EXERCISE 19(A)

1. Find the mean of 43, 51, 50, 57 and 54.

Solution:

The numbers given are 43, 51, 50, 57, 54

The mean of the given numbers will be

$$= \frac{43+51+50+57+54}{5}$$

$$= \frac{255}{5}$$

$$= 51$$

2. Find the mean of first six natural numbers.

Solution:

The first six natural numbers are $1, 2, 3, 4, 5, 6$

The mean of first six natural numbers

$$= \frac{1+2+3+4+5+6}{3}$$

$$= \frac{21}{3}$$

$$= 3.5$$

3. Find the mean of first ten odd natural number.

Solution:

The first ten odd natural numbers are $1, 3, 5, 7, 9, 11, 13, 15, 17, 19$

The mean of first ten odd numbers

$$= \frac{1+3+5+7+9+11+13+15+17+19}{10}$$

$$= \frac{100}{10}$$

$$= 10$$

4. Find the mean of all factors of 10.

Solution:

The all factors of 10 are $1, 2, 5, 10$

The mean of all factors of 10 are

$= \frac{1+2+5+10}{4}$

$= \frac{18}{4}$

$= 4.5$

5. Find the mean of x + 3, x + 5, x + 7, x + 9 and x + 11.

Solution:

The given values are $x+3, x+5, x+7, x+9, x+11$

The mean of the values are

$= \frac{x+3+x+5+x+7+x+9+x+11}{5}$

$= \frac{5x+35}{5}$

$= \frac{5(x+7)}{5}$

$= x+7$

6. If different values of variable x are 9.8, 5.4, 3.7, 1.7, 1.8, 2.6, 2.8, 8.6, 10.5 and 11.1; find

(i) the mean $\bar{x}$

(ii) **the value of** $\sum_{i=1}^{10}(x_i - \bar{x})$

Solution:

(i) The given numbers are 9.8, 5.4, 3.7, 1.7, 1.8, 2.6, 2.8, 8.6, 10.5 and 11.1

$\bar{x} = \frac{x1+x2+x3+x4+x5+\ldots\ldots+xn}{n}$

$= \frac{9.8+5.4+3.7+1.7+1.8+2.6+2.8+8.6+10.5+11.1}{10}$

$= 5.8$

(ii) The value of $\sum_{i=1}^{10}(x_i - \bar{x})$

$\sum_{i=1}^{n}(xi - \bar{x}) = (x1 - \bar{x}) + (x2 - \bar{x}) \ldots\ldots\ldots + (xn - \bar{x}) = 0$

$\bar{x} = 5.8$

$$\sum_{i=1}^{10}\left(x_i - \bar{x}\right)$$
$$= (9.8-5.8)+(5.4-5.8)+(3.7-5.8)+(1.7-5.8)+(1.8-5.8)$$
$$+(2.6-5.8)+(2.8-5.8)+(8.6-5.8)+(10.5-5.8)+(11.1-5.8)$$
$$= 4-.4-2.1-4.1-4-3.2-3+2.8+4.7+5.3$$
$$= 0$$

7. The mean of 15 observations is 32. Find the resulting mean, if each observation is :

(i) Increased by 3
(ii) Decreased by 7
(iii) Multiplied by 2
(iv) Divided by 0.5
(v) Increased by 60%
(vi) Decreased by 20%

Solution:

Given that the mean of 15 observations is 32

(i) resulting mean increased by 3
=32 + 3
=35

(ii) resulting mean decreased by 7
=32 - 7
= 25

(iii) resulting mean multiplied by 2
=32x2
=64

(iv) resulting mean divide by 0.5

$$= \frac{32}{.5}$$
$$= 64$$

(v) resulting mean increased by 60%

$$= 32 + \frac{60}{100} \times 32$$
$$= 32 + 19.2$$
$$= 51.2$$

(vi) resulting mean decreased by 20%

$$= 32 - \frac{20}{100} \times 32$$
$$= 32 - 6.4$$
$$= 25.6$$

8. The mean of 5 numbers is 18. If one number is excluded, the mean of remaining number becomes 16. Find the excluded number.

