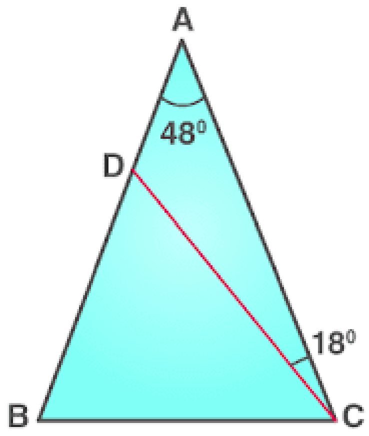
EXERCISE 10(A)

1. In the figure alongside,
AB=AC
$\angle A = 48^0$ and
$\angle ACD = 18^0$
Show that: BC=CD.



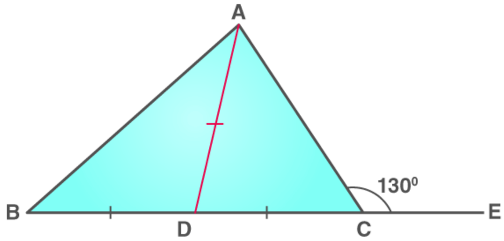
Solution:

In ΔABC,
$\angle BAC + \angle ACB + \angle ABC = 180^0$
$48^0 + \angle ACB + \angle ABC = 180^0$
But $\angle ACB = \angle ABC$ [AB = AC]
$2\angle ABC = 180^0 - 48^0$
$2\angle ABC = 132^0$
$\angle ABC = 66^0 = \angle ACB$ …… (i)

$\angle ACB = 66^0$
$\angle ACD + \angle DCB = 66^0$
$18^0 + \angle DCB = 66^0$
$\angle DCB = 48^0$ ……… (ii)

Now, In ΔDCB,
$\angle DBC = 66^0$ [From (i), Since $\angle ABC = \angle DBC$]
$\angle DCB = 48^0$ [From (ii)]
$\angle BDC = 180^0 - 48^0 - 66^0$
$\angle BDC = 66^0$
Since $\angle BDC = \angle DBC$
Hence, BC = CD
Equal angles have equal sides opposite to them.

2. Calculate:
(i) $\angle$ ADC
(ii) $\angle$ ABC
(iii) $\angle$ BAC



Solution:

Given: $\angle ACE = 130^0$; AD = BD = CD

Proof:

(i)

$\angle ACD + \angle ACE = 180^\circ$ [DCE is a st. line]

$\Rightarrow \angle ACD = 180^\circ - 130^\circ$

$\Rightarrow \angle ACD = 50^\circ$

Now, CD = AD

$\Rightarrow \angle ACD = \angle DAC = 50^\circ \ldots\ldots(i)$

[Since angles opposite to equal sides are equal]

In ΔADC,

$\angle ACD = \angle DAC = 50^\circ$

$\angle ACD + \angle DAC + \angle ADC = 180^\circ$

$50^\circ + 50^\circ + \angle ADC = 180^\circ$

$\angle ADC = 180^\circ - 100^\circ$

$\angle ADC = 80^\circ$

(ii)

$\angle ADC = \angle ABD + \angle DAB$ [Exterior angle is equal to sum of opp. interior angles]

But AD = BD

$\therefore \angle DAB = \angle ABD$

$\Rightarrow 80^\circ = \angle ABD + \angle ABD$

$\Rightarrow 2\angle BD = 80^\circ$

$\Rightarrow \angle ABD = 40^\circ = \angle DAB \ldots\ldots\ldots(ii)$

(iii)

$\angle BAC = \angle DAB + \angle DAC$

substituting the values from (i) and (ii)

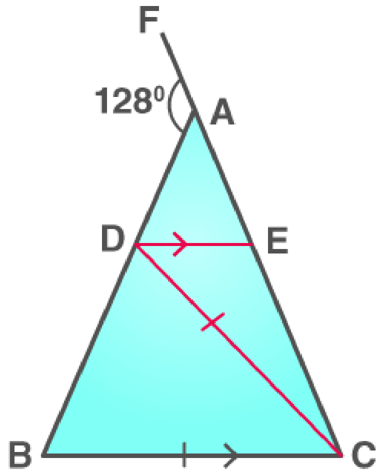
$\angle BAC = 40^\circ + 50^\circ$

$\Rightarrow \angle BAC = 90^\circ$

3. In the following figure, AB=AC; BC=CD and DE is parallel to BC. Calculate:

(i) $\angle$ CDE

(ii) $\angle$ DCE



Solution:

$\angle FAB = 128^\circ$ [Given]

$\angle BAC + \angle FAB = 180^\circ$ [FAC is a st. line]

$\Rightarrow \angle BAC = 180^\circ - 128^\circ$

$\Rightarrow \angle BAC = 52^\circ$

In ΔABC,

$\angle A = 52^\circ$

$\angle B = \angle C$ [Given AB = AC and angles opposite to equal sides are equal]

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + \angle B + \angle B = 180^\circ$

$\Rightarrow 52^\circ + 2\angle B = 180^\circ$

$\Rightarrow 2\angle B = 128^\circ$

$\Rightarrow \angle B = 64^\circ = \angle C$............(i)

$\angle B = \angle ADE$ [Given DE || BC]

(i)

Now,

$\angle ADE + \angle CDE + \angle B = 180^\circ$ [ADB is a st. line]

$\Rightarrow 64^\circ + \angle CDE + 64^\circ = 180^\circ$

$\Rightarrow \angle CDE = 180^\circ - 128^\circ$

$\Rightarrow \angle CDE = 52^\circ$

(ii)

Given DE||BC and DC is the transversal.

$\Rightarrow \angle CDE = \angle DCB = 52°$.......(ii)

Also, $\angle ECB = 64°$.........[From (i)]

But,

$\angle ECB = \angle DCE + \angle DCB$

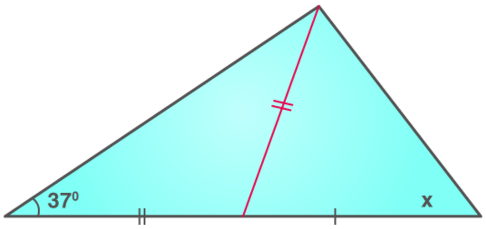
$\Rightarrow 64° = \angle DCE + 52°$

$\Rightarrow \angle DCE = 64° - 52°$

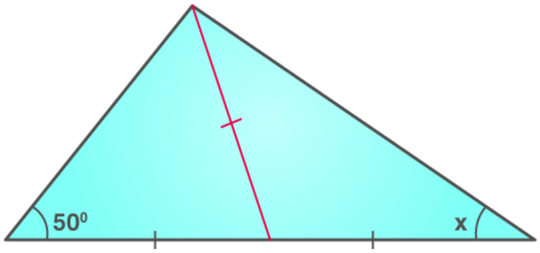
$\Rightarrow \angle DCE = 12°$

4. Calculate *x*:

(i)

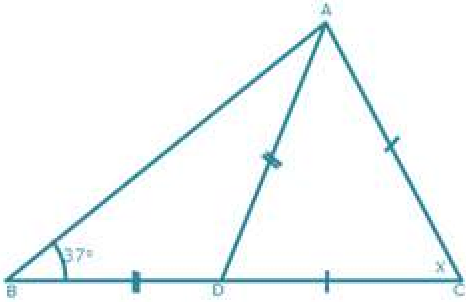


(ii)



Solution:

(i) Let the triangle be ABC and the altitude be AD.



In ΔABD,

$\angle DBA = \angle DAB = 37^\circ$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opp. interior angles]

$\therefore \angle CDA = 37^\circ + 37^\circ$

$\Rightarrow \angle CDA = 74^\circ$

Now in ΔADC,

$\angle CDA = \angle CAD = 74^\circ$ [Given CD = AC and angles opposite to equal sides are equal]

Now,

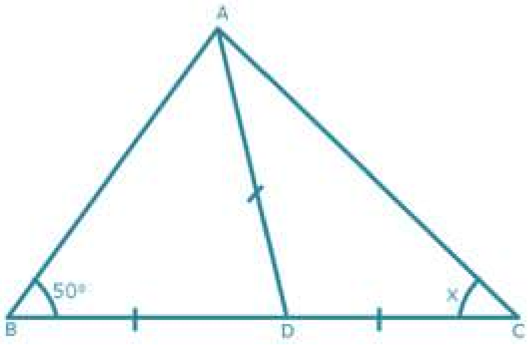
$\angle CAD + \angle CDA + \angle ACD = 180^\circ$

$\Rightarrow 74^\circ + 74^\circ + x = 180^\circ$

$\Rightarrow x = 180^\circ - 148^\circ$

$\Rightarrow x = 32^\circ$

(ii) Let triangle be ABC and altitude be AD.



