EXERCISE 20(A) PAGE:247

1. **Find the area of a triangle whose sides are 18 cm, 24 cm and 30 cm. Also, find the length of altitude corresponding to the largest side of the triangle.**

Solution:

$$s = \frac{18 + 24 + 30}{2}$$

$$= 36$$

Area of the triangle,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36(36-18)(36-24)(36-30)}$$

$$= \sqrt{36 \times 18 \times 12 \times 6}$$

$$= \sqrt{46656}$$

$$= 216\text{sqcm}$$

$$A = \frac{1}{2}\text{base} \times \text{altitude}$$

$$216 = \frac{1}{2} \times 30 \times h$$

$$h = 14.4cm$$

2. **The length of the sides of a triangle are in the ratio 3: 4: 5. Find the area of the triangle if its perimeter is 144 cm.**

Solution:

Let the sides of the triangle be
a=3x
b=4x
c=5x
Given that the perimeter is 144 cm.

$$3x + 4x + 5x = 144$$

$$\Rightarrow 12x = 144$$

$$\Rightarrow x = \frac{144}{12}$$

$$\Rightarrow x = 12$$

$$s = \frac{a+b+c}{2} = \frac{12x}{2} = 6x = 72$$

Therefore, the sides are a=36 cm, b=48 cm and c=60 cm
Then, Area of the triangle,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{72(72-36)(72-48)(72-60)}$$
$$= \sqrt{72 \times 36 \times 24 \times 12}$$
$$= \sqrt{746496}$$
$$= 864\ cm^2$$

3. ABC is a triangle in which AB = AC = 4 cm and $\angle$A = 90º. Calculate:

(i) The area of $\triangle$ABC,

(ii) The length of perpendicular from A to BC.

Solution:

(i)

Area of the triangle,

$$A = \frac{1}{2} \times AB \times AC$$
$$= \frac{1}{2} \times 4 \times 4$$
$$= 8\ \text{sq.cm}$$

(ii)

Area of the triangle,

$$A = \frac{1}{2} \times BC \times h$$
$$8 = \frac{1}{2} \times \left(\sqrt{4^2 + 4^2}\right) \times h$$
$$h = 2.83cm$$

4. The area of an equilateral triangle is $36\sqrt{3}$ sq. cm. Find its perimeter.

Solution:

$$\frac{\sqrt{3}}{4} \times (side)^2 = A$$
$$\frac{\sqrt{3}}{4} \times (side)^2 = 36\sqrt{3}$$
$$(side)^2 = 144$$
$$side = 12\ cm$$

Same time,

$$\text{perimeter} = 3 \times (\text{its side})$$
$$= 3 \times 12$$
$$= 36\ cm$$

5. Find the area of an isosceles triangle with perimeter is 36 cm and base is 16 cm.

Solution:

Given that the perimeter of the isosceles triangle is 36cm and base is 16cm.
Then the length of each of equal sides becomes,

$$\frac{36-16}{2} = 10cm$$

It is given that

$$a = \text{equa lsides} = 10cm$$

$$b = \text{base} = 16cm$$

Let 'h' be the altitude of the isosceles triangle.
Applying Pythagoras Theorem,
We get,

$$h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{4a^2 - b^2}$$

We know that

$$\text{Area of the triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\text{Area of the triangle} = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$
$$= \frac{1}{4} \times 16 \times \sqrt{400 - 256}$$
$$= 48\text{sq.cm}$$

6. The base of an isosceles triangle is 24 cm and its area is 192 sq. cm. Find its perimeter.

Solution:

$$A = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

Let 'a' be the length of one side which is equal.

$$\text{Area} = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$192 = \frac{1}{4} \times 24 \times \sqrt{4a^2 - 576}$$

$$192 = 6\sqrt{4a^2 - 576}$$

$$\sqrt{4a^2 - 576} = 32$$

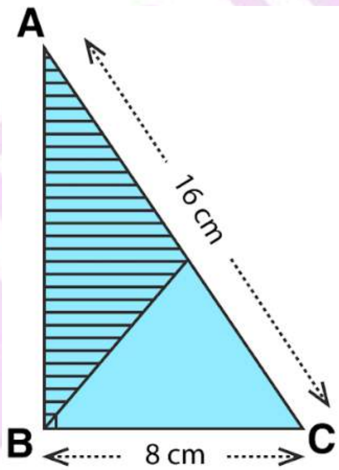
$$4a^2 - 576 = 1024$$

$$4a^2 = 1600$$

$$a = 20cm$$

Therefore, perimeter=20+20+24=64cm

7. The given figure shows a right-angled triangle ABC and an equilateral triangle BCD. Find the area of the shaded portion.