Solution:

Given the mean of 5 numbers is 18
Total sum of 5 numbers
=18x5
=90
On excluding an observation, the mean of remaining 4 observation is 16
=16x4
=64
Therefore sum of remaining 4 observations
= total of 5 observations-total of 4 observations
= 90 - 64
= 26

9. If the mean of observations x, x + 2, x + 4, x + 6 and x + 8 is 11, find:
(i) The value of x;
(ii) **The mean of first three observations.**

Solution:

(i) Given that the mean of observations x, x + 2, x + 4, x + 6 and x + 8 is 11

Mean= $\frac{observations}{n}$

$$11 = \frac{x + x + 2 + x + 4 + x + 6 + x + 8}{5}$$
$$11 = \frac{5x + 20}{5}$$
$$x = \frac{35}{7}$$
$$x = 7$$

(ii) The mean of first three observations are

$$= \frac{x + x + 2 + x + 4}{3}$$

$$= \frac{3x + 6}{3}$$

$$= \frac{3*7 + 6}{3} \qquad [\text{since } x=7]$$

$$= \frac{21 + 6}{3}$$

$$= 9$$

10. The mean of 100 observations is 40. It is found that an observation 53 was misread as 83. Find the correct mean.

Solution:

Given the mean of 100 observations is 40.

$$\frac{\sum x}{n} = \bar{x}$$

$$\Rightarrow \frac{\sum x}{n} = 40$$

$$\Rightarrow \quad x = 40*100$$

$$\Rightarrow \quad x = 4000$$

Incorrect value of x=4000

Correct value of x=Incorrect value of x-Incorrect observation + correct observation

=4000-83+53

=3970

Correct mean

$$= \frac{\text{correct value of } \sum x}{n}$$

$$= \frac{3970}{100}$$

$$= 39.7$$

11. The mean of 200 items was 50. Later on, it was discovered that two items were misread as 92 and 8 instead of 192 and 88. Find the correct mean.

Solution:

Given that the mean of 200 items was 50.

$$\text{Mean} = \frac{\sum x}{n}$$

$$\Rightarrow 50 = \frac{\sum x}{200}$$

$$\Rightarrow x = 10000$$

Incorrect value of $\sum x = 10000$

Correct value of

$$\sum x = 10000 - (92 + 8) + (192 + 88)$$

$$= 10000 - 100 + 280$$

$$= 10180$$

Correct mean

$$= \frac{\text{correct value of} \sum x}{n}$$

$$= \frac{10180}{200}$$

$$= 50.9$$

12. Find the mean of 75 numbers, if the mean of 45 of them is 18 and the mean of the remaining ones is 13.

Solution:

Mean of 45 numbers = 18

$\Rightarrow$ Sum of 45 numbers $= 18 \times 45 = 810$

Mean of remaining $(75 - 45) 30$ numbers $= 13$

$\Rightarrow$ Sum of remaining 30 numbers $= 13 \times 30 = 390$

$\Rightarrow$ Sum of all the 75 numbers $= 810 + 390 = 1200$

$\Rightarrow$ Mean of all the 75 numbers $= \frac{1200}{75} = 16$

13. The mean weight of 120 students of a school is 52.75 kg. If the mean weight of 50 of them is 51 kg, find the mean weight of the remaining students.

Solution:

Mean weight of 120 students = 52.75 kg
$\Rightarrow$ Sum of the weight of 120 students $= 120 \times 52.75 = 6330$ kg
Mean weight of 50 students = 51 kg
$\Rightarrow$ Sum of the weight of 50 students $= 50 \times 51 = 2550$ kg
$\Rightarrow$ Sum of the weight of remaining (120 - 50) 70 students
= Sum of the weight of 120 students - Sum of the weight of 50 students
$= (6330 - 2550)$ kg
$= 3780$ kg
$\Rightarrow$ Mean weight of remaining 70 students $= \frac{3780}{70} = 54$ kg

14. The mean marks (out of 100) of boys and girls in an examination are 70 and 73 respectively. If the mean marks of all the students in that examination is 71, find the ratio of the number of boys to the number of girls.

