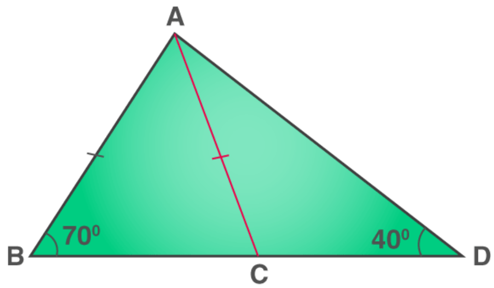
EXERCISE 11

1. **From the following figure, prove that:**
AB>CD

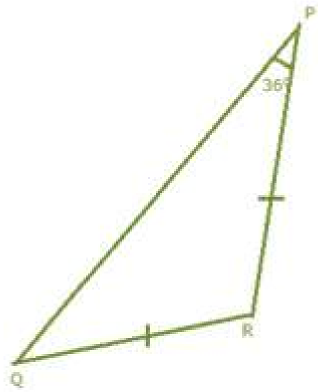


Solution:

In Δ ABC,
AB = AC[Given]
$\therefore \angle$ ACB = $\angle$ B[angles opposite to equal sides are equal]
$\angle$ B = 70^0[Given]
$\Rightarrow \angle$ ACB = 70^0 ……….(i)
Now,
$\angle$ ACB +$\angle$ ACD = 180^0[BCD is a straight line]
$\Rightarrow 70^0 + \angle$ ACD = 180^0
$\Rightarrow \angle$ ACD = 110^0 ………..(ii)
In Δ ACD,
$\angle$ CAD + $\angle$ ACD + $\angle$ D = 180^0
$\Rightarrow \angle$ CAD + 110^0 + $\angle$ D = 180^0 [From (ii)]
$\Rightarrow \angle$ CAD + $\angle$ D = 70^0
But $\angle$ D = 40^0 [Given]
$\Rightarrow \angle$ CAD + 40^0= 70^0
$\Rightarrow \angle$ CAD = 30^0 ………………(iii)
In Δ ACD,
$\angle$ ACD = 110^0[From (ii)]
$\angle$ CAD = 30^0[From (iii)]
$\angle$ D = 40^0 [Given]
$\therefore \angle D > \angle CAD$
$\Rightarrow AC > CD$
[Greater angle has greater side opposite to it]
Also,
AB = AC[Given]
Therefore, AB > CD.

2. **In a triangle PQR; QR = PR and $\angle$ P=360. Which is the largest side of the triangle?**

Solution:



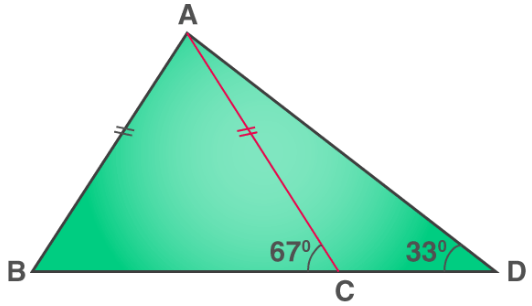
In $\triangle$PQR,
QR = PR[Given]
$\therefore \angle$P = $\angle$Q[angles opposite to equal sides are equal]
$\angle$P = 36^0[Given]
$\Rightarrow \angle$Q = 36^0
In $\triangle$PQR,
$\angle$P + $\angle$Q + $\angle$R = 180^0
$\Rightarrow 36^0 + 36^0 + \angle$R = 180^0
$\Rightarrow \angle$R + 72^0 = 180^0
$\Rightarrow \angle$R = 108^0
Now,
$\angle$R = 108^0
$\angle$P = 36^0
$\angle$Q = 36^0
Since $\angle$R is the greatest, therefore, PQ is the largest side.

3. If two sides of a triangle are 8 cm and 13 cm, then the length of the third side is between a cm and b cm. Find the values of a and b such that a is less than b.

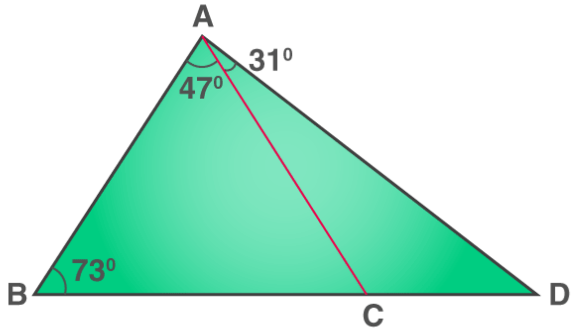
Solution:

We know that,
The sum of any two sides of the triangle is always greater than third side of the triangle.
Third side < 13 + 8 = 21 cm.
We also know that,
The difference between any two sides of the triangle is always less than the third side of the triangle.
Third side > 13 - 8 = 5 cm.
Therefore, the length of the third side is between 5 cm and 9 cm, respectively.
The value of a = 5 cm and b = 21 cm.