Solution

From triangle ABC,

$$AB = \sqrt{AC^2 - BC^2}$$

$$= \sqrt{16^2 - 8^2}$$

$$= \sqrt{192}$$

Area of triangle ABC,

$$\Delta ABC = \frac{1}{2} \times 8 \times \sqrt{192}$$

$$= 4\sqrt{192}$$

Area of triangle BCD,

$$\Delta BCD = \frac{\sqrt{3}}{4} \times 8^2$$

$$= 16\sqrt{3}$$

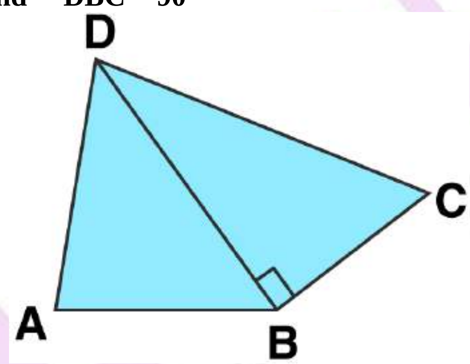
$$\Delta ABD = \Delta ABC - \Delta BDC$$

$$= 4\sqrt{192} - 16\sqrt{3}$$

$$= 32\sqrt{3} - 16\sqrt{3}$$

$$= 16\sqrt{3} \text{sq.cm}$$

8. Find the area and the perimeter of quadrilateral ABCD, given below; if AB = 8 cm, AD = 10 cm, BD = 12 cm, DC = 13 cm and ∠DBC = 90º



Solution:

Given , AB = 8 cm, AD = 10 cm, BD = 12 cm, DC = 13 cm and ∠DBC = 90º

$$BC = \sqrt{DC^2 - BD^2}$$

$$= \sqrt{13^2 - 12^2}$$

$$= 5cm$$

i.e., perimeter=8+10+13+5=36cm

Area of triangle ABD,

$$\Delta ABD = \sqrt{15(15-8)(15-10)(15-12)}$$

$$= \sqrt{15 \times 7 \times 5 \times 3}$$

$$= 15\sqrt{7}$$

$$= 39.7$$

Area of triangle DBC,

$$\Delta BDC = \frac{1}{2} \times 12 \times 5$$
$$= 30$$

$$\text{Area of } ABCD = area\ of\ \Delta ABD + area\ of\ \Delta BDC$$
$$= 39.7 + 30$$
$$= 69.7 \text{sq. cm}$$

9. The base of a triangular field is three times its height. If the cost of cultivating the field at ₹ 36.72 per 100 m² is ₹ 49,572; find its base and height.

Solution:

$$\text{Area of the rectangular field} = \frac{49572}{36.72} = 135000$$

Let the height of the triangle be x

$$135000 = \frac{1}{2} \times x \times 3x$$
$$\Rightarrow x^2 = 90000$$
$$\Rightarrow x = 300$$

i.e., Height = 300

Base = 3x=3x 300 = 900

10. The sides of a triangular field are in the ratio 5 : 3 : 4 and its perimeter is 180 m. Find:

(i) Its area.

(ii) Altitude of the triangle corresponding to its largest side.

(iii) The cost of levelling the field at the rate of Rs. 10 per square metre.

Solution:

(i) Ratio of sides = 5:3:4

Perimeter = 180m

Thus, we have, $5x + 4x + 3x = 180$

$$\Rightarrow 12x = 180$$
$$\Rightarrow x = \frac{180}{12}$$
$$\Rightarrow x = 15$$

Thus, the sides are 5×15, 3×15 and 4×15.