Solution:

Let the number of boys and girls be x and y respectively.
Now,
Given, Mean marks of x boys in the examination = 70
$\Rightarrow$ Sum of marks of x boys in the examination $= 70x$
Given, Mean marks of y girls in the examination = 73
$\Rightarrow$ Sum of marks of y girls in the examination $= 73y$
Given, Mean marks of all students $(x + y)$ in the examination = 71
$\Rightarrow$ Sum of marks of all students $(x + y)$ students in the examination $= 71(x + y)$
Now, Sum of marks of all students $(x + y)$ students in the examination
= Sum of marks of x boys in the examination
+ Sum of marks of y girls in the examination
$\Rightarrow 71(x + y) = 70x + 73y$
$\Rightarrow 71x + 71y = 70x + 73y$
$\Rightarrow x = 2y$
$\Rightarrow \frac{x}{y} = \frac{2}{1}$
$\Rightarrow x : y = 2 : 1$
Thus, the ratio of number of boys to the number of girls is 2 : 1.

15. Find x if 9, x, 14, 18 x, x, 8, 10 and 4 have a mean of 11.

Solution:

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$\Rightarrow 11 = \frac{9 + x + 14 + 18 + x + x + 8 + 10 + 4}{9}$$

$$\Rightarrow 11 \times 9 = 63 + 3x$$

$$\Rightarrow 3x + 63 = 99$$

$$\Rightarrow 3x = 99 - 63$$

$$\Rightarrow 3x = 36$$

$$\Rightarrow x = 12$$

16. In a series of tests, A appeared for 8 tests. Each test was marked out of 30 and averages 25. However, while checking his files, A could only find 7 of the 8 tests. For these he scored 29, 26, 18, 20, 27, 24 and 29. Determine how many marks he scored for the eighth test.

Solution:

Total number of tests = 8
Average score of A = 25
Let the score of 8^{th} test be x.
Then, total score of 8 tests = 29 + 26 + 18 + 20 + 27 + 24 + 29 + x
Now, we have

$$\text{Mean} = \frac{\text{Total score of 8 tests}}{\text{Total number of tests}}$$

$$\Rightarrow 25 = \frac{29 + 26 + 18 + 20 + 27 + 24 + 29 + x}{8}$$

$$\Rightarrow 25 \times 8 = 173 + x$$

$$\Rightarrow x + 173 = 200$$

$$\Rightarrow x = 200 - 173$$

$$\Rightarrow x = 27$$

Thus, A scored 27 marks in the eights test.

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1. Find the median of:

(i) 25, 16, 26, 16, 32, 31, 19, 28 and 35

(ii) 241, 243, 347, 350, 327, 299, 261, 292, 271, 258 and 257

(iii) 63, 17, 50, 9, 25, 43, 21, 50, 14 and 34

(iv) 233, 173, 189, 208, 194, 204, 194, 185, 200 and 220.

Solution:

(i) Firstly arrange the numbers in ascending order

16, 16, 19, 25, 26, 28, 31, 32, 35

Now since

n=9(odd)

Therefore Median

$$= \left(\frac{n+1}{2}\right)^{th}$$

$$= \left(\frac{9+1}{2}\right)^{th}$$

$$= 5^{th}$$

Thus the median is 26

(ii)

Firstly arrange the numbers in ascending order

241, 243, 257, 258, 261, 271, 292, 299, 327, 347, 350

Now since n=11(Odd)

$$\text{Median} = \text{value of} \left(\frac{n+1}{2}\right)^{th} \text{term}$$

$$= 6^{th}\text{term}$$

$$= 271$$

Thus median is 271.

