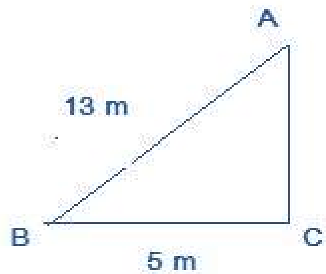
EXERCISE 13(A) PAGE: 158

1. **A ladder 13 m long rests against vertical wall. If the foot of the ladder is 5m from the foot of the wall, find the distance of the other end of the ladder from the ground.**

Solution:

The pictorial representation of the given problem is given below,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, AB is the hypotenuse. Therefore applying the Pythagoras theorem we get,

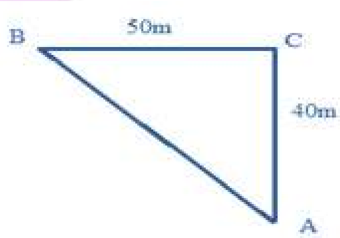
$$AB^2 = BC^2 + CA^2$$
$$13^2 = 5^2 + CA^2$$
$$CA^2 = 13^2 - 5^2$$
$$CA^2 = 144$$
$$CA = 12 \text{ m}$$

Therefore, the distance of the other end of the ladder from the ground is 12m

2. **A man goes 40m due north then 50 m due west. Find his distance from the starting point.**

Solution:

Here , we need to measure the distance AB as shown in the figure below,



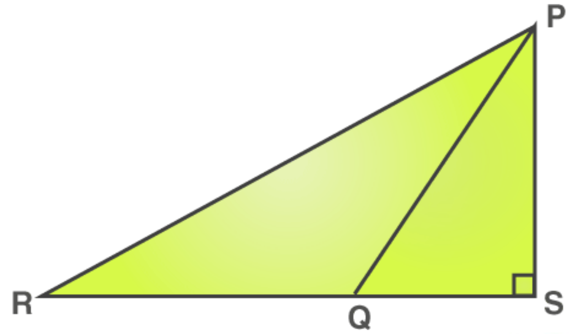
Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Therefore, in this case,

$$AB^2 = BC^2 + CA^2$$
$$AB^2 = 50^2 + 40^2$$
$$AB^2 = 2500 + 1600$$
$$AB^2 = 4100$$
$$AB = 64.03$$

Therefore the required distance is 64.03 m.

3. **In the figure: ∠PSQ = 90^0, PQ= 10 cm, QS = 6cm and RQ = 9 cm. Calculate the length of P.**



Solution:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔPQS and applying Pythagoras theorem we get,

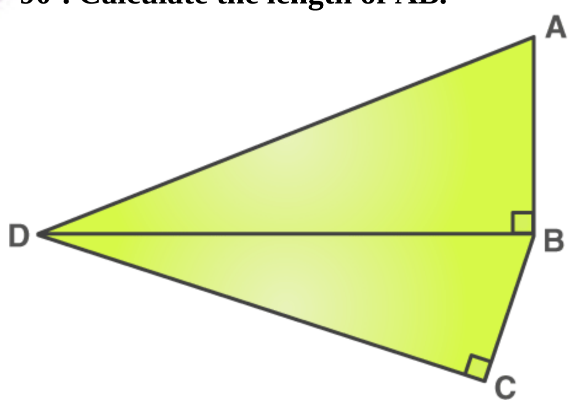
$$PQ^2 = PS^2 + QS^2$$
$$10^2 = PS^2 + 6^2$$
$$PS^2 = 100 - 36$$
$$PS = 8$$

Now, we consider the ΔPRS and applying Pythagoras theorem we get,

$$PR^2 = RS^2 + PS^2$$
$$PR^2 = 15^2 + 8^2$$
$$PR = 17$$

The length of PR= 17 cm

4. **The given figure shows a quadrilateral ABCD in which AD = 13cm, DC = 12 cm, BC = 3 cm and ∠ABD = ∠BCD = 90^0. Calculate the length of AB.**



Solution:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔBDC and applying Pythagoras theorem we get,

$DB^2 = DC^2 + BC^2$

$DB^2 = 12^2 + 3^2$

$DB^2 = 144 + 9$

$DB^2 = 153$

Now, we consider the ΔABD and applying Pythagoras theorem we get,

$DA^2 = DB^2 + BA^2$

$13^2 = 153 + BA^2$

$BA^2 = 169 - 153$

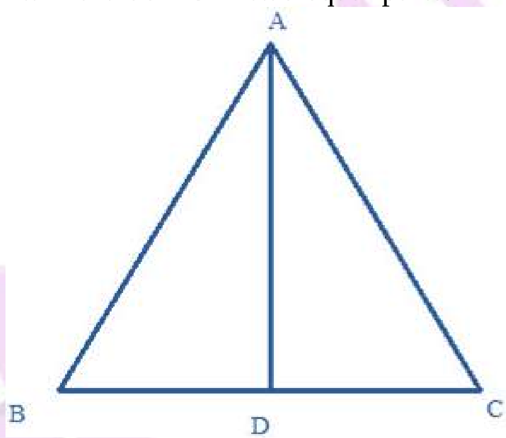
$BA = 4$

The length of AB is 4 cm.

5. AD is drawn perpendicular to base BC of an equilateral triangle ABC. Given BC = 10 cm, find the length of AD, correct to 1 place of decimal.