That is the sides are 75 m, 45 m and 60 m.

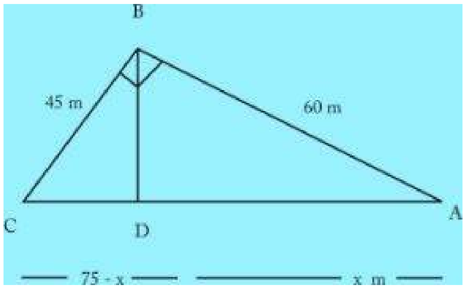
Since the sides are in the ratio, 5:3:4, it is a Pythagorean triplet.

Therefore, the triangle is a right angled triangle.

$$\text{Area of a right angled triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$
$$\Rightarrow = \frac{1}{2} \times 45 \times 60$$
$$\Rightarrow = 45 \times 30 = 1350 \text{ m}^2$$

(ii)

Consider the following figure.



In the above figure,

The largest side is AC = 75 m.

The altitude corresponding to AC is BD.

We need to find the value of BD.

Now consider the triangles ΔBCD and ΔBAD.

We have,

$\angle B = \angle B$ [common]

$BD = BD$ [common]

$\angle D = \angle D = 90°$

Thus, by Angle-Side-Angle criterion of congruence, we have $\Delta BCD - \Delta ABD$.

Similar triangles have similar proportionality.

Thus, we have,

$$\frac{CD}{BD} = \frac{BD}{AD}$$

$$\Rightarrow BD^2 = AD \times CD \ldots(1)$$

AC = 75 m, AB = 60 m and BC = 45 m

Let AD = x m $\Rightarrow$ CD $= (75 - x)$ m

Thus applying Pythgoras Theroem,

from right triangle ΔBCD, we have

$$45^2 = (75 - x)^2 + BD^2$$

$$\Rightarrow BD^2 = 45^2 - (75 - x)^2$$

$$\Rightarrow BD^2 = 2025 - (5625 + x^2 - 150x)$$

$$\Rightarrow BD^2 = 2025 - 5625 - x^2 + 150x$$

$$\Rightarrow BD^2 = -3600 - x^2 + 150x \ldots(2)$$

$60^2 = x^2 + BD^2$
$\Rightarrow BD^2 = 60^2 - x^2$
$\Rightarrow BD^2 = 3600 - x^2 \ldots(3)$
From equations (2) and (3), we have,
$-3600 - x^2 + 150x = 3600 - x^2$
$\Rightarrow 150x = 3600 + 3600$
$\Rightarrow 150x = 7200$
$\Rightarrow x = \frac{7200}{150}$
$\Rightarrow x = 48$
Thus, AD = 48 and CD = 75 – 48 = 27
Substituting the values AD=48 m
and CD=27 m in equation (1), we have
$BD^2 = 48 \times 27$
$\Rightarrow BD^2 = 1296$
$\Rightarrow BD = 36$ m
The altitude of the triangle corresponding to
its largest side is BD = 36 m

(iii)
The cost of levelling the field is
Rs.10 per square metre.
Thus, the total cost of
levelling the field = 1350×10 = Rs.13,500

11. Each of equal sides of an isosceles triangle is 4 cm greater than its height. IF the base of the triangle is 24 cm; calculate the perimeter and the area of the triangle.

Solution:
Let the height of the triangle be = x cm.
Given that equal sides are (x+4) cm.
Applying Pythagoras theorem,
$(x+4)^2 = x^2 + 12^2$
$8x = 128$
$x = 16cm$
Area of the isosceles triangle is given by
Here a=20cm
b=24cm

$$Area = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 24 \times \sqrt{1024}$$

$$= 192 sq.cm$$

12. Calculate the area and the height of an equilateral triangle whose perimeter is 60 cm.

Solution:

Side of triangle,

$$\frac{60}{3} = 20cm$$

Area of the equilateral triangle,

$$A = \frac{\sqrt{3}}{4} \times 20^2$$

$$= 100\sqrt{3}$$

$$= 173.2 sq.cm$$

The height h of the triangle is given by

$$\frac{1}{2} \times 20 \times h = 173.2$$

$$h = 17.32cm$$

13. In triangle ABC; angle A = 90º, side AB = x cm, AC = (x + 5) cm and area = 150 cm². Find the sides of the triangle.

Solution:

Area of the triangle = 150sq.cm

$$\frac{1}{2} \times x \times (x+5) = 150$$

$$x^2 + 5x - 300 = 0$$

$$(x+20)(x-15) = 0$$

$$x = 15$$

Hence, AB=15cm, AC=20cm and

$$BC = \sqrt{15^2 + 20^2}$$

$$= 25cm$$

14. If the difference between the sides of a right angled triangle is 3 cm and its area is 54 cm²; find its perimeter.

Solution:

Let the two sides = x cm and (x-3) cm.

$$\frac{1}{2} \times x \times (x-3) = 54$$

$$x^2 - 3x - 108 = 0$$

$$(x-12)(x+9) = 0$$

$$x = 12cm$$

Hence,

The sides are 12cm, 9cm and

$$\sqrt{12^2 + 9^2} = 15cm$$

The required perimeter is 12+9+15=36cm.

15. AD is altitude of an isosceles triangle ABC in which AB = AC = 30 cm and BC = 36 cm. A point O is marked on AD in such a way that $\angle$BOC = 90°. Find the area of quadrilateral ABOC.

Solution:

$$\text{Area of } \Delta ABC = \frac{1}{4} \times 36 \times \sqrt{4 \times 30^2 - 36^2}$$

$$= \frac{1}{4} \times 36 \times \sqrt{2304}$$

$$= \frac{1}{4} \times 36 \times 48$$

$$= 432$$

We know that,

AB=AC and angle BOC = 90^0

Angle BOD = COD = 45^0

So, angle OBD= 45^0

And OD = BD = 18cm

Now

$$\text{Area of } \Delta BOC = \frac{1}{2} \times 36 \times 18$$

$$= 324$$

Area of ABOC = Area of triangle ABC – Area of triangle BOC

= 432-324

= 108 cm^2

EXERCISE 20(B) PAGE:254

1. **Find the area of a quadrilateral one of whose diagonals is 30 cm long and the perpendiculars from the other two vertices are 19 cm and 11 cm respectively.**

Solution:

Area of a quadrilateral,

$$=\frac{1}{2}\times 30\times(11+19)$$

$$=450\text{sq.cm}$$

2. **The diagonals of a quadrilateral are 16 cm and 13 cm. If they intersect each other at right angles; find the area of the quadrilateral.**

Solution:

According to the question,

$$\text{Area of the quadriletaral} = \frac{1}{2}\times\text{the product of the diagonals.}$$

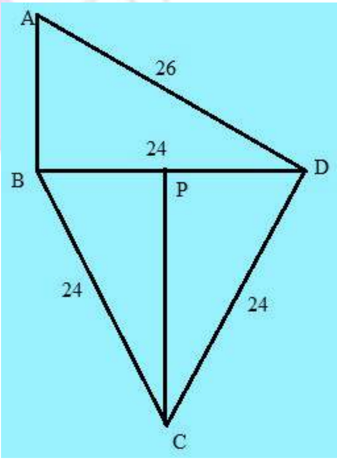
$$=\frac{1}{2}\times 16\times 13$$

$$=104cm^2$$

3. **Calculate the area of quadrilateral ABCD, in which $\angle$ABD = 90º, triangle BCD is an equilateral triangle of side 24 cm and AD = 26 cm.**

Solution:

Let the figure be:



From the right triangle ABD,

$$AB=\sqrt{26^2-24^2}$$

$$=2\sqrt{13^2-12^2}$$

$$=2(5)$$

$$=10$$

Area of right triangle ABD,

$$\Delta ABD = \frac{1}{2}(AB)(BD)$$
$$= \frac{1}{2}(10)(24)$$
$$= 120$$

From the equilateral triangle BCD we have CP perpendicular BD

$$PC = \sqrt{24^2 - 12^2}$$
$$= 12\sqrt{2^2 - 1^1}$$
$$= 12\sqrt{3}$$

Therefore,
Area of the triangle BCD,

$$\Delta BCD = \frac{1}{2}(BD)(PC)$$
$$= \frac{1}{2}(24)(12\sqrt{3})$$
$$= 144\sqrt{3}$$