(iii)

Firstly arrange the numbers in ascending order

9, 14, 17, 21, 25, 34, 43, 50, 50, 63

Since, n=10(even)

We find the median as:

$$\text{Median}=\frac{1}{2}\left[\text{value of }\left(\frac{n}{2}\right)^{th}\text{ term}+\text{value of }\left(\frac{n}{2}+1\right)^{th}\text{ term}\right]$$

$$=\frac{1}{2}\left[\text{value of }\left(\frac{10}{2}\right)^{th}\text{ term}+\text{value of }\left(\frac{10}{2}+1\right)^{th}\text{ term}\right]$$

$$=\frac{1}{2}[25+34]$$

$$=\frac{1}{2}[59]$$

$$=29.5$$

Thus the median is 29.5

(iv) Arrange the numbers in ascending order

173,185,189,194,194,200,204,208,220,223

$$\text{Median}=\frac{1}{2}\left[\text{value of }\left(\frac{n}{2}\right)^{th}\text{ term}+\text{value of }\left(\frac{n}{2}+1\right)^{th}\text{ term}\right]$$

$$=\frac{1}{2}\left[\text{value of }\left(\frac{10}{2}\right)^{th}\text{ term}+\text{value of }\left(\frac{10}{2}+1\right)^{th}\text{ term}\right]$$

$$=\frac{1}{2}[200+194]$$

$$=\frac{1}{2}[394]$$

$$=197$$

Thus the median is 197

2. The following data have been arranged in ascending order. If their median is 63, find the value of x. 34, 37, 53, 55, x, x + 2, 77, 83, 89 and 100.

Solution:

Given numbers are 34, 37, 53, 55, x, x+2, 77, 83, 89, 100

Here n = 10(even)

$$Median = \frac{1}{2}\left[value\ of\left(\frac{n}{2}\right)^{th} term + value\ of\left(\frac{n}{2}+1\right)^{th} term\right]$$

$$= \frac{1}{2}\left[value\ of\left(\frac{10}{2}\right)^{th} term + value\ of\left(\frac{10}{2}+1\right)^{th} term\right]$$

$$= \frac{1}{2}\left[value\ of\ (5)^{th} term + value\ of\ (5+1)^{th} term\right]$$

$$= \frac{1}{2}\left[value\ of\ (5)^{th} term + value\ of\ (6)^{th} term\right]$$

$$63 = \frac{1}{2}[x + x + 2]$$

$$\Rightarrow \frac{[2+2x]}{2} = 63$$

$$\Rightarrow x + 1 = 63$$

$$\Rightarrow x = 62$$

3. In 10 numbers, arranged in increasing order, the 7th number is increased by 8, how much will the median be changed?

Solution:

For any given set of data, the median is the value of its middle term.
Here, total observations = n = 10 (even)
If n is even, we have

$$\text{Median} = \frac{1}{2}\left[\text{value of}\left(\frac{n}{2}\right)^{th} \text{term} + \text{value of}\left(\frac{n}{2}+1\right)^{th} \text{term}\right]$$

Thus, for n = 10, we have

$$\text{Median} = \frac{1}{2}\left[\text{value of}\left(\frac{10}{2}\right)^{th} \text{term} + \text{value of}\left(\frac{10}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[\text{value of } 5^{th} \text{ term} + \text{value of } 6^{th} \text{ term}\right]$$

Hence, if 7th number is diminished by 8, there is no change in the median value.

4. Out of 10 students, who appeared in a test, three secured less than 30 marks and 3 secured more than 75 marks. The marks secured by the remaining 4 students are 35, 48, 66 and 40. Find the median score of the whole group.

