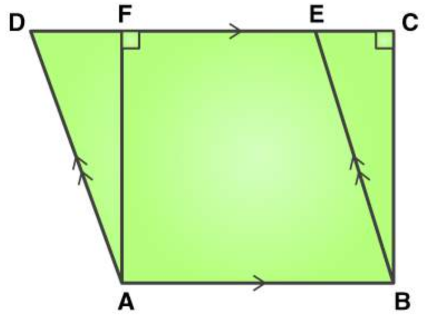
EXERCISE 16(A) PAGE:196

1. In the given figure, if area of triangle ADE is 60cm^2; state, giving reason, the area of:
(i) Parallelogram ABED;
(ii) Rectangle ABCF;
(iii) Triangle ABE.



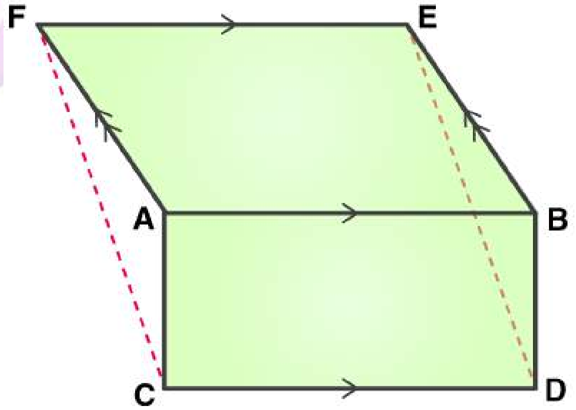
Solution:

(i) ΔADE and parallelogram ABED are on the same base AB and between the same parallels DE//AB, so area of the triangle ΔADE is half the area of parallelogram ABED.
Area of ABED = 2 (Area of ADE) = 120 cm^2

(ii) Area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between the same parallels
Area of ABCF = Area of ABED = 120 cm^2

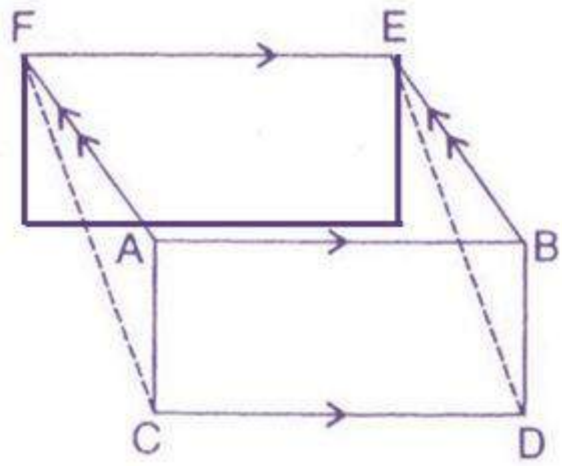
(iii) We know that area of triangles on the same base and between same parallel lines are equal
Area of ABE=Area of ADE =60 cm^2

2. The given figure shows a rectangle ABDC and a parallelogram ABEF; drawn on opposite of AB. Prove that:
(i) Quadrilateral CDEF is a parallelogram;
(ii) Area od quadrilateral CDEF=Area of rectangle ABDC + Area of //gm ABEF



Solution:

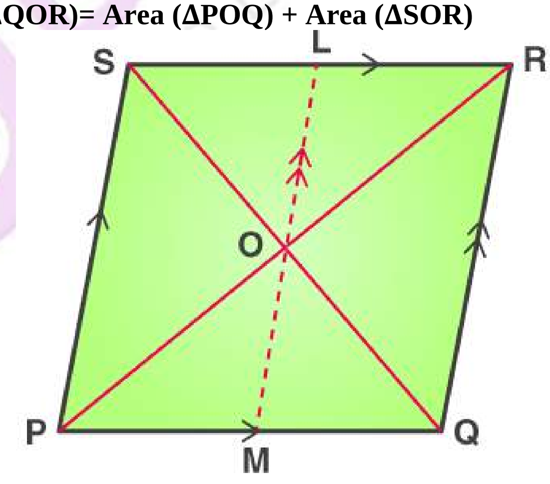
After drawing the opposite sides of AB, we get



Since from the figure, we get CD//FE therefore FC must parallel to DE. Therefore it is proved that the quadrilateral CDEF is a parallelogram.
Area of parallelogram on same base and between same parallel lines is always equal and area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between same parallel lines.
So Area of CDEF= Area of ABDC + Area of ABEF
Hence Proved

3. In the given figure, diagonals PR and QS of the parallelogram PQRS intersect at point O and LM is parallel to PS. Show that:

(i) 2 Area (ΔPOS)= Area (//gm PMLS)
(ii) Area (ΔPOS) + Area (ΔQOR)= Area (//gm PQRS)
(iii) Area (ΔPOS) + Area (ΔQOR)= Area (ΔPOQ) + Area (ΔSOR)



Solution:

(i) Since POS and parallelogram PMLS are on the same base PS and between the same parallels i.e. SP//LM.
As O is the center of LM and Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.
The area of the parallelogram is twice the area of the triangle if they lie on the same base and in between the same parallels.
So 2(Area of PSO) =Area of PMLS
Hence Proved.

(ii)

Consider the expression $Area(\triangle POS) + Area(QOR)$:

LM is parallel to PS and PS is parallel to RQ, therefore, LM is

Since triangle POS lie on the base PS and in between the parallels PS and LM, we have,

$$Area(\triangle POS) = \frac{1}{2} Area(\square PSLM)$$
,

Since triangle QOR lie on the base QR and in between the parallels LM and RQ, we have,

$$Area(\triangle QOR) = \frac{1}{2} Area(\square LMQR)$$

$$Area(\triangle POS) + Area(\triangle QOR) = \frac{1}{2} Area(\square PSLM) + \frac{1}{2} Area(\square LMQR)$$

$$= \frac{1}{2}[Area(\square PSLM) + Area(\square LMQR)]$$

$$= \frac{1}{2}[Area(\square PQRS)]$$

(iii)

In a parallelogram, the diagonals bisect each other.

Therefore, OS = OQ

Consider the triangle PQS, since OS = OQ, OP is the median of the triangle PQS.

We know that median of a triangle divides it into two triangles of equal area.

Therefore,

$$Area(\triangle POS) = Area(\triangle POQ)....(1)$$

Similarly, since OR is the median of the triangle QRS, we have,

$$Area(\triangle QOR) = Area(\triangle SOR)....(2)$$

Adding equations (1) and (2), we have,

$$Area(\triangle POS) + Area(\triangle QOR) = Area(\triangle POQ) + Area(\triangle SOR)$$

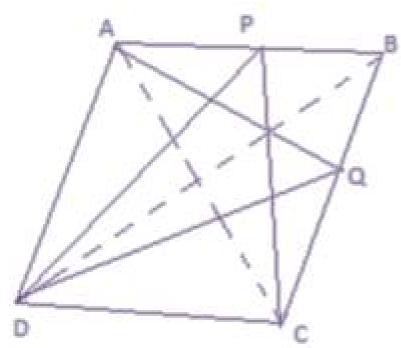
Hence Proved.

4. In parallelogram ABCD, P is a point on side AB and Q is a point on side BC. Prove that:

(i) ΔCOP and ΔAQD are equal in area.

(ii) Area (ΔAQD) = Area (ΔAPD) + Area (ΔCPB)

Solution:



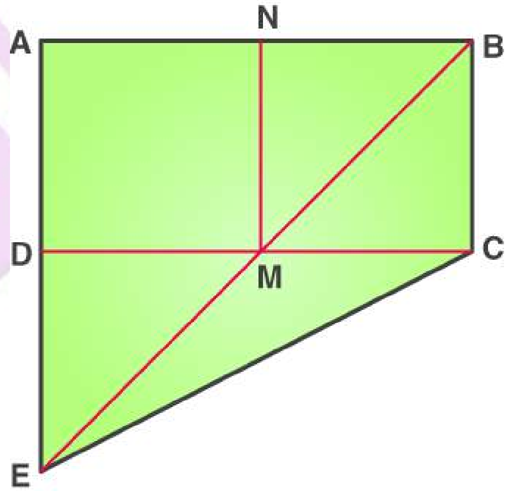
(i)

Given ABCD is a parallelogram. P and Q are any points on the sides AB and BC respectively, join diagonals AC and BD.
Proof:
Since triangles with same base and between same set of parallel lines have equal areas
Area (CPD) =area (BCD)…… (1)
And as the diagonals of the parallelogram bisects area in two equal parts
Area (BCD) = (1/2) area of parallelogram ABCD…… (2)
From (1) and (2)
Area (CPD) =1/2 area (ABCD)…… (3)
Similarly area (AQD) =area (ABD) =1/2 area (ABCD)…… (4)
From (3) and (4)
Area (CPD) =area (AQD),
Hence proved.

(ii)

We know that area of triangles on the same base and between same parallel lines are equal
So Area of AQD= Area of ACD= Area of PDC = Area of BDC = Area of ABC=Area of APD + Area of BPC
Hence Proved

5. In the given figure, M and N are the mid-points of the sides DC and AB respectively of the parallelogram ABCD.
If the area of parallelogram ABCD is 48 cm^2;
(i) State the area of the triangle BEC.
(ii) Name the parallelogram which is equal in area to the triangle BEC.



