EXERCISE 14(A)

1. **The sum of the interior angles of a polygon is four times the sum of its exterior angles. Find the number of sides in the polygon.**

Solution:

The sum of the interior angle=4 times the sum of the exterior angles.
Therefore the sum of the interior angles = 4×360° =1440°.
Now we have

$$(2n-4)\times 90^0 = 1440^0$$
$$2n-4 = 16$$
$$2n = 20$$
$$n = 10$$

Thus the number of sides in the polygon is 10.

2. **The angles of a pentagon are in the ratio 4:8:6:4:5. Find each angle of the pentagon.**

Solution:

Let the angles of the pentagon are 4x, 8x, 6x, 4x and 5x.
Thus we can write

$$4x+8x+6x+4x+5x = 540^0$$
$$27x = 540^0$$
$$x = 20^0$$

Hence the angles of the pentagon are:
$4\times20^0 = 80^0$
$8\times20^0 = 160^0$
$6\times20^0 = 120^0$
$4\times20^0 = 80^0$
$5\times20^0 = 100^0$

3. **One angle of a six-sided polygon is 140^0 and the other angles are equal. Find the measure of each equal angle.**

Solution:

Let the measure of each equal angles are x.
Then we can write

$$140^0 + 5x = (2\times6-4)\times 90^0$$
$$140^0 + 5x = 720^0$$
$$5x = 580^0$$
$$x = 116^0$$

Therefore the measure of each equal angles are 116^0

4. **In a polygon, there are 5 right angles and the remaining angles are equal to 195^0 each. Find the number of sides in the polygon.**

Solution:

Let the number of sides of the polygon is *n* and there are k angles with measure 195°.
Therefore we can write:

$$5\times90^0+k\times195^0=(2n-4)90^0$$
$$180^0n-195^0k=450^0-360^0$$
$$180^0n-195^0k=90^0$$
$$12n-13k=6$$

In this linear equation n and k must be integer. Therefore to satisfy this equation the minimum value of k must be 6 to get n as integer.
Hence the number of sides are: 5 + 6 = 11.

5. Three angles of a seven sided polygon are 132^0 each and the remaining four angles are equal. Find the value of each equal angle.

Solution:

Let the measure of each equal angles are x.
Then we can write:

$$3\times132^0+4x=(2\times7-4)90^0$$
$$4x=900^0-396$$
$$4x=504$$
$$x=126^0$$

Thus the measure of each equal angles are 126°.

6. Two angles of an eight sided polygon are 142^0 and 176^0. If the remaining angles are equal to each other; find the magnitude of each of the equal angles.

Solution:

Let the measure of each equal sides of the polygon is x.
Then we can write:

$$142^0+176^0+6x=(2\times8-4)90^0$$
$$6x=1080^0-318^0$$
$$6x=762^0$$
$$x=127^0$$

Thus the measure of each equal angles are 127°.

7. In a pentagon ABCDE, AB is parallel to DC and angles A: E: D=3:4:5. Find angle E.

Solution:

Let the measure of the angles are 3x, 4x and 5x.
Thus

$$\angle A+\angle B+\angle C+\angle D+\angle E=540^0$$
$$3x+(\angle B+\angle C)+4x+5x=540^0$$
$$12x+180^0=540^0$$
$$12x=360^0$$
$$x=30^0$$

Thus the measure of angle E will be $4\times30^0=120^0$

8. AB, BC and CD are the three consecutive sides of a regular polygon. If $\angle BAC=15^0$; Find,

(i) Each interior angle of the polygon.

(ii) Each exterior angle of the polygon.

(iii) Number of sides of the polygon.

Solution:

(i)

Let each angle of measure x degree.
Therefore measure of each angle will be:

$$x=180^0-2\times15^0=150^0$$

(ii)

Let each angle of measure x degree.
Therefore measure of each exterior angle will be:

$$x=180^0-150^0$$
$$=30^0$$

(iii)

Let the number of each sides is n.
Now we can write

$$n\cdot150^0=(2n-4)\times90^0$$
$$180^0n-150^0n=360^0$$
$$30^0n=360^0$$
$$n=12$$

Thus the number of sides are 12.

9. The ratio between an exterior angle and an interior angle of a regular polygon is 2:3. Find the number of sides in the polygon.

Solution:

Let measure of each interior and exterior angles are 3k and 2k.
Let number of sides of the polygon is n.
Now we can write:

$$n\cdot3k=(2n-4)\times90^0$$
$$3nk=(2n-4)90^0 \qquad \ldots(1)$$

Again

$n \cdot 2k = 360^0$

$nk = 180^0$

From (1)

$3 \cdot 180^0 = (2n-4)90^0$

$3 = n-2$

$n = 5$

Thus the number of sides of the polygon is 5.

10. The difference between an exterior angle of (n-1) sided regular polygon and an exterior angle of (n+2) sided regular polygon is 6^0. Find the value of n.

Solution:

For (n-1) sided regular polygon:
Let measure of each angle is x.
Therefore

$(n-1)x = (2(n-1)-4)90^0$

$x = \frac{n-3}{n-1}180^0$

For (n+1) sided regular polygon:
Let measure of each angle is y.
Therefore

$(n+2)y = (2(n+2)-4)90^0$

$y = \frac{n}{n+2}180^0$

Now we have

$y - x = 6^0$

$\frac{n}{n+2}180^0 - \frac{n-3}{n-1}180^0 = 6^0$

$\frac{n}{n+2} - \frac{n-3}{n-1} = \frac{1}{30}$

$30n(n-1) - 30(n-3)(n+2) = (n+2)(n-1)$

$-30n + 30n + 180 = n^2 + n - 2$

$n^2 + n - 182 = 0$

$(n-13)(n+14) = 0$

$n = 13, -14$

Thus the value of n is 13.

11. Two alternate sides of regular polygon, when produced, meet at right angle. Find:

(i) The value of each exterior angle of the polygon.

(ii) The number of sides in the polygon.

Solution:

(i) Let the measure of each exterior angle is x and the number of sides is n.
Therefore we can write:

$$n = \frac{360^0}{x}$$

Now we have

$$x + x + 90^0 = 180^0$$
$$2x = 90^0$$
$$x = 45^0$$

(ii) Thus the number of sides in the polygon is:

$$n = \frac{360^0}{45^0}$$
$$= 8$$

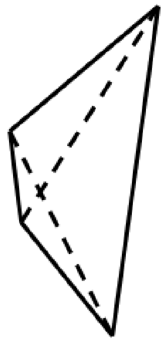
EXERCISE 14(B) PAGE: 175

1. State, 'true' or 'false'
(i) The diagonals of a rectangle bisect each other.
(ii) The diagonals of a quadrilateral bisect each other.
(iii) The diagonals of a parallelogram bisect each other at tight angle.
(iv) Each diagonal of a rhombus bisect it.
(v) The quadrilateral, whose four sides are equal, is a square.
(vi) Every rhombus is a parallelogram.
(vii) Every parallelogram is a rhombus.
(viii) Diagonals of a rhombus are equal.
(ix) If two adjacent sides of a parallelogram are equal, it is rhombus.
(x) If the diagonals of a quadrilateral bisect each other at right angle, the quadrilateral is a square.

Solution:

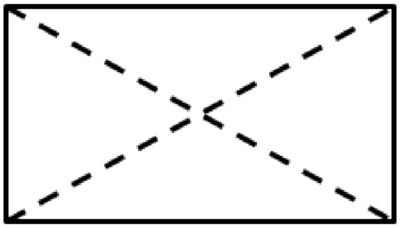
(i) True.
This is true, because we know that a rectangle is a parallelogram. So, all the properties of a parallelogram are true for a rectangle. Since the diagonals of a parallelogram bisect each other, the same holds true for a rectangle.