Solution:

Here, total observations = n = 10 (even)

$$\text{Median} = \frac{1}{2}\left[\text{value of } \left(\frac{10}{2}\right)^{th} \text{ term} + \text{value of } \left(\frac{10}{2}+1\right)^{th} \text{ term}\right]$$

$$= \frac{1}{2}\left[\text{value of } 5^{th} \text{ term} + \text{value of } 6^{th} \text{ term}\right]$$

$$\therefore \text{Median} = \frac{1}{2}(40+48) = \frac{88}{2} = 44$$

Therefore, the median score of the whole group is 44.

5. The median of observations 10, 11, 13, 17, x + 5, 20, 22, 24 and 53 (arranged in ascending order) is 18; find the value of x.

Solution:

$$\text{Total number of observations} = 9\ (\text{odd})$$

$$\text{Now, if } n = \text{odd}$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

$$\Rightarrow \text{Median} = \left(\frac{9+1}{2}\right)^{th} \text{ term} = 5^{th} \text{ term} = x + 5$$

$$\text{Now, Median} = 18\ (\text{given})$$

$$\therefore x + 5 = 18$$

$$\Rightarrow x = 13$$

EXERCISE 19(B) PAGE:241

1. Find the mean of 8, 12, 16, 22, 10 and 4. Find the resulting mean, if each of the observations, given above, be:

(i) Multiplied by 3.

(ii) Divided by 2.

(iii) Multiplied by 3 and then divided by 2.

(iv) Increased by 25%

(v) Decreased by 40%

Solution:

Mean of the given data $= \frac{8 + 12 + 16 + 22 + 10 + 4}{6}$

$= \frac{72}{6} = 12$

(i) Multiplied by 3

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$,

then mean of $ax_1, ax_2, ax_3, \ldots, ax_n$ is $a\bar{x}$.

Thus, when each of the given data is multiplied by 3,

the mean is also multiplied by 3.

Mean of the original data is 12.

Hence, the new mean = 12 x 3 = 36.

(ii) Divided by 2

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$,

then mean of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots, \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$.

Thus, when each of the given data is divided by 2,

the mean is also divided by 3.

Mean of the original data is 12.

Hence, the new mean $= \frac{12}{2} = 6$.

(iii) Multiplied by 3 and then divided by 2

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$,

then mean of $\frac{a}{b}x_1, \frac{a}{b}x_2, \frac{a}{b}x_3, \ldots, \frac{a}{b}x_n$ is $\frac{a}{b}\bar{x}$.

Thus, when each of the given data is multiplied by $\frac{3}{2}$,

the mean is also multiplied by $\frac{3}{2}$.

Mean of the original data is 12.

Hence, the new mean $= \frac{3}{2} \times 12 = \frac{36}{2} = 18$

(iv) Increased by 25%

New mean = Original mean + 25% of original mean

$\Rightarrow$ New mean = 12 + 25% of 12

$\Rightarrow$ New mean $= 12 + \frac{25}{100} \times 12$

$\Rightarrow$ New mean $= 12 + \frac{1}{4} \times 12$

$\Rightarrow$ New mean = 12 + 3

$\Rightarrow$ New mean = 15

(v) Decreased by 40%

New mean = Original mean − 40% of original mean

$\Rightarrow$ New mean = 12 − 40% of 12

$\Rightarrow$ New mean $= 12 - \frac{40}{100} \times 12$

$\Rightarrow$ New mean $= 12 - \frac{2}{5} \times 12$

$\Rightarrow$ New mean $= 12 - 0.4 \times 12$

$\Rightarrow$ New mean = 12 − 4.8

$\Rightarrow$ New mean = 7.2

2. The mean of 18, 24, 15, 2x + 1 and 12 is 21. Find the value of x.

Solution:

$$\text{Mean of given data} = \frac{18 + 24 + 15 + 2x + 1 + 12}{5}$$

$$\Rightarrow 21 = \frac{70 + 2x}{5}$$

$$\Rightarrow 5 \times 21 = 70 + 2x$$

$$\Rightarrow 105 = 70 + 2x$$

$$\Rightarrow 2x = 105 - 70$$

$$\Rightarrow 2x = 35$$

$$\Rightarrow x = \frac{35}{2}$$

$$\Rightarrow x = 17.5$$

3. The mean of 6 numbers is 42. If one number is excluded, the mean of remaining number is 45. Find the excluded number.

Solution:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

Mean of given data $= \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

Given that mean of 6 numbers is 42.