Solution:

(i)

Given that triangle BEC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC||AD.

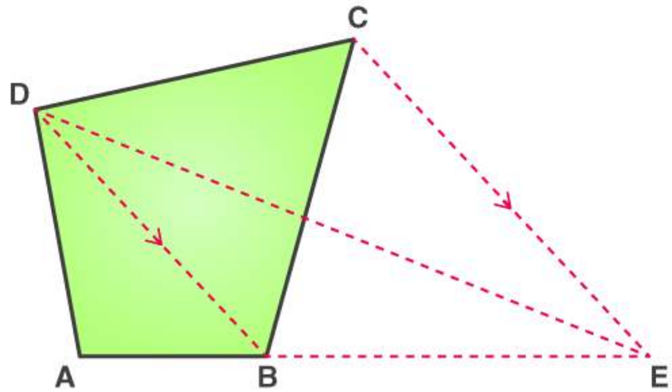
$$Area(\triangle BEC) = \frac{1}{2} \times Area(\square ABCD) = \frac{1}{2} \times 48 = 24\ cm^2$$

(ii)

$$Area(\square ANMD) = Area(\square BNMC)$$
$$= \frac{1}{2} Area(\square ABCD)$$
$$= \frac{1}{2} \times 2 \times Area(\triangle BEC)$$
$$= Area(\triangle BEC)$$

Hence, Parallelograms ANMD and NBCM have areas equal to triangle BEC

6. In the following figure, CE is drawn parallel to diagonal DB of the quadrilateral ABCD which meets AB produced at point E.
Prove that triangle ADE and Quadrilateral ABCD are equal in area.



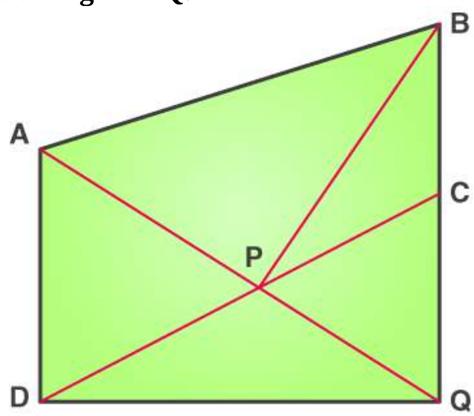
Solution:

Since $\triangle$DCB and $\triangle$DEB are on the same base DB and between the same parallels i.e. DB//CE, therefore we get

$$\text{Ar.}(\Delta DCB) = \text{Ar.}(\Delta DEB)$$
$$\text{Ar.}(\Delta DCB + \Delta ADB) = \text{Ar.}(\Delta DEB + \Delta ADB)$$
$$\text{Ar.}(ABCD) = \text{Ar.}(\Delta ADE)$$

Hence proved

7. ABCD is a parallelogram, a line through A cuts DC at point P and BC produced at Q. Prove that triangle BCP is equal in area to triangle DPQ.



Solution:

Δ APB and parallelogram ABCD are on the same base AB and between the same parallel lines AB and CD.

$$\therefore \text{Ar.}(\Delta APB) = \frac{1}{2}\text{Ar.}(\text{parallelogram ABCD}) \quad ......(i)$$

Δ ADQ and parallelogram ABCD are on the same base AD and between the same parallel lines AD and BQ.

$$\therefore \text{Ar.}(\Delta ADQ) = \frac{1}{2}\text{Ar.}(\text{parallelogram ABCD}) \quad ......(ii)$$

Adding equation (i) and (ii), we get

$$\therefore \text{Ar.}(\Delta APB) + \text{Ar.}(\Delta ADQ) = \text{Ar.}(\text{parallelogram ABCD})$$

$$\text{Ar.}(\text{quad.}ADQB) - \text{Ar.}(\Delta BPQ) = \text{Ar.}(\text{parallelogram ABCD})$$

$$\text{Ar.}(\text{quad.}ADQB) - \text{Ar.}(\Delta BPQ) = \text{Ar.}(\text{quad.}ADQB) - \text{Ar.}(\Delta DCQ)$$

$$\text{Ar.}(\Delta BPQ) = \text{Ar.}(\Delta DCQ)$$

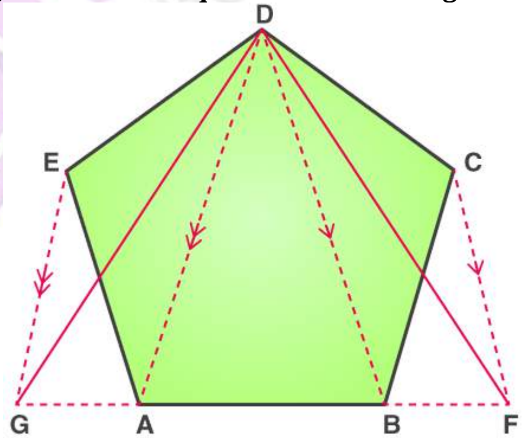
Subtracting Ar. Δ PCQ from both sides, we get

$$\text{Ar.}(\Delta BPQ) - \text{Ar.}(\Delta PCQ) = \text{Ar.}(\Delta DCQ) - \text{Ar.}(\Delta PCQ)$$

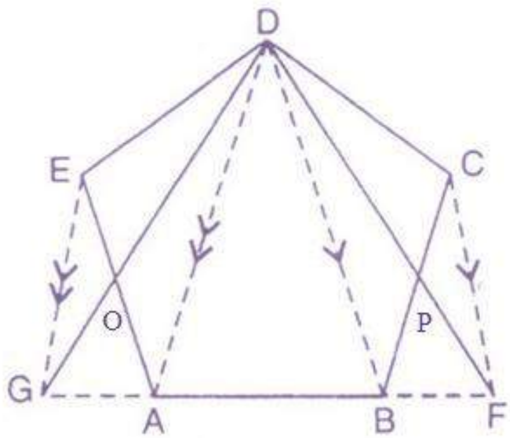
$$\text{Ar.}(\Delta BCP) = \text{Ar.}(\Delta DPQ)$$

Hence proved.

8. The given figure shows a pentagon ABCDE. EG drawn parallel to DA meets BA produced at G and CF drawn parallel to DB meets AB produced at F.
Prove that the area of pentagon ABCDE is equal to area of triangle GDF.



Solution:



Since triangle EDG and EGA are on the same base EG and between the same parallel lines EG and DA, therefore

$Ar.(\Delta EDG) = Ar.(\Delta EGA)$

Subtracting ΔEOG from both sides, we have

$Ar.(\Delta EOD) = Ar.(\Delta GOA)$ (i)

Similarly

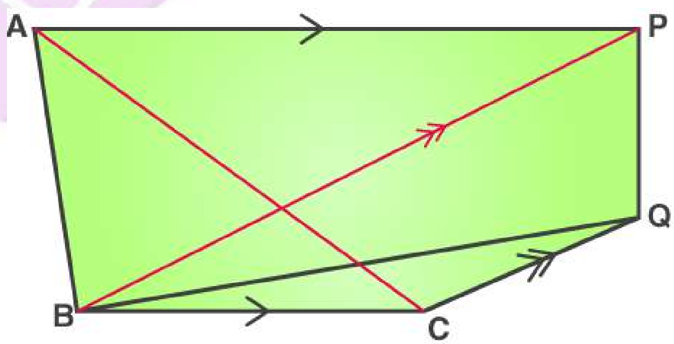
$Ar.(\Delta DPC) = Ar.(\Delta BPF)$ (ii)

Now

$$Ar.(\Delta GDF) = Ar.(\Delta GOA) + Ar.(\Delta BPF) + Ar.(\text{pen}.ABPDO)$$
$$= Ar.(\Delta EOD) + Ar.(\Delta DPC) + Ar.(\text{pen}.ABPDO)$$
$$= Ar.(\text{pen}.ABCDE)$$

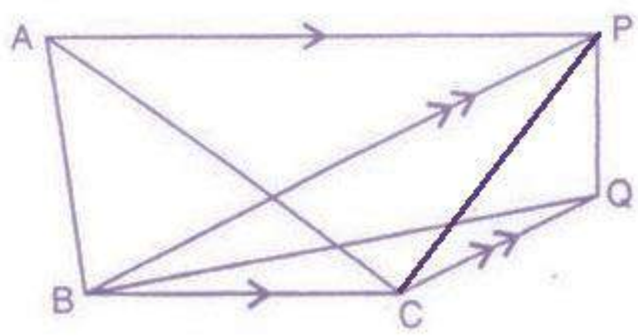
Hence proved

9. In the given figure, AP is parallel to BC, BP is parallel to CQ. Prove that the areas of triangles ABC and BQP are equal.



Solution:

Joining PC we get



Δ ABC and Δ BPC are on the same base BC and between the same parallel lines AP and BC.

$\therefore \text{Ar.}(\Delta ABC) = \text{Ar.}(\Delta BPC)$ ......(i)

Δ BPC and Δ BQP are on the same base BP and between the same parallel lines BP and CQ.

