

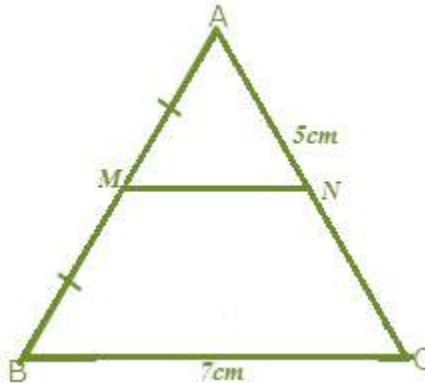
EXERCISE 12(A)

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1. In triangle ABC, M is mid-point of AB and a straight line through M and parallel to BC cuts AC at N. Find the lengths of AN and MN, if BC = 7cm and AC = 5cm.

Solution:

The triangle is shown below,



Since M is the midpoint of AB and $MN \parallel BC$ hence N is the midpoint of AC. Therefore

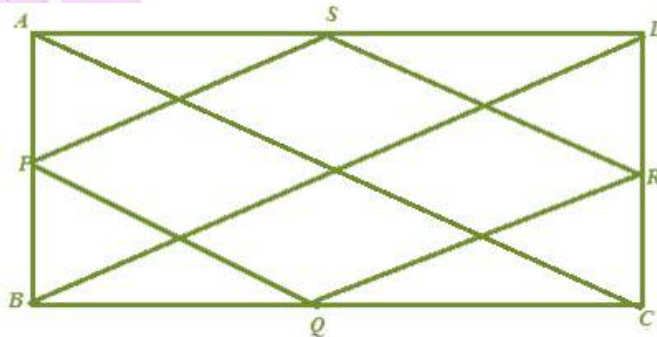
$$MN = \frac{1}{2} BC = \frac{1}{2} \times 7 = 3.5 \text{ cm}$$

$$\text{And } AN = \frac{1}{2} AC = \frac{1}{2} \times 5 = 2.5 \text{ cm}$$

2. Prove that the figure obtained by joining the mid-points of the adjacent sides of a rectangle is a rhombus.

Solution:

The figure is shown below,



Let ABCD be a rectangle where P, Q, R, S are the midpoint of AB, BC, CD, DA. We need to show that PQRS is a rhombus

For help we draw two diagonal BD and AC as shown in figure

Where $BD = AC$ (Since diagonal of rectangle are equal)

Proof:

From $\triangle ABD$ and $\triangle BCD$

$$PS = \frac{1}{2}BD = QR \text{ and } PS \parallel BD \parallel QR$$

$$2PS = 2QR = BD \text{ and } PS \parallel QR \text{ ----- (1)}$$

$$\text{Similarly } 2PQ = 2SR = AC \text{ and } PQ \parallel SR \text{ ----- (2)}$$

From (1) and (2) we get

$$PQ = QR = RS = PS$$

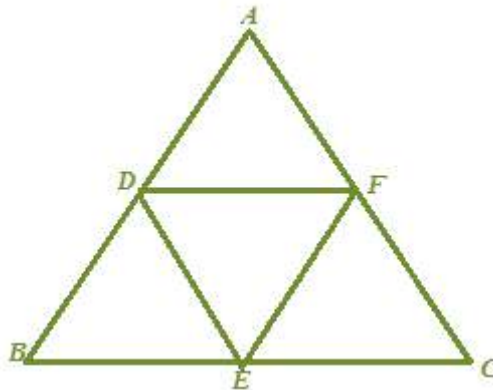
Therefore PQRS is a rhombus.

Hence proved

3. D, E and F are the mid-points of the sides AB, BC and CA of an isosceles triangle ABC in which AB=BC. Prove that triangle DEF is also isosceles.

Solution:

The figure is shown below



Given that ABC is an isosceles triangle where AB=AC.

Since D,E,F are midpoint of AB,BC,CA therefore

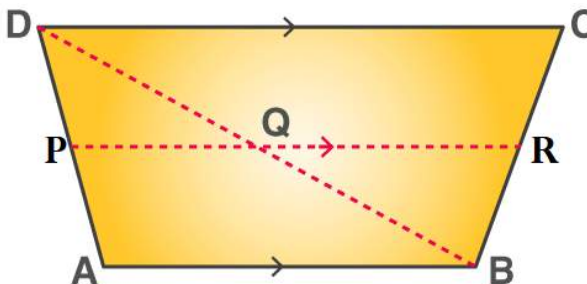
$2DE = AC$ and $2EF = AB$ this means $DE = EF$

Therefore DEF is an isosceles triangle as $DE = EF$.

Hence proved

4. The following figure shows a trapezium ABCD in which $AB \parallel DC$. P is the mid-point of AD and $PR \parallel AB$. Prove that:

$$PR = \frac{1}{2}(AB + CD)$$



Solution:

Here from triangle ABD P is the midpoint of AD and $PR \parallel AB$, therefore Q is the midpoint of BD

Similarly R is the midpoint of BC as $PR \parallel CD \parallel AB$

From triangle ABD $2PQ=AB$ (1)

From triangle BCD $2QR=CD$ (2)

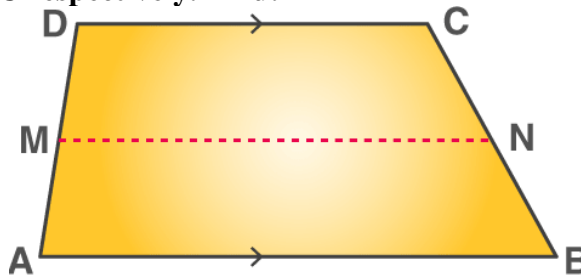
Now (1)+(2) \Rightarrow

$$2(PQ+QR)=AB+CD$$

$$PR = \frac{1}{2}(AB+CD)$$

Hence proved

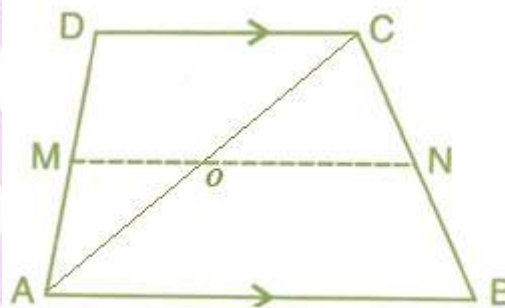
5. The figure given below, shows a trapezium ABCD. M and N are the mid-points of the non-parallel sides AD and BC respectively. Find:



- (i) MN, if $AB = 11$ cm and $DC = 8$ cm
- (ii) AB, if $DC = 20$ cm and $MN = 27$ cm
- (iii) DC, if $MN = 15$ cm and $AB = 23$ cm

Solution:

Let us draw a diagonal AC as shown in the figure below,



- (i) Given that $AB=11$ cm, $CD=8$ cm

From triangle ABC

$$ON = \frac{1}{2}AB = \frac{1}{2} \times 11 = 5.5 \text{ cm}$$

From triangle ACD

$$OM = \frac{1}{2}CD = \frac{1}{2} \times 8 = 4 \text{ cm}$$

$$\text{Hence } MN=OM+ON=(4+5.5)=9.5 \text{ cm}$$

- (ii) Given that $CD=20$ cm, $MN=27$ cm

From triangle ACD

$$OM = \frac{1}{2}CD = \frac{1}{2} \times 20 = 10 \text{ cm}$$

$$\text{Therefore } ON=27-10=17 \text{ cm}$$

From triangle ABC

$$AB = 2ON = 2 \times 17 = 34 \text{ cm}$$

(iii) Given that $AB=23\text{cm}, MN=15\text{cm}$

From triangle ABC

$$ON = \frac{1}{2} AB = \frac{1}{2} \times 23 = 11.5 \text{ cm}$$

$$\text{Therefore } OM = 15 - 11.5 = 3.5 \text{ cm}$$

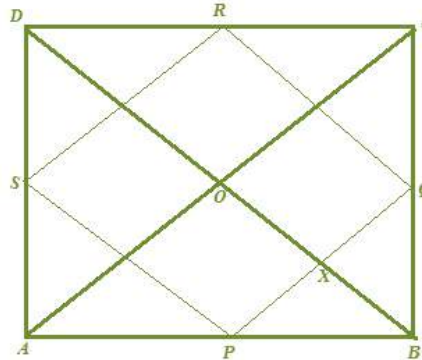
From triangle ACD

$$CD = 2OM = 2 \times 3.5 = 7 \text{ cm}$$

6. The diagonals of a quadrilateral intersect at right angles. Prove that the figure obtained by joining the mid-points of the adjacent sides of the quadrilateral is a rectangle.

Solution:

The figure is shown below



Let ABCD be a quadrilateral where P, Q, R, S are the midpoint of AB, BC, CD, DA. Diagonal AC and BD intersect at right angle at point O. We need to show that PQRS is a rectangle

Proof:

From $\triangle ABC$ and $\triangle ADC$

$$2PQ = AC \text{ and } PQ \parallel AC \dots (1)$$

$$2RS = AC \text{ and } RS \parallel AC \dots (2)$$

From (1) and (2) we get,

$$PQ = RS \text{ and } PQ \parallel RS$$

Similarly we can show that $PS = RQ$ and $PS \parallel RQ$

Therefore PQRS is a parallelogram.

$$\text{Now } PQ \parallel AC, \text{ therefore } \angle AOD = \angle PXO = 90^\circ \quad [\text{Corresponding angle}]$$

$$\text{Again } BD \parallel RQ, \text{ therefore } \angle PXO = \angle RQX = 90^\circ \quad [\text{Corresponding angle}]$$

$$\text{Similarly } \angle QRS = \angle RSP = \angle SPQ = 90^\circ$$

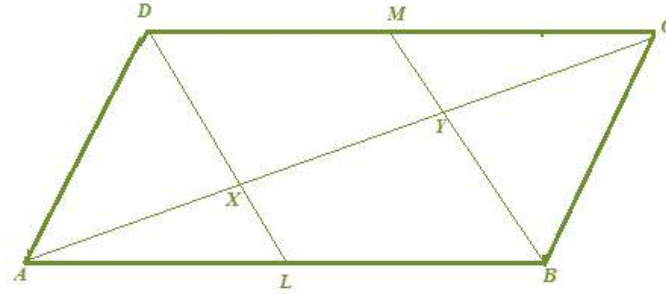
Therefore PQRS is a rectangle.

Hence proved

7. L and M are the mid-points of sides AB and DC respectively of parallelogram ABCD. Prove that segments DL and BM trisect diagonal AC.

Solution:

The required figure is shown below



From figure,

$BL = DM$ and $BL \parallel DM$ and $BLMD$ is a parallelogram, therefore $BM \parallel DL$

From triangle ABY

L is the midpoint of AB and $XL \parallel BY$, therefore x is the midpoint of AY .ie $AX = XY$ (1)

Similarly for triangle CDX

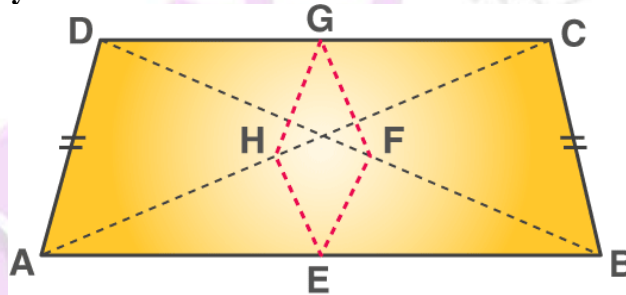
$CY = XY$ (2)

From (1) and (2)

$AX = XY = CY$ and $AC = AX + XY + CY$

Hence proved

8. $ABCD$ is a quadrilateral in which $AD = BC$. E, F, G and H are the mid-points of AB, BD, CD and AC respectively. Prove that $EFGH$ is a rhombus.