In ΔABD,

$\angle DBA = \angle DAB = 50^\circ$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opp. interior angles]

$\therefore \angle CDA = 50^\circ + 50^\circ$

$\Rightarrow \angle CDA = 100^\circ$

In ΔADC,

$\angle DAC = \angle DCA = x$ [Given AD = DC and angles opposite to equal sides are equal]

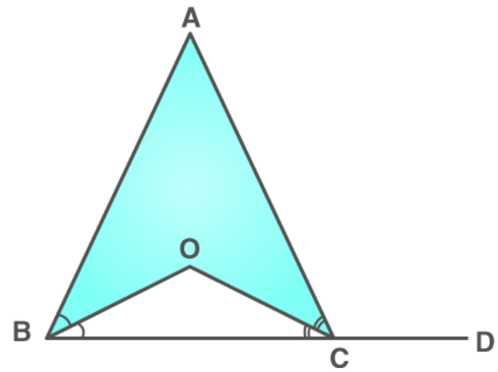
$\therefore \angle DAC + \angle DCA + \angle ADC = 180^\circ$

$\Rightarrow x + x + 100^\circ = 180^\circ$

$\Rightarrow 2x = 80^\circ$

$\Rightarrow x = 40^\circ$

5. In the figure, given below, AB=AC.
Prove that: $\angle$BOC = $\angle$ACD



Solution:

Let $\angle ABO = \angle OBC = x$ and $\angle ACO = \angle OCB = y$

In ΔABC,

$\angle BAC = 180^\circ - 2x - 2y$.........(i)

Since $\angle B = \angle C$ [AB = AC]

$\frac{1}{2}B = \frac{1}{2}C$

$\Rightarrow x = y$

Now,

$\angle ACD = 2x + \angle BAC$ [Exterior angle is equal to sum of opp. interior angles]

$= 2x + 180^\circ - 2x - 2y$ [From (i)]

$\angle ACD = 180^\circ - 2y$..........(ii)

In ΔOBC,

$\angle BOC = 180^\circ - x - y$

$\Rightarrow \angle BOC = 180^\circ - y - y$ [Already proved]

$\Rightarrow \angle BOC = 180^\circ - 2y....(iii)$

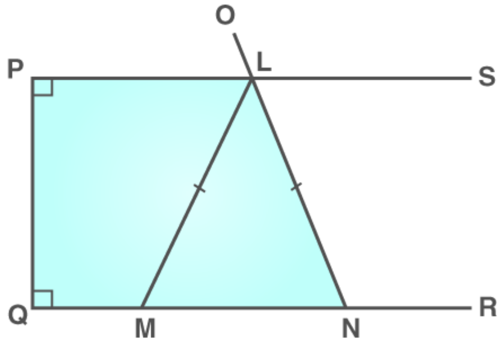
From (i) and (ii)

$\angle BOC = \angle ACD$

6. In the figure given below, LM=LN; angle PLN=110^0. Calculate:

(i) $\angle$ LMN

(ii) $\angle$ MLN



Solution:

Given: $\angle PLN = 110^\circ$

(i) We know that the sum of the measure of all the angles of a quadrilateral is 360°.

In quad. PQNL,

$\angle QPL + \angle PLN + \angle LNQ + \angle NQP = 360^\circ$

$\Rightarrow 90^\circ + 110^\circ + \angle LNQ + 90^\circ = 360^\circ$

$\Rightarrow \angle LNQ = 360^\circ - 290^\circ$

$\Rightarrow \angle LNQ = 70^\circ$

$\Rightarrow \angle LNM = 70^\circ..............(i)$

In ΔLMN,

$LM = LN$ [Given]

$\therefore \angle LNM = \angle LMN$ [angles opp. to equal sides are equal]

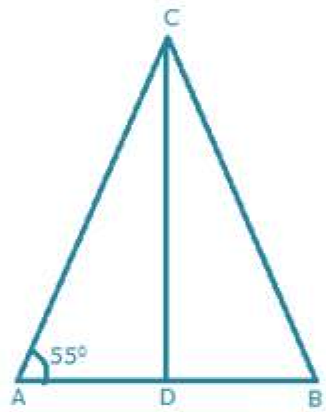
$\Rightarrow \angle LMN = 70^\circ.........(ii)$ [From (i)]

(ii)

In ΔLMN,
$\angle LMN + \angle LNM + \angle MLN = 180^\circ$
But, $\angle LNM = \angle LMN = 70^\circ$ [From (i) and (ii)]
$\therefore 70^\circ + 70^\circ + \angle MLN = 180^\circ$
$\Rightarrow \angle MLN = 180^\circ - 140^\circ$
$\Rightarrow \angle MLN = 40^\circ$

7. An isosceles triangle ABC has AC=BC. CD bisects AB at D and $\angle$CAB=55^0. Find:
(i) $\angle$DCB
(ii) $\angle$CBD.

Solution:



In ΔABC,
$AC = BC$ [Given]
$\therefore \angle CAB = \angle CBD$ [angles opp. to equal sides are equal]
$\Rightarrow \angle CBD = 55^\circ$
In ΔABC,
$\angle CBA + \angle CAB + \angle ACB = 180^\circ$
but, $\angle CAB = \angle CBA = 55^\circ$
$\Rightarrow 55^\circ + 55^\circ + \angle ACB = 180^\circ$
$\Rightarrow \angle ACB = 180^\circ - 110^\circ$
$\Rightarrow \angle ACB = 70^\circ$
Now,

In ΔACD and ΔBCD,

$AC = BC$ [Given]

$CD = CD$ [Common]

$AD = BD$ [Given : CD bisects AB]

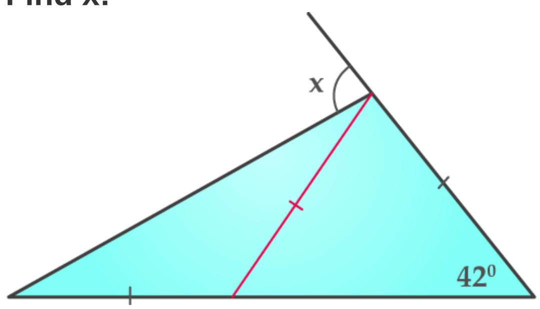
$\therefore \Delta ACD \cong \Delta BCD$

$\Rightarrow \angle DCA = \angle DCB$

$\Rightarrow \angle DCB = \dfrac{\angle ACB}{2} = \dfrac{70^\circ}{2}$

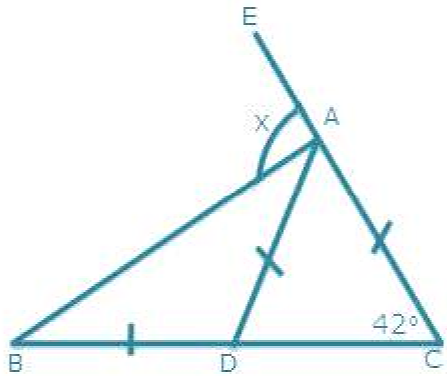
$\Rightarrow \angle DCB = 35^\circ$

8. Find x:



Solution:

Let us name the figure as following:



In ΔABC,

$AD = AC$ [Given]

$\therefore \angle ADC = \angle ACD$ [angles opp. to equal sides are equal]

$\Rightarrow \angle ADC = 42^\circ$

Now,

$\angle ADC = \angle DAB + \angle DBA$ [Exterior angle is equal to the sum of opp. interior angles]

But,

$\angle DAB = \angle DBA$ [Given : BD = DA]

$\therefore \angle ADC = 2\angle DBA$

$\Rightarrow 2\angle DBA = 42°$

$\Rightarrow \angle DBA = 21°$

For x:

$x = \angle CBA + \angle BCA$ [Exterior angle is equal to the sum of opp. interior angles]

We know that,

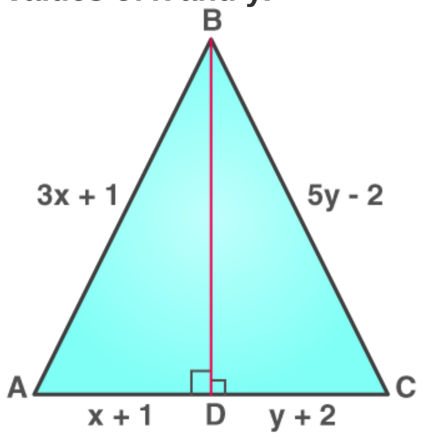
$\angle CBA = 21°$

$\angle BCA = 42°$

$\therefore x = 21° + 42°$

$\Rightarrow x = 63°$

9. In the triangle ABC, BD bisects angle B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.



Solution:

In ΔABD and ΔDBC,

$BD = BD$ [Common]

$\angle BDA = \angle BDC$ [each equal to 90°]

$\angle ABD = \angle DBC$ [BD bisects $\angle ABC$]

$\therefore \Delta ABD \cong \Delta DBC$ [ASA criterion]

Hence,

AD=DC

$x + 1 = y + 2$

$\Rightarrow x = y + 1.....(i)$

and AB = BC

$3x + 1 = 5y - 2$

Substituting the value of x from (i)

$3(y + 1) + 1 = 5y - 2$

$\Rightarrow 3y + 3 + 1 = 5y - 2$

$\Rightarrow 3y + 4 = 5y - 2$

$\Rightarrow 2y = 6$

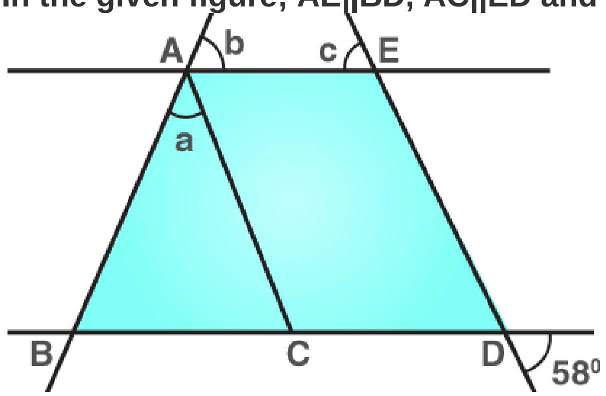
$\Rightarrow y = 3$

Putting y = 3 in (i)

x = 3 + 1

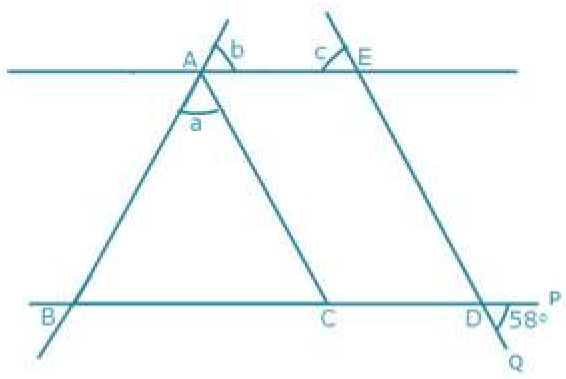
∴ x = 4

10. In the given figure; AE||BD, AC||ED and AB=AC. Find ∠a, ∠b and ∠c.



Solution:

Let P and Q be the points as shown below:



Given: $\angle PDQ = 58°$

$\angle PDQ = \angle EDC = 58°$ [Vertically opp. angles]

$\angle EDC = \angle ACB = 58°$ [Corresponding angles ∵ AC || ED]

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle ACB = \angle ABC = 58^\circ$ [angles opp. to equal sides are equal]

Now,

$\angle ACB + \angle ABC + \angle BAC = 180^\circ$

$\Rightarrow 58^\circ + 58^\circ + a = 180^\circ$

$\Rightarrow a = 180^\circ - 116^\circ$

$\Rightarrow a = 64^\circ$

Since $AE \parallel BD$ and AC is the transversal

$\angle ABC = b$ [Corresponding angles]

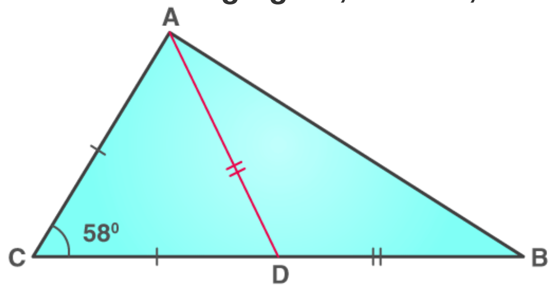
$\therefore b = 58^\circ$

Also since $AE \parallel BD$ and ED is the transversal

$\angle EDC = c$ [Corresponding angles]

$\therefore c = 58^\circ$

11. In the following figure; AC=CD, AD=BD and $\angle$C=58^0.



Find $\angle$CAB.