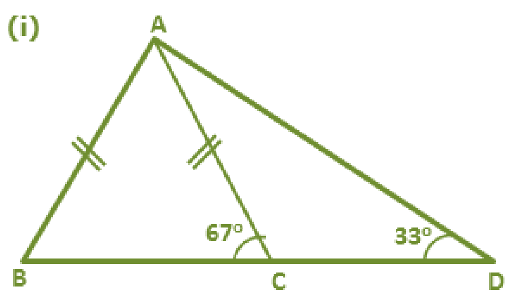
4. In each of the following figures write BC, AC and CD in ascending order of their lengths.
(i)



(ii)



Solution:



In ΔABC,

$AB = AC$

$\Rightarrow \angle ABC = \angle ACB$ (angles opposite to equal sides are equal)

$\Rightarrow \angle ABC = \angle ACB = 67°$

$\Rightarrow \angle BAC = 180° - \angle ABC - \angle ACB$ (angle sum property)

$\Rightarrow \angle BAC = 180° - 67° - 67° = 46°$

Since $\angle BAC < \angle ABC$, we have

$BC < AC$ ....(1)

Now, $\angle ACD = 180° - \angle ACB$ (linear pair)

$\Rightarrow \angle ACD = 180° - 67° = 113°$

Thus, in ΔACD,

$\angle CAD = 180° - \angle ACD - \angle ADC$

$\Rightarrow \angle CAD = 180° - 113° - 33° = 34°$

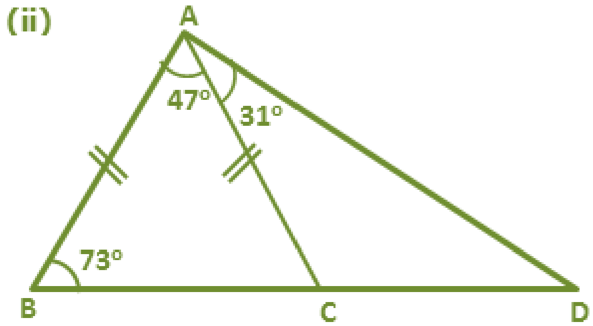
Since $\angle ADC < \angle CAD$, we have

$AC < CD$ ....(2)

From (1) and (2), we have

$BC < AC < CD$

(ii)



In ΔABC,

$\angle BAC < \angle ABC$

$\Rightarrow BC < AC \quad ....(1)$

Now, $\angle ACB = 180° - \angle ABC - \angle BAC$

$\Rightarrow \angle ACB = 180° - 73° - 47°$

$\Rightarrow \angle ACB = 60°$

Now, $\angle ACD = 180° - \angle ACB$

$\Rightarrow \angle ACD = 180° - 60° = 120°$

Now, in ΔACD,

$\angle ADC = 180° - \angle ACD - \angle CAD$

$\Rightarrow \angle ADC = 180° - 120° - 31°$

$\Rightarrow \angle ADC = 29°$

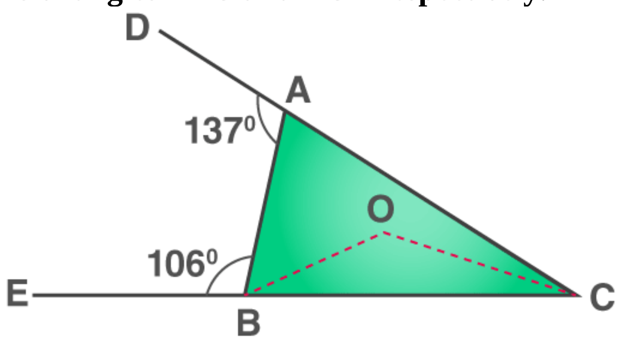
Since $\angle ADC < \angle CAD$, we have

