AC - CQ$
$\Rightarrow AP = AQ$

(iii)
In $\triangle APD$ and $\triangle AQD$,
$DP = DQ$ [proved]
AD = AD [common]
AP = AQ [Proved]
\[\therefore \triangle APD \cong \triangle AQD \quad \text{[SSS]}\]
$\Rightarrow \angle PAD = \angle QAD$ [cpct]
Hence, AD bisects angle $A$.

3. In triangle $ABC$, $AB=AC$; $BE \perp AC$ and $CF \perp AB$. Prove that:
   (i) $BE=CF$
   (ii) $AF=AE$
Solution:

(i)

In $\triangle AEB$ and $\triangle AFC$,

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC = 90^\circ$ [Given: $BE \perp AC$]

$AB = AC$ [Given]

$\Rightarrow \triangle AEB \cong \triangle AFC$ [AAS]

$\therefore BE = CF$ [cpct]

(ii)

Since $\triangle AEB \cong \triangle AFC$

$\angle ABE = \angle AFC$

$\therefore AF = AE$ [congruent angles of congruent triangles]

4. In isosceles triangle $ABC$, $AB=AC$. The side $BA$ is produced to $D$ such that $BA=AD$. Prove that: $\angle BCD=90^\circ$.

Solution:
Construction:
Join CD.

In \( \triangle ABC \),
\[
AB = AC \quad \text{[Given]}
\]
\[
\therefore \angle C = \angle B \quad \text{[angles opp. to equal sides are equal]}
\]

In \( \triangle ACD \),
\[
AC = AD \quad \text{[Given]}
\]
\[
\therefore \angle ADC = \angle ACD \quad \text{[(ii)]}
\]
Adding (i) and (ii)
\[
\angle B + \angle ADC = \angle C + \angle ACD
\]
\[
\angle B + \angle ADC = \angle BCD \quad \text{[(iii)]}
\]

In \( \triangle BCD \),
\[
\angle B + \angle ADC + \angle ECD = 180^\circ
\]
\[
\angle BCD + \angle BCD = 180^\circ \quad \text{[From (iii)]}
\]
\[
2\angle BCD = 180^\circ
\]
\[
\angle BCD = 90^\circ
\]

5.

(i) In a triangle ABC, AB = AC and \( \angle A = 36^\circ \). If the internal bisector of \( \angle C \) meets AB at point D, prove that AD = BC.

(ii) If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Solution:
6. Prove that the bisectors of the base angles of an isosceles triangle are equal.

**Solution:**

In \( \triangle ABC \),

\[ AB = AC \quad \text{[Given]} \]

\[ \therefore \angle C = \angle B \quad \text{[angles opp. to equal sides are equal]} \]

\[ \Rightarrow \frac{1}{2} \angle C = \frac{1}{2} \angle B \]

\[ \Rightarrow \angle BCF = \angle CBE \quad \text{[ii]} \]
In $\triangle ABC$ and $\triangle CBF$,

$\angle C = \angle B$ \hspace{1cm} \text{[From (i)]}

$\angle BCF = \angle CBE$ \hspace{1cm} \text{[From (ii)]}

$BC = BC$ \hspace{1cm} \text{[Common]}

$\therefore \triangle ABC \cong \triangle CBF$ \hspace{1cm} \text{[AAS]}

$\Rightarrow BD = CE$ \hspace{1cm} \text{[cpct]}

7. In the given figure, $AB = AC$ and $\angle DBC = \angle ECB = 90^0$.

Prove that:

(i) $BD = CE$

(ii) $AD = AE$

Solution:

In $\triangle ABC$,

$AB = AC$ \hspace{1cm} \text{[Given]}

$\therefore \angle ACB = \angle ABC$ \hspace{1cm} \text{[angles opp. to equal sides are equal]}

$\Rightarrow \angle ABC = \angle ACB$ \hspace{1cm} \text{(i)}

$\angle DBC = \angle ECB = 90^0$ \hspace{1cm} \text{[Given]}

$\Rightarrow \angle DBC = \angle ECB$ \hspace{1cm} \text{(ii)}

Subtracting (i) from (ii)

$\angle DBC - \angle ABC = \angle ECB - \angle ACB$

$\Rightarrow \angle DBA = \angle ECA$ \hspace{1cm} \text{(iii)}
8. ABC and DBC are two isosceles triangle on the same side of BC. Prove that:
   (i) DA (or AD) produced bisects BC at the right angle
   (ii) \( \angle BDA = \angle CDA \)