Solution:

In ΔACD,

$AC = CD$ [Given]

$\therefore \angle CAD = \angle CDA$

$\angle ACD = 58^\circ$ [Given]

$\angle ACD + \angle CDA + \angle CAD = 180^\circ$

$\Rightarrow 58^\circ + 2\angle CAD = 180^\circ$

$\Rightarrow 2\angle CAD = 122^\circ$

$\Rightarrow \angle CAD = \angle CDA = 61^\circ$..........(i)

Now,

$\angle CDA = \angle DAB + \angle DBA$ [Ext. angle is equal to sum of opp. int. angles]

But,

$\angle DAB = \angle DBA$ [Given : AD = DB]

$\therefore \angle DAB + \angle DAB = \angle CDA$

$\Rightarrow 2\angle DAB = 61^\circ$

$\Rightarrow \angle DAB = 30.5^\circ$..........(ii)

In ΔABC,

$\angle CAB = \angle CAD + \angle DAB$

$\therefore \angle CAB = 61^\circ + 30.5^\circ$

$\Rightarrow \angle AB = 91.5^\circ$

12. In the figure of Q.no.11, given above, if AC=AD=CD=BD; find $\angle$ ABC.

Solution:

In ΔACD,

AC = AD = CD [Given]

Hence, ACD is an equilateral triangle

$\therefore \angle ACD = \angle CDA = \angle CAD = 60^\circ$

$\angle CDA = \angle DAB + \angle ABD$ [Ext angle is equal to sum of opp. int. angles]

But,

$\angle DAB = \angle ABD$ [Given : AD = DB]

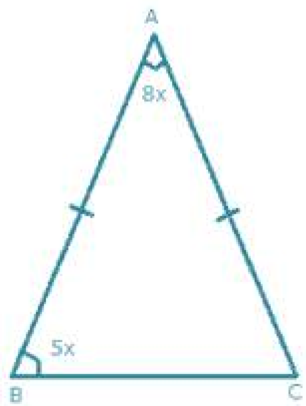
$\therefore \angle ABD + \angle ABD = \angle CDA$

$\Rightarrow 2\angle ABD = 60^\circ$

$\Rightarrow \angle ABD = \angle ABC = 30^\circ$

13. In triangle ABC; AB=AC and $\angle$ A: $\angle$ B= 85; find $\angle$ A.

Solution:

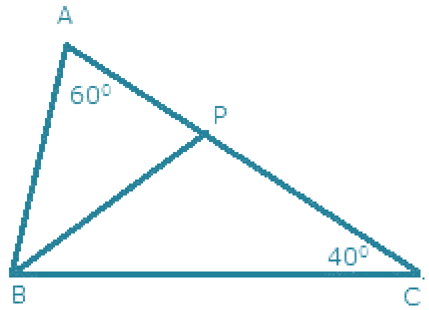


Let $\angle A = 8x$ and $\angle B = 5x$
Given: AB = AC
$\Rightarrow \angle B = \angle C = 5x$ [Angles opp. to equal sides are equal]
Now,
$\angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow 8x + 5x + 5x = 180^\circ$
$\Rightarrow 18x = 180^\circ$
$\Rightarrow x = 10^\circ$

Given that :
$\angle A = 8x$
$\Rightarrow \angle A = 8 \times 10^\circ$
$\Rightarrow \angle A = 80^\circ$

14. In triangle ABC; $\angle A = 60^0$, $\angle C = 40^0$ and bisector of $\angle ABC$ meets side AC at point P. Show that BP=CP.

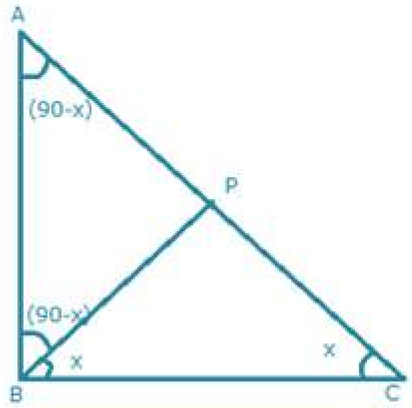
Solution:



In ΔABC,
$\angle A = 60^\circ$
$\angle C = 40^\circ$
$\therefore \angle B = 180^\circ - 60^\circ - 40^\circ$
$\Rightarrow \angle B = 80^\circ$
Now,
BP is the bisector of $\angle ABC$
$\therefore \angle PBC = \dfrac{\angle ABC}{2}$
$\Rightarrow \angle PBC = 40^\circ$
In ΔPBC
$\angle PBC = \angle PCB = 40^\circ$
$\therefore BP = CP$ [Sides opp. to equal angles are equal]

15. In triangle ABC; ∠ABC=90^0, and P is a point on AC such that ∠PBC=∠PCB. Show that PA=PB.

Solution:



Let ∠PBC = ∠PCB = x

In the right angled triangle ABC,

$\angle ABC = 90°$

$\angle ACB = x$

$\Rightarrow \angle BAC = 180° - (90° + x)$

$\Rightarrow \angle BAC = (90° - x)$.........(i)

And

$\angle ABP = \angle ABC - \angle PBC$

$\Rightarrow \angle ABP = 90° - x$........(ii)

Hence in the triangle ABP;

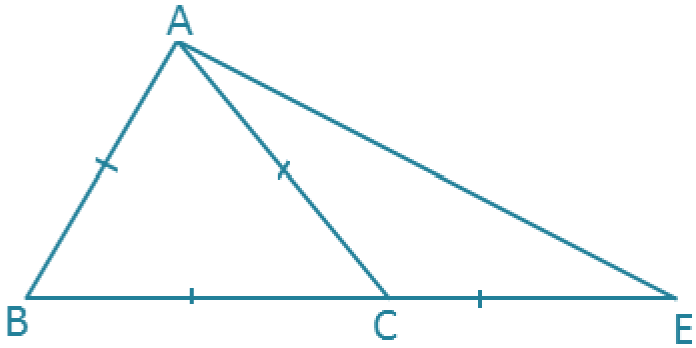
$\angle BAP = \angle ABP$

Hence,

PA = PB [sides opp. to equal angles are equal]

16. ABC is an equilateral triangle. Its side BC is produced upto point E such that C is midpoint of BE. Calculate the measure of angles ACE and AEC.

Solution:



ΔABC is an equilateral triangle

$\Rightarrow$ Side AB = Side AC

$\Rightarrow \angle ABC = \angle ACB$ [If two sides of a triangle are equal, then angles opposite to them are equal]

Similarly, Side AC = Side BC

$\Rightarrow \angle CAB = \angle ABC$ [If two sides of a triangle are equal, then angles opposite to them are equal]

Hence, $\angle ABC = \angle CAB = \angle ACB = y$ (say)

As the sum of all the angles of the triangle is $180°$

$\angle ABC + \angle CAB + \angle ACB = 180^0$

$\Rightarrow 3y = 180°$

$\Rightarrow y = 60°$

$\angle ABC = \angle CAB = \angle ACB = 60°$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$\Rightarrow \angle CAB + \angle CBA = \angle ACE$

$\Rightarrow 60° + 60^0 = \angle ACE$

$\Rightarrow \angle ACE = 120°$

Now ΔACE is an isosceles triangle with AC = CF

$\Rightarrow \angle EAC = \angle AEC$

Sum of all the angles of a triangle is $180°$

$\angle EAC + \angle AEC + \angle ACE = 180°$

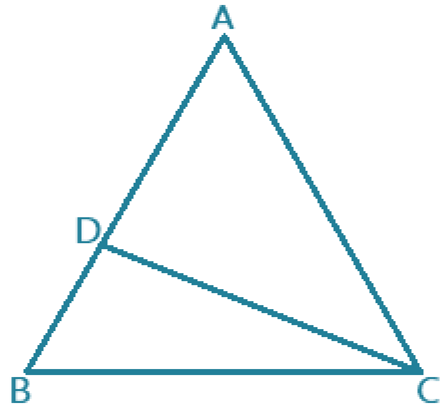
$\Rightarrow 2\angle AEC + 120° = 180°$

$\Rightarrow 2\angle AEC = 180° - 120°$

$\Rightarrow \angle AEC = 30°$

17. In triangle ABC, D is a point in AB such that AC=CD=DB. If $\angle$B=28^0, find the angle ACD.

Solution:



ΔDBC is an isosceles triangle

As, Side CD = Side DB

$\Rightarrow \angle DBC = \angle DCB$ $\left[\begin{array}{l}\text{If two sides of a triangle are equal, then angles} \\ \text{opposite to them are equal}\end{array}\right]$

And $\angle B = \angle DBC = \angle DCB = 28°$

As the sum of all the angles of the triangle is 180°

$\angle DCB + \angle DBC + \angle BCD = 180^0$

$\Rightarrow 28° + 28° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 180° - 56°$

$\Rightarrow \angle BCD = 124°$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$\Rightarrow \angle DBC + \angle DCB = \angle DAC$

$\Rightarrow 28° + 28° = 56°$

$\Rightarrow \angle DAC = 56°$

Now ΔACD is an isosceles triangle with AC = DC

$\Rightarrow \angle ADC = \angle DAC = 56°$

Sum of all the angles of a triangle is 180°

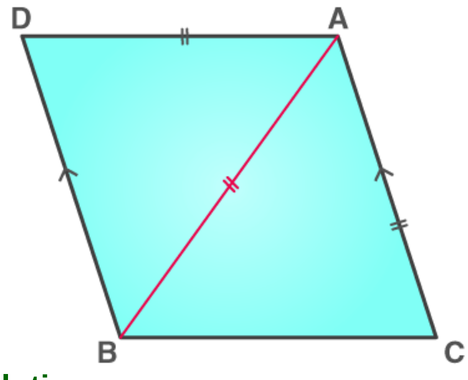
$\angle ADC + \angle DAC + \angle DCA = 180°$

$\Rightarrow 56° + 56° + \angle DCA = 180°$

$\Rightarrow \angle DCA = 180° - 112°$

$\Rightarrow \angle DCA = 64° = \angle ACD$

18. In the given alongside figure, AD=AB=AC, BD is parallel to CA and $\angle$ACB=65^0. Find the $\angle$DAC.



Solution:

We can see that the ΔABC is an isosceles triangle with Side AB = Side AC.