$AC < CD \quad ....(2)$

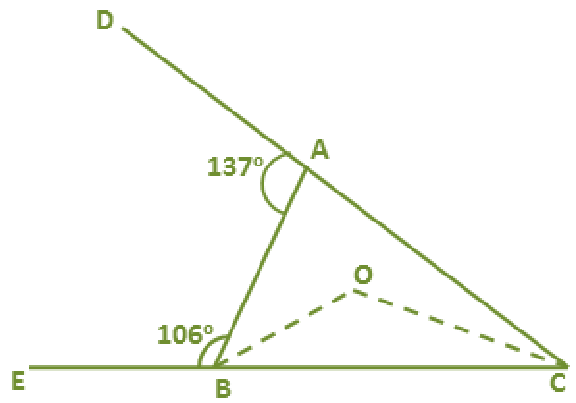
From (1) and (2), we have

$BC < AC < CD$

5. **Arrange the sides of triangle BOC in descending order of their lengths. BO and CO are bisectors of angles ABC and ACB respectively.**



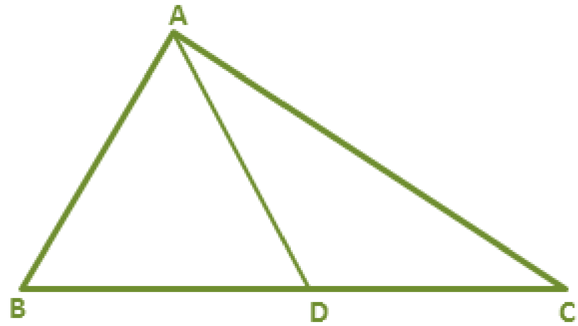
Solution:



$\angle BAC = 180° - \angle BAD = 180° - 137° = 43°$
$\angle ABC = 180 - \angle ABE = 180° - 106° = 74°$
Thus, in ΔABC,
$\angle ACB = 180° - \angle BAC - \angle ABC$
$\Rightarrow \angle ACB = 180° - 43° - 74° = 63°$
Now, $\angle ABC = \angle OBC + \angle ABO$
$\Rightarrow \angle ABC = 2\angle OBC$ (OB is biosector of $\angle ABC$)
$\Rightarrow 74° = 2\angle OBC$
$\Rightarrow \angle OBC = 37°$
Similarly,
$\angle ACB = \angle OCB + \angle ACO$
$\Rightarrow \angle ACB = 2\angle OCB$ (OC is bisector of $\angle ACB$)
$\Rightarrow 63° = 2\angle OCB$
$\Rightarrow \angle OCB = 31.5°$
Now, in ΔBOC,
$\angle BOC = 180° - \angle OBC - \angle OCB$
$\Rightarrow \angle BOC = 180° - 37° - 31.5°$
$\Rightarrow \angle BOC = 111.5°$
Since, $\angle BOC > \angle OBC > \angle OCB$, we have
$BC > OC > OB$

6. D is a point in side BC of triangle ABC. If AD>AC, show that AB>AC.

Solution:



AD > AC (given)

$\Rightarrow \angle C > \angle ADC$ ....(1)

Now, $\angle ADC > \angle B + \angle BAC$ (Exterior Angle Property)

$\Rightarrow \angle ADC > \angle B$ ....(2)

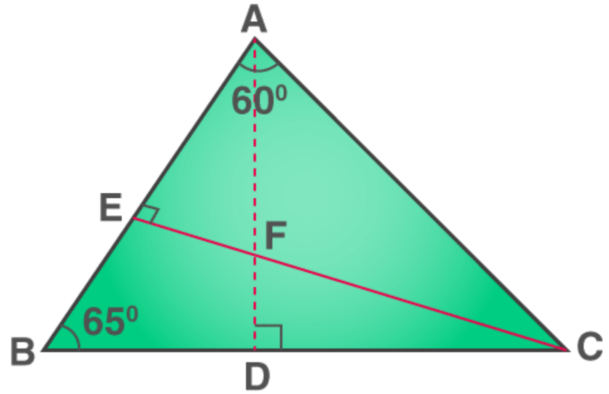
From (1) and (2), we have

$\angle C > \angle ADC > \angle B$

$\Rightarrow \angle C > \angle B$

$\Rightarrow AB > AC$

7. In the following figure, $\angle$ BAC = 60^0 and $\angle$ ABC = 65^0.
Prove that:
(i) CF > AF
(ii) DC > DF



Solution:

In ΔBEC,

$\angle B + \angle BEC + \angle BCE = 180^0$

$\angle B = 65^0$ [Given]