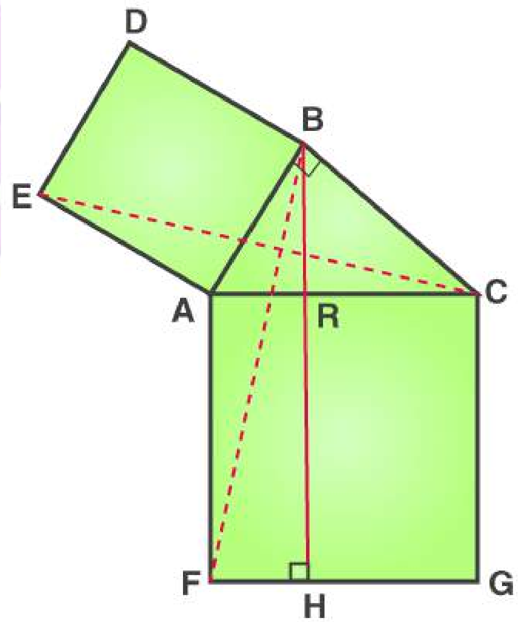
$\therefore \text{Ar.}(\Delta BPC) = \text{Ar.}(\Delta BQP)$ ......(ii)

From (i) and (ii), we get

$\therefore \text{Ar.}(\Delta ABC) = \text{Ar.}(\Delta BQP)$

Hence proved.

10. In the figure given alongside, squares ABDE and AFGC are drawn on the side AB and the hypotenuse AC of the right triangle ABC.
If BH is perpendicular to FG, prove that:
(i) $\Delta EAC \cong \Delta BAF$
(ii) Area of the square ABDE= Area of the rectangle ARHF.



Solution:

(i)

$\angle EAC = \angle EAB + \angle BAC$

$\angle EAC = 90° + \angle BAC$ ......(i)

$\angle BAF = \angle FAC + \angle BAC$

$\angle BAF = 90° + \angle BAC$ ......(ii)

From (i) and (ii), we get

$\angle EAC = \angle BAF$

In $\triangle$EAC and $\triangle$BAF, we have, EA=AB

$\angle EAC = \angle BAF$ and AC=AF

$\therefore$ $\triangle$EAC $\cong$ $\triangle$BAF (SAS axiom of congruency)

(ii)

Since $\triangle ABC$ is a right triangle, we have,

$AC^2 = AB^2 + BC^2$ [Using Pythagoras Theorem in $\triangle ABC$]

$\Rightarrow AB^2 = AC^2 - BC^2$

$\Rightarrow AB^2 = (AR + RC)^2 - (BR^2 + RC^2)$ [Since $AC = AR + RC$ and Using Pythagoras Theorem in $\triangle BRC$]

$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (BR^2 + RC^2)$ [Using the identity]

$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (AB^2 - AR^2 + RC^2)$ [Using Pythagoras Theorem in $\triangle ABR$]

$\Rightarrow 2AB^2 = 2AR^2 + 2AR \times RC$

$\Rightarrow AB^2 = AR(AR + RC)$

$\Rightarrow AB^2 = AR \times AC$

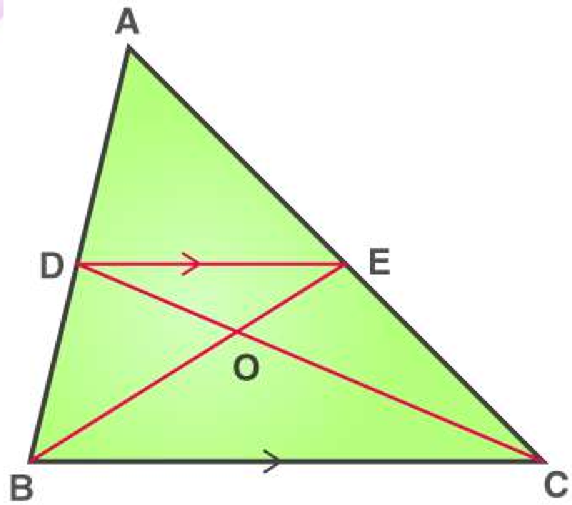
$\Rightarrow AB^2 = AR \times AF$

$\Rightarrow Area(\square ABDE) = Area(rectangle\ ARHF)$

11. In the following figure, DE is parallel to BC. Show that:

(i) Area (ΔADC) = Area (ΔAEB)

(ii) Area (ΔBOD) = Area (ΔCOE)



Solution:

(i)

In ΔABC, D is midpoint of AB and E is the midpoint of AC.

$$\frac{AD}{AB}=\frac{AE}{AC}$$

DE is parallel to BC.

$$\therefore\ Ar.(\Delta ADC)=Ar.(\Delta BDC)=\frac{1}{2}Ar.(\Delta ABC)$$

Again

$$\therefore\ Ar.(\Delta AEB)=Ar.(\Delta BEC)=\frac{1}{2}Ar.(\Delta ABC)$$

From the above two equations, we have

Area (ΔADC) = Area(ΔAEB).

Hence Proved

(ii)

We know that area of triangles on the same base and between same parallel lines are equal

Area(triangle DBC)= Area(triangle BCE)

Area(triangle DOB) + Area(triangle BOC) = Area(triangle BOC) + Area(triangle COE)

So Area(triangle DOB) = Area(triangle COE)

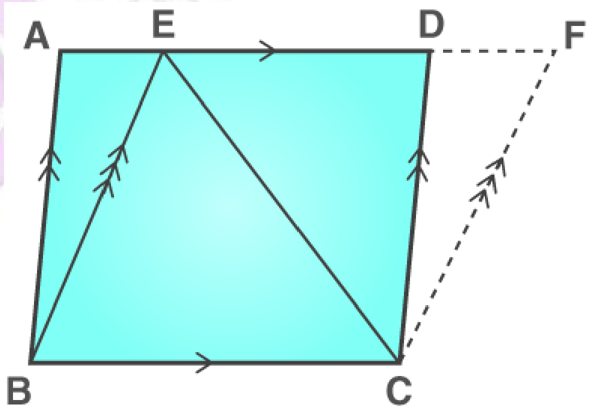
12. ABCD and BCFE are parallelograms. If area of triangle EBC=480cm^2, AB=30cm and BC=40cm; Calculate;

(i) Area of parallelogram ABCD;

(ii) Area of parallelogram BCFE;

(iii) Length of altitude from A on CD;

(iv) Area of triangle ECF.



Solution:

(i)

Since ΔEBC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC//AD.

$$\therefore \text{Ar.}(\Delta EBC)=\frac{1}{2}\times \text{Ar.}(\text{parallelogram } ABCD)$$

$$(\text{parallelogram } ABCD)=2\times \text{Ar.}(\Delta EBC)$$

$$=2\times 480\text{cm}^2$$

$$=960\text{cm}^2$$

(ii)

Parallelograms on same base and between same parallels are equal in area

Area of BCFE = Area of ABCD= 960 cm^2

(iii)

Area of triangle ACD=480 = (1/2) x 30 x Altitude

Altitude=32 cm

(iv)

The area of a triangle is half that of a parallelogram on the same base and between the same parallels.

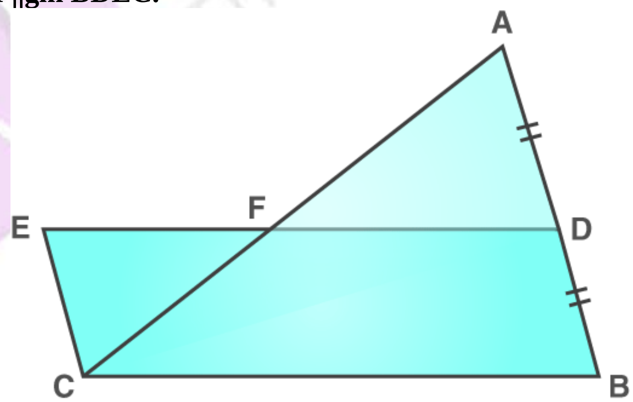
Therefore,

$$Area(\triangle ECF)=\frac{1}{2}Area(\square CBEF)$$

$$\text{Similarly, } Area(\triangle BCE)=\frac{1}{2}Area(\square CBEF)$$

$$\Rightarrow Area(\triangle ECF)=Area(\triangle BCE)=480\ cm^2$$

13. In the given figure, D is mid-point of side AB of ΔABC and BDEC is a parallelogram.
Prove that:
Area of ΔABC = Area of ||gm BDEC.