Solution:

In \( \triangle DBA \) and \( \triangle ECA \),

\[ \angle DBA = \angle ECA \] \hspace{1cm} \text{[From (iii)]}
\[ \angle DAB = \angle EAC \] \hspace{1cm} \text{[Vertically opposite angles]}
\[ AB = AC \] \hspace{1cm} \text{[Given]}
\[ \therefore \triangle DBA \cong \triangle ECA \] \hspace{1cm} \text{[ASA]}
\[ \Rightarrow BD = CE \] \hspace{1cm} \text{[cpct]}

Also,
\[ AD = AE \] \hspace{1cm} \text{[cpct]}

DA is produced to meet BC in L.

In \( \triangle ABC \),
\[ AB = AC \] \hspace{1cm} \text{[Given]}
\[ \therefore \angle ACB = \angle ABC \ldots \ldots (i) \] \hspace{1cm} \text{[angles opposite to equal sides are equal]}

In \( \triangle DBC \),
\[ DB = DC \] \hspace{1cm} \text{[Given]}
\[ \therefore \angle DCB = \angle DBC \ldots \ldots (ii) \] \hspace{1cm} \text{[angles opposite to equal sides are equal]}

Subtracting (i) from (ii)
\[ \angle DCA = \angle DBA \ldots \ldots (iii) \]
In \( \triangle DBA \) and \( \triangle DCA \),
\[
DB = DC \quad \text{[Given]}
\]
\[
\angle DBA = \angle DCA \quad \text{[From (iii)]}
\]
\[
AB = AC \quad \text{[Given]}
\]
\[
\therefore \triangle DBA \cong \triangle DCA \quad \text{[SAS]}
\]
\[
\Rightarrow \angle BDA = \angle CDA \quad \text{.........(iv)} \quad \text{[cpct]}
\]
In \( \triangle DCA \),
\[
\angle BAL = \angle DCA + \angle BDA \quad \text{.........(v)}
\]
[Ext. angle = sum of opp. int. angles]

From (iii), (iv) and (v)
\[
\angle BAL = \angle DCA + \angle CDA \quad \text{.........(vi)}
\]
In \( \triangle DCA \),
\[
\angle CAL = \angle DCA + \angle CDA \quad \text{.........(vii)}
\]
[Ext. angle = sum of opp. int. angles]

From (vi) and (vii)
\[
\angle BAL = \angle CAL \quad \text{.........(viii)}
\]
In \( \triangle BAL \) and \( \triangle CAL \),
\[
\angle BAL = \angle CAL \quad \text{[From (viii)]}
\]
\[
\angle BAL = \angle ACL \quad \text{[From (i)]}
\]
\[
AB = AC \quad \text{[Given]}
\]
\[
\therefore \triangle BAL \cong \triangle CAL \quad \text{[ASA]}
\]
\[
\Rightarrow \angle ALB = \angle ALC \quad \text{[cpct]}
\]
and \( BL = LC \quad \text{.........(x)} \quad \text{[cpct]}

Now,
\[
\angle ALB + \angle ALC = 180^\circ
\]
\[
\Rightarrow \angle ALB + \angle ALB = 180^\circ
\]
\[
\Rightarrow 2\angle ALB = 180^\circ
\]
\[
\Rightarrow \angle ALB = 90^\circ
\]
\[
\therefore \ AL \perp BC
\]
or \( DL \perp BC \) and \( BL = LC \)
\[
\therefore \ DA \text{ produced bisects } BC \text{ at right angle.}
\]

9. The bisectors of the equal angles B and C of an isosceles triangle ABC meet at O. Prove that AO bisects angle A.

**Solution:**
In $\triangle ABC$, we have $AB = AC$

$\Rightarrow \angle B = \angle C$ [angles opposite to equal sides are equal]

$\Rightarrow \frac{1}{2} \angle B = \frac{1}{2} \angle C$

$\Rightarrow \angle OBC = \angle OCB$ ................ (i)

$\Rightarrow OB = OC$ ............... (ii)

Now,

In $\triangle ABO$ and $\triangle ACO$,

$AB = AC$ [Given]

$\angle OBC = \angle OCB$ [From (i)]

$OB = OC$ [From (ii)]

$\triangle ABO \cong \triangle ACO$ [SAS criterion]

$\Rightarrow \angle BAO = \angle CAO$ [cpct]

Hence, AO bisects $\angle BAC$.