$\angle BEC = 90^0$[CE is perpendicular to AB]

$\Rightarrow 65^0 + 90^0 + \angle BCE = 180^0$

$\Rightarrow \angle BCE = 180^0 - 155^0$

$\Rightarrow \angle BCE = 25^0 = \angle DCF$ ....(i)

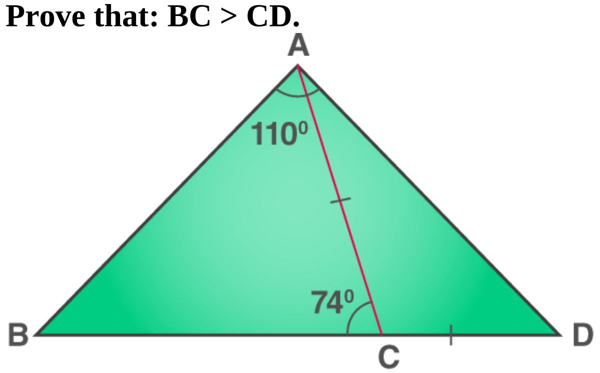
In ΔCDF,

$\angle DCF + \angle FDC + \angle CFD = 180^0$

$\angle DCF = 25^0$ [From (i)]

$\angle$ FDC = 90^0[AD is perpendicular to BC]
$\Rightarrow 25^0 + 90^0 + \angle$ CFD = 180^0
$\Rightarrow\angle$ CFD = $180^0 - 115^0$
$\Rightarrow\angle$ CFD = 65^0 …………(ii)
Now, $\angle$ AFC + $\angle$ CFD = 180^0[AFD is a straight line]
$\Rightarrow\angle$ AFC + $65^0 = 180^0$
$\Rightarrow\angle$ AFC = 115^0 ………(iii)
In Δ ACE,
$\angle$ ACE + $\angle$ CEA + $\angle$ BAC = 180^0
$\angle$ BAC = 60^0 [Given]
$\angle$ CEA = 90^0[CE is perpendicular to AB]
$\Rightarrow\angle$ ACE + $90^0 + 60^0 = 180^0$
$\Rightarrow\angle$ ACE = $180^0 - 150^0$
$\Rightarrow\angle$ ACE = 30^0 …………(iv)
In Δ AFC,
$\angle$ AFC + $\angle$ ACF + $\angle$ FAC = 180^0
$\angle$ AFC = 115^0 [From (iii)]
$\angle$ ACF = 30^0[From (iv)]
$\Rightarrow 115^0 + 30^0 + \angle$ FAC = 180^0
$\Rightarrow\angle$ FAC = $180^0 - 145^0$
$\Rightarrow\angle$ FAC = 35^0 …………(v)
In Δ AFC,
$\angle$ FAC = 35^0[From (v)]
$\angle$ ACF = 30^0[From (iv)]
$\therefore \angle FAC > \angle ACF$
$\Rightarrow CF > AF$
In Δ CDF,
$\angle$ DCF = 25^0[From (i)]
$\angle$ CFD = 65^0[From (ii)]
$\therefore \angle CFD > \angle DCF$
$\Rightarrow DC > DF$

8. In the following figure;
AC = CD; $\angle$ BAD=1100 and $\angle$ ACB=740.
Prove that: BC > CD.



Solution:

$\angle$ ACB = 74^0 …..(i)[Given]
$\angle$ ACB + $\angle$ ACD = 180^0[BCD is a straight line]
$\Rightarrow 74^0$ + $\angle$ ACD = 180^0
$\Rightarrow \angle$ ACD = 106^0 ……..(ii)
In Δ ACD,
$\angle$ ACD + $\angle$ ADC+ $\angle$ CAD = 180^0
Given that AC = CD
$\Rightarrow \angle$ ADC= $\angle$ CAD
$\Rightarrow 106^0$ + $\angle$ CAD + $\angle$ CAD = 180^0[From (ii)]
$\Rightarrow 2\angle$ CAD = 74^0
$\Rightarrow \angle$ CAD = 37^0 =$\angle$ ADC………..(iii)
Now,
$\angle$ BAD = 110^0[Given]
$\angle$ BAC + $\angle$ CAD = 110^0
$\angle$ BAC + 37^0 = 110^0
$\angle$ BAC = 73^0 ……..(iv)
In Δ ABC,
$\angle$ B + $\angle$ BAC+ $\angle$ ACB = 180^0
$\Rightarrow \angle$ B + 73^0 + 74^0 = 180^0[From (i) and (iv)]
$\Rightarrow \angle$ B + 147^0 = 180^0
$\Rightarrow \angle$ B = 33^0 ………..(v)
$\therefore \angle BAC > \angle B$ [From (iv) and (v)]
$\Rightarrow BC > AC$
But,
$AC = CD$ [Given]
$\Rightarrow BC > CD$