Solution:

Given that $AD = BC$ (1)

From the figure,

For triangle ADC and triangle ABD

$2GH = AD$ and $2EF = AD$, therefore $2GH = 2EF = AD$ (2)

For triangle BCD and triangle ABC

$2GF = BC$ and $2EH = BC$, therefore $2GF = 2EH = BC$ (3)

From (1),(2),(3) we get,

$2GH = 2EF = 2GF = 2EH$

$GH = EF = GF = EH$

Therefore $EFGH$ is a rhombus.

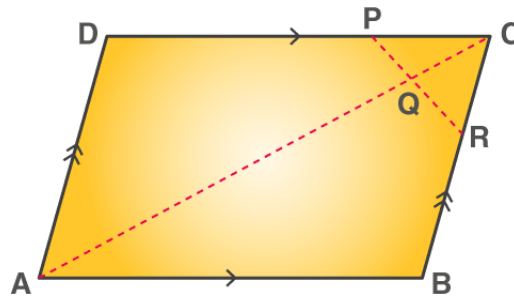
Hence proved

9. A parallelogram $ABCD$ has P the mid-point of DC and Q a point of AC such that $CQ = \frac{1}{4}AC$. PQ produced meets BC at R .

Prove that:

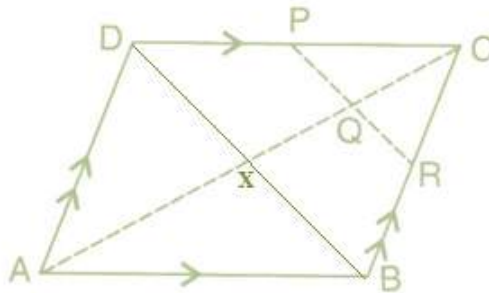
- (i) R is the mid-point of BC ,

(ii) $PR = \frac{1}{2}DB$



Solution:

For help we draw the diagonal BD as shown below



The diagonal AC and BD cuts at point X.

We know that the diagonal of a parallelogram intersects equally each other. Therefore $AX = CX$ and $BX = DX$

Given,

$$CQ = \frac{1}{4}AC$$

$$CQ = \frac{1}{4} \times 2CX$$

$$CQ = \frac{1}{2}CX$$

Therefore Q is the midpoint of CX.

(i) For triangle CDX $PQ \parallel DX$ or $PR \parallel BD$

Since for triangle CBX

Q is the midpoint of CX and $QR \parallel BX$. Therefore R is the midpoint of BC

(ii) For triangle BCD

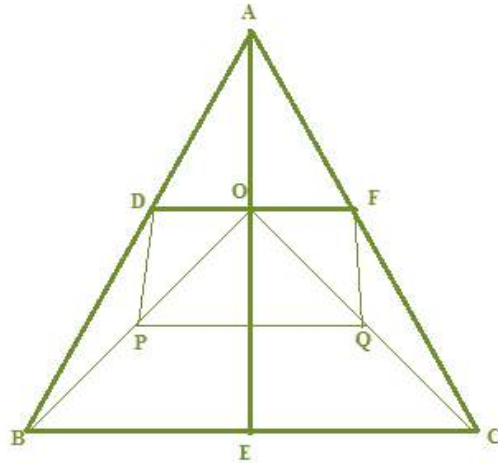
$$PR = \frac{1}{2}DB$$

As P and R are the midpoint of CD and BC, therefore

10. D, E and F are the mid-points of the sides AB, BC and CA respectively of triangle ABC. AE meets DF at O. P and Q are the mid-points of OB and OC respectively. Prove that DPQF is a parallelogram.

Solution:

The required figure is shown below



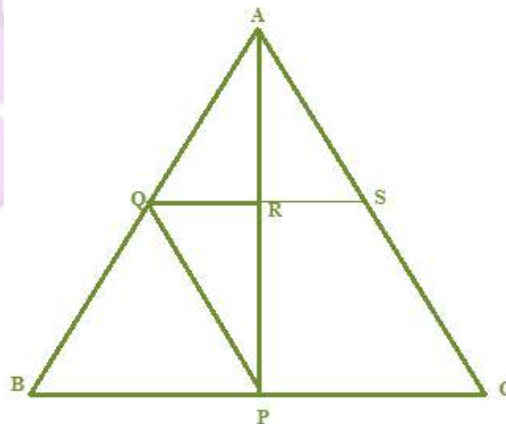
For triangle ABC and OBC
 $2DE=BC$ and $2PQ=BC$, therefore $DE=PQ$ (1)
 For triangle ABO and ACO
 $2PD=AO$ and $2FQ=AO$, therefore $PD=FQ$ (2)
 From (1),(2) we get that PQFD is a parallelogram.
 Hence proved

11. In a triangle ABC, P is the mid-point of side BC. A line through P and parallel to CA meets AB at point Q; and a line through Q and parallel to BC meets median AP at point R. Prove that:

- (i) $AP = 2AR$
- (ii) $BC = 4QR$

Solution:

The required figure is shown below



From the figure it is seen that P is the midpoint of BC and $PQ \parallel AC$ and $QR \parallel BC$
 Therefore Q is the midpoint of AB and R is the midpoint of AP