Solution:

Since ABC is an equilateral triangle therefore, all the sides of the triangle are of same measure and the perpendicular AD will divide BC in two equal parts.



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, we consider the ΔABD and applying Pythagoras theorem we get,

$AB^2 = AD^2 + BD^2$

$AD^2 = 100^2 - 5^2$ $\left[\begin{array}{l}\text{Given}, BC = 10 \text{ cm} = AB, \\ BD = \frac{1}{2} BC\end{array}\right]$

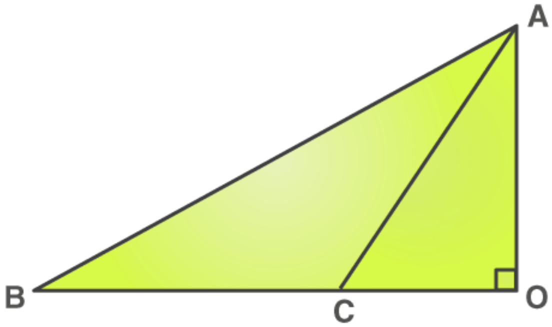
$AD^2 = 100 - 25$

$AD^2 = 75$

$AD = 8.7$

Therefore, the length of AD is 8.7 cm

6. In triangle ABC, given below, AB = 8cm, BC = 6cm and Ac = 3cm. Calculate the length of OC.



Solution:

We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the $\Delta ABO,$ and applying Pythagoras theorem we get,

$$AB^2 = AO^2 + OB^2$$
$$AO^2 = AB^2 - OB^2$$
$$AO^2 = AB^2 - (BC + OC)^2$$
$$[\text{Let}, OC = x]$$
$$AO^2 = AB^2 - (BC + x)^2 \qquad \ldots\ldots(i)$$

First, we consider the $\Delta ACO,$ and applying Pythagoras theorem we get,

$$AC^2 = AO^2 + x^2$$
$$AO^2 = AC^2 - x^2 \qquad \ldots\ldots(ii)$$

Now, from (i) and(ii),

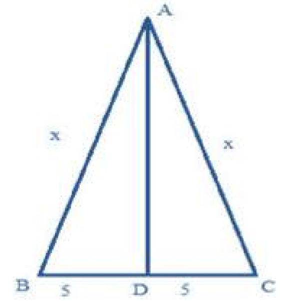
$$AB^2 - (BC + x)^2 = AC^2 - x^2$$
$$8^2 - (6 + x)^2 = 3^2 - x^2 \qquad [Given, AB = 8cm, BC = 8cm \text{ and } AC = 3cm]$$
$$x = 1\frac{7}{12} cm$$

Therefore, the length of OC will be $1\frac{7}{12}$ cm

7. **In triangle ABC,**
   AB = AC = x; BC = 10cm and the area of the triangle is 60 cm^2. Find x.

Solution:

Here, the diagram will be,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Since, ABC is an isosceles triangle, therefore perpendicular from vertex will cut the base in two equal segments.
First, we consider the $\Delta ABD,$ and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$
$$AD^2 = x^2 - 5^2$$
$$AD^2 = x^2 - 25$$
$$AD = \sqrt{x^2 - 25} \qquad \ldots\ldots(i)$$

Now,

$$\text{Area}=60$$
$$\frac{1}{2}\times 10\times \text{AD}=60$$
$$\frac{1}{2}\times 10\times \sqrt{x^2-25}=60$$
$$x=13$$

Therefore, x is 13cm

8. If the sides of a triangle are in the ratio 1: $\sqrt{2}$: 1, show that it is a right angles triangle.

Solution:

Let, the sides of the triangle be, $x, \sqrt{2}x \text{ and } x$

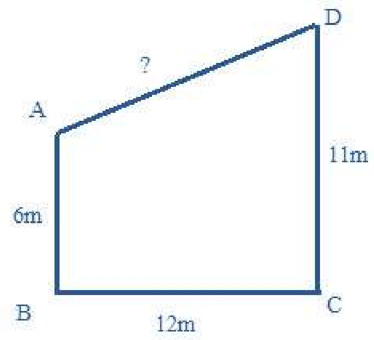
Now, $x^2 + x^2 = 2x^2 = \left(\sqrt{2}x\right)^2 \qquad \ldots\ldots(i)$

Here, in (i) it is shown that, square of one side of the given triangle is equal to the addition of square of other two sides. This is nothing but Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Therefore, the given triangle is a right angled triangle.

9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12m; find the distance between their tips

Solution:

The diagram of the given problem is given below,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, $11 - 6 = 5\text{m}$ (Since DC is perpendicular to BC)

base = 12m

Applying Pythagoras theorem we get,

$$\text{hypotenuse}^2 = 5^2 + 12^2$$

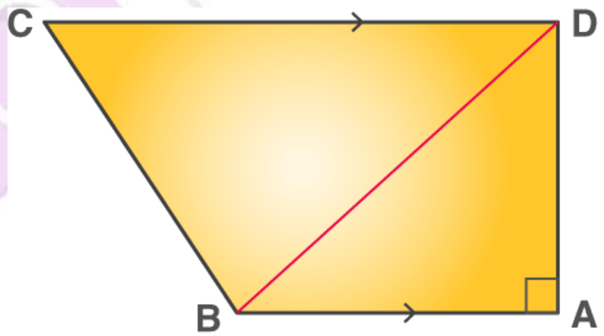
$$h^2 = 25 + 144$$

$$h^2 = 169$$

$$h = 13$$

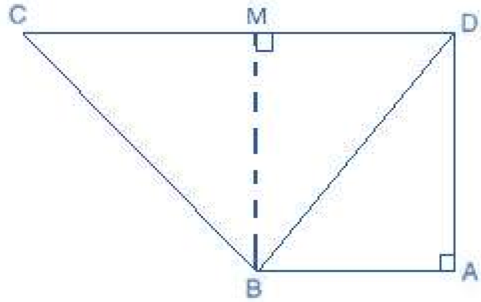
Therefore, the distance between the tips will be 13m