Solution:

Here AD=DB and EC=DB, therefore EC=AD

Again, $\angle EFC = \angle AFD$ (opposite angles)

Since ED and CB are parallel lines and AC cut this line, therefore

$\angle ECF = \angle FAD$

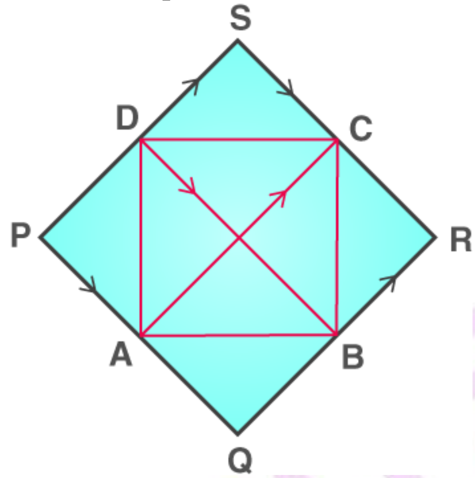
From the above conditions, we have

$\Delta EFC = \Delta AFD$

Adding quadrilateral CBDF in both sides, we have

Area of // gm BDEC= Area of ΔABC

14. In the following figure, AC||PS||QR and PQ||DB||SR.
Prove that:
Area of Quadrilateral PQRS = 2 x Area of quad. ABCD.



Solution:

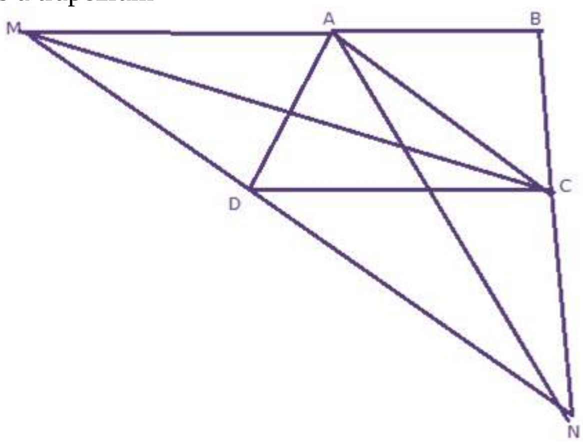
In Parallelogram PQRS, AC // PS // QR and PQ // DB // SR.
Similarly, AQRC and APSC are also parallelograms.
Since $\triangle$ABC and parallelogram AQRC are on the same base AC and between the same parallels, then

Ar.($\triangle$ABC)= $\frac{1}{2}$ Ar.(AQRC)......(i)
Similarly,
Ar.($\triangle$ADC)= $\frac{1}{2}$ Ar.(APSC).......(ii)
Adding (i) and (ii), we get
Area of quadrilateral PQRS = 2 $\times$ Area of quad. ABCD

15. ABCD is a trapezium with AB||DC. A line parallel to AC intersects AB at point M and BC at point N. Prove that:
Area of triangle ADM = area of triangle ACN

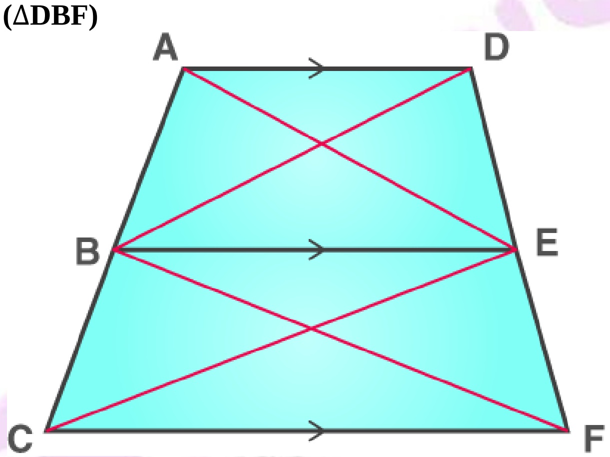
Solution:

Given: ABCD is a trapezium



AB || CD, MN || AC
Join C and M
We know that area of triangles on the same base and between same parallel lines are equal.
So Area of Δ AMD = Area of Δ AMC
Similarly, consider AMNC quadrilateral where MN || AC.
Δ ACM and Δ ACN are on the same base and between the same parallel lines. So areas are equal.
So, Area of Δ ACM = Area of Δ CAN
From the above two equations, we can say
Area of Δ ADM = Area of Δ CAN
Hence Proved.

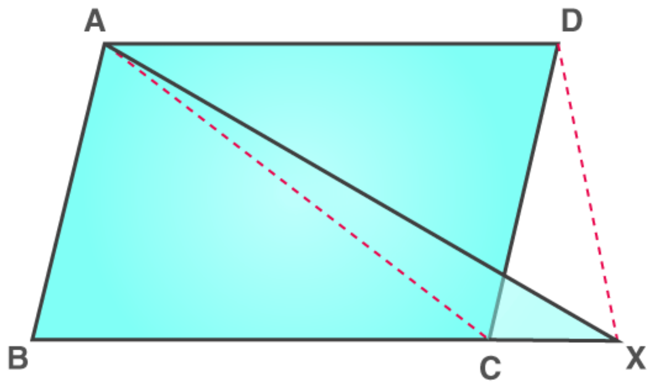
16. In the given figure,
AD||BE||CF.
Prove that:
Area (ΔAEC) = Area (ΔDBF)



Solution:

We know that area of triangles on the same base and between same parallel lines are equal.
Consider ABED quadrilateral; AD||BE
With common base, BE and between AD and BE parallel lines, we have
Area of ΔABE = Area of ΔBDE
Similarly, in BEFC quadrilateral, BE||CF
With common base BC and between BE and CF parallel lines, we have
Area of ΔBEC = Area of ΔBEF
Adding both equations, we have
Area of ΔABE + Area of ΔBEC = Area of ΔBEF + Area of ΔBDE
=> Area of AEC = Area of DBF
Hence Proved

17. In the given figure, ABCD is a parallelogram BC is produced to point X. Prove that:
Area of triangle ABX = area of quadrilateral ACXD.



Solution:

Given: ABCD is a parallelogram.

Area of ΔABC = Area of ΔACD

Consider ΔABX,

Area of ΔABX = Area of ΔABC + Area of ΔACX

We also know that area of triangles on the same base and between same parallel lines are equal.

Area of ΔACX = Area of ΔCXD

From above equations, we can conclude that

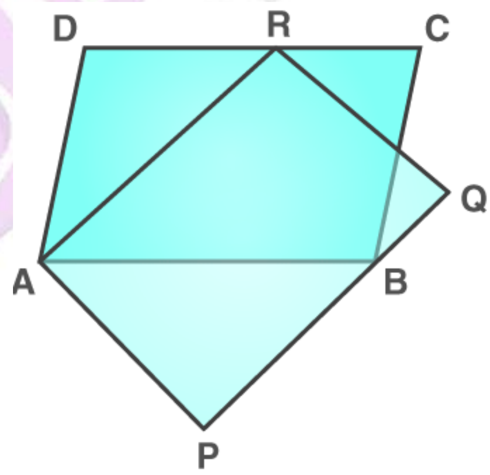
Area of ΔABX = Area of ΔABC + Area of ΔACX

= Area of ΔACD+ Area of ΔCXD

= Area of ACXD Quadrilateral

Hence Proved

18. The given figure shows parallelograms ABCD and APQR. Show that these parallelograms are equal in area.
[Join B and R]



Solution:

Join B and R and P and R.

We know that the area of the parallelogram is equal to twice the area of the triangle, if the triangle and the parallelogram are on the same base and between the parallels

Consider ABCD parallelogram:

Since the parallelogram ABCD and the triangle ABR lie on AB and between the parallels AB and DC, we have

$Area(\square ABCD) = 2 \times Area(\triangle ABR)$ ....(1)

We know that the area of triangles with same base and between the same parallel lines are equal. Since the triangles ABR and APR lie on the same base AR and between the parallels AR and QP, we have,

$Area(\triangle ABR) = Area(\triangle APR)$ ....(2)

From equations (1) and (2), we have,

$Area(\square ABCD) = 2 \times Area(\triangle APR)$....(3)

Also, the triangle APR and the parallelogram ARQP lie on the same base AR band between the parallels, AR and QP,

$Area(\triangle APR) = \frac{1}{2} \times Area(\square ARQP)$....(4)

Using (4) in equation (3), we have,

$Area(\square ABCD) = 2 \times \frac{1}{2} \times Area(\square ARQP)$

$Area(\square ABCD) = Area(\square ARQP)$

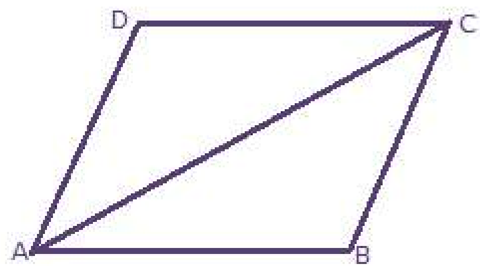
Hence proved.

EXERCISE 16(B) PAGE:201

1. Show that:
(i) A diagonal divides a parallelogram into two triangles of equal area.
(ii) The ratio of the areas of two triangles of the same height is equal to the ratio of their bases.
(iii) The ratio of the areas of two triangles of the same base is equal to the ratio of their heights.