$\Rightarrow \angle ACB = \angle ABC$

As $\angle ACB = 65°$

hence $\angle ABC = 65°$

Sum of all the angles of a triangle is 180°

$\angle ACB + \angle CAB + \angle ABC = 180°$

$65° + 65° + \angle CAB = 180°$

$\angle CAB = 180° - 130°$

$\angle CAB = 50°$

As BD is parallel to CA

Therefore, $\angle CAB = \angle DBA$ since they are alternate angles.

$\angle CAB = \angle DBA = 50°$

We see that ΔADB is an isosceles triangle with Side AD = Side AB.

$\Rightarrow \angle ADB = \angle DBA = 50°$

Sum of all the angles of a trianlge is 180°

$\angle ADB + \angle DAB + \angle DBA = 180°$

$50° + \angle DAB + 50° = 180°$

$\angle DAB = 180^0 - 100° = 80°$

$\angle DAB = 80°$

The angle DAC is sum of angle DAB and CAB.

$\angle DAC = \angle CAB + \angle DAB$

$\angle DAC = 50° + 80°$

$\angle DAC = 130°$

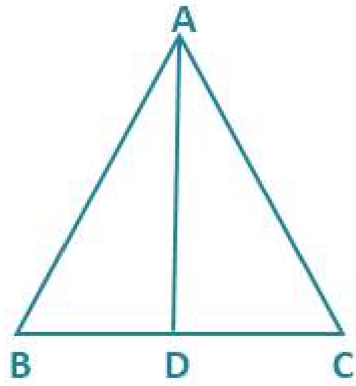
19. Prove that a triangle ABC is isosceles, if:

(i) Altitude AD bisects $\angle$BAC, or

(ii) Bisector of $\angle$BAC is perpendicular to base BC.

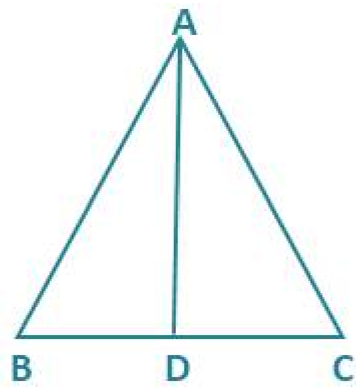
Solution:

(i) In ΔABC, let the altitude AD bisects $\angle BAC$.
Then we have to prove that the ΔABC is isosceles.



In triangles ADB and ADC,
∠BAD = ∠CAD (AD is bisector of ∠BAC)
AD = AD (common)
∠ADB = ∠ADC (Each equal to 90°)
⇒ ΔADB ≅ ΔADC (by ASA congruence criterion)
⇒ AB = AC (cpct)
Hence, ΔABC is an isosceles.

(ii) In Δ ABC, the bisector of ∠ BAC is perpendicular to the base BC. We have to prove that the ΔABC is isosceles.



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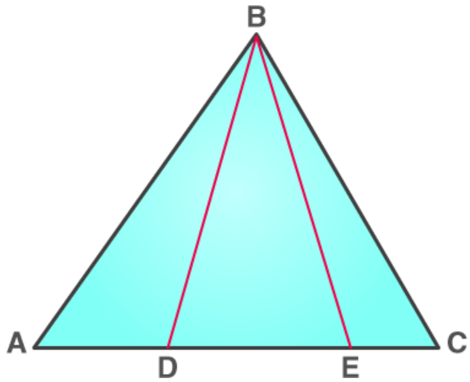
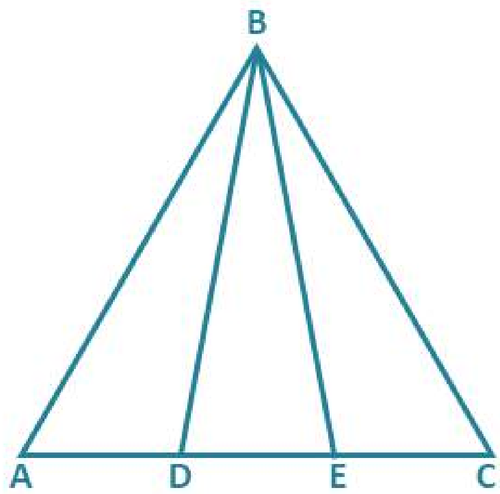
∠ADB = ∠ADC (Each equal to 90°)
⇒ ΔADB ≅ ΔADC (by ASA congruence criterion)
⇒ AB = AC (cpct)
Hence, ΔABC is an isosceles.

**20. In the given figure;
AB=BC and AD=EC.
Prove that: BD=BE.**



Solution:

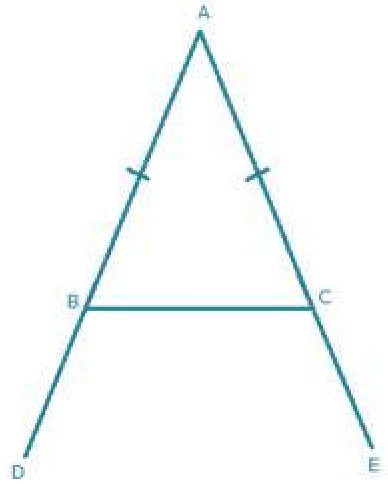


In ΔABC,
AB = BC (given)
⇒ ∠BCA = ∠BAC (Angles opposite to equal sides are equal)
⇒ ∠BCD = ∠BAE ….(i)
Given, AD = EC
⇒ AD + DE = EC + DE (Adding DE on both sides)
⇒ AE = CD ….(ii)
Now, in triangles ABE and CBD,
AB = BC (given)
∠BAE = ∠BCD [From (i)]
AE = CD [From (ii)]
⇒ ΔABE ≅ ΔCBD
⇒ BE = BD (cpct)

EXERCISE 10(B) PAGE: 135

1. If the equal sides of an isosceles triangle are produced, prove that the exterior angles so formed are obtuse and equal.

Solution:



Construction:

AB is produced to D and AC is produced to E so that exterior angles $\angle DBC$ and $\angle ECB$ is formed.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$.......(i) [angles opp. to equal sides are equal]

Since angle B and angle C are acute they cannot be right angles or obtuse angles.

$\angle ABC + \angle DBC = 180°$ [ABD is a st. line]

$\Rightarrow \angle DBC = 180° - \angle ABC$

$\Rightarrow \angle DBC = 180° - \angle B$........(ii)

Similarly,

$\angle ACB + \angle ECB = 180°$ [ABD is a st. line]

$\Rightarrow \angle ECB = 180° - \angle ACB$

$\Rightarrow \angle ECB = 180° - \angle C$..........(iii)

$\Rightarrow \angle ECB = 180° - \angle B$.........(iv) [from (i) and (iii)]

$\Rightarrow \angle DBC = \angle ECB$ [from (ii) and (iv)]

Now,

$\angle DBC = 180^\circ - \angle B$

But $\angle B$ = Acute angle

$\therefore \angle DBC = 180^\circ$ – Acute angle = obtuse angle

Similarly,

$\angle ECB = 180^\circ - \angle C$.

But $\angle C$ = Acute angle

$\therefore \angle ECB = 180^\circ$ – Acute angle = obtuse angle

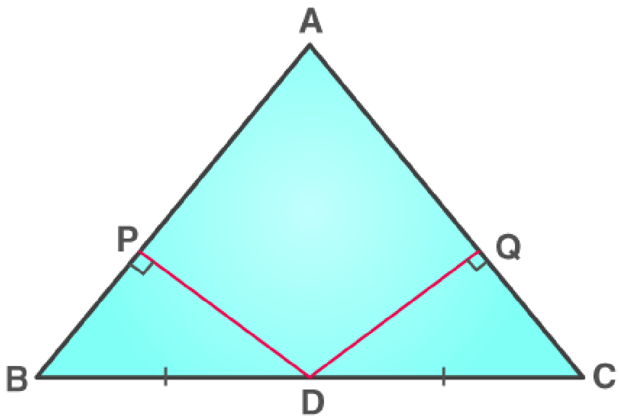
Hence, exterior angles formed are obtuse and equal.