10. In the given figure, AB||CD, AB = 7cm, BD = 25cm and CD = 17 cm; find the length of side BC.



Solution:

Let M be the point on CD such that AB = DM.
So DM = 7cm and MC = 10 cm
Join points B and M to form the line segment BM.
So BM || AD also BM = AD.



In right-angled ΔBAD

$BD^2 = AD^2 + BA^2$

$(25)^2 = AD^2 + (7)^2$

$AD^2 = (25)^2 - (7)^2$

$AD^2 = 576$

$AD = 24$

In right-angled ΔCMB

$CB^2 = CM^2 + MB^2$

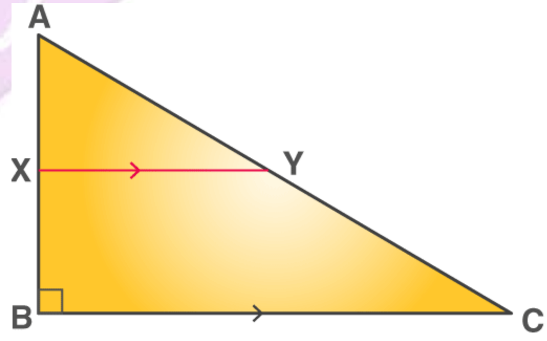
$CB^2 = (10)^2 + (24)^2$ [$MB = AD$]

$CB^2 = 100 + 576$

$CB^2 = 676$

$CB = 26cm$

11. In the given figure, ∠B=90⁰, XY||BC, AB=12cm, AY = 8cm and AX : XB = 1: 2 = AY : YC. Find the lengths of AC and BC.



Solution:

Given that AX:XB = 1:2.

Let n be the common multiple for which this proportion gets satisfied.

So, AX = 1(n) and XB = 2(n)

$$AX + XB = 1(n) + 2(n)$$
$$\Rightarrow AB = n + 2n$$
$$\Rightarrow 12 = 3n$$
$$\Rightarrow n = 4$$

AX = 1(n) = 4 and XB = 2(n) = 8

In ΔABC,

$XY \parallel BC$

$$\frac{AB}{AX} = \frac{AC}{AY} = \frac{BC}{XY}$$
$$\Rightarrow \frac{AB}{AX} = \frac{AC}{AY}$$
$$\Rightarrow \frac{12}{4} = \frac{AC}{8}$$
$$\Rightarrow AC = 24cm$$

In right-angled ΔABC

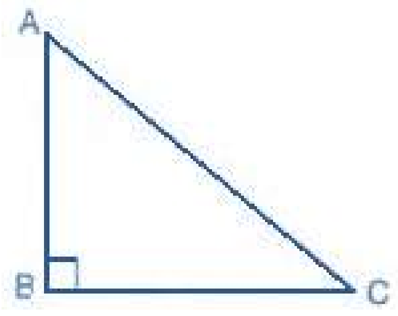
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow (24)^2 = (12)^2 + BC^2$$
$$\Rightarrow BC^2 = (24)^2 - (12)^2$$
$$\Rightarrow BC^2 = 576 - 144$$
$$\Rightarrow BC^2 = 432$$
$$\Rightarrow BC = 12\sqrt{3}\,cm$$

12. In ΔABC, ∠B = 90^0. Find the sides of the triangle, if:

(i) AB = (x-3) cm, BC = (x+4) cm and AC = (x+6)cm

(ii) AB = x cm, BC = (4x+4)cm and AC = (4x+5) cm

Solution:



(i) In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (x+6)^2 = (x-3)^2 + (x+4)^2$

$\Rightarrow (x^2 + 12x + 36) = (x^2 - 6x + 9) + (x^2 + 8x + 16)$

$\Rightarrow x^2 - 10x - 11 = 0$

$\Rightarrow (x-11)(x+1) = 0$

$\Rightarrow x = 11$ or $x = -1$

But length of the side of a triangle can not be negative.

$\Rightarrow x = 11cm$

$\therefore AB = (x-3) = (11-3) = 8cm$

$BC = (x+4) = (11+4) = 15cm$

$AC = (x+6) = (11+6) = 17cm$

(ii) In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (4x+5)^2 = (x)^2 + (4x+4)^2$

$\Rightarrow (16x^2 + 40x + 25) = (x^2) + (16x^2 + 32x + 16)$

$\Rightarrow x^2 - 8x - 9 = 0$

$\Rightarrow (x-9)(x+1) = 0$

$\Rightarrow x = 9$ or $x = -1$

But length of the side of a triangle can not be negative.