(ii) False
This is not true for any random quadrilateral. Observe the quadrilateral shown below:



Clearly the diagonals of the given quadrilateral do not bisect each other. However, if the quadrilateral was a special quadrilateral like a parallelogram, this would hold true.

(iii) False
Consider a rectangle as shown below.

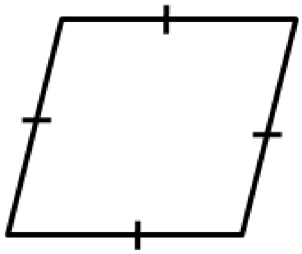


It is a parallelogram. However, the diagonals of a rectangle do not intersect at right angles, even though they bisect each other.

(iv) True
Since a rhombus is a parallelogram, and we know that the diagonals of a parallelogram bisect each other, hence the diagonals of a rhombus too, bisect other.

(v) False
This need not be true, since if the angles of the quadrilateral are not right angles, the quadrilateral would be a rhombus rather than a square.

(vi) True



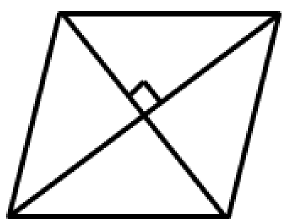
A parallelogram is a quadrilateral with opposite sides parallel and equal.
Since opposite sides of a rhombus are parallel, and all the sides of the rhombus are equal, a rhombus is a parallelogram.

(vii) False
This is false, since a parallelogram in general does not have all its sides equal. Only opposite sides of a parallelogram are equal. However, a rhombus has all its sides equal. So, every parallelogram cannot be a rhombus, except those parallelograms that have all equal sides.

(viii) False
This is a property of a rhombus. The diagonals of a rhombus need not be equal.

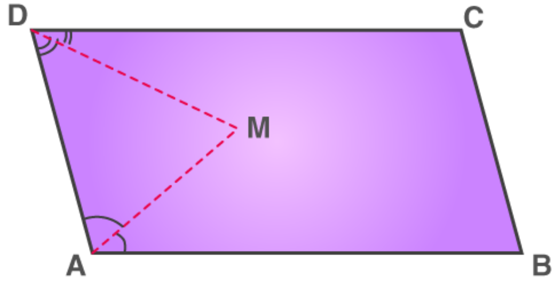
(ix) True
A parallelogram is a quadrilateral with opposite sides parallel and equal.
A rhombus is a quadrilateral with opposite sides parallel, and all sides equal.
If in a parallelogram the adjacent sides are equal, it means all the sides of the parallelogram are equal, thus forming a rhombus.

(x) False



Observe the above figure. The diagonals of the quadrilateral shown above bisect each other at right angles, however the quadrilateral need not be a square, since the angles of the quadrilateral are clearly not right angles.

2. **In the figure given below, AM bisects angle A and DM bisects angle D of parallelogram ABCD. Prove that:$\angle$AMD = 90^0.**



Solution:

From the given figure we conclude that

$\angle A+\angle D=180^\circ$ [since consecutive angles are supplementary]

$\frac{\angle A}{2}+\frac{\angle D}{2}=90^\circ$

Again from the ΔADM

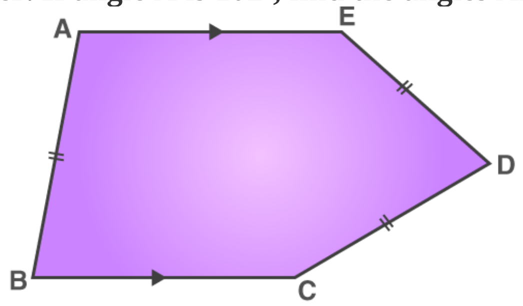
$\frac{\angle A}{2}+\frac{\angle D}{2}+\angle M=180^\circ$

$\Rightarrow 90^\circ+\angle M=180^\circ \quad \left[\sin ce \frac{\angle A}{2}+\frac{\angle D}{2}=90^\circ\right]$

$\Rightarrow \angle M=90^\circ$

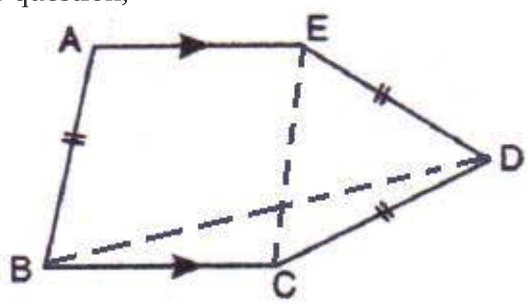
Hence $\angle AMD=90^\circ$

3. **In the following figure, AE and BC are equal and parallel and the three sides AB, CD and DE are equal to one another. If angle A is 102^0, find the angles AEC ad BCD.**



Solution:

According to the question,



Given that AE = BC
We have to find ∠AEC ∠BCD
Let us join EC and BD.
In the quadrilateral AECB
AE = BC and AB = EC
also AE || BC
⇒ AB || EC
So quadrilateral is a parallelogram.

In parallelogram consecutive angles are supplementary
$\Rightarrow \angle A + \angle B = 180^\circ$
$\Rightarrow 102^\circ + \angle B = 180^\circ$
$\Rightarrow \angle B = 78^\circ$

In parallelogram opposite angles are equal
$\Rightarrow \angle A = \angle BEC$ and $\angle B = \angle AEC$
$\Rightarrow \angle BEC = 102^\circ$ and $\angle AEC = 78^\circ$

Now consider ΔECD
EC = ED = CD [Since AB = EC]
Therefore ΔECD is an equilateral triangle.
$\Rightarrow \angle ECD = 60^\circ$

$\angle BCD = \angle BEC + \angle ECD$
$\Rightarrow \angle BCD = 102^\circ + 60^\circ$
$\Rightarrow \angle BCD = 162^\circ$

Therefore $\angle AEC = 78^\circ$ and $\angle BCD = 162^\circ$

4. In a square ABCD, diagonals meet at O. P is a point on BC, such that OB=BP. Show that:

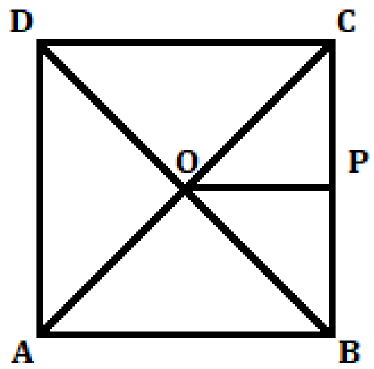
(i) $\angle POC = (22\frac{1}{2})$

(ii) $\angle BDC = 2\angle POC$

(iii) $\angle BOP = 3\angle COP$

Solution:

Given ABCD is a square and diagonals meet at O.P is a point on BC such that OB=BP



In the
ΔBOC and ΔDOC

$\Rightarrow BD = BD$ [common side]

$\Rightarrow BO = CO$

Þ$OD = OC$ [since diagonals cuts at O]

$\Delta BOC \cong \Delta DOC$ [by SSS]

Therefore
$\angle BOC = 90°$
NOW
$\angle POC = 22.5$

$\angle BOP = 67.5$ [since $\angle BOC = 67.5° + 22.5°$]