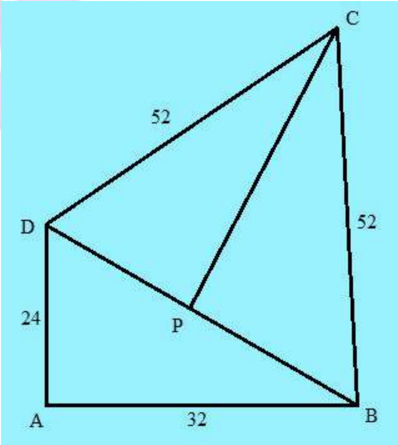
Hence, Area of the quadrilateral,

$$\Delta ABD + \Delta BCD = 120 + 144\sqrt{3}$$
$$= 369.41 \text{ cm}^2$$

4. Calculate the area of quadrilateral ABCD in which AB = 32 cm, AD = 24 cm $\angle$A = 90º and BC = CD = 52 cm.

Solution:

Let the figure be:



Here, ABD is a right triangle.
Area of triangle ABD,

$$= \frac{1}{2}(24)(32)$$
$$= 384$$

$$BD = \sqrt{24^2 + 32^2}$$
$$= 8\sqrt{3^2 + 4^2}$$
$$= 8(5)$$
$$= 40$$

BCD is an isosceles triangle
BP is perpendicular to BD
Therefore,

$$DP = \frac{1}{2}BD$$
$$= \frac{1}{2}(40)$$
$$= 20$$

From the right triangle DPC,

$$PC = \sqrt{52^2 - 20^2}$$
$$= 4\sqrt{13^2 - 5^2}$$
$$= 4(12)$$
$$= 48$$

So,

$$\Delta DPC = \frac{1}{2}(40)(48)$$
$$= 960$$

Hence,
Area of the quadrilateral,

$$\Delta ABD + \Delta DPC = 960 + 384$$
$$= 1344 \text{ cm}^2$$

5. The perimeter of a rectangular field is $\frac{3}{5}$ km. If the length of the field is twice its width; find the area of the rectangle in sq. metres.

Solution:

Let the width and length be x and 2x respectively.

$$2(x + 2x) = \frac{3}{5}$$
$$x = \frac{1}{10}km$$
$$= 100m$$

So, the width is 100m and length is 200m.
Area,

$$A = \text{length} \times \text{width}$$
$$= 100 \times 200$$
$$= 20{,}000 \text{sq.m}$$

6. **A rectangular plot 85 m long and 60 m broad is to be covered with grass leaving 5 m all around. Find the area to be laid with grass.**

Solution:

Length of the laid with grass = 85-5-5 =75m
Width of the laid with grass = 60-5-5 =50m
Hence,
Area of the laid with grass,
A = 75x50
= 3750 m^2

7. **The length and the breadth of a rectangle are 6 cm and 4 cm respectively. Find the height of a triangle whose base is 6 cm and area is 3 times that of the rectangle.**

Solution:

Area of the rectangle,

$$A = l \times b$$
$$= 6 \times 4$$
$$= 24 \text{sq.cm}$$

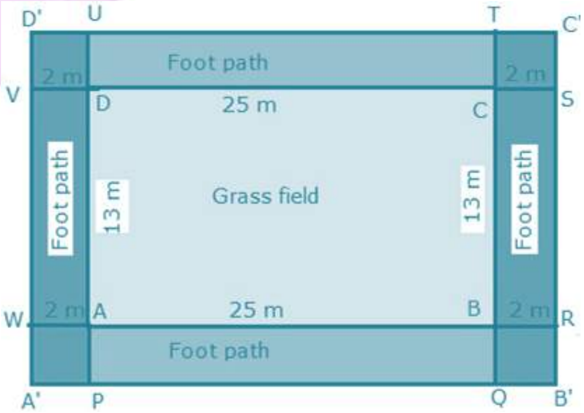
Let h be the height of the triangle, then

$$\frac{1}{2} \times base \times h = 3A$$
$$\frac{1}{2} \times 6 \times h = 3 \times 24$$
$$h = 24cm$$