$\Rightarrow x = 9cm$

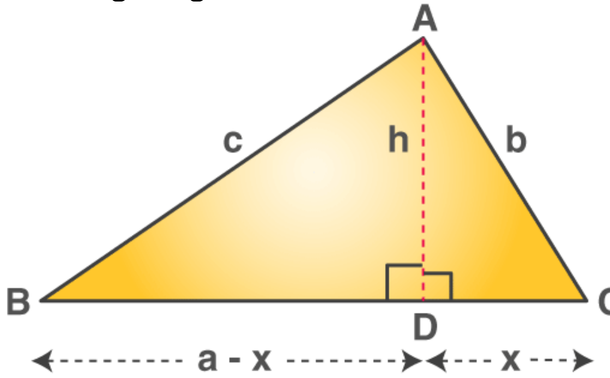
$\therefore AB = x = 9cm$

$BC = (4x+4) = (36+4) = 40cm$

$AC = (4x+5) = (36+5) = 41cm$

EXERCISE 13(B)

1. In the figure, given below, AD⊥BC. Prove that: $c^2=a^2+b^2-2ax$.



Solution:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔABD and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$
$$c^2 = h^2 + (a-x)^2$$
$$h^2 = c^2 - (a-x)^2 \qquad \ldots\ldots(i)$$

First, we consider the ΔACD and applying Pythagoras theorem we get,

$$AC^2 = AD^2 + CD^2$$
$$b^2 = h^2 + x^2$$
$$h^2 = b^2 - x^2 \qquad \ldots\ldots(ii)$$

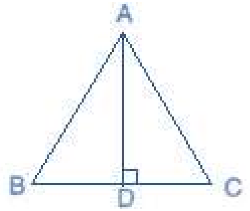
From (*i*) and (*ii*) we get,

$$c^2 - (a-x)^2 = b^2 - x^2$$
$$c^2 - a^2 - x^2 + 2ax = b^2 - x^2$$
$$c^2 = a^2 + b^2 - 2ax$$

Hence Proved.

2. In equilateral triangle ABC, AD⊥BC and BC = x cm. Find, in terms of x, the length of AD.

Solution:



In equilateral Δ ABC, AD ⊥ BC.
Therefore, BD = DC = x/2 cm.
In right - angled ΔADC

$$AC^2 = AD^2 + DC^2$$

$$\Rightarrow (x)^2 = AD^2 + \left(\frac{x}{2}\right)^2$$

$$\Rightarrow AD^2 = (x^2) - \left(\frac{x}{2}\right)^2$$

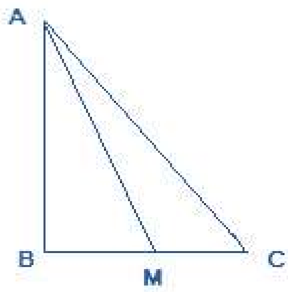
$$\Rightarrow AD^2 = \left(\frac{x}{2}\right)^2$$

$$\Rightarrow AD = \left(\frac{x}{2}\right) cm$$

3. ABC is a triangle, ight angles at B. M is a point on BC. Prove that:
$AM^2 + BC^2 = AC^2 + BM^2$

Solution:

The pictorial form of the given problem is as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
First, we consider the ΔABM and applying Pythagoras theorem we get,

$$AM^2 = AB^2 + BM^2$$

$$AB^2 = AM^2 - BM^2 \quad \ldots\ldots(i)$$

Now, we consider the ΔABC and applying Pythagoras theorem we get,

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2 \quad \ldots\ldots(ii)$$

From (*i*) and (*ii*) we get,

$$AM^2 - BM^2 = AC^2 - BC^2$$

$$AM^2 + BC^2 = AC^2 + BM^2$$

Hence Proved

4. M and N are the mid-points of the sides QR and PQ respectively of a triangle PQR, right angled at Q. Prove that:

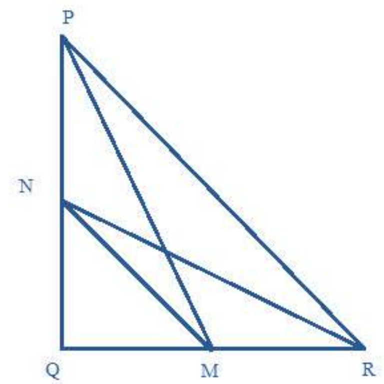
(i) $PM^2 + RN^2 = 5\ MN^2$

(ii) $4\ PM^2 = 4\ PQ^2 + QR^2$

(iii) $4\ RN^2 = PQ^2 + 4\ QR^2$

(iv) $4\ (PM^2 + RN^2) = 5\ PR^2$

Solution:



We draw , PM,MN,NR

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, M andN are the mid-points of the sides QR and PQ respectively, therefore,

PN=NQ,QM=RM

(i)

First, we consider the ΔPQM, and applying Pythagoras theorem we get,

$$\begin{aligned} PM^2 &= PQ^2 + MQ^2 \\ &= (PN + NQ)^2 + MQ^2 \\ &= PN^2 + NQ^2 + 2PN.NQ + MQ^2 \\ &= MN^2 + PN^2 + 2PN.NQ \qquad \begin{bmatrix} \text{From, } \Delta MNQ, \\ MN^2 = NQ^2 + MQ^2 \end{bmatrix} \ldots\ldots(i) \end{aligned}$$