That is,

$\frac{x_1 + x_2 + x_3 + \ldots + x_6}{6} = 42$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_6 = 6 \times 42$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 252 - x_6 \ldots (1)$

Also, given that the mean of 5 numbers is 45.

That is,

$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 45$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 5 \times 45$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 225 \ldots (2)$

From equations (1) and (2), we have,

$x_1 + x_2 + x_3 + x_4 + x_5 = 252 - x_6 = x_1 + x_2 + x_3 + x_4 + x_5 = 225$

$252 - x_6 = 225$

$\Rightarrow x_6 = 252 - 225 = 27$

4. The mean of 10 numbers is 24. If one more number is included, the new mean is 25. Find the included number.

Solution:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

Mean of given data $= \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

Given that mean of 10 numbers is 24.

That is,

$\frac{x_1 + x_2 + x_3 + \ldots + x_{10}}{10} = 24$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} = 10 \times 24$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} = 240$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 240 + x_{11} \ldots (1)$

Also, given that mean of 11 numbers is 25.

$$\frac{x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11}}{11} = 25$$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 11 \times 25$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 275 \ldots (2)$

From equations (1) and (2), we have:

$x_1 + x_2 + x_3 + \ldots + x_{10} + x_{11} = 240 + x_{11} = 275$

$240 + x_{11} = 275$

$\Rightarrow x_{11} = 275 - 240 = 35$

5. The following observations have been arranged in ascending order. If the median of the data is 78, find the value of x.
44, 47, 63, 65, x + 13, 87, 93, 99, 110.

Solution:

Consider the given data:

44, 47, 63, 65, x+13, 87, 93, 99, 110

Here the number of observations is 9, which is odd.

Thus, the median of the given data is $\left(\frac{n+1}{2}\right)^{th}$ observation.

From the given data, $\left(\frac{9+1}{2} = 5\right)^{th}$ observation is $x + 13$

Also, given that the median is 78.

Thus, we have

$x + 13 = 78$

$\Rightarrow x = 78 - 13$

$\Rightarrow x = 65$

6. The following observations have been arranged in ascending order. If the median of these observations is 58, find the value of x.
24, 27, 43, 48, x - 1, x + 3, 68, 73, 80, 90.

Solution:

Consider the given data:

24, 27, 43, 48, x − 1, x + 3, 68, 73, 80, 90.

Here the number of observations is 10, which is even.

Thus, the median of given data is $\frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$.

From the given data, $\left(\frac{10}{2} = 5\right)^{th}$ observation is x − 1

and $\left(\frac{10}{2} + 1 = 6\right)^{th}$ observation is x + 3.

Also, given that the median is 58.

Thus, we have

$\frac{1}{2}[x - 1 + x + 3] = 116$

$\Rightarrow 2x + 2 = 116$

$\Rightarrow 2x = 116 - 2$

$\Rightarrow 2x = 114$

$\Rightarrow x = \frac{114}{2}$

$\Rightarrow x = 57$

7. Find the mean of the following data:
30, 32, 24, 34, 26, 28, 30, 35, 33, 25
(i) Show that the sum of the deviations of all the given observation from the mean is zero.
(ii) Find the median of the given data.

Solution:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

$\text{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

Therefore,

$\text{Mean of given data} = \frac{30 + 32 + 24 + 34 + 26 + 28 + 30 + 35 + 33 + 25}{10}$

$= \frac{297}{10}$

$= 29.7$

(i)

Let us tabulate the observations and their deviations from the mean

Observations x_i	Devaiations $x_i - \overline{x}$
30	0.3
32	2.3
24	−5.7
34	4.3
26	−3.7
28	−1.7
30	0.3
35	5.3
33	3.3
25	−4.7
Total	0

(ii)

Consider the given data:

30, 32, 24, 34, 26, 28, 30, 35, 33, 25

Let us rewrite the above data in ascending order.