2. In the given figure, AB=AC. Prove that:

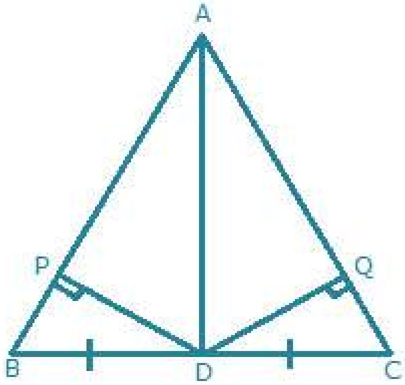
(i) DP=DQ

(ii) AP=AQ

(iii) AD bisects angle A



Solution:



Construction:

Join AD.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

(i)

In ΔBPD and ΔCQD,

$\angle BPD = \angle CQD$ [Each = 90°]

$\angle B = \angle C$ [proved]

$BD = DC$ [Given]

$\therefore \Delta BPD \cong \Delta CQD$ [AAS criterion]

$\therefore DP = DQ$ [cpct]

(ii)

We have already proved that $\Delta BPD \cong \Delta CQD$
Hence,BP = CQ[cpct]
Now,
AB = AC[Given]
$\Rightarrow$AB - BP = AC - CQ
$\Rightarrow$AP = AQ
(iii)

In ΔAPD and ΔAQD,

$DP = DQ$ [proved]

$AD = AD$ [common]

$AP = AQ$ [Proved]

$\therefore \Delta APD \cong \Delta AQD$ [SSS]

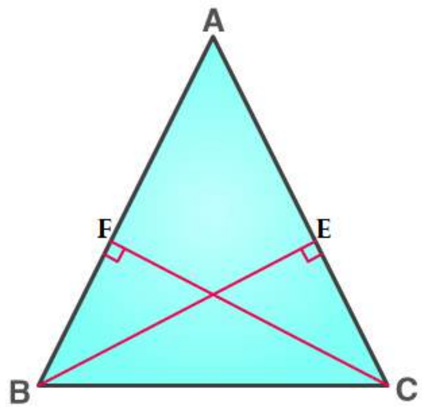
$\Rightarrow \angle PAD = \angle QAD$ [cpct]

Hence, AD bisects angle A.

3. In triangle ABC, AB=AC; BE⊥ AC and CF⊥AB. Prove that:

(i) BE=CF

(ii) AF=AE



Solution:

(i)

In ΔAEB and ΔAFC,

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC = 90^\circ$ [Given: $BE \perp AC$]

[Given: $CF \perp AB$]

$AB = AC$ [Given]

$\Rightarrow \Delta AEB \cong \Delta AFC$ [AAS]

$\therefore BE = CF$ [cpct]

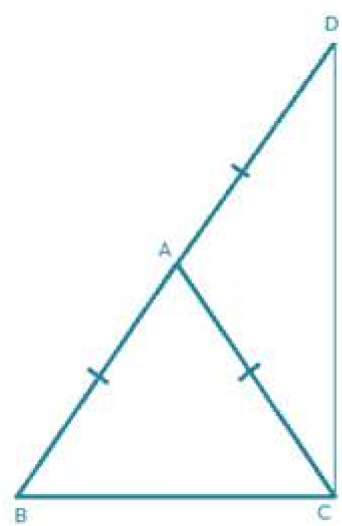
(ii) Since $\Delta AEB \cong \Delta AFC$

$\angle ABE = \angle AFC$

$\therefore AF = AE$ [congruent angles of congruent triangles]

4. In isosceles triangle ABC, AB=AC. The side BA is produced to D such that BA=AD. Prove that: $\angle BCD=90^0$.

Solution:



Construction:

Join CD.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

In ΔACD,

$AC = AD$ [Given]

$\therefore \angle ADC = \angle ACD$..........(ii)

Adding (i) and (ii)

$\angle B + \angle ADC = \angle C + \angle ACD$

$\angle B + \angle ADC = \angle BCD$........(iii)

In ΔBCD,

$\angle B + \angle ADC + \angle BCD = 180^\circ$

$\angle BCD + \angle BCD = 180^\circ$ [From (iii)]

$2\angle BCD = 180^\circ$

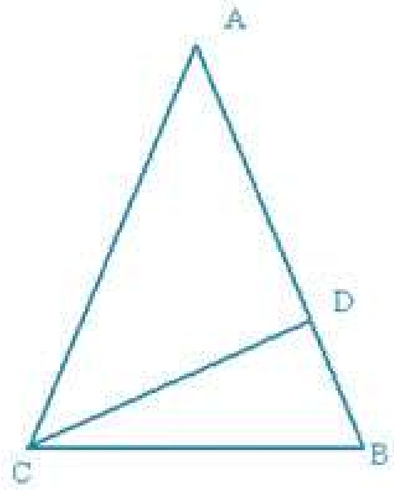
$\angle BCD = 90^\circ$

5.

(i) **In a triangle ABC, AB=AC and $\angle A=36^0$. If the internal bisector of $\angle C$ meets AB at point D, prove that AD=BC.**

(ii) **If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.**

Solution:



AB = AC

ΔABC is an isosceles triangle.

$\angle A = 36^\circ$

$\angle B = \angle C = \dfrac{180^\circ - 36^\circ}{2} = 72^\circ$

$\angle ACD = \angle BCD = 36^\circ$ [$\because$ CD is the angle bisector of $\angle C$]

ΔADC is an isosceles triangle since $\angle DAC = \angle DCA = 36^\circ$

$\therefore$ AD = CD......(i)

In ΔDCB,

$\angle CDB = 180^\circ - (\angle DCB + \angle DBC)$

$= 180^\circ - (36^\circ + 72^\circ)$

$= 180^\circ - 108^\circ$

$= 72^\circ$

ΔDCB is an isosceles triangle since $\angle CDB = \angle CBD = 72^\circ$

$\therefore$ DC = BC.......(ii)

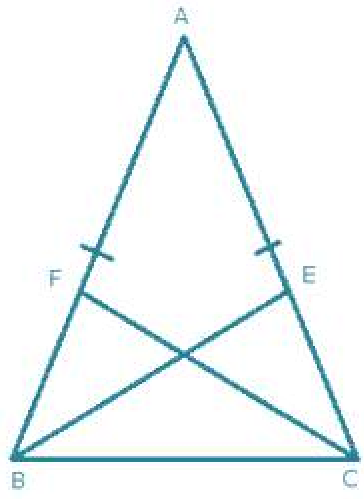
From (i) and (ii), we get

AD = BC

Hence proved

6. Prove that the bisectors of the base angles of an isosceles triangles are equal.

Solution:



In ΔABC,

AB = AC [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle B$

$\Rightarrow \angle BCF = \angle CBE$........(ii)

In ΔBCE and ΔCBF,

$\angle C = \angle B$ [From (i)]

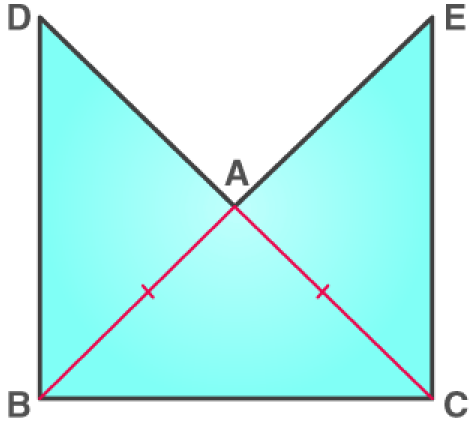
$\angle BCF = \angle CBE$ [From (ii)]

$BC = BC$ [Common]

$\therefore \Delta BCE \cong \Delta CBF$ [AAS]

$\Rightarrow BE = CF$ [cpct]

7. In the given figure, AB=AC and $\angle$DBC= $\angle$ECB= 90^0.
Prove that:
(i) BD=CE
(ii) AD=AE



Solution:

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle ACB = \angle ABC$ [angles opp. to equal sides are equal]

$\Rightarrow \angle ABC = \angle ACB......(i)$

$\angle DBC = \angle ECB = 90^o$[Given]

$\Rightarrow \angle DBC = \angle ECB$ ....(ii)

Subtracting (i) from (ii)

$\angle DCB - \angle ABC = \angle ECB - \angle ACB$

$\Rightarrow \angle DBA = \angle ECA.........(iii)$

In ΔDBA and ΔECA,

$\angle DBA = \angle ECA$ [From (iii)]

$\angle DAB = \angle EAC$ [Vertically opposite angles]

$AB = AC$ [Given]

$\therefore \Delta DBA \cong \Delta ECA$ [ASA]

$\Rightarrow BD = CE$ [cpct]

Also,

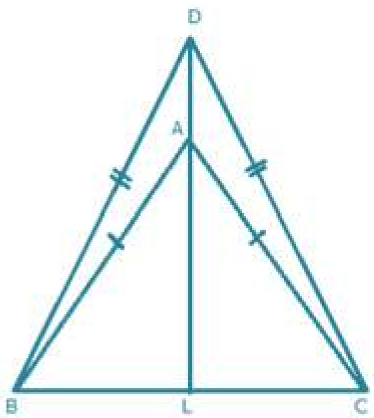
$AD = AE$ [cpct]

8. ABC and DBC are two isosceles triangle on the same side of BC. Prove that:

(i) DA (or AD) produced bisects BC at the right angle

(ii) $\angle BDA = \angle CDA$

Solution:



DA is produced to meet BC in L.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle ACB = \angle ABC$........(i) [angles opposite to equal sides are equal]

In ΔDBC,

$DB = DC$ [Given]

$\therefore \angle DCB = \angle DBC$.......(ii) [angles opposite to equal sides are equal]

Subtracting (i) from (ii)

$\angle DCB - \angle ACB = \angle DBC - \angle ABC$

$\Rightarrow \angle DCA = \angle DBA$.......(iii)

In ΔDBA and ΔDCA,

$DB = DC$ [Given]

$\angle DBA = \angle DCA$ [From (iii)]

$AB = AC$ [Given]

$\therefore \Delta DBA \cong \Delta DCA$ [SAS]

$\Rightarrow \angle BDA = \angle CDA$.........(iv) [cpct]

In ΔDBA,

$\angle BAL = \angle DBA + \angle BDA$.........(v)

[Ext. angle = sum of opp. int. angles]

From (iii), (iv) and (v)

$\angle BAL = \angle DCA + \angle CDA$.......(vi)

In ΔDCA,

$\angle CAL = \angle DCA + \angle CDA$.........(vii)

[Ext. angle = sum of opp. int. angles]

From (vi) and (vii)

$\angle BAL = \angle CAL$.......(viii)

In ΔBAL and ΔCAL,

$\angle BAL = \angle CAL$ [From (viii)]

$\angle ABL = \angle ACL$ [From (i)]

$AB = AC$ [Given]

$\therefore \Delta BAL \cong \Delta CAL$ [ASA]

$\Rightarrow \angle ALB = \angle ALC$ [cpct]

and $BL = LC$..........(ix) [cpct]

Now,

$\angle ALB + \angle ALC = 180°$

$\Rightarrow \angle ALB + \angle ALB = 180°$

$\Rightarrow 2\angle ALB = 180°$

$\Rightarrow \angle ALB = 90°$

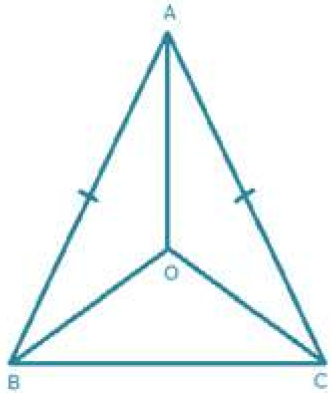
$\therefore AL \perp BC$

or $DL \perp BC$ and $BL = LC$

$\therefore$ DA produced bisects BC at right angle.