Now, we consider the ΔRNQ, and applying Pythagoras theorem we get,

$$\begin{aligned} RN^2 &= NQ^2 + RQ^2 \\ &= NQ^2 + (QM + RM)^2 \\ &= NQ^2 + QM^2 + RM^2 + 2QM.RM \\ &= MN^2 + RM^2 + 2QM.RM \end{aligned}$$

$$\ldots\ldots(ii)$$

Adding (*i*) and (*ii*) we get,

$$PM^2+RN^2 = MN^2+PN^2+2PN.NQ+MN^2+RM^2+2QM.RM$$
$$PM^2+RN^2 = 2MN^2+PN^2+RM^2+2PN.NQ+2QM.RM$$
$$PM^2+RN^2 = 2MN^2+NQ^2+QM^2+2(QN^2)+2(QM^2)$$
$$PM^2+RN^2 = 2MN^2+MN^2+2MN^2$$
$$PM^2+RN^2 = 5MN^2$$

Hence Proved

(ii)

We consider the ΔPQM, and applying Pythagoras theorem we get,

$$PM^2 = PQ^2+MQ^2$$
$$4PM^2 = 4PQ^2+4MQ^2 \quad \begin{bmatrix}\text{Multiplying both}\\ \text{sides by 4}\end{bmatrix}$$
$$4PM^2 = 4PQ^2+4\cdot\left(\frac{1}{2}QR\right)^2 \quad \left[MQ=\frac{1}{2}QR\right]$$
$$4PM^2 = 4PQ^2+4\cdot\frac{1}{4}QR^2$$
$$4PM^2 = 4PQ^2+QR^2$$

Hence Proved.

(iii)

We consider the ΔRQN, and applying Pythagoras theorem we get,

$$RN^2 = NQ^2+RQ^2$$
$$4RN^2 = 4NQ^2+4QR^2 \quad \begin{bmatrix}\text{Multiplying both}\\ \text{sides by 4}\end{bmatrix}$$
$$4RN^2 = 4QR^2+4\cdot\left(\frac{1}{2}PQ\right)^2 \quad \left[NQ=\frac{1}{2}PQ\right]$$
$$4RN^2 = 4QR^2+4\cdot\frac{1}{4}PQ^2$$
$$4RN^2 = PQ^2+4QR^2$$

Hence Proved

(iv)

First, we consider the ΔPQM, and applying Pythagoras theorem we get,

$$PM^2 = PQ^2+MQ^2$$
$$=(PN+NQ)^2+MQ^2$$
$$=PN^2+NQ^2+2PN.NQ+MQ^2$$
$$=MN^2+PN^2+2PN.NQ \quad \begin{bmatrix}\text{From, } \Delta MNQ,\\ MN^2=NQ^2+MQ^2\end{bmatrix} \ldots\ldots(i)$$

Now, we consider the $\Delta RNQ,$ and applying Pythagoras theorem we get,

$$\begin{aligned} RN^2 &= NQ^2 + RQ^2 \\ &= NQ^2 + (QM + RM)^2 \\ &= NQ^2 + QM^2 + RM^2 + 2QM.RM \\ &= MN^2 + RM^2 + 2QM.RM \end{aligned}$$

$$\ldots\ldots(ii)$$

Adding (i) and (ii) we get,

$$\mathrm{PM}^2 + \mathrm{RN}^2 = MN^2 + PN^2 + 2PN.NQ + MN^2 + RM^2 + 2QM.RM$$

$$\mathrm{PM}^2 + \mathrm{RN}^2 = 2MN^2 + PN^2 + RM^2 + 2PN.NQ + 2QM.RM$$

$$\mathrm{PM}^2 + \mathrm{RN}^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN^2) + 2(QM^2)$$

$$\mathrm{PM}^2 + \mathrm{RN}^2 = 2MN^2 + MN^2 + 2MN^2$$

$$\mathrm{PM}^2 + \mathrm{RN}^2 = 5MN^2$$

$$4\left(\mathrm{PM}^2 + \mathrm{RN}^2\right) = 4 \cdot 5 \cdot (NQ^2 + MQ^2)$$

$$4\left(\mathrm{PM}^2 + \mathrm{RN}^2\right) = 4 \cdot 5 \cdot \left[\left(\frac{1}{2}PQ\right)^2 + \left(\frac{1}{2}QR\right)^2\right]$$

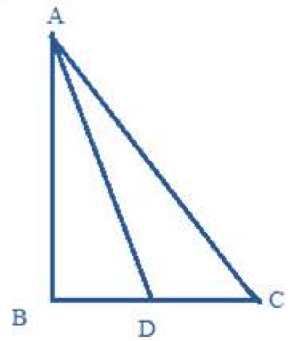
$$\left[\because NQ = \frac{1}{2}PQ, MQ = \frac{1}{2}QR\right]$$

$$4\left(\mathrm{PM}^2 + \mathrm{RN}^2\right) = 5PR^2$$

$$\text{Hence Proved}$$

5. In triangle ABC, ∠B = 90⁰ and D is the midpoint of BC. Prove that:
$AC^2 = AD^2 + 3CD^2$.

Solution:



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

In triangle ABC, $\angle B = 90^{o}$ and D is the mid-point of BC. Join AD. Therefore, BD=DC