- (i) Therefore $AP=2AR$
- (ii) Here we increase QR so that it cuts AC at S as shown in the figure.
- (iii) From triangle PQR and triangle ARS

$$\angle PQR = \angle ARS \quad (\text{Opposite angle})$$

$$PR = AR$$

$$PQ = AS \quad \left[PQ = AS = \frac{1}{2} AC \right]$$

$$\triangle PQR \cong \triangle ARS \quad (\text{SAS Postulate})$$

Therefore $QR = RS$

Now

$$BC = 2QS$$

$$BC = 2 \times 2QR$$

$$BC = 4QR$$

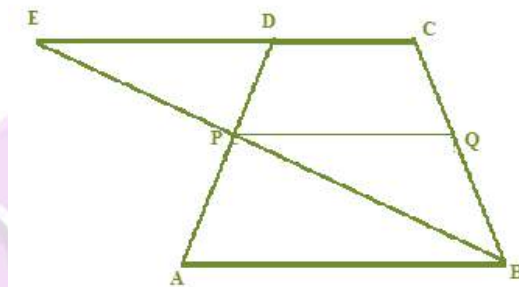
Hence proved

12. In trapezium ABCD, AB is parallel to DC; P and Q are the mid-points of AD and BC respectively. BP produced meets CD produced at point E. Prove that:

- (i) Point P bisects BE,
- (ii) PQ is parallel to AB.

Solution:

The required figure is shown below



(i)

From $\triangle PED$ and $\triangle ABP$

$$PD = AP \quad [P \text{ is the midpoint of } AD]$$

$$\angle DPE = \angle APB \quad (\text{Opposite angle})$$

$$\angle PED = \angle PBA \quad [AB \parallel CE]$$

$$\therefore \triangle PED \cong \triangle ABP \quad [ASA \text{ postulate}]$$

$$\therefore EP = BP$$

(ii) For triangle ECB $PQ \parallel CE$

Again $CE \parallel AB$

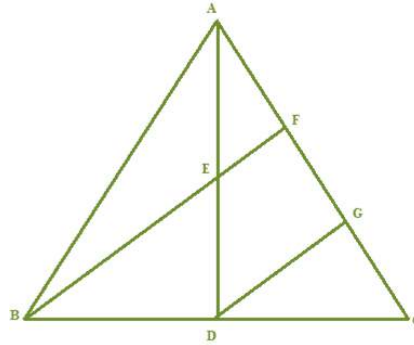
Therefore $PQ \parallel AB$

Hence proved

13. In a triangle ABC, AD is a median and E is mid-point of median AD. A line through B and E meets AC at point F. Prove that: $AC = 3AF$

Solution:

The required figure is shown below



For help we draw a line $DG \parallel BF$

Now from triangle ADG, $DG \parallel BF$ and E is the midpoint of AD

Therefore F is the midpoint of AG, i.e. $AF = GF$ (1)

From triangle BCF, $DG \parallel BF$ and D is the midpoint of BC

Therefore G is the midpoint of CF, i.e. $GF = CF$... (2)

$$AC = AF + GF + CF$$

$$AC = 3AF \text{ (From (1) and (2))}$$

Hence proved

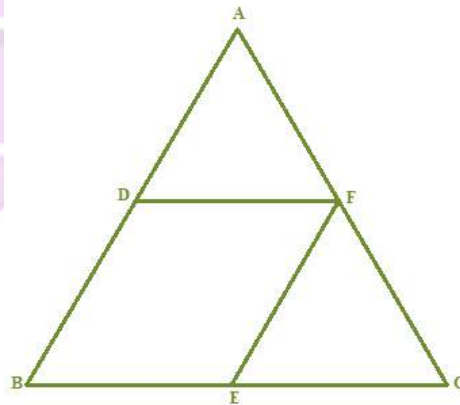
14. D and F are mid-points of sides AB and AC of a triangle ABC. A line through F and parallel to AB meets BC at point E.

(i) Prove that BDFE is a parallelogram.

(ii) Find AB, if $EF = 4.8$ cm.

Solution:

The required figure is shown below



(i) Since F is the midpoint and $EF \parallel AB$.

Therefore E is the midpoint of BC

$$\text{So } BE = \frac{1}{2} BC \quad \text{and} \quad EF = \frac{1}{2} AB \quad \dots (1)$$

Since D and F are the midpoint of AB and AC

Therefore $DE \parallel BC$

So $DF = \frac{1}{2} BC$ and $DB = \frac{1}{2} AB$ (2)

From (1),(2) we get

$BE=DF$ and $BD=EF$

Hence BDEF is a parallelogram.

(ii) Since

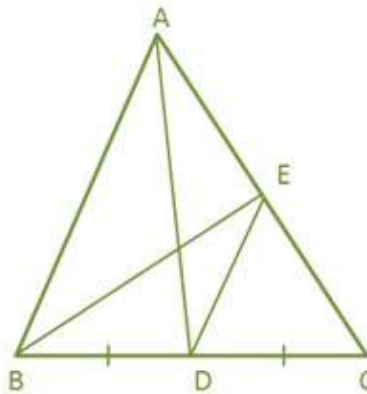
$AB = 2EF$

$= 2 \times 4.8$

$= 9.6cm$

15. In triangle ABC, AD is the median and DE, drawn parallel to side BA, meets AC at point E. Show that BE is also a median.

Solution:



In $\triangle ABC$,

AD is the median of BC

\Rightarrow D is the mid-point of BC.

Given that $DE \parallel BA$

By the Converse of the Mid-point theorem,

\Rightarrow DE bisects AC

\Rightarrow E is the mid-point of AC

\Rightarrow BE is the median of AC,

that is BE is also a median.

16. In triangle ABC, AD is the median and DE, drawn parallel to side BA, meets AC at point E. Show that BE is also a median.