10. Prove that the medians corresponding to equal sides of an isosceles triangle are equal.

Solution:
11. Use the given figure to prove that, \( AB = AC \).

**Solution:**

In \( \triangle APQ \),

\( AP = AQ \) \quad [Given]

\( \therefore \angle APQ = \angle AQP \ldots \ldots(i) \)

[angles opposite to equal sides are equal]

In \( \triangle ABP \),

\( \angle APQ = \angle BAP + \angle ABP \ldots \ldots(ii) \)

[Ext. angle is equal to sum of opp. int. angles]
In $\triangle AQC$,
$\angle AQP = \angle CAQ + \angle ACQ$ .......(iii)

[Ext. angle is equal to sum of opp. int. angles]

From (i), (ii) and (iii)
$\angle BAP + \angle AQP = \angle CAQ + \angle ACQ$

But, $\angle BAP = \angle CAQ$  .......[Given]
$\Rightarrow \angle CAQ + \angle ABP = \angle CAQ + \angle ACQ$
$\Rightarrow \angle ABP = \angle ACQ$  
$\Rightarrow \angle E = \angle C$ .......(iv)

In $\triangle ABC$,
$\angle B = \angle C$  
$\Rightarrow AB = AC$  .......[Sides opposite to equal angles are equal]

12. In the given figure; AE bisects exterior angle CAD and AE is parallel to BC. Prove that: $AB=AC$.

Solution:
Since $AE \parallel BC$ and DAB is the transversal
$\therefore \angle CAE = \angle ABC = \angle B$  .......[Corresponding angles]

Since $AE \parallel BC$ and AC is the transversal
$\angle CAE = \angle ACB = \angle C$  .......[Alternate Angles]

But AE bisects $\angle CAD$
$\therefore \angle DAE = \angle CAE$
$\Rightarrow \angle B = \angle C$
$\Rightarrow AB = AC$  .......[Sides opposite to equal angles are equal]

13. In an equilateral triangle ABC; points P, Q and R are taken on the sides AB, BC and CA respectively such that AP= BQ= CR. Prove that triangle PQR is equilateral.
Solution:

AB = BC = CA……..(i) [Given]
AP = BQ = CR……..(ii) [Given]
Subtracting (ii) from (i)
AB - AP = BC - BQ = CA - CR
BP = CQ = AR ………..(iii)
\[\angle A = \angle B = \angle C \]……..(iv) [angles opp. to equal sides are equal]
In \( \triangle BQP \) and \( \triangle CQR \),
BP = CQ \[\text{[From (iii)]}\]
\[\angle B = \angle C \] \[\text{[From (iv)]}\]
BQ = CR \[\text{[Given]}\]
\[\therefore \triangle BQP \cong \triangle CQR \] \[\text{[SAS criterion]}\]
\[\Rightarrow PQ = QR \]……..(v)
In \( \triangle CQR \) and \( \triangle APR \),
CQ = AR \[\text{[From (iii)]}\]
\[\angle C = \angle A \] \[\text{[From (iv)]}\]
CR = AP \[\text{[Giver]}\]
\[\therefore \triangle CQR \cong \triangle APR \] \[\text{[SAS criterion]}\]
\[\Rightarrow QR = PR \]……..(vi)
From (v) and (vi)
PQ = QR = PR
Hence, PQR is an equilateral triangle.

14. In triangle ABC, altitudes BE and CF are equal. Prove that the triangle is isosceles.
Solution:
In \( \triangle ABE \) and \( \triangle ACF \),
\[
\angle A = \angle A [\text{Common}]
\]
\[
\angle AEB = \angle AFC = 90^\circ [\text{Given: } BE \perp AC; CF \perp AB]
\]

BE = CF [Given]

\[
\therefore \triangle ABE \cong \triangle ACF \quad [\text{AAS criterion}]
\]

\[
\Rightarrow AB = AC
\]

Hence, \( \triangle ABC \) is an isosceles triangle.

15. Through any point in the bisector of an angle, a straight line is drawn parallel to either arm of the angle. Prove that the triangle so formed is isosceles.

**Solution:**

AL is bisector of angle A. Let D is any point on AL. From D, a straight line DE is drawn parallel to AC.