Again
ΔBDC

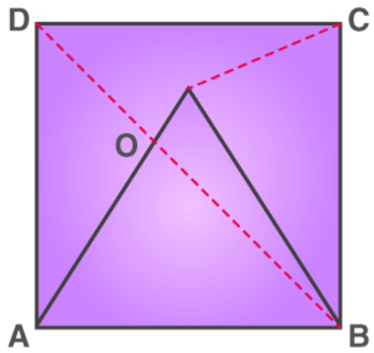
$\angle BDC = 45°$ [since $\angle B = 45°, \angle C = 90°$]

Therefore
$\angle BDC = 2\angle POC$
$\angle BOP = 67.5°$

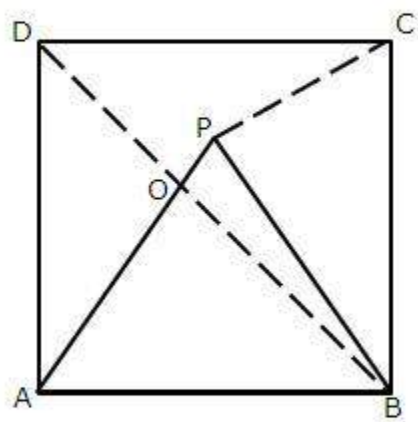
$\Rightarrow \angle BOP = 2\angle POC$
Hence proved

5. The given figure shows a square ABCD and an equilateral triangle ABP. Calculate:

(i) ∠AOB
(ii) ∠BPC
(iii) ∠PCD
(iv) Reflex ∠APC



Solution:



In the given figure ΔAPB is an equilateral triangle

Therefore all its angles are $60°$

Again in the

ΔADB

$\angle ABD = 45°$

$\angle AOB = 180° - 60° - 45°$

$= 75°$

Again

$\triangle BPC$

$\Rightarrow \angle BPC = 75°$ [Since BP = CB]

Now

$\angle C = \angle BCP + \angle PCD$

$\Rightarrow \angle PCD = 90° - 75°$

$\Rightarrow \angle PCD = 15°$

Therefore

$\angle APC = 60° + 75°$

$\Rightarrow \angle APC = 135°$

$\Rightarrow Reflex\ \angle APD = 360° - 135° = 225°$

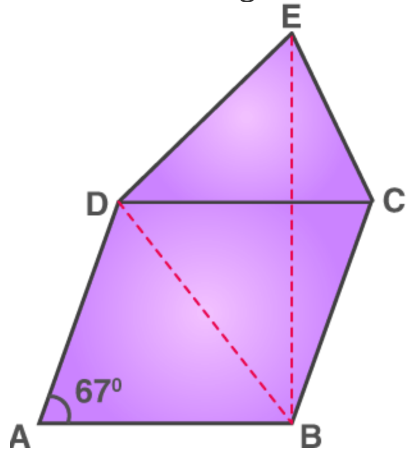
(i) $\angle AOB = 75^\circ$

(ii) $\angle BPC = 75^\circ$

(iii) $\angle PCD = 15^\circ$

(iv) Reflex $\angle APD = 225^\circ$

6. In the given figure; ABCD is a rhombus with angle A=67^0.



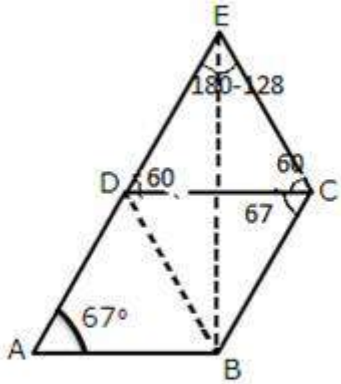
If DEC is an equilateral triangle, calculate:

(i) ∠CBE

(ii) ∠DBE

Solution:

Given that the figure ABCD is a rhombus with angle A = 67^o



In the rhombus We have

$\angle A = 67^\circ = \angle C$ [Opposite angles]

$\angle A + \angle D = 180^\circ$ [Consecutive angles are supplementary]

$\Rightarrow \angle D = 113^\circ$

$\Rightarrow \angle ABC = 113^\circ$

Consider $\triangle DBC$,

$DC = CB$ [Sides of rhombous]

$So \triangle DBC$ is an isoscales triangle

$\Rightarrow \angle CDB = \angle CBD$

$Also$,

$\angle CDB + \angle CDB + \angle BCD = 180^\circ$

$\Rightarrow 2\angle CBD = 113^\circ$

$\Rightarrow \angle CDB = \angle CBD = 56.5^\circ$............$(i)$

Consider $\triangle DCE$,

$EC = CB$

$So \triangle DCE$ is an isoscales triangle

$\Rightarrow \angle CBE = \angle CEB$

$Also$,

$\angle CBE + \angle CEB + \angle BCE = 180^\circ$

$\Rightarrow 2\angle CBE = 53^\circ$

$\Rightarrow \angle CDE = 26.5^\circ$

$From\ (i)$

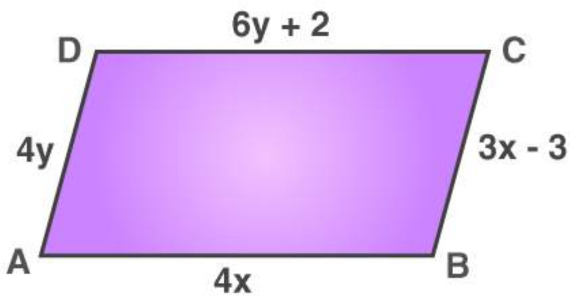
$\angle CBD = 56.5^\circ$

$\Rightarrow \angle CBE + \angle DBE = 56.5^\circ$

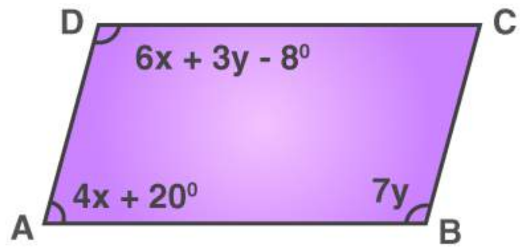
$\Rightarrow 26.5^\circ + \angle DBE = 56.5^\circ$

$\Rightarrow \angle DBE = 30.5^\circ$

7. **In each of the following figures, ABCD is a parallelogram.**
   In each case, find the value of x and y.
   (i)



(ii)



Solution:

(i) ABCD is a parallelogram

Therefore

AD=BC

AB=DC

Thus

$4y = 3x - 3$ [since AD=BC]

$\Rightarrow 3x - 4y = 3$ (i)

$6y + 2 = 4x$ [since AB=DC]

$4x - 6y = 2$ (ii)

Solving equations (i) and (ii) we have

x=5

y=3

(ii)

In the figure ABCD is a parallelogram

$\angle A = \angle C$

$\angle B = \angle D$ [since opposite angles are equal]

Therefore

$7y = 6y + 3y - 8^\circ$ (i) [Since $\angle A = \angle C$]

$4x + 20^\circ = 0$ (ii)

Solving (i), (ii) we have

$X = 12^\circ$

$Y = 16^\circ$

8. The angles of quadrilateral are in the ratio 3:4:5:6. Show that the quadrilateral is a trapezium.

Solution:

Given that the angles of a quadrilateral are in the ratio $3:4:5:6$ Let the angles be $3x, 4x, 5x, 6x$