Solution:

Construction : Draw $DY \parallel BQ$

In $\triangle BCQ$ and $\triangle DCY$,

$\angle BCQ = \angle DCY$ (Common)

$\angle BQC = \angle DYC$ (Corresponding angles)

So, $\triangle BCQ \sim \triangle DCY$ (AA Similarity criterion)

$$\Rightarrow \frac{BQ}{DY} = \frac{BC}{DC} = \frac{CQ}{CY} \text{ (Corresponding sides are proportional)}$$

$$\Rightarrow \frac{BQ}{DY} = \frac{2CD}{CD} \text{ (D is the mid-point of BC)}$$

$$\Rightarrow \frac{BQ}{DY} = 2 \dots (i)$$

Similarly, $\triangle AEQ \sim \triangle ADY$

$$\Rightarrow \frac{EQ}{DY} = \frac{AE}{ED} = \frac{1}{2} \text{ (E is the mid-point of AD)}$$

$$\text{that is } \frac{EQ}{DY} = \frac{1}{2} \dots (ii)$$

Dividing (i) by (ii), we get

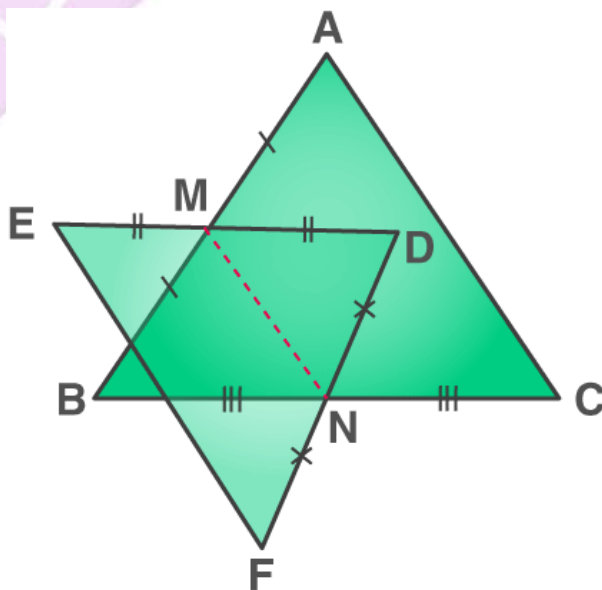
$$\Rightarrow \frac{BQ}{EQ} = 4$$

$$\Rightarrow BE + EQ = 4EQ$$

$$\Rightarrow BE = 3EQ$$

$$\Rightarrow \frac{BE}{EQ} = \frac{3}{1}$$

17. In the given figure, M is mid-point of AB and DE, whereas N is mid-point of BC and DF. Show that: $EF = AC$.



Solution:

In $\triangle EDF$,

M is the mid-point of AB and N is the mid-point of DE.

$$\Rightarrow MN = \frac{1}{2}EF \text{ (Mid-point theorem)}$$

$$\Rightarrow EF = 2MN \dots\dots (i)$$

In $\triangle ABC$,

M is the mid-point of AB and N is the mid-point of BC.

$$\Rightarrow MN = \frac{1}{2}AC \text{ (Mid-point theorem)}$$

$$\Rightarrow AC = 2MN \dots\dots (ii)$$

From (i) and (ii), we get

$$\Rightarrow EF = AC$$

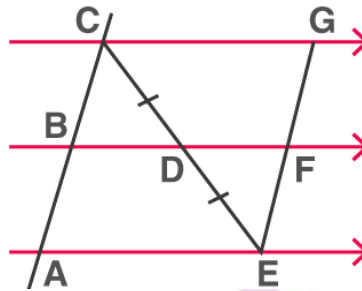


EXERCISE 12(B)

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1. Use the following figure to find:

- (i) BC, if AB = 7.2 cm.
- (ii) GE, if FE = 4 cm.
- (iii) AE, if BD = 4.1 cm.
- (iv) DF, if CG = 11 cm.



Solution:

According to equal intercept theorem since $CD = DE$
Therefore $AB = BC$ and $EF = GF$

(i) $BC = AB = 7.2 \text{ cm}$

(ii) $GE = EF + GF = 2EF = 2 \times 4 = 8 \text{ cm}$

Since B, D, F are the midpoint and $AE \parallel BF \parallel CG$

Therefore $AE = 2BD$ and $CG = 2DF$

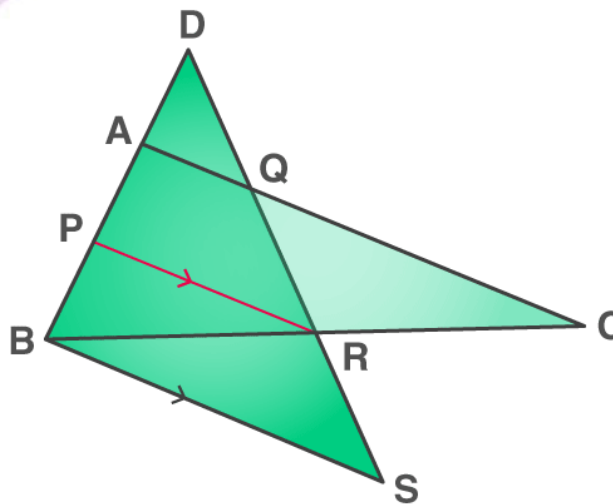
(iii) $AE = 2BD = 2 \times 4.1 = 8.2$

(iv)

$$DF = \frac{1}{2} CG = \frac{1}{2} \times 11 = 5.5 \text{ cm}$$

2. In the figure, given below, $2AD = AB$, P is mid-point of AB, Q is mid-point of DR and $PR \parallel BS$. Prove that:

- (i) $AQ \parallel BS$,
- (ii) $DS = 3RS$



Solution:

Given that $AD=AP=PB$ as $2AD=AB$ and p is the midpoint of AB

(i) From triangle DPR , A and Q are the midpoint of DP and DR .