9. The bisectors of the equal angles B and C of an isosceles triangle ABC meet at O. Prove that AO bisects angle A.

Solution:



In ΔABC, we have AB = AC

$\Rightarrow \angle$B = $\angle$C [angles opposite to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\Rightarrow \angle OBC = \angle OCB$..........(i)

$\Rightarrow OB = OC$.............(ii)

[angles opposite to equal sides are equal]

Now,

In ΔABO and ΔACO,

AB = AC [Given]

$\angle$OBC = $\angle$OCB [From (i)]

OB = OC [From (ii)]

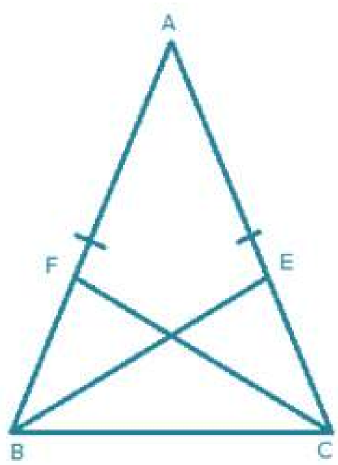
$\Delta ABO \cong \Delta ACO$ [SAS criterion]

$\Rightarrow \angle BAO = \angle CAO$ [cpct]

Hence, AO bisects $\angle$BAC.

10. Prove that the medians corresponding to equal sides of an isosceles triangle are equal.

Solution:



In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BF = CE$........(ii)

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BF = CE$........(ii)

In ΔBCE and ΔCBF,

$\angle C = \angle B$ [From (i)]

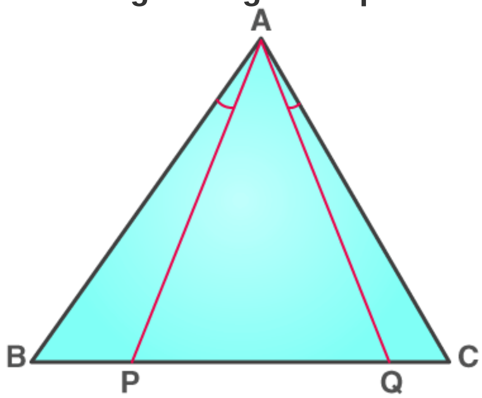
$BF = CE$ [From (ii)]

$BC = BC$ [Common]

$\therefore \Delta BCE \cong \Delta CBF$ [SAS]

$\Rightarrow BE = CF$ [cpct]

11. Use the given figure to prove that, AB=AC.



Solution:

In ΔAPQ,

$AP = AQ$ [Given]

$\therefore \angle APQ = \angle AQP$........(i)

[angles opposite to equal sides are equal]

In ΔABP,

$\angle APQ = \angle BAP + \angle ABP$.......(ii)

[Ext. angle is equal to sum of opp. int. angles]

In ΔAQC,

$\angle AQP = \angle CAQ + \angle ACQ$.......(iii)

[Ext. angle is equal to sum of opp. int. angles]

From (i), (ii) and (iii)

$\angle BAP + \angle ABP = \angle CAQ + \angle ACQ$

But, $\angle BAP = \angle CAQ$ [Given]

$\Rightarrow \angle CAQ + \angle ABP = \angle CAQ + \angle ACQ$

$\Rightarrow \angle ABP = \angle CAQ + \angle ACQ - \angle CAQ$

$\Rightarrow \angle ABP = \angle ACQ$

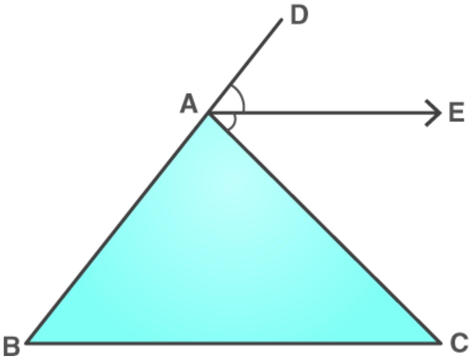
$\Rightarrow \angle B = \angle C$........(iv)

In ΔABC,

$\angle B = \angle C$

$\Rightarrow AB = AC$ [Sides opposite to equal angles are equal]

12. In the given figure; AE bisects exterior angle CAD and AE is parallel to BC. Prove that: AB=AC.



Solution:

Since AE || BC and DAB is the transversal

$\therefore \angle DAE = \angle ABC = \angle B$ [Corresponding angles]

Since AE || BC and AC is the transversal

$\angle CAE = \angle ACB = \angle C$ [Alternate Angles]

But AE bisects $\angle CAD$

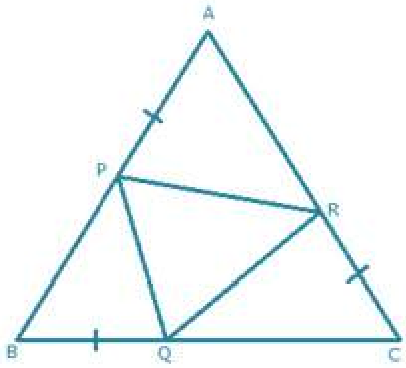
$\therefore \angle DAE = \angle CAE$

$\Rightarrow \angle B = \angle C$

$\Rightarrow$ AB = AC [Sides opposite to equal angles are equal]

13. In an equilateral triangle ABC; points P, Q and R are taken on the sides AB, BC and CA respectively such that AP= BQ= CR. Prove that triangle PQR is equilateral.

Solution:



AB = BC = CA…….(i) [Given]
AP = BQ = CR…….(ii) [Given]
Subtracting (ii) from (i)
AB - AP = BC - BQ = CA - CR
BP = CQ = AR …………(iii)
$\therefore \angle A = \angle B = \angle C$ ……..(iv) [angles opp. to equal sides are equal]

In ΔBPQ and ΔCQR,

BP = CQ [From (iii)]

$\angle B = \angle C$ [From (iv)]

BQ = CR [Given]

$\therefore \Delta BPQ \cong \Delta CQR$ [SAS criterion]

$\Rightarrow PQ = QR$……..(v)

In ΔCQR and ΔAPR,

CQ = AR [From (iii)]

$\angle C = \angle A$ [From (iv)]

CR = AP [Given]

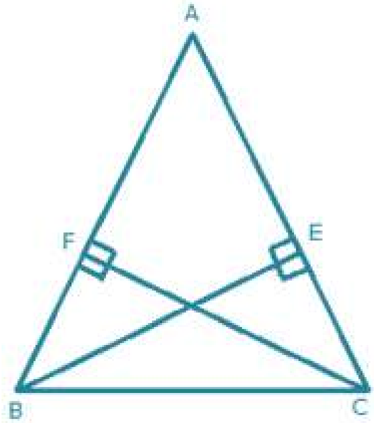
$\therefore \Delta CQR \cong \Delta APR$ [SAS criterion]

$\Rightarrow QR = PR$……..(vi)

From (v) and (vi)
PQ = QR = PR
Hence, PQR is an equilateral triangle.

14. In triangle ABC, altitudes BE and CF are equal. Prove that the triangle is isosceles.

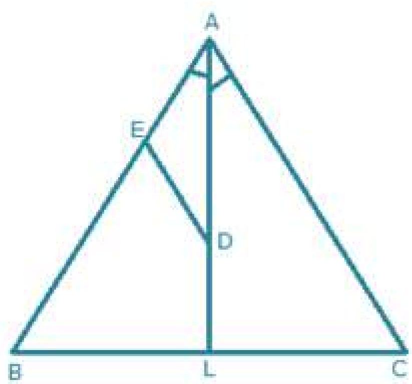
Solution:



In ΔABE and ΔACF,
$\angle$A = $\angle$A[Common]
$\angle$AEB = $\angle$AFC = 90^0[Given: BE $\perp$AC; CF $\perp$AB]
BE = CF[Given]
$\therefore \Delta ABE \cong \Delta ACF$ [AAS criterion]
$\Rightarrow AB = AC$
Hence, ABC is an isosceles triangle.

15. Through any point in the bisector of an angle, a straight line is drawn parallel to either arm of the angle. Prove that the triangle so formed is isosceles.

Solution:



AL is bisector of angle A. Let D is any point on AL. From D, a straight line DE is drawn parallel to AC.
DE || AC [Given]
$\therefore$ $\angle$ADE = $\angle$DAC….(i) [Alternate angles]
$\angle$DAC = $\angle$DAE…….(ii) [AL is bisector of $\angle$A]
From (i) and (ii)
$\angle$ADE = $\angle$DAE
$\therefore$ AE = ED [Sides opposite to equal angles are equal]
Hence, AED is an isosceles triangle.