9. From the following figure; prove that:
(i) AB > BD
(ii) AC > CD
(iii) AB + AC > BC

Solution:

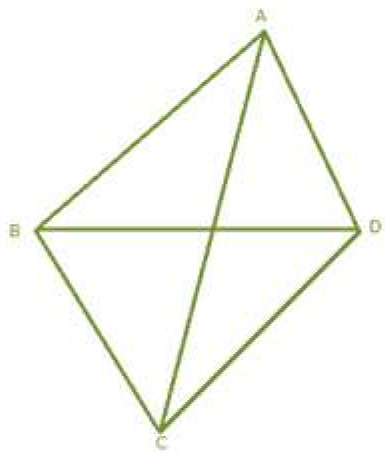
(i) $\angle$ ADC + $\angle$ ADB = 180^0[BDC is a straight line]
$\angle$ ADC = 90^0[Given]
90^0 + $\angle$ ADB = 180^0
$\angle$ ADB = 90^0 …………(i)
In Δ ADB,
$\angle$ ADB = 90^0[From (i)]
$\therefore \angle$ B + $\angle$ BAD = 90^0
Therefore, $\angle$ B and $\angle$ BAD are both acute, that is less than 90^0.
$\therefore$ AB > BD …….(ii)[Side opposite 90^0 angle is greater than

side opposite acute angle]

(ii) In △ADC,
∠ADB = 90^0
∴ ∠C + ∠DAC = 90^0
Therefore, ∠C and ∠DAC are both acute, that is less than 90^0.
∴ AC > CD ……..(iii)[Side opposite 90^0 angle is greater than side opposite acute angle]
Adding (ii) and (iii)
AB + AC > BD + CD
⇒ AB + AC > BC

10. In a quadrilateral ABCD; prove that:
(i) AB + BC + CD > DA
(ii) AB + BC + CD + DA > 2AC
(iii) AB + BC + CD + DA > 2BD

Solution:

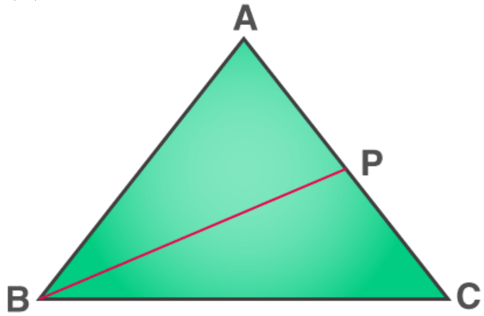


Construction:
Join AC and BD.
(i) In △ABC,
AB + BC > AC….(i)[Sum of two sides is greater than the third side]
In △ACD,
AC + CD > DA….(ii)[Sum of two sides is greater than the third side]
Adding (i) and (ii)
AB + BC + AC + CD > AC + DA
AB + BC + CD > AC + DA - AC
AB + BC + CD > DA …….(iii)

(ii) In △ACD,
CD + DA > AC….(iv)[Sum of two sides is greater than the third side]
Adding (i) and (iv)
AB + BC + CD + DA > AC + AC
AB + BC + CD + DA > 2AC

(iii) In △ABD,
AB + DA > BD….(v)[Sum of two sides is greater than the third side]
In △BCD,
BC + CD > BD….(vi)[Sum of two sides is greater than the third side]
Adding (v) and (vi)
AB + DA + BC + CD > BD + BD
AB + DA + BC + CD > 2BD

11. In the following figure, ABC is an equilateral triangle and P is any point in AC; prove that:
(i) BP > PA
(ii) BP > PC



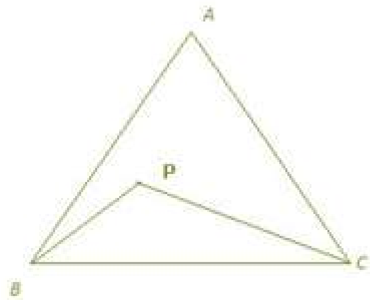
Solution:

(i) In △ABC,
AB = BC = CA[ABC is an equilateral triangle]
$\therefore \angle A = \angle B = \angle C$
$\therefore \angle A = \angle B = \angle C = \frac{180^\circ}{3}$
$\Rightarrow \angle A = \angle B = \angle C = 60^\circ$
In △ABP,
$\angle A = 60^0$
$\angle ABP < 60^0$
$\therefore \angle A > \angle ABP$
$\Rightarrow BP > PA$
[Side opposite to greater side is greater]

(ii) In △BPC,
$\angle C = 60^0$
$\angle CBP < 60^0$
$\therefore \angle C > \angle CBP$
$\Rightarrow BP > PC$
[Side opposite to greater side is greater]

12. P is any point inside the triangle ABC. Prove that: ∠BPC > ∠BAC.

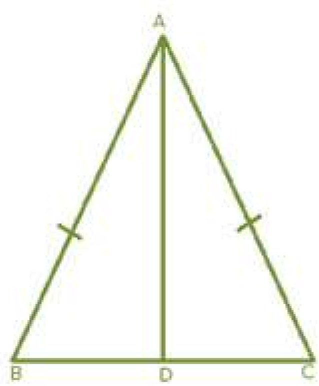
Solution:



Let ∠PBC = x and ∠PCB = y
then,
∠BPC = 180^0 - (x + y) ………(i)
Let ∠ABP = a and ∠ACP = b
then,
∠BAC = 180^0 - (x + a) - (y + b)
⇒∠BAC = 180^0 - (x + y) - (a + b)
⇒∠BAC =∠BPC - (a + b)
⇒∠BPC = ∠BAC + (a + b)
⇒∠BPC > ∠BAC

13. Prove that the straight line joining the vertex of an isosceles triangle to any point in the base is smaller than either of the equal sides of the triangle.

Solution:



We know that exterior angle of a triangle is always greater than each of the interior opposite angles.
∴ In △ABD,
∠ADC > ∠B ……..(i)
In △ABC,
AB = AC
∴ ∠B = ∠C …..(ii)
From (i) and (ii)
∠ADC > ∠C
(i) In △ADC,
∠ADC > ∠C
∴ AC > AD ………(iii) [side opposite to greater angle is greater]

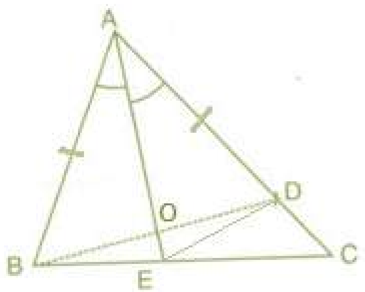
(ii) In $\triangle$ABC,
AB = AC
$\Rightarrow$AB > AD[From (iii)]

14. In the following diagram; AD = AB and AE bisects angle A. Prove that:

(i) BE = DE

(ii) $\angle$ ABD > $\angle$ C

Solution:



Construction:

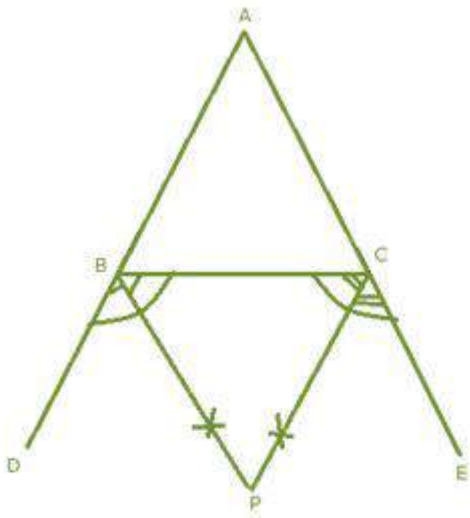
Join ED.
In $\triangle$AOB and $\triangle$AOD,
AB = AD[Given]
AO = AO[Common]
$\angle$ BAO = $\angle$ DAO[AO is bisector of $\angle$ A]
$\therefore$ $\Delta AOB \cong \Delta AOD$ [SAS criterion]
Hence,
BO = OD………(i)[cpct]
$\angle$ AOB = $\angle$ AOD …….(ii)[cpct]
$\angle$ ABO = $\angle$ ADO $\Rightarrow\angle$ ABD = $\angle$ ADB ………(iii)[cpct]
Now,
$\angle$ AOB = $\angle$ DOE[Vertically opposite angles]
$\angle$ AOD = $\angle$ BOE[Vertically opposite angles]
$\Rightarrow\angle$ BOE = $\angle$ DOE ……(iv)[From (ii)]