DE \( \parallel \) AC [Given]

\[
\therefore \angle ADE = \angle DAC \ldots (i) \quad [\text{Alternate angles}]
\]

\[
\angle DAC = \angle DAE \ldots (ii) \quad [\text{AL is bisector of } \angle A]
\]

From (i) and (ii)

\[
\angle ADE = \angle DAE
\]

\[
\therefore AE = ED \quad [\text{Sides opposite to equal angles are equal}]
\]

Hence, \( \triangle AED \) is an isosceles triangle.
16. In triangle $ABC$; $AB = AC$. $P$, $Q$ and $R$ are mid-points of sides $AB$, $AC$ and $BC$ respectively. Prove that:

(i) $PR = QR$

(ii) $BQ = CP$

**Solution:**

(i)

In $\triangle ABC$,

$AB = AC$

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AC$

$\Rightarrow AP = AQ$ ....(i) [Since $P$ and $Q$ are mid-points]

In $\triangle BCA$,

$PR = \frac{1}{2} AC$ [PR is line joining the mid-points of $AB$ and $BC$]

$\Rightarrow PR = AQ$ ....(ii)

In $\triangle CAB$,

$QR = \frac{1}{2} AB$ [QR is line joining the mid-points of $AC$ and $BC$]

$\Rightarrow QR = AP$ ....(iii)

From (i), (ii) and (iii)

$PR = QR$

(ii)

$AB = AC$
17. From the following figure, prove that:
   (i) \( \angle ACD = \angle CBE \)
   (ii) \( AD = CE \)

Solution:
(i) In \( \triangle ACB \),
   \( AC = AC \) [Given]
   \[ \therefore \angle ABC = \angle ACB \] \( \ldots (i) \) [angles opposite to equal sides are equal]
   \( \angle ACD + \angle ACB = 180^0 \) \( \ldots (ii) \) [DCB is a straight line]
   \( \angle ABC + \angle CBE = 180^0 \) \( \ldots (iii) \) [ABE is a straight line]
   Equating (ii) and (iii)
   \[ \angle ACD + \angle ACB = \angle ABC + \angle CBE \]
   \[ \therefore \angle ACD = \angle CBE \] [From (i)]

(ii)
18. Equal sides AB and AC of an isosceles triangle ABC are produced. The bisectors of the exterior angles so formed meet at D. Prove that AD bisects angle A.

Solution:

In $\triangle ACD$ and $\triangle CBE$,

- $DC = CB$ [Given]
- $AC = BE$ [Given]
- $\angle ACD = \angle CBE$ [Proved Earlier]
- $\triangle ACD \cong \triangle CBE$ [SAS criterion]

$\Rightarrow AD = CE$ [cpct]

AB is produced to E and AC is produced to F. BD is bisector of angle CBE and CD is bisector of angle BCF. BD and CD meet at D.

In $\triangle ABC$,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$ [angles opposite to equal sides are equal]

$\angle CBE = 180^\circ - \angle B$ [ABE is a straight line]

$\Rightarrow \angle CBD = \frac{180^\circ - \angle B}{2}$ [BD is bisector of $\angle CBE$]

$\Rightarrow \angle CBD = 90^\circ - \frac{\angle B}{2}$ ...............(i)

Similarly,

$\angle BCF = 180^\circ - \angle C$ [ACF is a straight line]

$\Rightarrow \angle BCD = \frac{180^\circ - \angle C}{2}$ [CD is bisector of $\angle BCF$]

$\Rightarrow \angle BCD = 90^\circ - \frac{\angle C}{2}$ ...............(ii)
Now,
\[
\Rightarrow \angle CBD = 90° - \frac{\angle C}{2} \quad [\therefore \angle B = \angle C]
\]
\[
\Rightarrow \angle CBD = \angle BCD
\]
In \(\triangle BCD\),
\[
\angle CBD = \angle BCD
\]
\[
\therefore BD = CD
\]
In \(\triangle ABD\) and \(\triangle ACD\),
\[
AB = AC[\text{Given}]
\]
\[
AD = AD[\text{Common}]
\]
\[
BD = CD[\text{Proved}]
\]
\[
\therefore \triangle ABD \cong \triangle ACD \quad [\text{SSS criterion}]
\]
\[
\Rightarrow \angle BAD = \angle CAD \quad [\text{cpct}]
\]
Hence, \(AD\) bisects \(\angle A\).