24, 25, 26, 28, 30, 30, 32, 33, 34, 35

There are 10 observations, which is even.

$$\text{Therefore, median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th}\text{term} + \left(\frac{n}{2}+1\right)^{th}\text{term}\right]$$

$$= \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th}\text{term} + \left(\frac{10}{2}+1\right)^{th}\text{term}\right]$$

$$= \frac{1}{2}\left[(5)^{th}\text{term} + (5+1)^{th}\text{term}\right]$$

$$= \frac{1}{2}\left[5^{th}\text{term} + 6^{th}\text{term}\right]$$

$$= \frac{1}{2}[30+30]$$

$$= \frac{1}{2}[60]$$

$$= 30$$

8. Find the mean and median of the data:
35, 48, 92, 76, 64, 52, 51, 63 and 71.
If 51 is replaced by 66, what will be the new median?

Solution:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Therefore,

$$\text{Mean of given data} = \frac{35 + 48 + 92 + 76 + 64 + 52 + 51 + 63 + 71}{9}$$

$$= \frac{552}{9}$$

$$= 61.33$$

Rewriting the given data in ascending order:
35, 48, 51, 52, 63, 64, 71, 76, 92

$$\text{Therefore, median} = \left(\frac{n+1}{2}\right)^{th} \text{observation}$$

$$\Rightarrow \text{Median} = \left(\frac{9+1}{2}\right)^{th} \text{observation}$$

$$\Rightarrow \text{Median} = \left(\frac{10}{2}\right)^{th} \text{observation}$$

$$\Rightarrow \text{Median} = 5^{th} \text{observation}$$

$$\Rightarrow \text{Median} = 63$$

If 51 is replaced by 66, the new set of data in ascending order is:
35, 48, 52, 63, 64, 66, 71, 76, 92

Since median $= 5^{th}$ observation,

We have, new median $= 64$

9. The mean of x, x + 2, x + 4, x + 6 and x + 8 is 11, find the mean of the first three observations.

Solution:

Let $\bar{x}$ be the mean of n number of observations $x_1, x_2, x_3, \ldots, x_n$

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Therefore,

$$\text{Mean of given data} = \frac{x + x + 2 + x + 4 + x + 6 + x + 8}{5}$$

$$= \frac{5x + 20}{5}$$

$$= x + 4$$

Also, it's given that mean of the given data is 11.

$\Rightarrow x + 4 = 11$

$\Rightarrow x = 7$

Hence the mean of the first three observations $= \frac{x + x + 2 + x + 4}{3}$

$= \frac{3x + 6}{3}$

$= x + 2$

$= 7 + 2$

$= 9$

10. Find the mean and median of all the positive factors of 72.

Solution:

Let us find the factors of 72:

$72 = 1 \times 72$
$= 2 \times 36$
$= 3 \times 24$
$= 4 \times 18$
$= 6 \times 12$
$= 8 \times 9$
$= 9 \times 8$
$= 12 \times 6$
$= 18 \times 4$
$= 24 \times 3$
$= 36 \times 2$
$= 72 \times 1$

Hence, the data becomes,

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Mean of the above data set $= \frac{1+2+3+4+6+8+9+12+18+24+36+72}{12}$

$= \frac{195}{12}$

$= 16.25$

Since the number of observations is 12, which is even, median is given by

$$\text{Median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[\left(\frac{12}{2}\right)^{th} \text{term} + \left(\frac{12}{2}+1\right)^{th} \text{term}\right]$$

$$= \frac{1}{2}\left[6^{th} \text{term} + 7^{th} \text{term}\right]$$

$$= \frac{1}{2}[8+9]$$

$$= \frac{1}{2} \times 17$$

$$= 8.5$$

11. The mean weight of 60 students in a class is 40 kg. The mean weight of boys is 50 kg while that of girls is 30 kg. Find the number of boys and girls in the class.