Therefore $AQ \parallel PR$

Since $PR \parallel BS$, hence $AQ \parallel BS$

(ii) From triangle ABC , P is the midpoint and $PR \parallel BS$

Therefore R is the midpoint of BC

From $\triangle BRS$ and $\triangle QRC$

$$\angle BRS = \angle QRC$$

$$BR = RC$$

$$\angle RBS = \angle RCQ$$

$$\therefore \triangle BRS \cong \triangle QRC$$

$$\therefore QR = RS$$

$$DS = DQ + QR + RS = QR + QR + RS = 3RS$$

3. The side AC of a triangle ABC is produced to point E so that $CE = \frac{1}{2} AC$. D is the mid-point of BC and ED produced meets AB at F . Lines through D and C are drawn parallel to AB which meet AC at point P and EF at point R respectively. Prove that:

(i) $3DF = EF$

(ii) $4CR = AB$

Solution:

Given that D is the midpoint of BC and DP is parallel to AB , therefore P is the midpoint of AC

$$PD = \frac{1}{2} AB$$

(i) Again from the triangle AEF we have $AE \parallel PD \parallel CR$ and $AP = \frac{1}{3} AE$
 $DF = \frac{1}{3} EF$
 Therefore or we can say that $3DF = EF$.
 Hence it is shown.

(ii) From the triangle PED we have $PD \parallel CR$ and C is the midpoint of PE therefore $CR = \frac{1}{2} PD$
 Now
 $PD = \frac{1}{2} AB$
 $\frac{1}{2} PD = \frac{1}{4} AB$
 $CR = \frac{1}{4} AB$
 $4CR = AB$
 Hence it is shown.

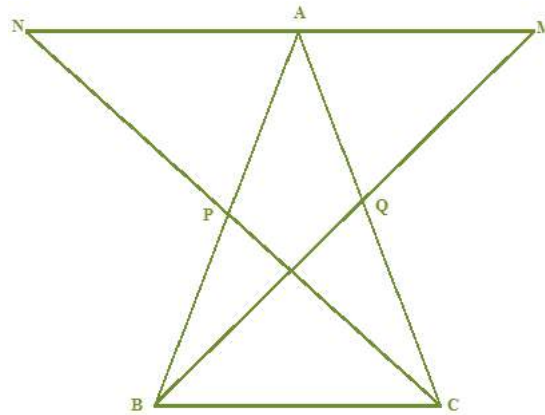
4. In triangle ABC , the medians BP and CQ are produced upto points M and N respectively such that $BP = PM$ and $CQ = QN$. Prove that:

(i) M , A and N are collinear.

(ii) A is the mid-point of MN .

Solution:

The figure is shown below



(i)

From triangle BPC and triangle APN

$$\angle BPC = \angle APN \quad [\text{Opposite angle}]$$

$$BP = AP$$

$$PC = PN$$

$$\therefore \triangle BPC \cong \triangle APN \quad [\text{SAS postulate}]$$

$$\therefore \angle PBC = \angle PAN \quad \dots\dots (1)$$

$$\text{And } BC = AN \quad \dots\dots (3)$$

$$\text{Similarly } \angle QCB = \angle QAN \quad \dots\dots (2)$$

$$\text{And } BC = AM \quad \dots\dots (4)$$

Now

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$\angle PAN + \angle QAM + \angle BAC = 180^\circ \quad [(1), (2) \text{ we get}]$$

Therefore M, A, N are collinear

(ii)

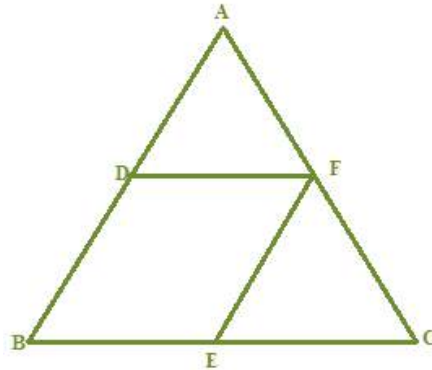
From (3) and (4) $MA = NA$

Hence A is the midpoint of MN

5. In triangle ABC, angle B is obtuse. D and E are mid-points of sides AB and BC respectively and F is a point on side AC such that EF is parallel to AB. Show that BEFD is a parallelogram.

Solution:

The figure is shown below



From the figure $EF \parallel AB$ and E is the midpoint of BC.
Therefore F is the midpoint of AC.
Here $EF \parallel BD$, $EF = BD$ as D is the midpoint of AB
 $BE \parallel DF$, $BE = DF$ as E is the midpoint of BC.
Therefore BEFD is a parallelogram.

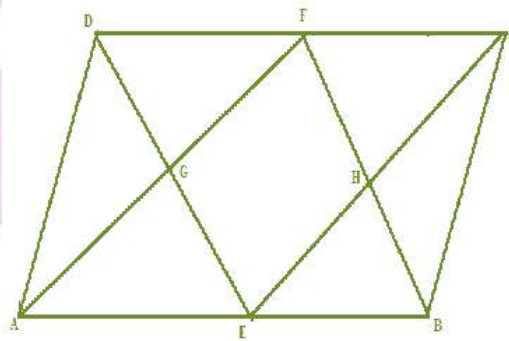
6. In parallelogram ABCD, E and F are mid-points of the sides AB and CD respectively. The line segments AF and BF meet the line segments ED and EC at points G and H respectively.

Prove that:

- (i) Triangles HEB and FHC are congruent;
- (ii) GEHF is a parallelogram.