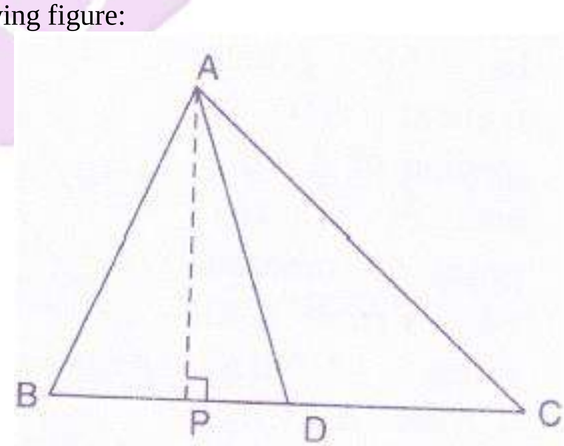
Solution:

(i) Suppose ABCD is a parallelogram (given)



Consider the triangles ABC and ADC:
AB = CD [ABCD is a parallelogram]
AD = BC [ABCD is a parallelogram]
AD = AD [common]
By Side – Side – Side criterion of congruence, we have,
$\triangle ABC \cong \triangle ADC$
Area of congruent triangles are equal.
Therefore, Area of ABC = Area of ADC

(ii) Consider the following figure:



Here $AP \perp BC$

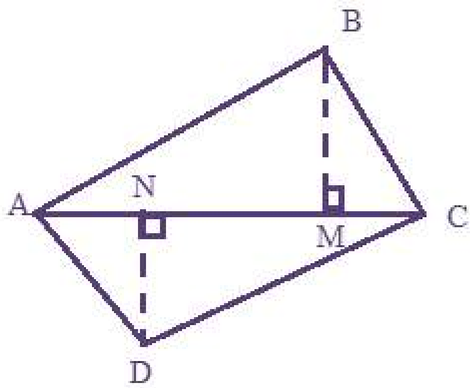
Since Ar.($\triangle ABD$)= $\frac{1}{2} BD \times AP$

Ar.(ΔADC)= $\frac{1}{2}DC \times AP$

$\therefore \frac{Area(\Delta ABD)}{Area(\Delta ADC)} = \frac{\frac{1}{2}BD \times AP}{\frac{1}{2}DC \times AP} = \frac{BD}{DC}$,

Hence proved

(iii) Consider the following figure:



Here

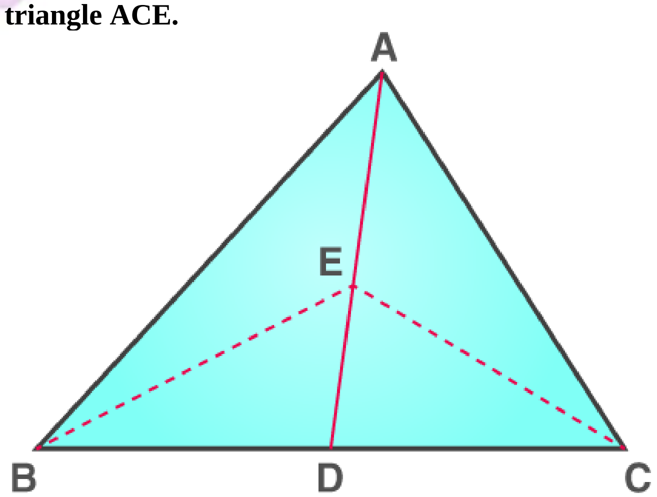
Ar.(ΔABC)= $\frac{1}{2}BM \times AC$

And, Ar.(ΔADC)= $\frac{1}{2}DN \times AC$

$\therefore \frac{Area(\Delta ABC)}{Area(\Delta ADC)} = \frac{\frac{1}{2}BM \times AC}{\frac{1}{2}DN \times AC} = \frac{BM}{DN}$,

Hence proved

2. In the given figure; AD is median of ΔABC and E is any point on median AD. Prove that area of triangle ABE = area of triangle ACE.



Solution:

AD is the median of $\triangle$ABC. Therefore it will divide $\triangle$ABC into two triangles of equal areas.

$\therefore$ Area($\triangle$ABD)= Area($\triangle$ACD) (i)

ED is the median of $\triangle$EBC

$\therefore$ Area($\triangle$EBD)= Area($\triangle$ECD) (ii)

Subtracting equation (ii) from (i), we obtain

Area($\triangle$ABD)- Area($\triangle$EBD)= Area($\triangle$ACD)- Area($\triangle$ECD)

Area ($\triangle$ABE) = Area ($\triangle$ACE). Hence proved

3. In the figure of question 2, if E is the mid-point of median AD, then prove that: Area of triangle ABE= ¼ area of triangle ABC.

Solution:

AD is the median of $\triangle$ABC. Therefore it will divide $\triangle$ABC into two triangles of equal areas.

$\therefore$ Area($\triangle$ABD)= Area($\triangle$ACD)

Area ($\triangle$ABD)= $\frac{1}{2}$ Area($\triangle$ABC) (i)

In $\triangle$ABD, E is the mid-point of AD. Therefore BE is the median.

$\therefore$ Area($\triangle$BED)= Area($\triangle$ABE)

Area($\triangle$BED)= $\frac{1}{2}$ Area($\triangle$ABD)

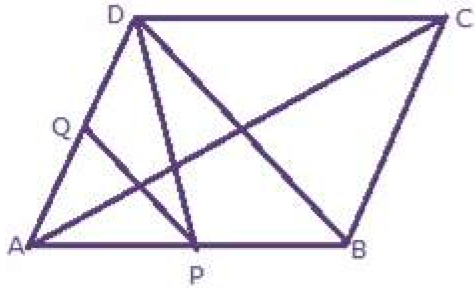
Area($\triangle$BED)= $\frac{1}{2}\times\frac{1}{2}$ Area($\triangle$ABC)[from equation (i)]

Area($\triangle$BED)= $\frac{1}{4}$ Area($\triangle$ABC)

4. ABCD is a parallelogram. P and Q are the mid-points of sides AB and AD respectively. Prove that area of triangle APQ= 1/8 of the parallelogram ABCD.

Solution:

We have to join PD and BD.



BD is the diagonal of the parallelogram ABCD. Therefore it divides the parallelogram into two equal parts.

$\therefore$ Area($\triangle$ABD)= Area($\triangle$DBC)

= $\frac{1}{2}$ Area (parallelogram ABCD) (i)

DP is the median of $\triangle$ABD. Therefore it will divide $\triangle$ABD into two triangles of equal areas.

$\therefore$ Area($\triangle$APD)= Area($\triangle$DPB)

$= \frac{1}{2}$ Area ($\triangle$ABD)

$= \frac{1}{2} \times \frac{1}{2}$ Area(parallelogram ABCD)[from equation (i)]

$= \frac{1}{4}$ Area (parallelogram ABCD) (ii)

In $\triangle$APD, Q is the mid-point of AD. Therefore PQ is the median.

$\therefore$ Area($\triangle$APQ)= Area($\triangle$DPQ)

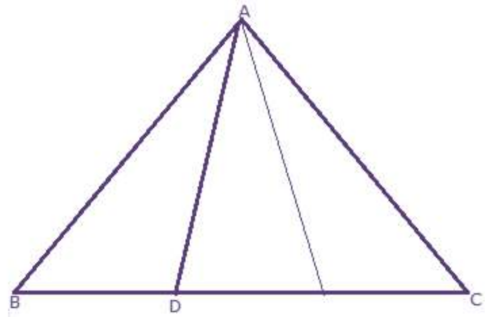
$= \frac{1}{2}$ Area($\triangle$APD)

$= \frac{1}{2} \times \frac{1}{4}$ Area (parallelogram ABCD) [from equation (ii)]

Area ($\triangle$APQ)= $\frac{1}{8}$ Area (parallelogram ABCD),hence proved

5. **The base BC of triangle ABC is divided at D so that BD=1/2 DC. Prove that area of triangle ABD=1/3 of the area of triangle ABC.**

Solution:



In $\triangle$ABC, $\because$ BD = $\frac{1}{2}$ DC $\Rightarrow \frac{BD}{DC} = \frac{1}{2}$

$\therefore$ Ar.($\triangle$ABD):Ar.($\triangle$ADC)=1:2

But Ar.($\triangle$ABD)+Ar.($\triangle$ADC)=Ar.($\triangle$ABC)

Ar.($\triangle$ABD)+2Ar.($\triangle$ABD)=Ar.($\triangle$ABC)

3 Ar.($\triangle$ABD)= Ar.($\triangle$ABC)

Ar.($\triangle$ABD)= $\frac{1}{3}$ Ar.($\triangle$ABC)

6. **In a parallelogram ABCD, point P lies in DC such that DP: PC=3:2. If area of triangle DPB=30 sq. cm, find the area of the parallelogram ABCD.**

Solution:

Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.

$$\frac{\text{Area of DPB}}{\text{Area of PCB}} = \frac{DP}{PC} = \frac{3}{2}$$

Given: Area of ΔDPB = 30 sq. cm

Let 'x' bet the area of the triangle PCB

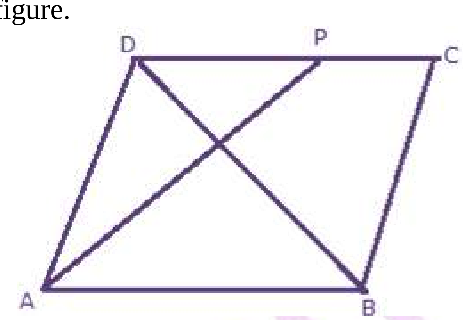
Therefore, we have,

$$\frac{30}{x} = \frac{3}{2}$$

$$\Rightarrow x = \frac{30}{3} \times 2 = 20 \; sq.\, cm.$$

So area of ΔPCB = 20 sq. cm

Consider the following figure.