$3x+4x+5x+6x=360°$

$\Rightarrow x=\frac{360°}{18}$

$\Rightarrow x=20°$

Therefore the angles are

$3\times 20=60°,$

$4\times 20=80°,$

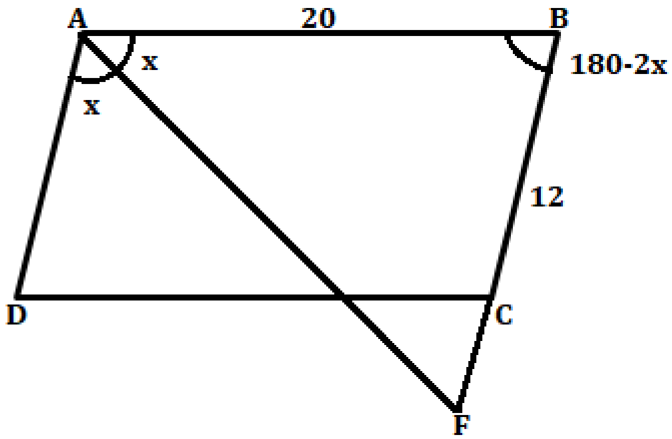
$5\times 20=100°,$

$6\times 20=120°$

Since all the angles are of different degrees thus forms a trapezium

9. In a parallelogram ABCD, AB=20 cm and AD=12cm. The bisector of angle A meets DC at E nd BC produced at F. Find the length of CF.

Solution:



Given AB = 20 cm and AD = 12 cm.

From the above figure,

it's evident that ABF is an isosceles triangle with angle BAF = angle BFA = x

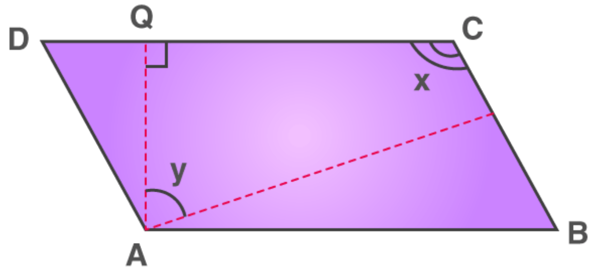
So AB = BF = 20

BF = 20

BC + CF = 20

CF = 20 - 12 = 8 cm

10. In parallelogram ABCD, AP and AQ are perpendiculars from vertex of obtuse angle A as shown. If angles x:y=2:1; find the angles of the parallelogram.



Solution:

We know that AQCP is a quadrilateral. So sum of all angles must be 360.

∴ x + y + 90 + 90 = 360

x + y = 180

Given x:y = 2:1

So substitute x = 2y

3y = 180

y = 60

x = 120

We know that angle C = angle A = x = 120

Angle D = Angle B = 180 - x = 180 - 120 = 60

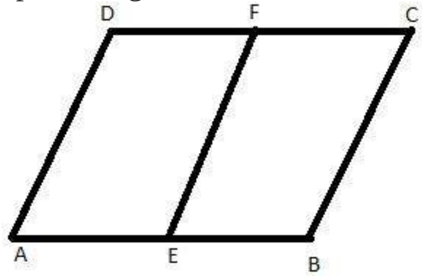
Hence, angles of parallelogram are 120, 60, 120 and 60.

EXERCISE 14(C) PAGE: 181

1. **E is mid-point of sides AB and F is the midpoint of side DC pf parallelogram ABCD. Prove that AEFD is a parallelogram.**

Solution:

Let us draw a parallelogram ABCD Where F is the midpoint
Of side DC of parallelogram ABCD
To prove: AEFD is a parallelogram

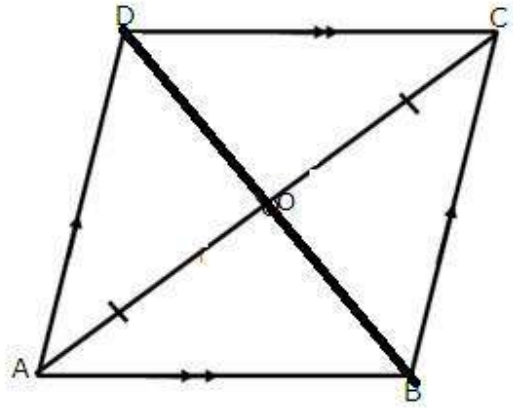


Proof:
Therefore ABCD
$AB \parallel DC$
$BC \parallel AD$
$AB = DC$
$\frac{1}{2}AB = \frac{1}{2}DC$
$AE = DF$
Also AD|| EF
Therefore, AEFC is a parallelogram.

2. **The diagonal BD of a parallelogram ABCD bisects angles B and D. Prove that ABCD is a rhombus.**

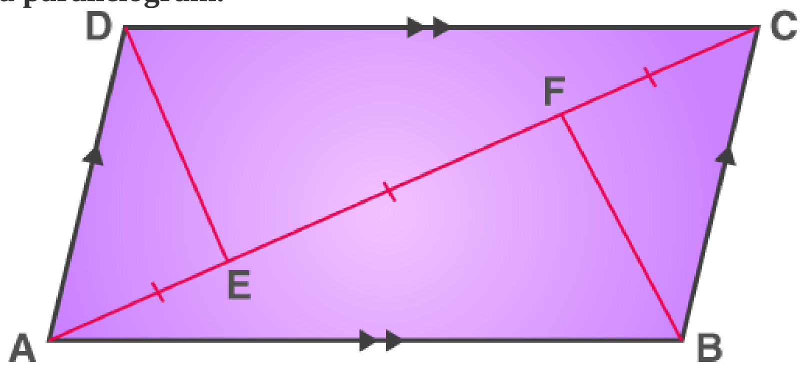
Solution:

Given:
ABCD is a parallelogram where the diagonal BD bisects
Parallelogram ABCD at angle B and D
To prove:
ABCD is a rhombus
Proof:
Let us draw a parallelogram ABCD where the diagonal BD bisects the parallelogram at angle B and D.
Construction:
Let us join AC as a diagonal of the parallelogram ABCD



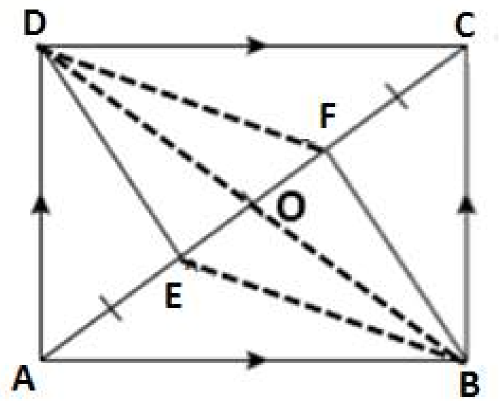
Since ABCD is a parallelogram
Therefore,
AB=DC
AD=BC
Diagonal BD bisects angle b and D
So $\angle COD = \angle DOA$
Again AC also bisects at A and C
Therefore $\angle AOB = \angle BOC$
Thus ABCD is a rhombus.
Hence proved

3. The given figure shows a parallelogram ABCD in which AE=EF=FC. Prove that:
(i) DE is parallel to FB
(ii) DE=FB
(iii) DEBF is a parallelogram.



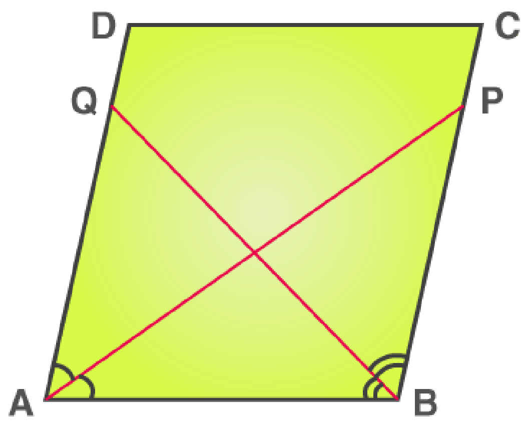
Solution:

Construction:
Join DF and EB.
Join diagonal BD.