First, we consider the $\Delta ADB,$ and applying Pythagoras theorem we get,

$AD^2 = AB^2 + BD^2$

$AB^2 = AD^2 - BD^2 \quad ......(i)$

Similarly, we get from rt. angle triangles ABC we get,

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2 \quad ......(ii)$

From (i) and (ii) ,

$AC^2 - BC^2 = AD^2 - BD^2$

$AC^2 = AD^2 - BD^2 + BC^2$

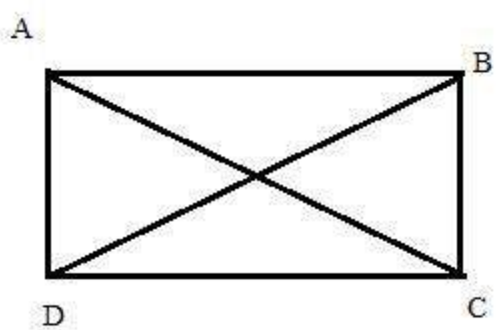
$AC^2 = AD^2 - CD^2 + 4CD^2 \quad \left[BD = CD = \frac{1}{2}BC\right]$

$AC^2 = AD^2 + 3CD^2$

Hence proved.

6. In a rectangle ABCD, prove that:
$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Solution:



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, ABCD is a rectangle angles A,B,C and D are rt. angles.

First, we consider the ΔACD, and applying Pythagoras theorem we get,

$AC^2 = DA^2 + CD^2 \quad ......(i)$

Similarly, we get from rt. angle triangle BDC we get,

$BD^2 = BC^2 + CD^2$

$= BC^2 + AB^2 \quad \left[\begin{array}{l}\text{In a rectangle , opposite} \\ \text{sides are equal,} \therefore CD = AB\end{array}\right] \quad ......(ii)$

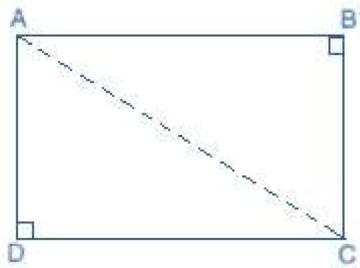
Adding (i) and (ii) ,

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Hence proved.

7. In a quadrilateral ABCD, ∠B = 90^0 and ∠D = 90^0. Prove that:
$2AC^2 - AB^2 = BC^2 + CD^2 + DA^2$.

Solution:



In quadrilateral ABCD, ∠B = 90° and ∠D = 90°.
So, ΔABC and ΔADC are right-angled triangles.

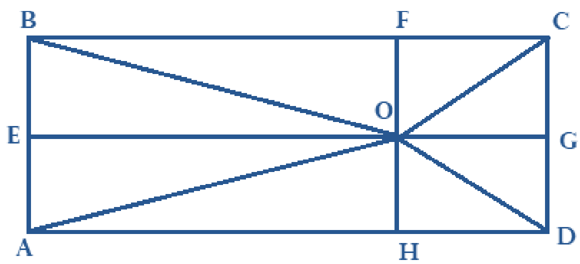
In ΔABC using Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$\Rightarrow AB^2 = AC^2 - BC^2 \ldots\ldots\ldots(i)$

In ΔADC, using Pythagoras theorem,
$AC^2 = AD^2 + DC^2 \ldots\ldots\ldots(ii)$

$LHS = 2AC^2 - AB^2$
$= 2AC^2 - (AC^2 - BC^2)$ [from(i)]
$= 2AC^2 - AC^2 + BC^2$
$= AC^2 + BC^2$
$= AD^2 + DC^2 + BC^2$ [from(ii)]
$= RHS$

8. O is any point inside a rectangle ABCD. Prove that: $OB^2 + OD^2 = OC^2 + OA^2$

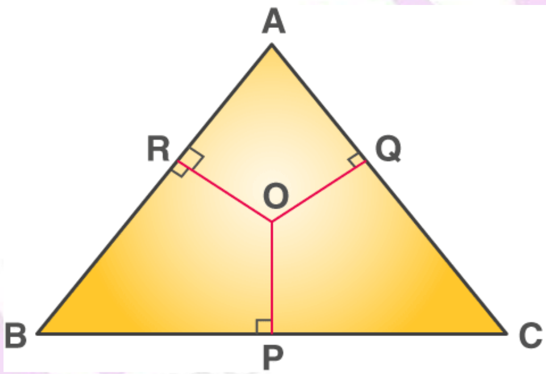
Solution:



Draw rectangle ABCD with arbitrary point O within it, and then draw lines OA, OB, OC, OD.
Then draw lines from point O perpendicular to the sides: OE, OF, OG, OH.
Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to

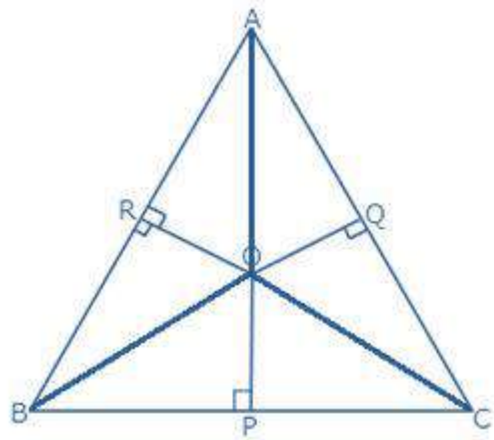
the sum of the squares on the remaining two sides.
Using Pythagorean theorem we have from the above diagram:
$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$
$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$
$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$
$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$
Adding these equalities we get:
$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$
$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$
From which we prove that for any point within the rectangle there is the relation
$OA^2 + OC^2 = OB^2 + OD^2$
Hence Proved.