From the diagram, it is clear that,

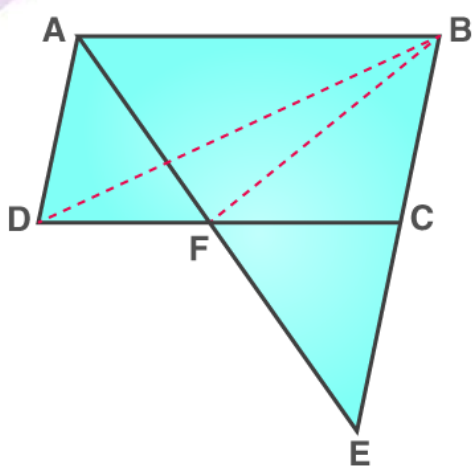
$$Area(\triangle CDB) = Area(\triangle DPB) + Area(\triangle CPB)$$
$$= 30 + 20$$
$$= 50 \; sq.\, cm$$

Diagonal of the parallelogram divides it into two triangles ABD and CBD of equal area.

Therefore,

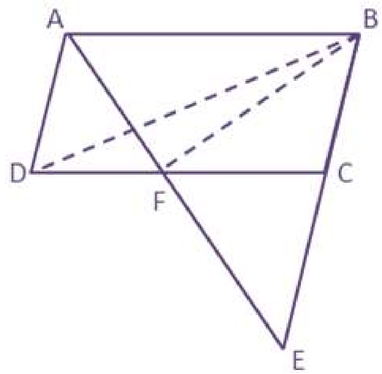
$$Area(||gm\; ABCD) = 2 \times \triangle CDB$$
$$= 2 \times 50 = 100 \; sq.\, cm$$

7. ABCD is a parallelogram in which BC is produced to E such that CE=BC and AE intersects CD at F.



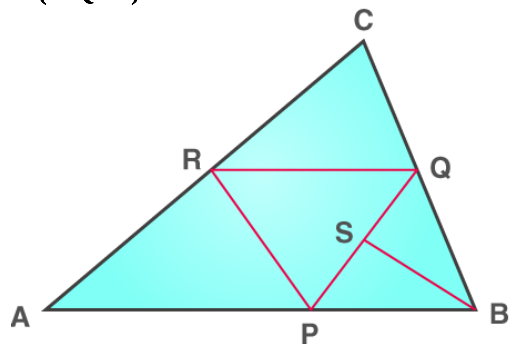
If area of triangle DFB=30cm^2; find the area of parallelogram

Solution:



BC = CE (given)
Also, in parallelogram ABCD, BC = AD
$\Rightarrow$ AD = CE
Now, in ΔADF and ΔECF, we have
AD = CE
$\angle$ADF = $\angle$ECF (Alternate angles)
$\angle$DAF = $\angle$CEF (Alternate angles)
$\therefore$ $\Delta ADF \cong \Delta ECF$ (ASA Criterion)
$\Rightarrow$ Area(ΔADF) = Area(ΔECF) ....(1)
Also, in ΔFBE, FC is the median (Since BC = CE)
$\Rightarrow$ Area(ΔBCF) = Area(ΔECF) ....(2)
From (1) and (2),
Area(ΔADF) = Area(ΔBCF) ....(3)
Again, ΔADF and ΔBDF are on the base DF and between parallels DF and AB.
$\Rightarrow$ Area(ΔBDF) = Area(ΔADF) ....(4)
From (3) and (4),
Area(ΔBDF) = Area(ΔBCF) = 30 cm^2
Þ Area(ΔBCD) = Area(ΔBDF) + Area(ΔBCF) = 30 + 30 = 60 cm^2
Hence, Area of parallelogram ABCD = 2 $\times$ Area(ΔBCD) = 2 $\times$ 60 = 120 cm^2

8. The following figure shows a triangle ABC in which P, Q and R are mid-points of sides AB, BC and CA respectively. S is mid-point of PQ.
Prove that: ar.(Δ ABC) = 8 x ar.(Δ QSB)



Solution:

In ΔABC,

R and Q are the mid-points of AC and BC respectively.

$\Rightarrow RQ \parallel AB$

that is $RQ \parallel PB$

So, area(ΔPBQ) = area(ΔAPR)...(i)..(Since AP = PB and triangles on the same base and between the same parallels are equal in area)

Since P and R are the mid-points of AB and AC respectively.

$\Rightarrow PR \parallel BC$

that is $PR \parallel BQ$

So, quadrilateral PMQR is a parallelogram.

Also, area(ΔPBQ) = area(ΔPQR)...(ii)...(diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

from (i) and (ii),

area(ΔPQR) = area(ΔPBQ) = area(ΔAPR)...(iii)

Similarly, P and Q are the mid-points of AB and BC respectively.

$\Rightarrow PQ \parallel AC$

that is $PQ \parallel RC$

So, quadrilateral PQCR is a parallelogram.

Also, area(ΔRQC) = area(ΔPQR)...(iv)...(diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

From (iii) and (iv),

area(ΔPQR) = area(ΔPBQ) = area(ΔRQC) = area(ΔAPR)

So, area(ΔPBQ) $= \frac{1}{4}$ area(ΔABC)...(v)

Also, since S is the mid-point of PQ,

BS is the median of ΔPBQ

So, area(ΔQSB) $= \frac{1}{2}$ area(ΔPBQ)

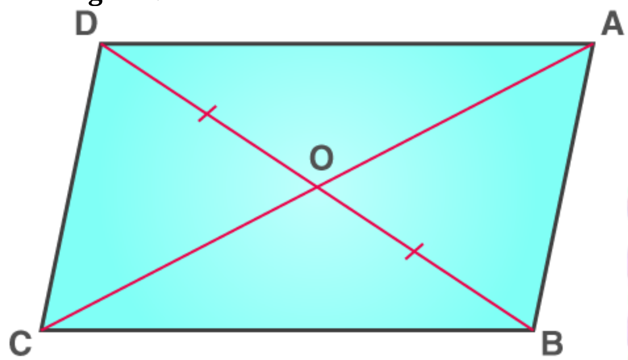
from (v),

area(ΔQSB) $= \frac{1}{2} \times \frac{1}{4}$ area(ΔABC)

$\Rightarrow$ area(ΔABC) = 8 area(ΔQSB)

EXERCISE 16(C) PAGE:201

1. **In the given figure, the diagonal AC and BD intersect at point O. If OB = OD and AB||DC, Prove that:**
   (i) Area (Δ DOC) = Area (Δ AOB)
   (ii) Area (Δ DOC) = Area (Δ ACB)
   (iii) ABCD is a parallelogram.



Solution:

(i)

Ratio of area of triangles with same vertex and bases along the same line is equal to the ratio of their respective bases. So, we have:

$$\frac{\text{Area of } \Delta \text{ DOC}}{\text{Area of } \Delta \text{ BOC}} = \frac{DO}{BO} = 1 \quad \text{----1}$$

Similarly

$$\frac{\text{Area of } \Delta \text{ DOA}}{\text{Area of } \Delta \text{ BOA}} = \frac{DO}{BO} = 1 \quad \text{------2}$$

We know that area of triangles on the same base and between same parallel lines are equal.
Area of Δ ACD = Area of Δ BCD
Area of Δ AOD + Area of Δ DOC = Area of Δ DOC + Area of Δ BOC
=> Area of Δ AOD = Area of Δ BOC ------3
From 1, 2 and 3 we have
Area (Δ DOC) = Area (Δ AOB)
Hence Proved.

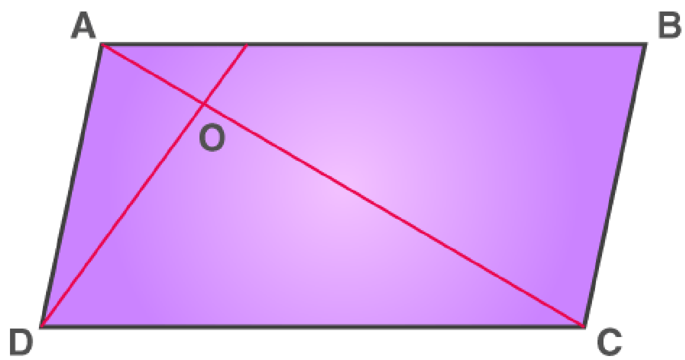
(ii)

Similarly, from 1, 2 and 3, we also have
Area of Δ DCB = Area of Δ DOC + Area of Δ BOC = Area of Δ AOB + Area of Δ BOC = Area of Δ ABC
So Area of Δ DCB = Area of Δ ABC
Hence Proved.