Solution:

The figure is shown below



- (i)

From $\triangle HEB$ and $\triangle FHC$

$$BE = FC$$

$$\angle EHB = \angle FHC \quad [\text{Opposite angle}]$$

$$\angle HBE = \angle HFC$$

$$\therefore \triangle HEB \cong \triangle FHC$$

$$\therefore EH = CH, BH = FH$$

- (ii)

Similarly $AG = GF$ and $EG = DG$ (1)

For triangle ECD, F and H are the midpoint of CD and EC.

$$HF = \frac{1}{2} DE$$

Therefore $HF \parallel DE$ and(2)

(1),(2) we get, $HF = EG$ and $HF \parallel EG$

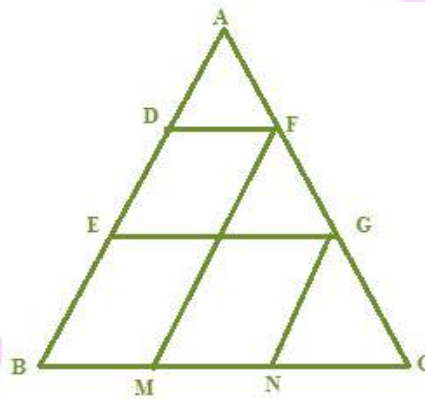
Similarly we can show that $EH = GF$ and $EH \parallel GF$

Therefore $GEHF$ is a parallelogram.

7. In triangle ABC , D and E are points on side AB such that $AD = DE = EB$. Through D and E , lines are drawn parallel to BC which meet side AC at points F and G respectively. Through F and G , lines are drawn parallel to AB which meet side BC at points M and N respectively. Prove that: $BM = MN = NC$.

Solution:

The figure is shown below



For triangle AEG

D is the midpoint of AE and $DF \parallel EG \parallel BC$

Therefore F is the midpoint of AG .

$AF = GF$ (1)

Again $DF \parallel EG \parallel BC$ $DE = BE$, therefore $GF = GC$ (2)

(1),(2) we get $AF = GF = GC$.

Similarly $GN \parallel FM \parallel AB$ and $AF = GF$,

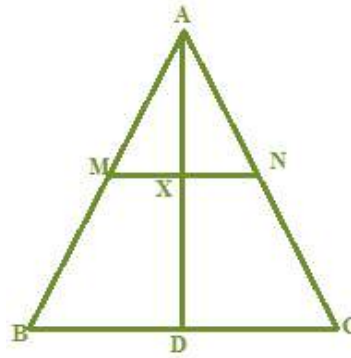
therefore $BM = MN = NC$

Hence proved

8. In triangle ABC ; M is mid-point of AB , N is mid-point of AC and D is any point in base BC . Use Intercept Theorem to show that MN bisects AD .

Solution:

The figure is shown below

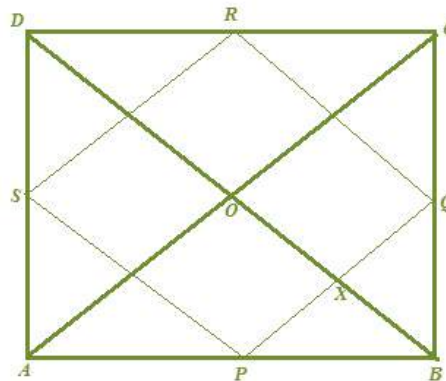


Since M and N are the midpoint of AB and AC, $MN \parallel BC$
According to intercept theorem Since $MN \parallel BC$ and $AM = BM$,
Therefore $AX = DX$. Hence proved

9. If the quadrilateral formed by joining the mid-points of the adjacent sides of quadrilateral ABCD is a rectangle, show that the diagonals AC and BD intersect at right angle.

Solution:

The figure is shown below



Let ABCD be a quadrilateral where P, Q, R, S are the midpoint of AB, BC, CD, DA. PQRS is a rectangle. Diagonal AC and BD intersect at point O. We need to show that AC and BD intersect at right angle.

Proof:

$PQ \parallel AC$, therefore $\angle AOD = \angle PXO$ [Corresponding angle] (1)

|| Again $BD \parallel RQ$,

therefore $\angle PXO = \angle RQX = 90^\circ$ [Corresponding angle and angle of rectangle] ... (2)

From (1) and (2) we get,

$$\angle AOD = 90^\circ$$

Similarly $\angle AOB = \angle BOC = \angle DOC = 90^\circ$

Therefore diagonals AC and BD intersect at right angle

Hence proved

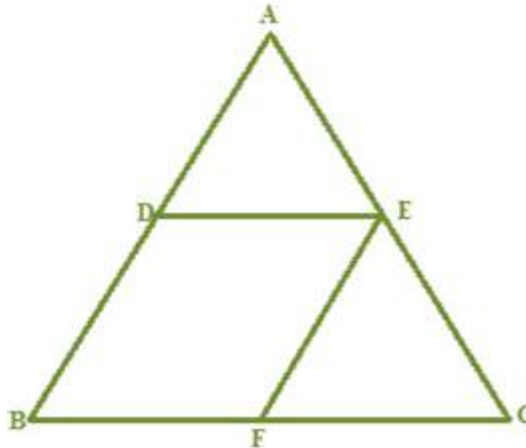
10. In triangle ABC; D and E are mid-points of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a

parallelogram.

If $AB = 16$ cm, $AC = 12$ cm and $BC = 18$ cm, find the perimeter of the parallelogram BDEF.

Solution:

The figure is shown below



From figure since E is the midpoint of AC and $EF \parallel AB$
Therefore F is the midpoint of BC and $2DE = BC$ or $DE = BF$
Again D and E are midpoint, therefore $DE \parallel BF$ and $EF = BD$
Hence BDEF is a parallelogram.