(i) In $\triangle$BOE and $\triangle$DOE,
BO = CD[From (i)]
OE = OE[Common]
$\angle$ BOE = $\angle$ DOE[From (iv)]
$\therefore$ $\Delta BOE \cong \Delta DOE$ [SAS criterion]
Hence, BE = DE[cpct]

(ii) In $\triangle$BCD,
$\angle$ ADB = $\angle$ C + $\angle$ CBD[Ext. angle = sum of opp. int. angles]
$\Rightarrow\angle$ ADB > $\angle$ C
$\Rightarrow\angle$ ABD > $\angle$ C[From (iii)]

15. The sides AB and AC of a triangle ABC are produced; and the bisectors of the external angles at B and C meet at P. Prove that if AB>AC, then PC > PB.

Solution:



In Δ ABC,

AB > AC,

$\Rightarrow \angle ABC < \angle ACB$

$\therefore 180^0 - \angle ABC > 180^0 - \angle ACB$

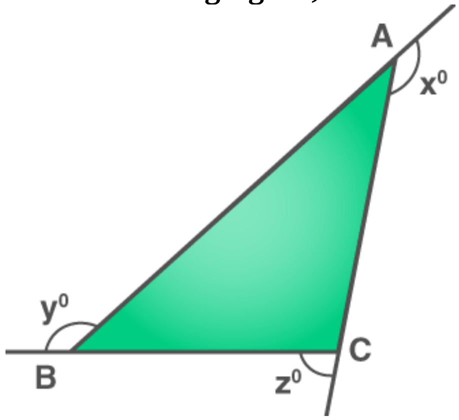
$$\Rightarrow \frac{180^0 - \angle ABC}{2} > \frac{180^0 - \angle ACB}{2}$$

$$\Rightarrow 90^\circ - \frac{1}{2}\angle ABC > 90^\circ - \frac{1}{2}\angle ACB$$

$\Rightarrow \angle CBP > \angle BCP$ [BP is bisector of $\angle CBD$ and CP is bisector of $\angle BCE$]

$\Rightarrow PC > PB$ [side opposite to greater angle is greater]

16. In the following figure; AB is the largest side and BC is the smallest side of triangle ABC.



Write the angles x^0, y^0 and z^0 in ascending order of their values.

Solution:

Since AB is the largest side and BC is the smallest side of the triangle ABC

$AB > AC > BC$

$\Rightarrow 180^\circ - z^\circ > 180^\circ - y^\circ > 180^\circ - x^\circ$

$\Rightarrow -z^\circ > -y^\circ > -x^\circ$

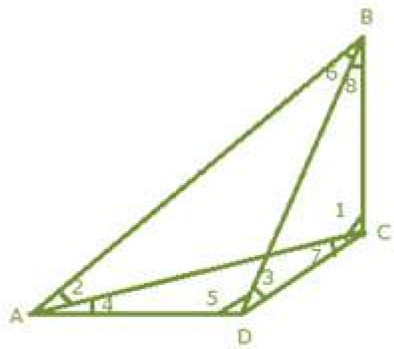
$\Rightarrow z^\circ < y^\circ < x^\circ$

17. In quadrilateral ABCD, side AB is the longest and side DC is the shortest. Prove that:

(i) $\angle$C > $\angle$A

(ii) $\angle$D > $\angle$B

Solution:



In the quad. ABCD,

Since AB is the longest side and DC is the shortest side.

(i) $\angle 1 > \angle 2$[AB > BC]

$\angle 7 > \angle 4$[AD > DC]

$\therefore \angle 1 + \angle 7 > \angle 2 + \angle 4$

$\Rightarrow \angle C > \angle A$

(ii) $\angle 5 > \angle 6$[AB > AD]

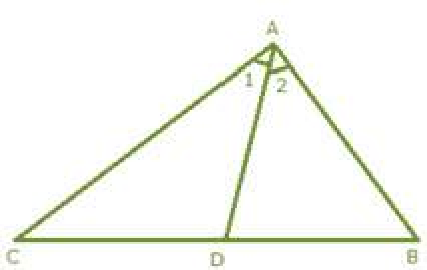
$\angle 3 > \angle 8$[BC > CD]

$\therefore \angle 5 + \angle 3 > \angle 6 + \angle 8$

$\Rightarrow \angle D > \angle B$

18. In triangle ABC, side AC is greater than side AB. If the internal bisector of angle A meets the opposite side at point D, prove that: $\angle$ADC is greater than $\angle$ADB.