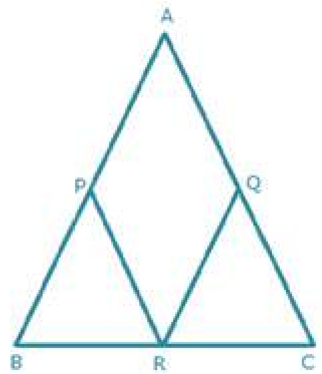
16. **In triangle ABC; AB=AC. P, Q and R are mid-points of sides AB, AC and BC respectively. Prove that:**

(i) PR= QR

(ii) BQ = CP

Solution:

(i)



In ΔABC,

AB = AC

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow$AP = AQ …….(i)[Since P and Q are mid - points]

In ΔBCA,

PR = $\frac{1}{2}AC$ [PR is line joining the mid - points of AB and BC]

$\Rightarrow$PR = AQ……..(ii)

In ΔCAB,

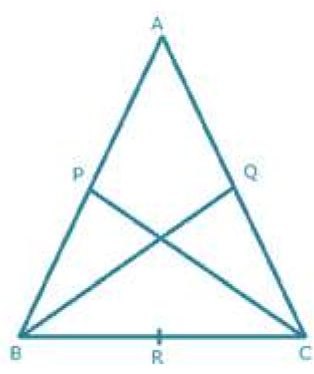
QR = $\frac{1}{2}AB$ [QR is line joining the mid - points of AC and BC]

$\Rightarrow$QR = AP……(iii)

From (i), (ii) and (iii)

PR = QR

(ii)



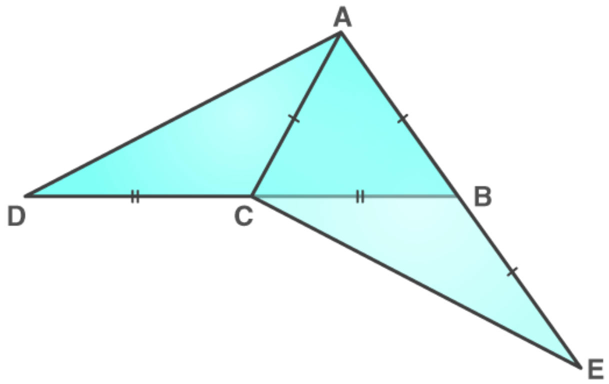
AB = AC

$\Rightarrow \angle B = \angle C$
Also,
$\frac{1}{2}AB = \frac{1}{2}AC$
$\Rightarrow BP = CQ$ [P and Q are mid-points of AB and AC]
In ΔBPC and ΔCQB,
BP = CQ
$\angle B = \angle C$
BC = BC
Hence, $\Delta BPC \cong \Delta CQB$ [SAS]
BP = CP

17. From the following figure, prove that:

(i) $\angle$ACD = $\angle$CBE

(ii) AD=CE



Solution:

(i) In ΔACB,
AC = AC[Given]
$\therefore \angle ABC = \angle ACB$ …….(i)[angles opposite to equal sides are equal]
$\angle ACD + \angle ACB = 180^0$ …….(ii)[DCB is a straight line]
$\angle ABC + \angle CBE = 180^0$ ……..(iii)[ABE is a straight line]
Equating (ii) and (iii)
$\angle ACD + \angle ACB = \angle ABC + \angle CBE$
$\Rightarrow \angle ACD + \angle ACB = \angle ACB + \angle CBE$[From (i)]
$\Rightarrow \angle ACD = \angle CBE$

(ii)

In ΔACD and ΔCBE,

$DC = CB$ [Given]

$AC = BE$ [Given]

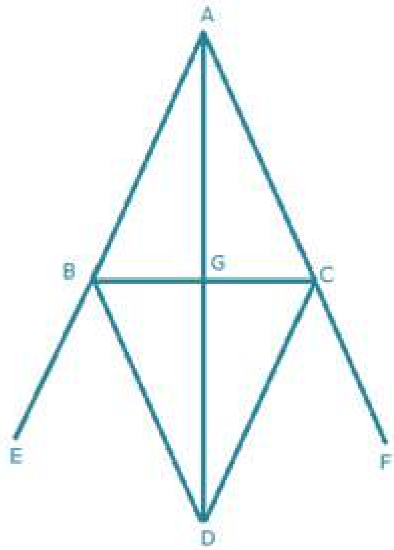
$\angle ACD = \angle CBE$ [Proved Earlier]

$\therefore \Delta ACD \cong \Delta CBE$ [SAS criterion]

$\Rightarrow AD = CE$ [cpct]

18. Equal sides AB and AC of an isosceles triangle ABC are produced. The bisectors of the exterior angles so formed meet at D. Prove that AD bisects angle A.

Solution:



AB is produced to E and AC is produced to F. BD is bisector of angle CBE and CD is bisector of angle BCF. BD and CD meet at D.

In ΔABC,

AB = AC[Given]

$\therefore \angle C = \angle B$[angles opposite to equal sides are equal]

$\angle CBE = 180^0 - \angle B$[ABE is a straight line]

$\Rightarrow \angle CBD = \dfrac{180^\circ - \angle B}{2}$ [BD is bisector of $\angle CBE$]

$\Rightarrow \angle CBD = 90^\circ - \dfrac{\angle B}{2}$.............(i)

Similarly,

$\angle BCF = 180^0 - \angle C$[ACF is a straight line]

$\Rightarrow \angle BCD = \dfrac{180^\circ - \angle C}{2}$ [CD is bisector of $\angle BCF$]

$\Rightarrow \angle BCD = 90^\circ - \dfrac{\angle C}{2}$.............(ii)

Now,

$\Rightarrow \angle CBD = 90° - \dfrac{\angle C}{2}$ $[\because \angle B = \angle C]$

$\Rightarrow \angle CBD = \angle BCD$

In ΔBCD,

$\angle CBD = \angle BCD$

$\therefore$ BD = CD

In ΔABD and ΔACD,

AB = AC[Given]

AD = AD[Common]

BD = CD[Proved]

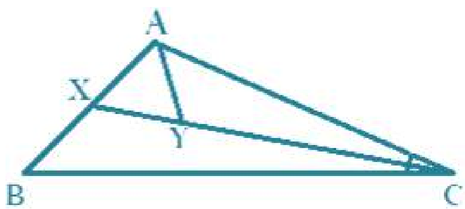
$\therefore \Delta ABD \cong \Delta ACD$ [SSS criterion]

$\Rightarrow \angle BAD = \angle CAD$ [cpct]

Hence, AD bisects $\angle$ A.

19. ABC is a triangle. The bisector of the angle BCA meets AB in X. A point Y lies on CX such that AX=AY.
Prove that: $\angle$ CAY = $\angle$ ABC

Solution:



In ΔABC,

CX is the angle bisector of $\angle$ C

$\Rightarrow \angle$ ACY = $\angle$ BCX....... (i)

In ΔAXY,

AX = AY [Given]

$\angle$ AXY = $\angle$ AYX …….(ii) [angles opposite to equal sides are equal]

Now $\angle$ XYC = $\angle$ AXB = 180° [straight line]

$\Rightarrow \angle$ AYX + $\angle$ AYC = $\angle$ AXY + $\angle$ BXY

$\Rightarrow \angle$ AYC = $\angle$ BXY........ (iii) [From (ii)]

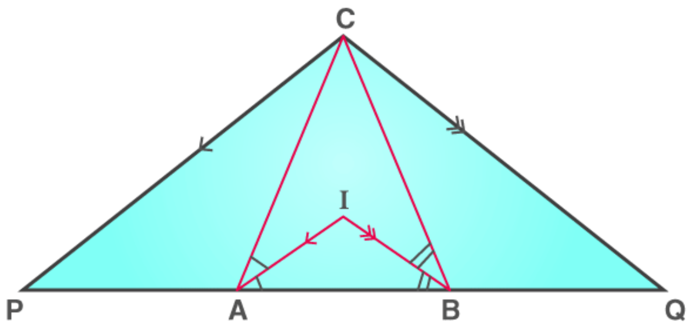
In ΔAYC and ΔBXC

$\angle$ AYC + $\angle$ ACY + $\angle$ CAY = $\angle$ BXC + $\angle$ BCX + $\angle$ XBC = 180°

$\Rightarrow \angle$ CAY = $\angle$ XBC [From (i) and (iii)]

$\Rightarrow$ $\angle$ CAY = $\angle$ ABC

20. In the following figures; IA and IB are bisector of angles CAB and CBA respectively. CP is parallel to IA and CQ is parallel to IB.



Prove that:

PQ= The perimeter of the triangle ABC.

Solution:

Since IA || CP and CA is a transversal

∴ ∠CAI = ∠PCA [Alternate angles]

Also, IA || CP and AP is a transversal

∴ ∠IAB = ∠APC [Corresponding angles]

But ∴ ∠CAI = ∠IAB [Given]

∴ ∠PCA = ∠APC

⇒AC = AP

Similarly,

BC = BQ

Now,

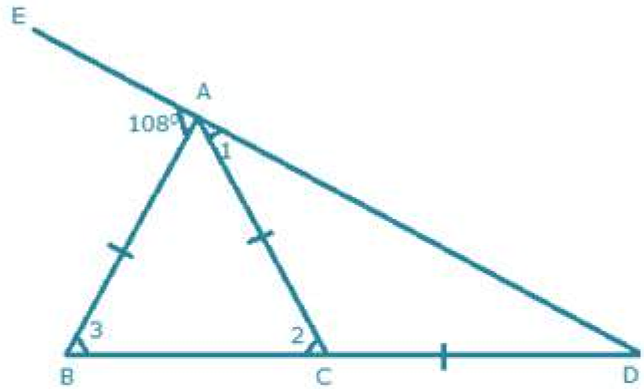
PQ = AP + AB + BQ

= AC + AB + BC

= Perimeter of △ABC

21. **Sides AB and AC of a triangle ABC are equal. BC is produced through C upto point D such that AC=CD. D and A are joined and produced (through vertex A) upto point E. If angle BAE=108^0; find angle ADB.**

Solution:



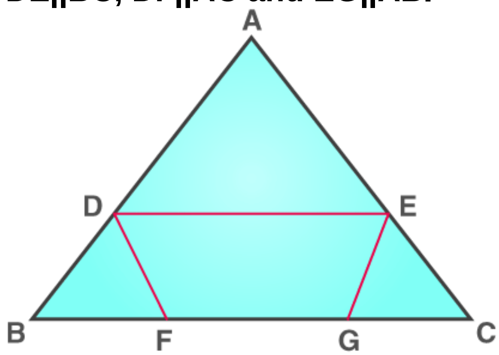
In △ABD,

∠BAE = ∠3 + ∠ADB

⇒108^0 = ∠3 + ∠ADB

But AB = AC
⇒∠3 = ∠2
⇒108^0 = ∠2 + ∠ADB ……(i)
Now,
In ΔACD,
∠2=∠1+ ∠ADB
But AC = CD
⇒∠1 = ∠ADB
⇒∠2 = ∠ADB + ∠ADB
⇒∠2 = 2∠ADB
Putting this value in (i)
⇒108^0 = 2∠ADB + ∠ADB
⇒3∠ADB = 108^0
⇒∠ADB = 36^0

22. The given figure shows an equilateral triangle ABC with each side 15 cm. Also DE||BC, DF||AC and EG||AB.



If DE+DF+EG=20cm, find FG.