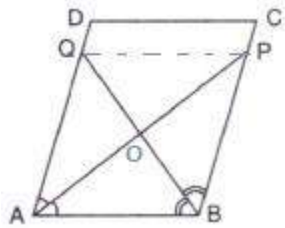


Since diagonals of a parallelogram bisect each other.
$\therefore$ OA = OC and OB = OD
Also, AE = EF = FC
Now, OA = OC and AE = FC
$\Rightarrow$ OA – AE = OC – FC
$\Rightarrow$ OE = OF
Thus, in quadrilateral DEFB, we have
OB = OD and OE = OF
$\Rightarrow$ Diagonals of a quadrilateral DEFB bisect each other.
$\Rightarrow$ DEFB is a parallelogram.
$\Rightarrow$ DE is parallel to FB
$\Rightarrow$ DE = FB (Opposite sides are equal)

4. In the given figure, ABCD is a parallelogram in which AP bisects angle A and BQ bisects angle B. Prove that:
(i) AQ = BP
(ii) PQ = CD
(iii) ABPQ is a parallelogram



Solution:



Join PQ.

Consider the $\triangle AOQ$ and $\triangle BOP$

$\angle AOQ = \angle BOP$ [opposite angles]

$\angle OAQ = \angle BPO$ [alternate angles]

$\Rightarrow \triangle AOQ \cong \triangle BOP$ [AA test]

Hence $AQ = BP$

Consider the $\triangle QOP$ and $\triangle AOB$

$\angle AOB = \angle QOP$ [opposite angles]

$\angle OAB = \angle APQ$ [alternate angles]

$\Rightarrow \triangle QOP \cong \triangle AOB$ [AA test]

Hence $PQ = AB = CD$

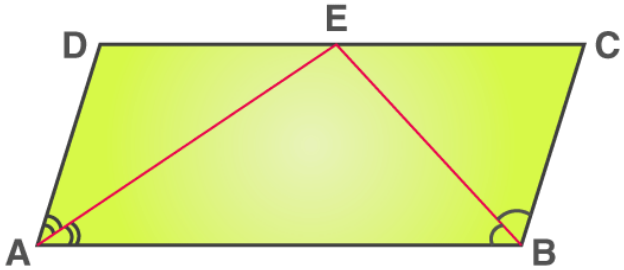
Consider the quadrilateral $QPCD$

$DQ = CP$ *and* $DQ \parallel CP$ [Since $AD = BC$ *and* $AD \parallel BC$]

Also $QP = DC$ *and* $AB \parallel QP \parallel DC$

Hence quadrilateral $QPCD$ is a parallelogram.

5. In the given figure, ABCD is a parallelogram. Prove that: AB = 2BC.



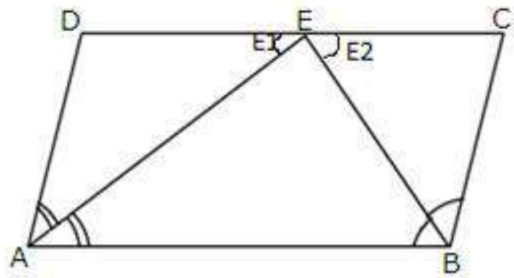
Solution:

Given:

ABCD is a parallelogram

To prove:

AB=2BC



Proof:

ABCD is a parallelogram

$\angle A+\angle D=\angle B+\angle C=180°$

From the ΔAEB we have

$\Rightarrow \frac{\angle A}{2}+\frac{\angle B}{2}+\angle E=180°$

$\Rightarrow \angle A-\frac{\angle A}{2}+\angle D+\angle E1=180°$ [taking E1 as new angle]

$\Rightarrow \angle A+\angle D+\angle E1=180°+\frac{\angle A}{2}$

$\Rightarrow \angle E1=\frac{\angle A}{2}$ [Since $\angle A+\angle D=180°$]

Similarly,

$\angle E2=\frac{\angle B}{2}$

AB=DE+EC

=AD+BC

=2BC [since AD=BC]

Hence proved

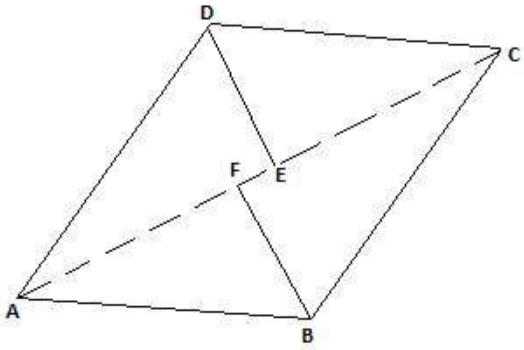
6. Prove that the bisectors of opposite angles of a parallelogram are parallel.

Solution:

Given ABCD is a parallelogram. The bisectors of $\angle ADC$ and $\angle BCD$ meet at E.

The bisectors of $\angle ABC$ and $\angle BCD$ meet at F

From the parallelogram ABCD we have



$\angle ADC + \angle BCD = 180°$ [sum of adjacent angles of a parallelogram]

$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90°$

$\Rightarrow \angle EDC + \angle ECD = 90°$

In triangle ECD sum of angles $= 180°$

$\Rightarrow \angle EDC + \angle ECD + \angle CED = 180°$

$\Rightarrow \angle CED = 90°$

Similarly taking triangle BCF it can be prove that $\angle BFC = 90°$

$\angle BFC = \angle CED = 90°$

Therefore the lines DE and BF are parallel

Hence proved

7. Prove that the bisectors of interior angles of a parallelogram form a rectangle.

Solution:

Given: ABCD is a parallelogram

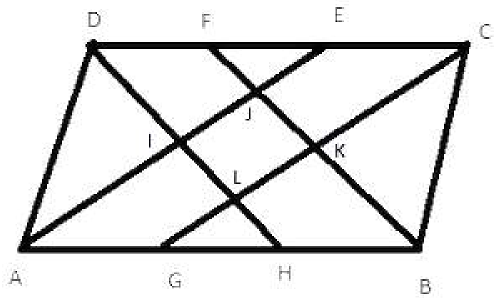
AE bisects $\angle BAD$

BF bisects $\angle ABC$

CG bisects $\angle BCD$

DH bisecsts $\angle ADC$

To prove: LKJI is a rectangle



Proof :

$\angle BAD + \angle ABC = 180°$ [adjacent angles of a parallelogram are supplementary]

$\angle BAJ = \frac{1}{2}\angle BAD$ [AE bisects ĐBAD]

$\angle ABJ = \frac{1}{2}\angle ABC$ [DH bisect ĐABC]

$\angle BAJ + \angle ABJ = 90°$ [halves of supplementary angles are complementary]

ABJ is a right triangle because its acute interior angles are complementary.
Similarly