9. In the following figure, OP, OQ and OR are drawn perpendicular to the isdes BC, CA and AB respectively of triangle ABC. Prove that:
$AR^2+BP^2+CQ^2 = AQ^2 + CP^2 + BR^2$



Solution:

Here, we first need to join OA, OB, and OC after which the figure becomes as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides. First, we consider the ΔARO and applying Pythagoras theorem we get,

$AO^2 = AR^2 + OR^2$

$AR^2 = AO^2 - OR^2 \quad \ldots\ldots (i)$

Similarly, from triangles, BPO, COQ, AOQ, CPO and BRO we get the following results,

$BP^2 = BO^2 - OP^2 \quad \ldots\ldots \quad (ii)$

$CQ^2 = OC^2 - OQ^2 \quad \ldots\ldots \quad (iii)$

$AQ^2 = AO^2 - OQ^2 \quad \ldots\ldots \quad (iv)$

$CP^2 = OC^2 - OP^2 \quad \ldots\ldots \quad (v)$

$BR^2 = OB^2 - OR^2 \quad \ldots\ldots \quad (vi)$

Adding (*i*), (*ii*) and (*iii*),we get

$$AR^2 + BP^2 + CQ^2 = AO^2 - OR^2 + BO^2 - OP^2 + OC^2 - OQ^2 \quad \ldots\ldots (vii)$$

Adding (*iv*), (*v*) and (*vi*),we get ,

$$AQ^2 + CP^2 + BR^2 = AO^2 - OR^2 + BO^2 - OP^2 + OC^2 - OQ^2 \quad \ldots\ldots (viii)$$

From (*vii*) and (*viii*), we get,

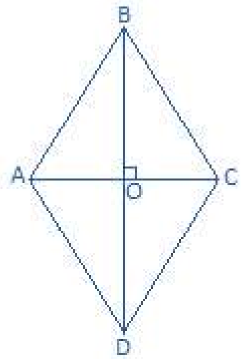
$AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$

Hence proved.

10. Diagonals of rhombus ABCD intersect each other at point O. Prove that:

$OA^2 + OC^2 = 2AD^2 - \frac{BD^2}{2}$

Solution:



Diagonals of the rhombus are perpendicular to each other.

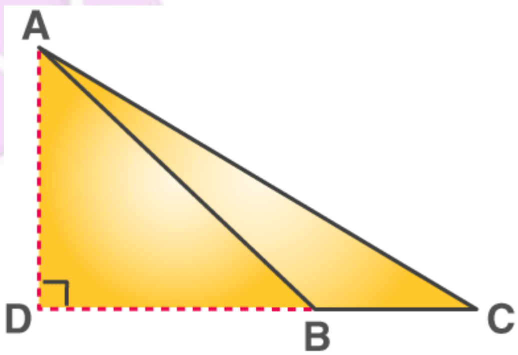
In quadrilateral ABCD, $\angle AOD = \angle COD = 90°$.
So, ΔAOD and ΔCOD are right-angled triangles.

In ΔAOD using Pythagoras theorem,
$AD^2 = OA^2 + OD^2$
$\Rightarrow OA^2 = AD^2 - OD^2 ....(i)$

In ΔCOD using Pythagoras theorem,
$CD^2 = OC^2 + OD^2$
$\Rightarrow OC^2 = CD^2 - OD^2 ....(ii)$

$LHS = OA^2 + OC^2$
$= AD^2 - OD^2 + CD^2 - OD^2 \qquad [from (i) and (ii)]$
$= AD^2 + CD^2 - 2OD^2$
$= AD^2 + AD^2 - 2\left(\frac{BD}{2}\right)^2 \qquad \left[AD = CD \text{ and } OD = \frac{BD}{2}\right]$
$= 2AD^2 - \frac{BD^2}{2}$
$= RHS$

11. In the figure AB = BC and AD is perpendicular to CD. Prove that:
$AC^2 = 2.BC.DC$



Solution:

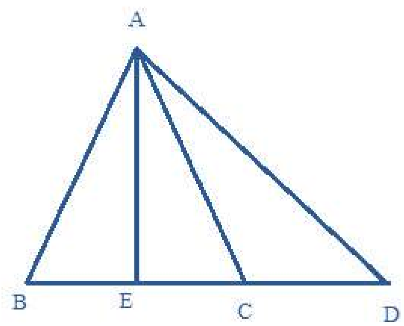
Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
We consider the ΔACD and applying Pythagoras theorem we get,

$$AC^2 = AD^2 + DC^2$$
$$= (AB^2 - DB^2) + (DB + BC)^2$$
$$= BC^2 - DB^2 + DB^2 + BC^2 + 2DB \cdot BC \quad (\text{Given}, AB = BC)$$
$$= 2BC^2 + 2DB \cdot BC$$
$$= 2BC(BC + DB)$$
$$= 2BC \cdot DC$$

Hence Proved.

12. In an isosceles triangle ABC; AB = AC and D is a point on BC produced. Prove that: $AD^2 = AC^2 + BD.CD$

Solution:



In an isosceles triangle ABC; AB = AC and D is point on BC produced. Construct AE perpendicular BC.