Solution:



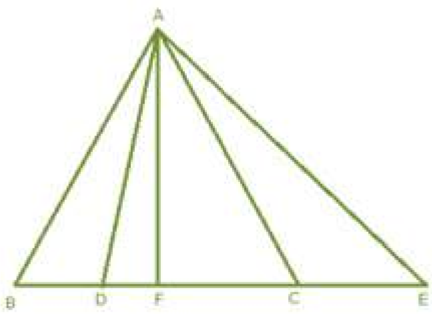
In $\triangle$ADC,

$\angle$ADB = $\angle 1 + \angle$C.............(i)

In $\triangle$ ADB,
$\angle$ ADC = $\angle$ 2 + $\angle$ B.................(ii)
But AC > AB[Given]
$\Rightarrow\angle$ B > $\angle$ C
Also given, $\angle$ 2 = $\angle$ 1[AD is bisector of $\angle$ A]
$\Rightarrow\angle$ 2 + $\angle$ B > $\angle$ 1 + $\angle$ C …….(iii)
From (i), (ii) and (iii)
$\Rightarrow\angle$ ADC > $\angle$ ADB

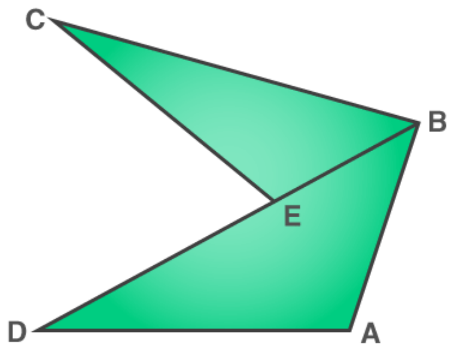
19. In isosceles triangle ABC, sides AB and AC are equal. If point D lies in base BC and point E lies on BC produced (BC being produced through vertex C), prove that:
(i) AC > AD
(ii) AE > AC
(iii) AE > AD

Solution:



We know that the bisector of the angle at the vertex of an isosceles triangle bisects the base at right angle.
Using Pythagoras theorem in $\triangle$ AFB,
$AB^2 = AF^2 + BF^2$…………..(i)
In $\triangle$ AFD,
$AD^2 = AF^2 + DF^2$…………..(ii)
We know ABC is isosceles triangle and AB = AC
$AC^2 = AF^2 + BF^2$ ……..(iii)[From (i)]
Subtracting (ii) from (iii)
$AC^2 - AD^2 = AF^2 + BF^2 - AF^2 - DF^2$
$AC^2 - AD^2 = BF^2 - DF^2$
Let 2DF = BF
$AC^2 - AD^2 = (2DF)^2 - DF^2$
$AC^2 - AD^2 = 4DF^2 - DF^2$
$AC^2 = AD^2 + 3DF^2$
$\Rightarrow AC^2 > AD^2$
$\Rightarrow AC > AD$
Similarly, AE > AC and AE > AD.

20. Given: ED = EC
Prove: AB + AD > BC



Solution:

The sum of any two sides of the triangle is always greater than the third side of the triangle.

In ΔCEB,

$CE + EB > BC$

$\Rightarrow DE + EB > BC \quad [CE = DE]$

$\Rightarrow DB > BC$........(i)

In ΔADB,

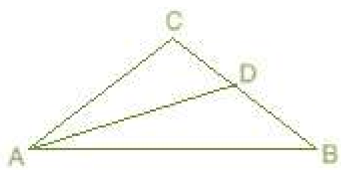
$AD + AB > BD$

$\Rightarrow AD + AB > BD > BC \quad$ [from(i)]

$\Rightarrow AD + AB > BC$

21. In triangle ABC, AB > AC and D is a point in side BC. Show that: AB>AD.

Solution:



Given that, AB > AC

$\Rightarrow \angle C > \angle B$.......(i)

Also in ΔADC

$\angle ADB = \angle DAC + \angle C \quad$ [Exterior angle]

$\Rightarrow \angle ADB > \angle C$

$\Rightarrow \angle ADB > \angle C > \angle B \quad$ [From(i)]

$\Rightarrow \angle ADB > \angle B$

$\Rightarrow AB > AD$