$\angle DLC = 90°$

$\angle AID = 90°$

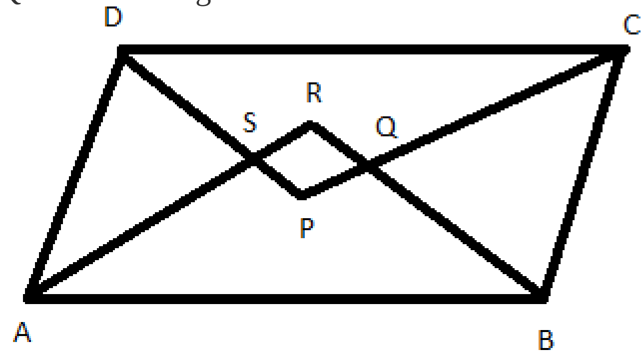
Then $\angle JIL = 90°$ because $\angle AID$ and $\angle JIL$ are vertical angles

LJKI is a rectangle, since its interior angles are all right angles
Hence proved

8. Prove that the bisectors of the interior angles of a rectangle form a square.

Solution:

Given: A parallelogram ABCD in which AR, BR, CP, DP
Are the bisectors of angles A, B, C, D respectively forming quadrilaterals PQRS.
To prove: PQRS is a rectangle



Proof :

$\angle DCB + \angle ABC = 180°$ [co-interior angles of parallelogram are supplementary]

$\Rightarrow \frac{1}{2}\angle DCB + \frac{1}{2}\angle ABC = 90°$

$\Rightarrow \angle 1 + \angle 2 = 90°$

$\Delta CQB, \angle 1 + \angle 2 + \angle CQB = 180°$

$\angle CQB = 180° - 90° = 90°$

$\angle RQP = 90°$ [$\angle CQB = \angle RQP$, *vertically* opposite angles]

$\angle QRP = \angle RSP = \angle SPQ = 90°$

Hence PQRS is a rectangle

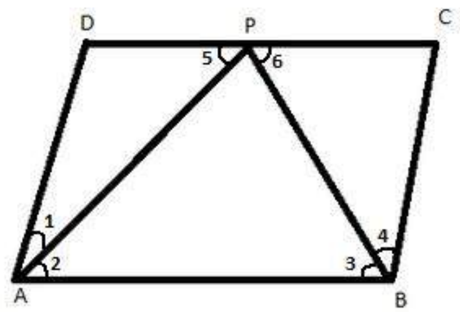
9. In parallelogram ABCD, the bisector of angle A meets DC at P and AB=2AD.

Prove that:

(i) BP bisects angle B.

(ii) Angle APB = 90^0.

Solution:



(i) Let $AD = x$

$AB = 2AD = 2x$

Also AP is the bisector $\angle A$

$\angle 1 = \angle 2$

Now ,

$\angle 2 = \angle 5$ [alternate angles]

Therefore $\angle 1 = \angle 5$

Now

$AP = DP = x$ [sides opposite to equal angles are also equal]

Therefore

$AB = CD$ [opposite sides of parallelogram are equal]

$CD = 2x$

$\Rightarrow DP + PC = 2x$

$\Rightarrow x + PC = 2x$

$\Rightarrow PC = x$

Also , $BC = x$

ΔBPC

$\Rightarrow \angle 6 = \angle 4$ [*angles* opposite to equal sides are equal]

In $\Rightarrow \angle 6 = \angle 3$

Therefore $\angle 3 = \angle 4$

Hence BP bisect $\angle B$

(ii)

Opposite angles are supplementary

Therefore

$\angle 1+\angle 2+\angle 3+\angle 4=180°$

$\Rightarrow 2\angle 2+2\angle 3=180° \begin{bmatrix}\angle 1=\angle 2\\ \angle 3=\angle 4\end{bmatrix}$

$\Rightarrow \angle 2+\angle 3=90°$

ΔAPB

$\angle 2+\angle 3\angle APB=180°$

$\Rightarrow \angle APB=180°-90°$ [by angle sum property]

$\Rightarrow \angle APB=90°$

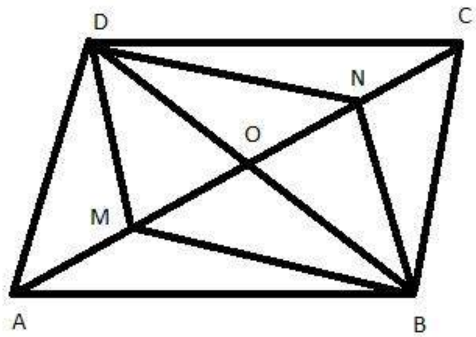
Hence proved

10. Points M and N are taken on the diagonal AC of a parallelogram ABCD such that AM=CN. Prove that BMDN is a parallelogram.

Solution:

Points M and N are taken on the diagonal AC of a parallelogram ABCD such that AM=CN.

Prove that BMDN is a parallelogram



Construction: Join B to D to meet AC in O.

Proof: We know that the diagonals of parallelogram bisect each other.

Now, AC and BD bisect each other at O.

OC=OA

AM=CN

$\Rightarrow$ OA-AM=OC-CN

$\Rightarrow$ OM=ON

Thus in a quadrilateral BDMN, diagonal BD and MN are such that OM=ON and OD=OB

Therefore, the diagonals AC and PQ bisect each other.

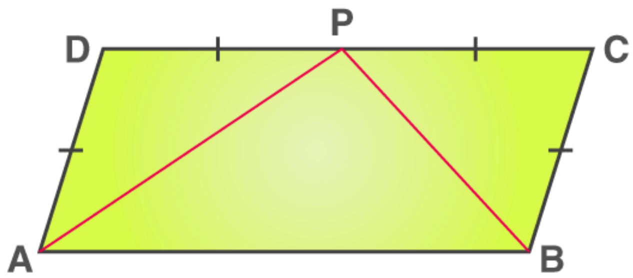
Hence, BMDN is a parallelogram

11. In the following figure, ABCD is a parallelogram. Prove that:

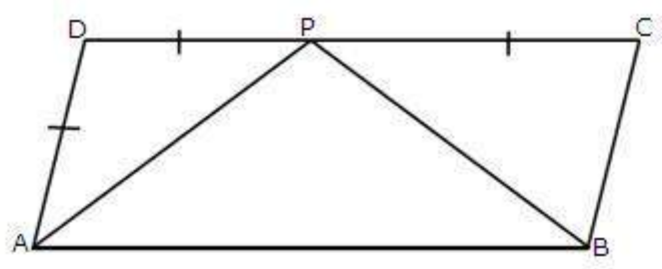
(i) AP bisects angle A

(ii) BP bisects angle B

(iii) Angle DAP + Angle CBP = Angle APB



Solution:



Consider ΔADP and ΔBCP

AD=BC [since ABCD is a parallelogram]

DC=AB [since ABCD is a parallelogram]

$\angle A = \angle C$ [opposite angles]

$\Delta ADP \cong \Delta BCP$ [SAS]

Therefore AP=BP

AP bisects $\angle A$

BP bisects $\angle B$

In ΔAPB

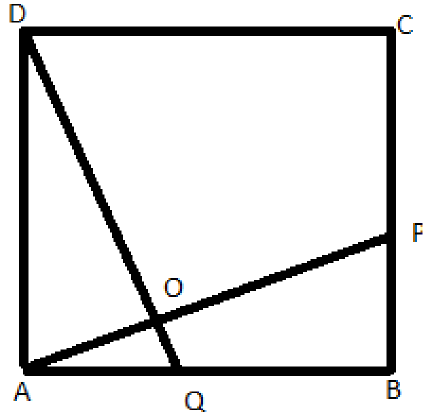
AP=PB

$\angle APB = \angle DAP + \angle BCP$

Hence proved

12. ABCD is a square. A is joined to a point P on BC and D is joined to a point Q on AB. If AP=DQ; prove that AP and DQ are perpendicular to each other.