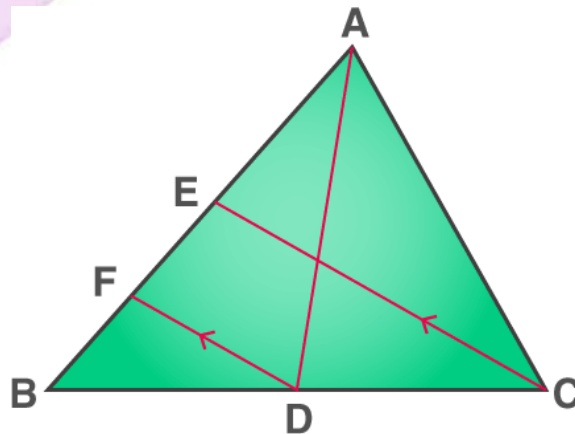
Now

$$BD = EF = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8 \text{ cm}$$

$$BF = DE = \frac{1}{2} BC = \frac{1}{2} \times 18 = 9 \text{ cm}$$

$$\text{Therefore perimeter of BDEF} = 2(BF + EF) = 2(9 + 8) = 34 \text{ cm}$$

11. In the given figure, AD and CE are medians and $DF \parallel CE$. Prove that: $FB = \frac{1}{4} AB$.



Solution:

Given AD and CE are medians and $DF \parallel CE$.

We know that from the midpoint theorem, if two lines are parallel and the starting point of

segment is at the midpoint on one side, then the other point meets at the midpoint of the other side.

Consider triangle BEC. Given $DF \parallel CE$ and D is midpoint of BC. So F must be the midpoint of BE.

$$\text{So } FB = \frac{1}{2} BE \text{ but } BE = \frac{1}{2} AB$$

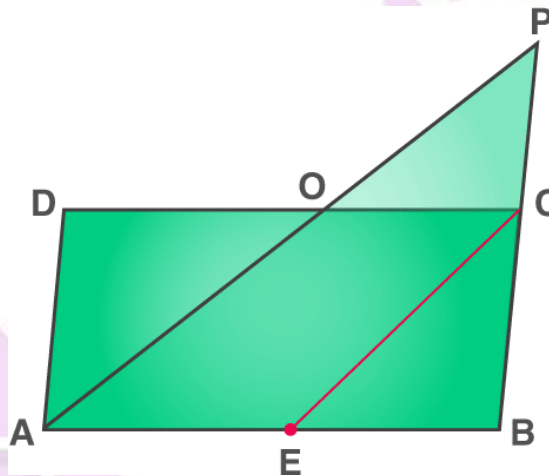
Substitute value of BE in first equation, we get

$$FB = \frac{1}{4} AB$$

Hence Proved

12. In parallelogram ABCD, E is the mid-point of AB and AP is parallel to EC which meets DC at point O and BC produced at P. Prove that:

- (i) **$BP = 2AD$**
- (ii) **O is mid-point of AP.**



Solution:

Given ABCD is parallelogram, so $AD = BC$, $AB = CD$.

Consider triangle APB, given EC is parallel to AP and E is midpoint of side AB. So by midpoint theorem, C has to be the midpoint of BP.

So $BP = 2BC$, but $BC = AD$ as ABCD is a parallelogram.

Hence $BP = 2AD$

Consider triangle APB, $AB \parallel OC$ as ABCD is a parallelogram. So by midpoint theorem, O has to be the midpoint of AP.

Hence Proved

13. In trapezium ABCD, sides AB and DC are parallel to each other. E is mid-point of AD and F is the mid-point of BC.

Prove that: $AB + DC = 2EF$

Solution:

Consider trapezium ABCD.

Given E and F are midpoints on sides AD and BC, respectively.



We know that $AB = GH = IJ$

From midpoint theorem,

$$EG = \frac{1}{2}DI, HF = \frac{1}{2}JC$$

Consider LHS,

$$AB + CD = AB + CJ + JI + ID = AB + 2HF + AB + 2EG$$

$$\text{So } AB + CD = 2(AB + HF + EG) = 2(EG + GH + HF) = 2EF$$

$$AB + CD = 2EF$$

Hence Proved

- 14. In triangle ABC, AD is the median and DE is parallel to BA, where E is a point in AC. Prove that BE is also a median.**

Solution:

Given $\triangle ABC$

AD is the median. So D is the midpoint of side BC.

Given $DE \parallel AB$. By the midpoint theorem, E has to be midpoint of AC.

So line joining the vertex and midpoint of the opposite side is always known as median. So BE is also median of $\triangle ABC$.

- 15. Adjacent sides of a parallelogram are equal and one of the diagonals is equal to any one of the sides of this parallelogram. Show that its diagonals are in the ratio $\sqrt{3}:1$.**

Solution:

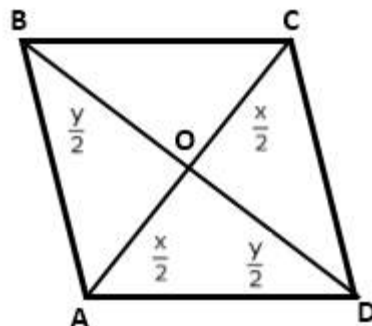
If adjacent sides of a parallelogram are equal, then it is rhombus.

Now, the diagonals of a rhombus bisect each other and are perpendicular to each other.

Let the lengths of the diagonals be x and y.

Diagonal of length y be equal to the sides of rhombus.

Thus, each side of rhombus = y



Now, in right-angles $\triangle BOC$, by Pythagoras theorem

$$OB^2 + OC^2 = BC^2$$

$$\Rightarrow \left(\frac{y}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = y^2$$

$$\Rightarrow \frac{x^2}{4} = y^2 - \frac{y^2}{4}$$

$$\Rightarrow \frac{x^2}{4} = \frac{4y^2 - y^2}{4}$$

$$\Rightarrow \frac{x^2}{4} = \frac{3y^2}{4}$$

$$\Rightarrow x^2 = 3y^2$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{3}{1}$$

$$\Rightarrow \frac{x}{y} = \frac{\sqrt{3}}{1}$$

Thus, the diagonals are in the ratio $\sqrt{3} : 1$.