(iii)

Area of triangles on the same base and between same parallel lines are equal.
Given: triangles are equal in area on the common base, so it indicates AD|| BC.
So, ABCD is a parallelogram.
Hence Proved

2. **The given figure shows a parallelogram ABCD with area 324 sq. cm. P is a point in AB such that AP: PB=1:2. Find the area of ΔAPD.**



Solution:

Ratio of area of triangles with the same vertex and bases along the same line is equal to the ratio of their respective bases.

$$\frac{\text{Area of } \Delta \text{ APD}}{\text{Area of } \Delta \text{ BPD}} = \frac{AP}{BP} = \frac{1}{2}$$

Area of parallelogram ABCD = 324 sq.cm

Area of the triangles with the same base and between the same parallels are equal.

Area of the triangle is half the area of the parallelogram if they lie on the same base and between the parallels.

Hence, we have,

$$Area(\triangle ABD) = \frac{1}{2} \times Area(||gm\ ABCD)$$
$$= \frac{324}{2}$$
$$= 162\ sq.\ cm$$

From the diagram it is clear that,

$$Area(\triangle ABD) = Area(\triangle APD) + Area(\triangle BPD)$$
$$\Rightarrow 162 = Area(\triangle APD) + 2Area(\triangle APD)$$
$$\Rightarrow 162 = 3Area(\triangle APD)$$
$$\Rightarrow Area(\triangle APD) = \frac{162}{3}$$
$$\Rightarrow Area(\triangle APD) = 54\ sq.\ cm$$

Consider the triangles $\triangle AOP$ *and* $\triangle COD$

$\angle AOP = \angle COD$ [*vertically opposite angles*]

$\angle CDO = \angle APD$ [*AB and DC are parallel and DP is the transversal, alternate interior angles are equal*]

Thus, by Angle – Angle similarity, $\triangle AOP \sim \triangle COD$.

Hence the corresponding sides are proportional.

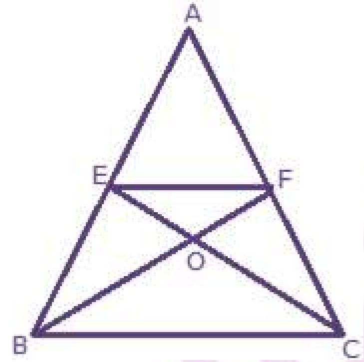
$$\frac{AP}{CD} = \frac{OP}{OD} = \frac{AP}{AB}$$
$$= \frac{AP}{AP + PB}$$
$$= \frac{AP}{3AP}$$
$$= \frac{1}{3}$$

Hence OP:OD = 1:3

3. **In triangle ABC, E and F are mid-points of sides AB and AC respectively. If BF and CE intersect each other at point O, prove that the triangle OBC and quadrilateral AEOF are equal in area.**

Solution:

E and F are the midpoints of the sides AB and AC.
Consider the following figure.



Therefore, by midpoint theorem, we have, EF || BC
Triangles BEF and CEF lie on the common base EF and between the parallels, EF and BC

$Therefore,\ Ar.(\triangle BEF) = Ar.(\triangle CEF)$

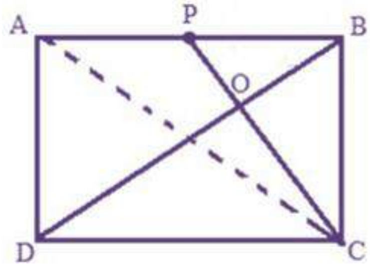
$\Rightarrow Ar.(\triangle BOE) + Ar.(\triangle EOF) = Ar.(\triangle EOF) + Ar.(\triangle COF)$

$\Rightarrow Ar.(\triangle BOE) = Ar.(\triangle COF)$

Now BF and CE are the medians of the triangle ABC
Medians of the triangle divides it into two equal areas of triangles.
Thus, we have, Ar.$\triangle$ABF=Ar.$\triangle$CBF
Subtracting Ar.$\triangle$BOE on the both the sides, we have
Ar.$\triangle$ABF - Ar.$\triangle$BOE=Ar.$\triangle$CBF - Ar.$\triangle$BOE
Since, Ar.($\triangle$BOE)= Ar.($\triangle$COF),
Ar.$\triangle$ABF- Ar.$\triangle$BOE=Ar.$\triangle$CBF- Ar.$\triangle$COF
Ar. (quad. AEOF)=Ar.($\triangle$OBC), hence proved

4. **In parallelogram ABCD, P is mid-point of AB. CP and BD intersect each other at point O. If area of triangle POB = 40 cm^2 and OP:OC = 1:2, find:**
   (i) Areas of triangle BOC and PBC
   (ii) Areas of triangle ABC and parallelogram ABCD.

Solution:



Consider the triangles $\triangle POB$ and $\triangle COD$

$\angle POB = \angle DOC$ [vertically opposite angles]

$\angle OPB = \angle ODC$ [AB and DC are parallel, CP and BD are the transversals, alternate interior angles are equal]

Therefore, by Angle – Angle similarity criterion of congruence,

$\triangle POB \sim \triangle COD$

Since P is the midpoint $AP = BP$, and $AB = CD$, we have $CD = 2BP$

Therefore, we have,

$$\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2}$$

$\Rightarrow OP : OC = 1 : 2$

5. The medians of a triangle ABC intersect each other at point G. If one of its medians is AD, prove that:

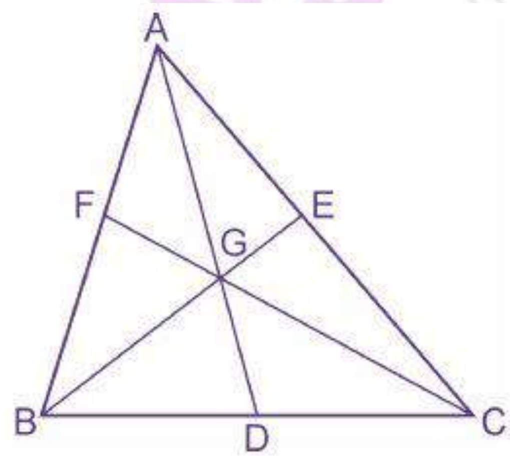
(i) Area (ΔABD) = 3 × Area (ΔBGD)

(ii) Area (ΔACD) = 3 × Area (ΔCGD)

(iii) Area (ΔBGC) = 1/3 × Area (ΔABC)

Solution:

(i) The figure is shown below



Medians intersect at centroid.
Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.
Centroid divides the medians in the ratio 2:1
That is $AG:GD=2:1$
Since BG divides AD in the ratio 2:1, *we have,*

$$\frac{Area(\triangle AGB)}{Area(\triangle BGD)}=\frac{2}{1}$$
$$\Rightarrow Area(\triangle AGB)=2Area(\triangle BGD)$$

From the figure, it is clear that,

$$Area(\triangle ABD)=Area(\triangle AGB)+Area(\triangle BGD)$$
$$\Rightarrow Area(\triangle ABD)=2Area(\triangle BGD)+Area(\triangle BGD)$$
$$\Rightarrow Area(\triangle ABD)=3Area(\triangle BGD)....(1)$$

(ii)

Medians intersect at centroid.
Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.
Centroid divides the medians in the ratio 2:1
That is $AG:GD=2:1$
Similarly CG divides AD in the ratio 2:1, *we have,*

$$\frac{Area(\triangle AGC)}{Area(\triangle CGD)}=\frac{2}{1}$$
$$\Rightarrow Area(\triangle AGC)=2Area(\triangle CGD)$$

From the figure, it is clear that,

$$Area(\triangle ACD)=Area(\triangle AGC)+Area(\triangle CGD)$$
$$\Rightarrow Area(\triangle ACD)=2Area(\triangle CGD)+Area(\triangle CGD)$$
$$\Rightarrow Area(\triangle ACD)=3Area(\triangle CGD)....(2)$$

(iii)

Adding equations (1) *and* (2), *we have,*

$$Area(\triangle ABD)+Area(\triangle ACD)=3Area(\triangle BGD)+3Area(\triangle CGD)$$
$$\Rightarrow Area(\triangle ABC)=3[Area(\triangle BGD)+Area(\triangle CGD)]$$
$$\Rightarrow Area(\triangle ABC)=3[Area(\triangle BGC)]$$
$$\Rightarrow \frac{Area(\triangle ABC)}{3}=[Area(\triangle BGC)]$$
$$\Rightarrow Area(\triangle BGC)=\frac{1}{3}Area(\triangle ABC)$$

6. The perimeter of a triangle ABC is 37 cm and the ratio between the lengths of its altitudes be 6:5:4. Find the lengths of its sides.

Solution:

Consider that the sides be x cm, y cm and (37-x-y) cm.

Let the lengths of altitudes be 6a cm, 5a cm and 4a cm.

∴ Area of a triangle= $\frac{1}{2}\times$ base $\times$ altitude

∴ $\frac{1}{2}\times x\times 6a=\frac{1}{2}\times y\times 5a=\frac{1}{2}\times(37-x-y)\times 4a$

$6x=5y=148-4x-4y$

$6x=5y$ and $6x=148-4x-4y$

$6x-5y=0$ and $10x+4y=148$

Solving both the equations, we have

X=10 cm, y=12 cm and (37-x-y)cm=15 cm

7. In parallelogram ABCD, E is a point in AB and DE meets diagonal AC at point F. If DF : FE = 5:3 and area of ΔADF is 60cm²; find:

(i) Area of ΔADE

(ii) If AE : EB = 4:5, find the area of ΔADB

(iii) Also, find the area of parallelogram ABCD.