Solution:



ABCD is a square and AP=PQ

Consider $\triangle DAQ$ and $\triangle ABP$

$\angle DAQ = \angle ABP = 90^\circ$

$DQ = AP$

$AD = AB$

$\triangle DAQ \cong \triangle ABP$

$\Rightarrow \angle PAB = \angle QDA$

Now,

$\angle PAB + \angle APB = 90^\circ$

also $\angle QDA + \angle APB = 90^\circ$ $[\angle PAB = \angle QDA]$

Consider $\triangle AOQ$ By ASP

$\angle QDA + \angle APB + \angle AOD = 180^\circ$

$\Rightarrow 90^\circ + \angle AOD = 180^\circ$

$\Rightarrow \angle AOD = 90^\circ$

Hence AP and DQ are perpendicular.

13. In a quadrilateral ABCD, AB=AD and CB=CD. Prove that:

(i) AC bisects angle BAD.

(ii) AC is perpendicular bisector of BD.

Solution:

Given: ABCD is quadrilateral,

AB=AD

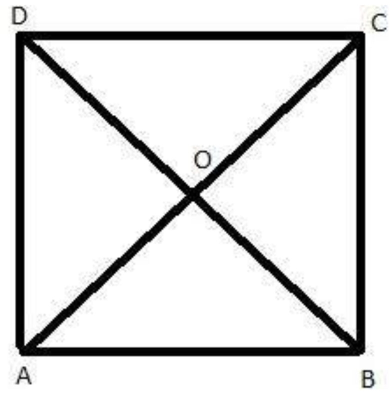
CB=CD

To prove:

(i) AC bisects angle BAD.

(ii) AC is perpendicular bisector of BD.

Proof:



In ΔABC and ΔADC

$AB=AD$ [given]

$CB=CD$ [given]

$AC=AC$ [common side]

$\Delta ABC \cong \Delta ADC$ [SSS]

Therefore, AC bisects $\angle BAD$

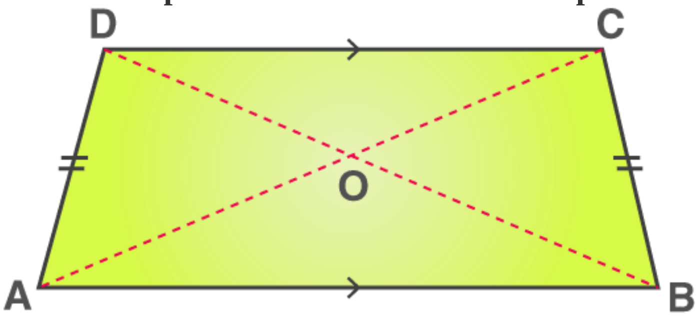
$OD=OB$

$OA=OA$ [diagonals bisect each other at O]

Thus AC is perpendicular bisector of BD.

Hence proved

14. The following figure shows a trapezium ABCD in which AB is parallel to DC and AD=BC.



Prove that:

(i) $\angle$**DAB =** $\angle$**CBA**

(ii) $\angle$**ADC =** $\angle$**BCD**

(iii) **AC = BD**

(iv) **OA = OB and OC = OD**

Solution:

Given ABCD is a trapezium, AB||DC and AD=BC

To prove:

(i) $\angle$DAB = $\angle$CBA

(ii) $\angle$ADC = $\angle$BCD

(iii) AC = BD

(iv) OA = OB and OC = OD

Proof:

(i) Since $AD \parallel CE$ and transversal AE cuts them at A and E respectively.

Therefore, $\angle A + \angle B = 180°$

Since $AB \parallel CD$ and $AD \parallel BC$

Therefore $ABCD$ is a parallelogram

$\angle A = \angle C$

$\angle B = \angle D$ [since ABCD is a parallelogram]

Therefore $\angle DAB = \angle CBA$

$\angle ADC = \angle BCD$

In ΔABC and ΔBAD ,we have

$BC = AD$ [given]

$AB = BA$ [common]

$\angle A = \angle B$ [proved]

$\Delta ABC \cong \Delta BAD$ [SAS]

Since $\Delta ABC \cong \Delta BAD$

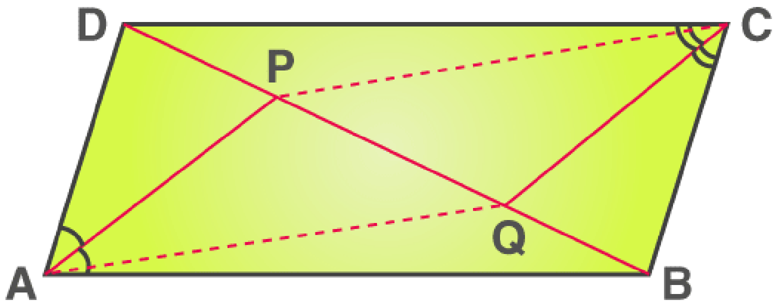
Therefore $AC = BD$ [corresponding parts of congruent triangles are equal]

Again $OA = OB$

$OC = OD$ [since diagonals bisect each other at O]

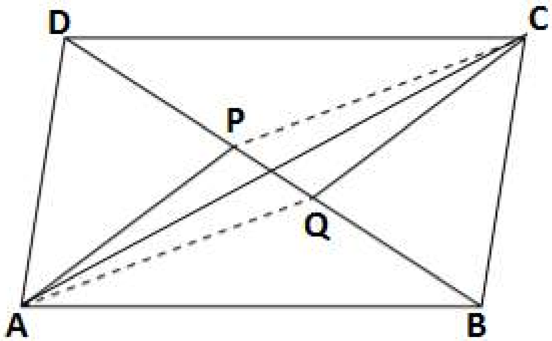
Hence proved

15. In the given figure, AP is bisector of ∠A and CQ is the bisector of ∠C of parallelogram ABCD. Prove that APCQ is a parallelogram.



Solution:

Construction: Join AC.



Proof :

$\angle BAP = \frac{1}{2}\angle A$ ....(AP is the bisector of ∠A)

$\angle DCQ = \frac{1}{2}\angle C$ ....(CQ is the bisector of $\angle C$)

$\Rightarrow \angle BAP = \angle DCQ$ (i)....[$\angle A = \angle R$ (Opposite angles of a parallelogram)]

Now, $\angle BAC = \angle DCA$ (ii)....[Alternate angles since AB $\parallel$ DC)

Subtracting (ii) from (i), we get

$\angle BAP - \angle BAC = \angle DCQ - \angle DCA$

$\Rightarrow \angle CAP = \angle ACQ$

$\Rightarrow$ AP $\parallel$ QC (Alternate angles are equal)

Similarly, PC $\parallel$ AQ

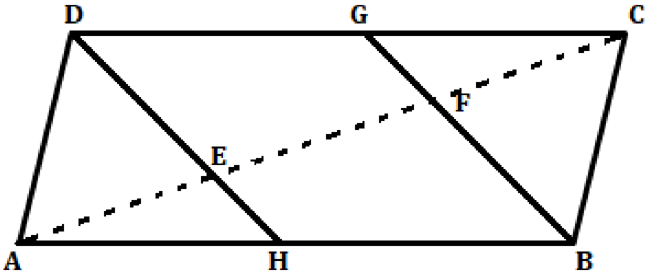
Hence, APCQ is a parallelogram.

16. In case of a parallelogram, prove that:

(i) The bisector of any two adjacent angles intersect at 90^0.

(ii) The bisectors of opposite angles are parallel to each other.