19. \(\triangle ABC\) is a triangle. The bisector of the angle \(BCA\) meets \(AB\) in \(X\). A point \(Y\) lies on \(CX\) such that \(AX=AY\).
Prove that: \(\angle CAY = \angle ABC\)

**Solution:**

![Diagram of \(\triangle ABC\) with bisectors and parallel lines]

In \(\triangle ABC\),
\[
CX \text{ is the angle bisector of } \angle C
\]
\[
\Rightarrow \angle ACY = \angle BCX \ldots \ldots \text{(i)}
\]
In \(\triangle AXY\),
\[
AX = AY \text{ [Given]}
\]
\[
\angle AXY = \angle AYX \ldots \ldots \text{(ii)} \text{ [angles opposite to equal sides are equal]}
\]
Now \(\angle XYC = \angle AXB = 180° \text{ [straight line]}
\]
\[
\Rightarrow \angle AYX + \angle AYC = \angle AXY + \angle BXY
\]
\[
\Rightarrow \angle AYC = \angle BXY \ldots \ldots \text{(iii) [From (ii)]}
\]
In \(\triangle AYC\) and \(\triangle BXC\),
\[
\angle AYC + \angle ACY + \angle CAY = \angle BXC + \angle BCX + \angle XBC = 180°
\]
\[
\Rightarrow \angle CAY = \angle XBC \text{ [From (i) and (iii)]}
\]
\[
\Rightarrow \angle CAY = \angle ABC
\]

20. In the following figures; \(IA\) and \(IB\) are bisector of angles \(CAB\) and \(CBA\) respectively. \(CP\) is parallel to \(IA\) and \(CQ\) is parallel to \(IB\).
Prove that:
\( PQ = \text{The perimeter of the triangle ABC.} \)

**Solution:**

Since \( IA \parallel CP \) and \( CA \) is a transversal
\[ \therefore \quad \angle CAI = \angle PCA \quad [\text{Alternate angles}] \]

Also, \( IA \parallel CP \) and \( AP \) is a transversal
\[ \therefore \quad \angle IAB = \angle APC \quad [\text{Corresponding angles}] \]

But \[ \angle CAI = \angle IAB \quad [\text{Given}] \]
\[ \therefore \quad \angle PCA = \angle APC \]
\[ \Rightarrow AC = AP \]

Similarly,
\( BC = BQ \)

Now,
\[ PQ = AP + AB + BQ \]
\[ = AC + AB + BC \]
\[ = \text{Perimeter of } \triangle ABC \]

21. Sides \( AB \) and \( AC \) of a triangle \( ABC \) are equal. \( BC \) is produced through \( C \) upto point \( D \) such that \( AC = CD \). \( D \) and \( A \) are joined and produced (through vertex \( A \)) upto point \( E \). If angle \( BAE = 108^\circ \); find angle \( ADB \).

**Solution:**

\[ \text{In } \triangle ABD, \]
\[ \angle BAE = \angle 3 + \angle ADB \]
\[ \Rightarrow 108^\circ = \angle 3 + \angle ADB \]
But AB = AC
⇒ ∠3 = ∠2
⇒ 108° = ∠2 + ∠ADB ......(i)

Now,
In ΔACD,
∠2 = ∠1 + ∠ADB
But AC = CD
⇒ ∠1 = ∠ADB
⇒ ∠2 = ∠ADB + ∠ADB
⇒ ∠2 = 2∠ADB
Putting this value in (i)
⇒ 108° = 2∠ADB + ∠ADB
⇒ 3∠ADB = 108°
⇒ ∠ADB = 36°

22. The given figure shows an equilateral triangle ABC with each side 15 cm. Also DE || BC, DF || AC and EG || AB.

If DE + DF + EG = 20 cm, find FG.

Solution:

ABC is an equilateral triangle.
Therefore, AB = BC = AC = 15 cm
∠A = ∠B = ∠C = 60°

In ΔADE, DE || BC [Given]
∠AED = 60° [∴ ∠ACB = 60°]
∠ADE = 60° [∴ ∠ABC = 60°]
∠DAE = 180° - (60° + 60°) = 60°

Similarly, ΔBDF & ΔGEC are equilateral triangles.
∴ ∠C = 60°

Let AD = x, AE = x, DE = x [∴ ΔADE is an equilateral triangle]
Let BD = y, FD = y, FB = y [∴ ΔBDF is an equilateral triangle]
Let EC = z, GC = z, GE = z [∴ ΔGEC is an equilateral triangle]

https://byjus.com
23. If all the three altitudes of a triangle are equal, the triangles are equilateral. Prove it.