Solution:

(i)

Consider the triangles $\triangle AFE$ and $\triangle DFC$.

$\angle AFE=\angle DEC$ [Vertically opposite angles]

$\angle FAE=\angle DCF$ [AB and DC are parallel lines, AC is a transversal, alternate interior angles are equal]

Thus, by Angle – Angle similarity, we have,

$\triangle AFE\sim\triangle DFC$

Therefore, we have,

$\frac{DF}{FE}=\frac{DC}{AE}=\frac{CF}{AF}=\frac{2}{1}$

$\Rightarrow DF:FE=2:1$

(ii)

Since from part(i) we have $DF:FE=2:1$, therefore,

$Area(\triangle DCF)=4Area(\triangle AFE)$...(1)

Also we know that,

$Area(\triangle ADF)+Area(\triangle AFE)=Area(\triangle ADE)$

$\Rightarrow 60+Area(\triangle AFE)=Area(\triangle ADE)$ $[Area(\triangle ADF)=60\ cm^2]$

$\Rightarrow 2Area(\triangle ADE)=2[60+Area(\triangle AFE)]$

Median divides the triangle into two equal areas of triangle.

Therefore, $2Area(\triangle ADE)=Area(\triangle ABD)$

$\Rightarrow Area(\triangle ABD)=2[60+Area(\triangle AFE)]$

$\Rightarrow Area(\triangle ABD)=120+2Area(\triangle AFE)$...(2)

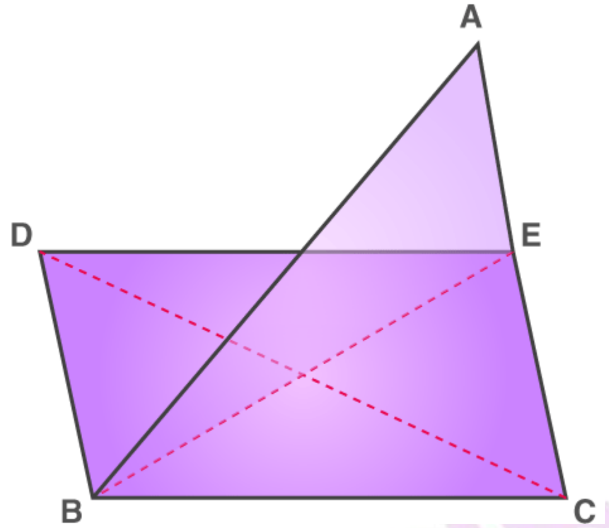
Triangles with equal bases and between the parallels are of equal area.

$Area(\triangle ABD) = Area(\triangle ACD)$
Thus, Equation (2), becomes,
$Area(\triangle ACD) = 120 + 2Area(\triangle AFE)...(3)$
From the figure, it is clear that,
$Area(\triangle ACD) = Area(\triangle DCF) + Area(\triangle ADF)$
$\Rightarrow Area(\triangle ACD) = Area(\triangle DCF) + 60$
$\Rightarrow Area(\triangle ACD) = 4Area(\triangle AEF) + 60...(4)$
Equating equations (3) and (4), we have,
$120 + 2Area(\triangle AFE) = 4Area(\triangle AEF) + 60$
$\Rightarrow 2Area(\triangle AFE) = 60$
$\Rightarrow Area(\triangle AFE) = \frac{60}{2}$
$\Rightarrow Area(\triangle AFE) = 30$
$\Rightarrow Area\ (\triangle ADE) = Area(\triangle ADF) + Area(\triangle AFE)$
$\Rightarrow Area\ (\triangle ADE) = 60 + 30$
$\Rightarrow Area\ (\triangle ADE) = 90\ cm^2$

(iii)
Median of a trianlge divides it intot two equal areas of triangle.
$Area\ (\triangle ADB) = 2Arear(\triangle ADE)$
$\Rightarrow Area\ (\triangle ADB) = 2Arear(\triangle ADE)$
$\Rightarrow Area\ (\triangle ADB) = 2 \times 90\ cm^2$
$\Rightarrow Area\ (\triangle ADB) = 180\ cm^2$

(iv) DB divides the parallelogram ABCD into 2 equal triangles.
So, Area of triangle DBC = Area of triangle ADB = 180 cm^2
Therefore,
Area of the parallelogram ABCD = Area of triangle ADB + Area of triangle DBC
= 180 cm^2 + 180 cm^2
= 360 cm^2

8. In the following figure, BD is parallel to CA, E is midpoint of CA and BD = ½ CA.
Prove that:
Ar.(ΔABC) = 2 x ar.(ΔDBC)



Solution:

Here BCED is a parallelogram, since BD = CE and BD || CE.

ar.(ΔDBC) = ar.(ΔEBC)...(Since they have the same base and are between the same parallels)

In ΔABC,

BE is the median,

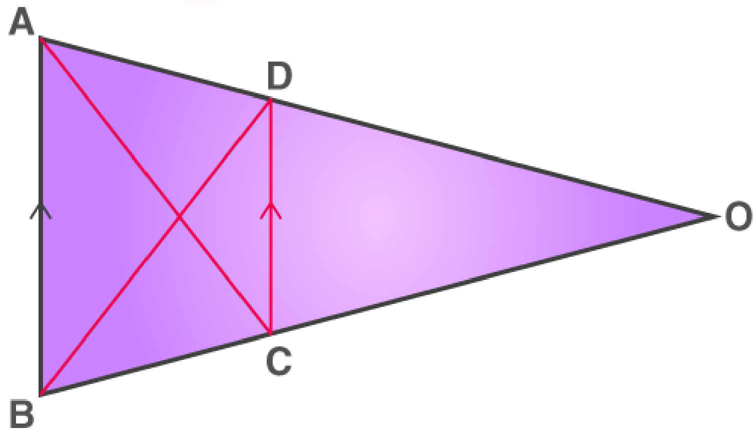
So, ar.(ΔEBC) = $\frac{1}{2}$ ar.(ΔABC)

Now, ar.(ΔABC) = ar.(ΔEBC) + ar.(ΔABE)

Also, ar.(ΔABC) = 2ar.(ΔEBC)

⇒ ar.(ΔABC) = 2ar.(ΔDBC)

9. In the following figure, OAB is a triangle and AB||DC.
If the area od triangle CAD = 140 cm^2 and the area of triangle ODC = 172 cm^2, find
(i) The area of triangle DBC
(ii) The area of triangle OAC
(iii) The area of triangle ODB



Solution:

Given :

$\Delta CAD = 140\ cm^2$

$\Delta ODC = 172\ cm^2$

AB || CD

As Triangle DBC and ΔCAD have same base CD and between the same parallel lines Hence,

Area of ΔDBC = Area of $\Delta CAD = 140\ cm^2$

Area of ΔOAC = Area of ΔCAD + Area of $\Delta ODC = 140\ cm^2 + 172\ cm^2 = 312\ cm^2$

Area of ΔODB = Area of ΔDBC + Area of $\Delta ODC = 140\ cm^2 + 172\ cm^2 = 312\ cm^2$

10. E, F, G and H are the midpoints of the sides of a parallelogram ABCD. SHow that area of quadrilateral EFGH is half of the area of parallelogram ABCD.

Solution:

Given that ABCD is a ||gm .

Also given that,
E, F G and H are respectively the midpoints of the sides AB, BC, CD and AD of a ||gm ABCD.
To Prove: ar(EFGH) = ½ ar(ABCD).
Proof:

Join FH , such that FH || AB || CD .
Since, FH || AB and AH || BF.
So, ABFH is a ||gm.
ΔEFH and ||gm ABFH are on same base FH and between the same parallel lines.
Hence, ar(Δ EFH) = ½ ar(||gm ABFH)............. (1).

Since, FH || CD and DH || FC .
DCFH is a ||gm .
We know that,
ΔFGH and ||gm DCFH are on same base FH and between the same parallel lines .
ar(Δ FGH) = ½ ar(||gm DCFH)............. (2) .

Adding equation (1) and (2), we get
ar(Δ EFH) + ar(Δ FGH) = ½ ar(||gm ABFH) + ½ ar(||gm DCFH) .
ar(Δ EFH) + ar(Δ FGH) = ½ [ar(||gm ABFH) + ar(||gm DCFH)] .
ar(EFGH) = ½ ar(ABCD).
Hence Proved.

11. ABCD is a trapezium with AB parallel to DC. A line parallel to AC intersects AB at X and BC at Y. Prove that area of triangle ADX= area of triangle ACY.

Solution:

Two Triangles on the same base and between the same parallels are equal in area.
Given that,

ABCD is a trapezium with AB || DC & XY || AC

Construction,

Join CX.

To Prove,

ar(ADX) = ar(ACY)

Proof:

ar(△ADX) = ar(△AXC) — (i)

(Since, they are on the same base AX and between the same parallels AB and CD)

ar(△ AXC)=ar(△ ACY) — (ii)

(Since, they are on the same base AC and between the same parallels XY and AC.)

From (i) and (ii),

We get,

ar(△ADX)=ar(△ACY)

Hence Proved