Solution:



ABCD is a parallelogram, the bisectors of $\angle ADC$ and $\angle BCD$ meet at a point E and the bisectors of $\angle BCD$ AND $\angle ABC$ meet at F.

We have to prove that the $\angle CED = 90°$ and $\angle CFG = 90°$

Proof: In the parallelogram ABCD

$\angle ADC + \angle BCD = 180°$ [sum of adjacent angles of a parallelogram]

$\Rightarrow \frac{\angle ADC}{2} + \frac{\angle BCD}{2} = 90°$

$\Rightarrow \angle EDC + \angle ECD + \angle CED = 180°$

$\Rightarrow \angle CED = 90°$

Similarly taking triangle BCF it can be proved that $\angle BFC = 90°$

$\angle BFC + \angle CFG = 180°$ [adjacent angles on a line]

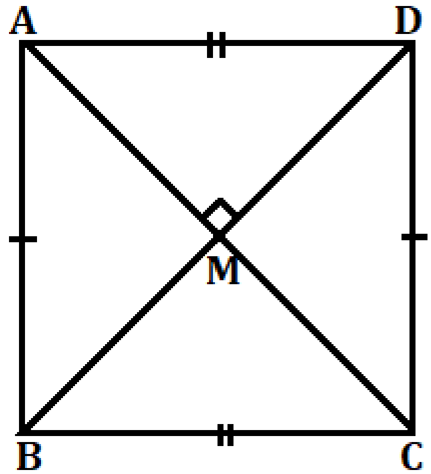
Also $\Rightarrow \angle CFG = 90°$

Now since $\angle CFG = \angle CED = 90°$ [it means that the lines DE and BG are parallel]

Hence proved

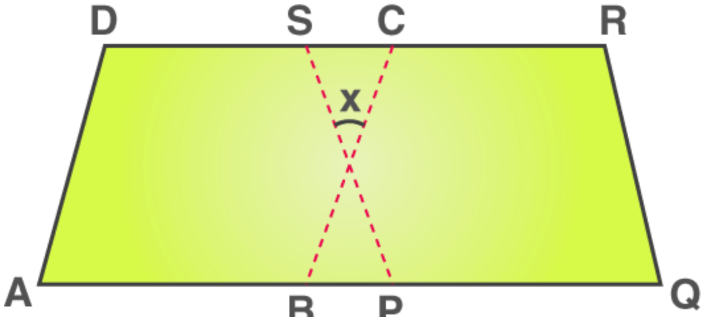
17. The diagonals of a rectangle intersect each other at right angles, prove that the rectangle is a square.

Solution:



To prove : ABCD is a square,
that is, to prove that sides of the quadrilateral are equal
and each angle of the quadrilateral is 90°.
ABCD is a rectangle,
$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90°$ and diagonals bisect each other
that is, MD = BM...(i)
Consider ΔAMD and ΔAMB,
MD = BM (from (i))
$\angle AMD = \angle AMB = 90°$ (given)
AM = AM (common side)
$\Delta AMD \cong \Delta AMB$ (SAS congruence criterion)
$\Rightarrow$ AD = AB (cpctc)
Since ABCD is a rectangle, AD = BC and AB = CD
Thus, AB = BC = CD = AD and $\angle A = \angle B = \angle C = \angle D = 90°$
$\Rightarrow$ ABCD is a square.

18. In the following figure, ABCD and PQRS are two parallelograms such that $\angle D=120^0$ and $\angle Q=70^0$. Find the value of x.



Solution:

ABCD is a parallelogram

$\Rightarrow$ opposite angles of a parallelogram are congruent

$\Rightarrow \angle DAB = \angle BCD$ and $\angle ABC = \angle ADC = 120°$

In ABCD,

$\angle DAB + \angle BCD + \angle ABC + \angle ADC = 360°$

.......(sum of the measures of angles of a quadrilateral)

$\Rightarrow \angle BCD + \angle BCD + 120° + 120° = 360°$

$\Rightarrow 2\angle BCD = 360° - 240°$

$\Rightarrow 2\angle BCD = 120°$

$\Rightarrow \angle BCD = 60°$

PQRS is a parallelogram

$\Rightarrow \angle PQR = \angle PSR = 70°$

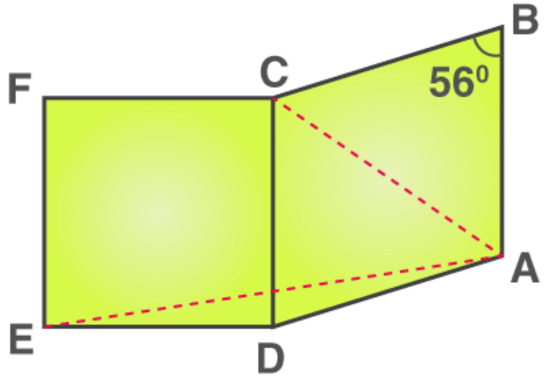
In ΔCMS,

$\angle CMS + \angle CSM + \angle MCS = 180°$ ....(angle sum property)

$\Rightarrow x + 70° + 60° = 180°$

$\Rightarrow x = 50°$

19. In the following figure, ABCD is a rhombus and DCFE is a square.



If $\angle ABC=56^0$, find:

(i) $\angle DAE$

(ii) $\angle FEA$

(iii) $\angle EAC$

(iv) $\angle AEC$

Solution:

ABCD is a rhombus $\Rightarrow AD = CD$ and $\angle ADC = \angle ABC = 56^\circ$

DCFE is a square $\Rightarrow ED = CD$ and $\angle FED = \angle EDC = \angle DCF = \angle CFE = 90^\circ$

$\Rightarrow AD = CD = ED$

In ΔADE,

$AD = ED \Rightarrow \angle DAE = \angle AED \ldots(i)$

$\angle DAE + \angle AED + \angle ADE = 180^\circ$

$\Rightarrow 2\angle DAE + 146^\circ = 180^\circ \quad \ldots(\text{Since } \angle ADE = \angle EDC + \angle ADC = 90^\circ + 56^\circ = 146^\circ)$

$\Rightarrow 2\angle DAE = 34^\circ$

$\Rightarrow \angle DAE = 17^\circ$

$\Rightarrow \angle DEA = 17^\circ \ldots(ii)$

In ABCD,

$\angle ABC + \angle BCD + \angle ADC + \angle DAB = 360^\circ$

$\Rightarrow 56^\circ + 56^\circ + 2\angle DAB = 360^\circ$ ($\because$ opposite angles of a rhombus are equal)

$\Rightarrow 2\angle DAB = 248^\circ$

$\Rightarrow \angle DAB = 124^\circ$

We know that diagonals of a rhombus, bisect its angles.

$$\Rightarrow \angle DAC = \frac{124^\circ}{2} = 62^\circ$$

$\Rightarrow \angle EAC = \angle DAC - \angle DAE = 62^\circ - 17^\circ = 45^\circ$

Now, $\angle FEA = \angle FED - \angle DEA$

$= 90^\circ - 17^\circ \quad \ldots$(from (ii) and each angle of a square is 90°)

$= 73^\circ$

We know that diagonals of a square bisect its angles.

$$\Rightarrow \angle CED = \frac{90^\circ}{2} = 45^\circ$$

So, $\angle AEC = \angle CED - \angle DEA$

$= 45^\circ - 17^\circ$

$= 28^\circ$

Hence, $\angle DAE = 17^\circ$, $\angle FEA = 73^\circ$, $\angle EAC = 45^\circ$ and $\angle AEC = 28^\circ$.