**Solution:**

In right \( \triangle BEC \) and \( \triangle BFC \),
- \( BE = CF \) [Given]
- \( BC = BC \) [Common]
- \( \angle BEC = \angle BFC \) [each = 90°]

\[ \Rightarrow \triangle BEC \cong \triangle BFC \] [RHS]

\[ \Rightarrow \angle B = \angle C \]

Similarly,

\[ \angle A = \angle B \]

Hence, \( \angle A = \angle B = \angle C \)

\[ \Rightarrow AB = BC = AC \]

Hence, \( \triangle ABC \) is an equilateral triangle.
24. In a triangle ABC, the internal bisector of angle A meets opposite side BC at point D. Through vertex C, line CE is drawn parallel to DA which meets BA produced at point E. Show that triangle ACE is isosceles.

Solution:

DA || CE [Given]
⇒ ∠1 = ∠4 ........ (i) [Corresponding angles]
∠2 = ∠3 .......... (ii) [Alternate angles]
But ∠1 = ∠2 .......... (iii) [AD is the bisector of ∠A]
From (i), (ii) and (iii)
∠3 = ∠4
⇒ AC = AE
⇒ ΔACE is an isosceles triangle.

25. In triangle ABC, bisector of angle BAC meets opposite side BC at point D. If BD = CD, prove that triangle ABC is isosceles.

Solution:

Produce AD upto E such that AD = DE.
26. In triangle ABC, D is a point on BC such that AB = AD = BD= DC. Show that: 
\[ \angle ADC : \angle C = 4 : 1 \]

**Solution:**

Since \( AB = AD = BD \)

\[ \therefore \triangle ABD \text{ is an equilateral triangle.} \]

\[ \therefore \angle ADB = 60^\circ \]

\[ \Rightarrow \angle ADC = 180^\circ - \angle ADB \]
\[ = 180^\circ - 60^\circ \]
\[ = 120^\circ \]

Again in \( \triangle ADC \),

\( AD = DC \)
Concise Selina Solutions for Class 9 Maths Chapter 10 - Isosceles Triangle

27. Using the information given in each of the following figures, find the value of a and b.

(i) 

\[ \therefore \angle 1 = \angle 2 \]

But,
\[ \angle 1 + \angle 2 + \angle ADC = 180^\circ \]
\[ \Rightarrow 2\angle 1 + 120^\circ = 180^\circ \]
\[ \Rightarrow 2\angle 1 = 60^\circ \]
\[ \Rightarrow \angle 1 = 30^\circ \]
\[ \Rightarrow \angle C = 30^\circ \]
\[ \therefore \angle ADC : \angle C = 120^\circ : 30^\circ \]
\[ \Rightarrow \angle ADC : \angle C = 4 : 1 \]

(ii) 

Solution:

(i) 

In \( \triangle CAE \), \( \angle CAE = \angle AEC = \frac{180^\circ - 68^\circ}{2} = 56^\circ \) \([\because CE=AC]\)

In \( \angle BCA \), \( a = 180^\circ - 56^\circ = 124^\circ \)

In \( \triangle ABE \), \( \angle ABE = 180^\circ - (a + \angle BAE) \)
\[ = 180^\circ - (124^\circ + 14^\circ) \]
\[ = 180^\circ - 138^\circ = 42^\circ \]
(ii)

In \( \triangle AEB \) & \( \triangle CAD \),

\( \angle EAB = \angle CAD \) [Given]

\( \angle ADC = \angle AEB \) \( \because \) \( \angle ADE = \angle AED \) \{AE = AD\}

\( 180^\circ - \angle ADE = 180^\circ - \angle AED \)

\( \angle ADC = \angle AEB \) \[Given\]

\( \therefore \triangle AEB \cong \triangle CAD \) [ASA]

\( AC = AB \) [By C.F.C.T.]

\[ 2a + 2 = 7b - 1 \]

\( \Rightarrow 2a - 7b = -3 \ldots \text{(i)} \)

\( CD = EB \)

\( \Rightarrow a = 3b \ldots \text{(ii)} \)

Solving (i) & (ii), we get

\( a = 9, b = 3 \)