Solution:

Total number of students = 60
Mean weight of 60 students = 40
Let the number of boys = x
Then, number of girls = 60 - x

$$\text{Mean weight of boys} = \frac{\text{Total weight of boys}}{\text{Total number of boys}}$$

$$\Rightarrow 50 = \frac{\text{Total weight of boys}}{x}$$

$$\Rightarrow \text{Total weight of boys} = 50x$$

$$\text{Mean weight of girls} = \frac{\text{Total weight of girls}}{\text{Total number of girls}}$$

$$\Rightarrow 30 = \frac{\text{Total weight of girls}}{60-x}$$

$$\Rightarrow \text{Total weight of girls} = 30(60-x)$$

Now,

$$\text{Mean weight of 60 students} = \frac{\text{Total weight of boys} + \text{Total weight of girls}}{\text{Total number of students}}$$

$$\Rightarrow 40 = \frac{50x + 30(60-x)}{60}$$

$$\Rightarrow 2400 = 50x + 1800 - 30x$$

$$\Rightarrow 20x = 600$$

$$\Rightarrow x = 30$$

$$\Rightarrow 60 - x = 60 - 30 = 30$$

Hence, the number of boys is 30 and the number of girls is also 30.

12. The average of n numbers x_1, x_2, x_3 ….. x_n is A.
If x_1 is replaced by $(x+ \alpha)\ x_1$, x_2, is replaced by $(x+ \alpha)x_2$ and so on. Find the new average.

Solution:

Mean of n numbers = A

$$\Rightarrow A = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$$

$$\Rightarrow x_1 + x_2 + \ldots\ldots + x_n = n \times A \qquad \ldots.(i)$$

$$\text{New Mean} = \frac{(x+a)x_1 + (x+a)x_2 + \ldots\ldots + (x+a)x_n}{n}$$

$$= \frac{(x+a)(x_1 + x_2 + \ldots\ldots + x_n)}{n}$$

$$= \frac{(x+a)(n \times A)}{n} \qquad \text{[From (i)]}$$

$$= \frac{(x+a) \times n \times A}{n}$$

$$= (x+a)A$$

13. The heights (in cm) of the volley- ball players from team A and team B were recorded as:

Team A: 180, 178, 176, 181, 190, 175, 187

Team B: 174, 175, 190, 179, 178, 185, 177

Which team had the greater average height?

Find the median of team A and team B.

Solution:

Total number of players in each team = 7

$$\text{Mean height of team A} = \frac{\text{Sum of heights of players of team A}}{\text{Total number of team A players}}$$

$$= \frac{180+178+176+181+190+175+187}{7}$$

$$= \frac{1267}{7}$$

$$= 181 \text{ cm}$$

$$\text{Mean height of team B} = \frac{\text{Sum of heights of players of team B}}{\text{Total number of team B players}}$$

$$= \frac{174+175+190+179+178+185+177}{7}$$

$$= \frac{1258}{7}$$

$$= 179.7 \text{ cm}$$

Therefore, team A has greater average height.

Median of team A:

Arranging heights in ascending order, we get

175, 176, 178, 180, 181, 187, 190

Total number of observations = n = 7 (odd)

$\therefore$ Median = $\left(\frac{n+1}{2}\right)^{th}$ observation = $\left(\frac{7+1}{2}\right)^{th}$ observation = 4^{th} observation = 180 cm

Median of team B:

Arranging heights in ascending order, we get

174, 175, 177, 178, 179, 185, 190

Total number of observations = n = 7 (odd)

$\therefore$ Median = $\left(\frac{n+1}{2}\right)^{th}$ observation = $\left(\frac{7+1}{2}\right)^{th}$ observation = 4^{th} observation = 178 cm