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

We consider the rt. angled ΔAED and applying Pythagoras theorem we get,

$$AD^2 = AE^2 + ED^2$$
$$AD^2 = AE^2 + (EC + CD)^2 \quad \ldots\ldots (i)$$
$$[\because ED = EC + CD]$$

Similarly, in ΔAEC,

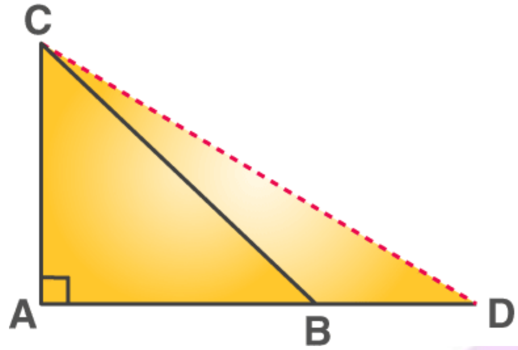
$$AC^2 = AE^2 + EC^2$$
$$AE^2 = AC^2 - EC^2 \quad \ldots\ldots (ii)$$

putting $AE^2 = AC^2 - EC^2$ in (i), we get,

$$AD^2 = AC^2 - EC^2 + (EC + CD)^2$$
$$= AC^2 + CD(CD + 2EC)$$
$$AD^2 = AC^2 + BD \cdot CD \quad [\because 2EC + CD = BD]$$

Hence Proved

13. In an isosceles triangle ABC; angle A = 90⁰, CA = AB and D is a point on AB produced.
Prove that:
$DC^2 - BD^2 = 2AB.AD$



Solution:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

We consider the rt. angled ΔACD and applying Pythagoras theorem we get,

$$CD^2 = AC^2 + AD^2$$

$$CD^2 = AC^2 + (AB + BD)^2 \qquad [\because AD = AB + BD]$$

$$CD^2 = AC^2 + AB^2 + BD^2 + 2AB \cdot BD \qquad \ldots\ldots(i)$$

Similarly, in ΔABC,

$$BC^2 = AC^2 + AB^2$$

$$BC^2 = 2AB^2 \qquad [AB = AC]$$

$$AB^2 = \frac{1}{2}BC^2 \qquad \ldots\ldots(ii)$$

Putting, AB^2 from (ii) in (i) we get,

$$CD^2 = AC^2 + \frac{1}{2}BC^2 \quad + BD^2 + 2AB \cdot BD$$

$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot (AD - AB)$$

$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot AD - 2AB^2$$

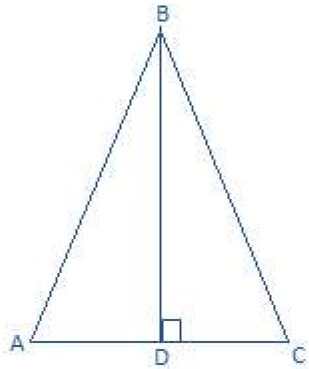
$$CD^2 - BD^2 = 2AB \cdot AD$$

$$DC^2 - BD^2 = 2AB \cdot AD$$

Hence Proved.

14. In triangle ABC, AB=AC and BD is perpendicular to AC. Prove that:
$BD^2 - CD^2 = 2CD \times AD$

Solution:



In right angled ΔADB

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2 ....(i)$

$AC = AD + DC$

$\Rightarrow AC^2 = (AD + DC)^2$

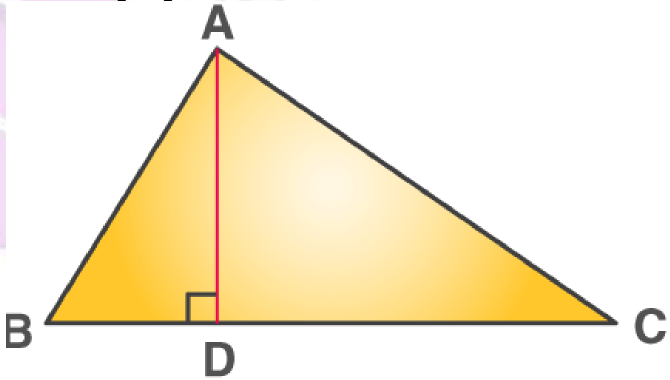
$\Rightarrow AC^2 = AD^2 + DC^2 + 2AD \times DC$

$\Rightarrow AC^2 = AB^2 - BD^2 + DC^2 + 2AD \times DC \quad [from(i)]$

$\Rightarrow AC^2 = AC^2 - BD^2 + DC^2 + 2AD \times DC \quad [AB = AC]$

$\Rightarrow BD^2 - DC^2 = 2AD \times DC$

15. In the following figure, AD is perpendicular to BC and D divides BC in the ratio 1:3.



Prove that: $2AC^2 = 2AB^2 + BC^2$

Solution:

According to the question,

$BD : DC = 1 : 3$

$\Rightarrow BD = \frac{1}{4} BC$ and $CD = \frac{3}{4} BC$

$AC^2 = AD^2 + CD^2$ and $AB^2 = AD^2 + BD^2$

Therefore,

$$AC^2 - AB^2 = CD^2 - BD^2$$
$$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$
$$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2$$
$$= \frac{1}{2}BC^2$$

$\therefore 2AC^2 - 2AB^2 = BC^2$

$$2AC^2 = 2AB^2 + BC^2$$

Hence proved