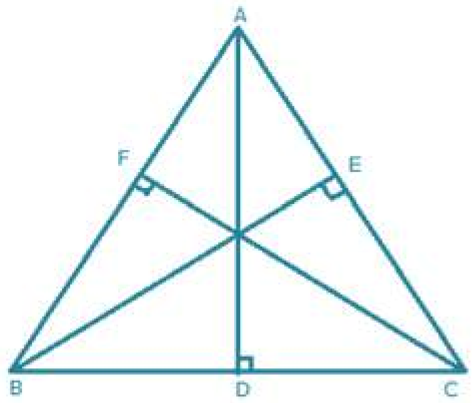
Solution:

ABC is an equilateral triangle.
Therefore, AB = BC = AC = 15 cm
∠A = ∠B = ∠C = 60°
In ΔADE, DE || BC[Given]
∠AED = 60° [∵ ∠ACB=60°]
∠ADE = 60° [∵ ∠ABC=60°]
∠DAE = 180° − (60° + 60°) = 60°
Similarly, ΔBDF & ΔGEC are equilateral triangles.
=60° [∵ ∠C = 60°]
Let AD = x, AE = x, DE = x [∵ ΔADE is an equilateral triangle]
Let BD = y, FD = y, FB = y [∵ ΔBDF is an equilateral triangle]
Let EC = z, GC = z, GE = z [∵ ΔGEC is an equilateral triangle]

Now, AD + DB = 15 $\Rightarrow x + y = 15$......(*i*)
AE + EC = 15 $\Rightarrow x + z = 15$.....(*ii*)
Given, DE + DF + EG = 20
$\Rightarrow x + y + z = 20$
$\Rightarrow 15 + z = 20$ [from (i)]
$\Rightarrow z = 5$
From (ii), we get $x = 10$
$\therefore y = 5$
Also, BC = 15
BF + FG + GC = 15
$\Rightarrow y$ + FG + z = 15
$\Rightarrow$ 5 + FG + 5 = 15
$\Rightarrow$ FG = 5

23. If all the three altitudes of a triangle are equal, the triangles are equilateral. Prove it.

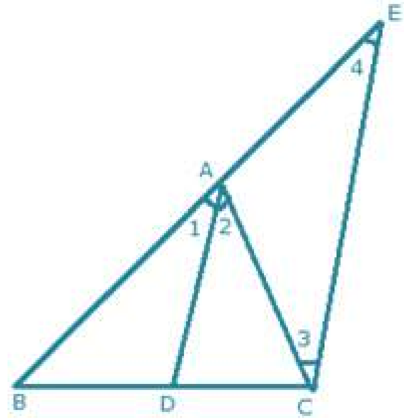
Solution:



In right $\triangle$BEC and $\triangle$BFC,
BE = CF[Given]
BC = BC[Common]
$\angle$BEC = $\angle$BFC[each = 90^0]
$\therefore \triangle BEC \cong \triangle BFC$ [RHS]
$\Rightarrow \angle B = \angle C$
Similarly,
$\angle A = \angle B$
Hence, $\angle A = \angle B = \angle C$
$\Rightarrow AB = BC = AC$
Hence, ABC is an equilateral triangle.

24. In a triangle ABC, the internal bisector of angle A meets opposite side BC at point D. Through vertex C, line CE is drawn parallel to DA which meets BA produced at point E. Show that triangle ACE is isosceles.

Solution:



DA || CE[Given]

$\Rightarrow \angle 1 = \angle 4 \ldots\ldots\ldots(i)$ [Corresponding angles]

$\angle 2 = \angle 3 \ldots\ldots\ldots(ii)$ [Alternate angles]

But $\angle 1 = \angle 2 \ldots\ldots\ldots(iii)$ [AD is the bisector of $\angle$ A]

From (i), (ii) and (iii)

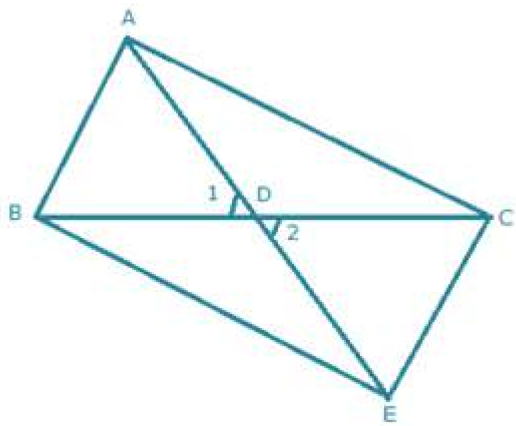
$\angle 3 = \angle 4$

$\Rightarrow$ AC = AE

$\Rightarrow$ ΔACE is an isosceles triangle.

25. In triangle ABC, bisector of angle BAC meets opposite side BC at point D. If BD = CD, prove that triangle ABC is isosceles.

Solution:



Produce AD upto E such that AD = DE.

In ΔABD and ΔEDC,

$AD = DE$ [by construction]

$BD = CD$ [Given]

$\angle 1 = \angle 2$ [vertically opposite angles]

$\therefore \Delta ABD \cong \Delta EDC$ [SAS]

$\Rightarrow AB = CE$.......(i)

and $\angle BAD = \angle CED$

But, $\angle BAD = \angle CAD$ [AD is bisector of $\angle BAC$]

$\therefore \angle CED = \angle CAD$

$\Rightarrow AC = CE$......(ii)

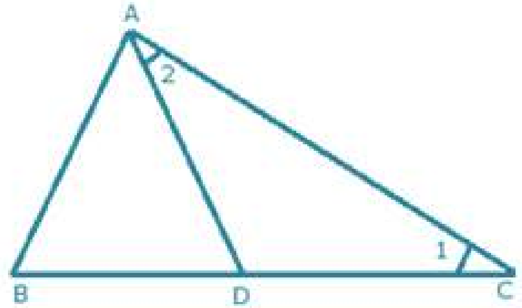
From (i) and (ii)

$AB = AC$

Hence, ABC is an isosceles triangle.

26. In triangle ABC, D is a point on BC such that AB = AD = BD= DC. Show that: $\angle$ADC: $\angle$C= 4 : 1

Solution:



Since AB = AD = BD

$\therefore$ ΔABD is an equilateral triangle.

$\therefore \angle ADB = 60^\circ$

$\Rightarrow \angle ADC = 180^\circ - \angle ADB$

$= 180^\circ - 60^\circ$

$= 120^\circ$

Again in ΔADC,

AD = DC

$\therefore \angle 1 = \angle 2$

But,

$\angle 1 + \angle 2 + \angle ADC = 180^\circ$

$\Rightarrow 2\angle 1 + 120^\circ = 180^\circ$

$\Rightarrow 2\angle 1 = 60^\circ$

$\Rightarrow \angle 1 = 30^\circ$

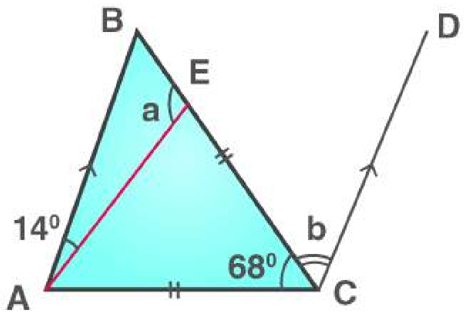
$\Rightarrow \angle C = 30^\circ$

$\therefore \angle ADC : \angle C = 120^\circ : 30^\circ$

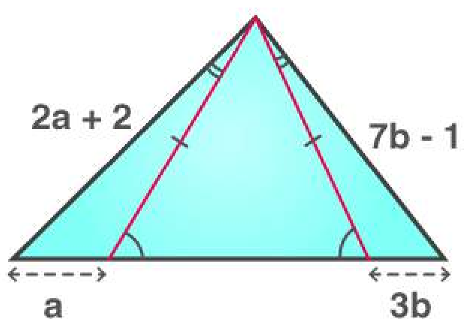
$\Rightarrow \angle ADC : \angle C = 4 : 1$

27. Using the information given in each of the following figures, find the value of a and b.

(i)



(ii)



[Given: CE = AC]

Solution:

(i)

In ΔCAE, $\angle CAE = \angle AEC = \dfrac{180^\circ - 68^\circ}{2} = 56^\circ$ [$\because$ CE=AC]

In $\angle BEA$, $a = 180^\circ - 56^\circ = 124^\circ$

In ΔABE, $\angle ABE = 180^\circ - (a + \angle BAE)$

$= 180^\circ - (124^\circ + 14^\circ)$

$= 180^\circ - 138^\circ = 42^\circ$

(ii)

In ΔAEB & ΔCAD,

$\angle EAB = \angle CAD$ [Given]

$\angle ADC = \angle AEB$ [$\because \angle ADE = \angle AED$ {AE=AD}

$180^\circ - \angle ADE = 180^\circ - \angle AED$

$\angle ADC = \angle AEB$]

AE=AD [Given]

$\therefore \Delta AEB \cong \Delta CAD$ [ASA]

AC=AB [By C.P.C.T.]

$2a+2=7b-1$

$\Rightarrow 2a-7b=-3$.....(i)

CD=EB

$\Rightarrow a=3b$.......(ii)

Solving (i) & (ii), we get

$a=9$, $b=3$