8. **How many tiles, each of area 400 cm^2, will be needed to pave a footpath which is 2 m wide and surrounds a grass plot 25 m long and 13 m wide?**

Solution:

Let the figure be:



Thus the required area = area shaded in blue + area shaded in red
= Area ABPQ + Area TUDC + Area A'PUD' + Area QB'C'T
= 2Area ABPQ + 2Area QB'C'T
=2(Area ABPQ +Area QB'C'T)

Area of the footpath,
A = 2 x (25+25+17+17)
= 168 m^2
= 1680000m^2
Therefore,
No.of tiles required = 1680000/400 = 4200

9. The cost of enclosing a rectangular garden with a fence all round, at the rate of 75 paise per metre, is Rs. 300. If the length of the garden is 120 metres, find the area of the field in square metres.

Solution:

Perimeter of the garden, s = 300/0.75
= 400 m^2
Length of the garden = 120 m.
Breadth of the garden b is,
2(l+b) = S
2(120+b) = 400
b=80m
Area of the field,
A = 120 x 80
= 9600 m^2

10. The width of a rectangular room is 4/7of its length, x, and its perimeter is y. Write an equation connecting x and y. Find the length of the room when the perimeter is 4400 cm.

Solution:

Length of the rectangle = x
Width of the rectangle = $\frac{4}{7}$x
Hence, perimeter implies,

$$2\left(x+\frac{4}{7}x\right)=y$$

$$2\left(\frac{11x}{7}\right)=y$$

$$\frac{22x}{7}=y$$

Perimeter = 4400cm.
Hence,

$$\frac{22x}{7}=4400$$

x = 1400
Length of the rectangle=1400 cm = 14 m

11. The length of a rectangular verandah is 3 m more than its breadth. The numerical value of its area is equal to the numerical value of its perimeter.

(i) Taking x as the breadth of the verandah, write an equation in x that represents the above statement.

(ii) Solve the equation obtained in (i) above and hence find the dimensions of the verandah.

Solution:

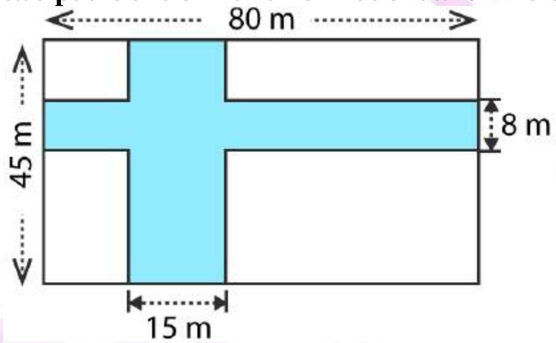
(i)

Breadth of the verandah=x
Length of the verandah=x+3
$2(x + (x+3)) = x(x+3)$
$4x+6 = x^2+3x$
$x^2-x-6 = 0$

(ii)

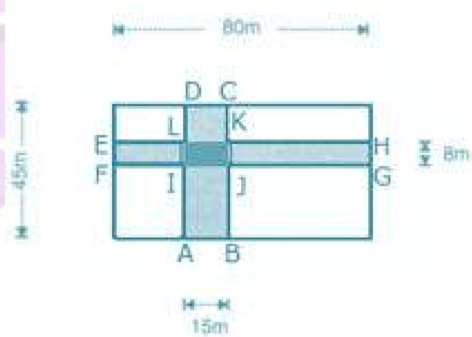
$x^2-x-6 = 0$
$(x-3)(x+2) = 0$
[x = -2 is not possible as length can never be negative]
$x=3$
Hence breadth=3m
Length =3+3=6m

12. The diagram, given below, shows two paths drawn inside a rectangular field 80 m long and 45 m wide. The widths of the two paths are 8 m and 15 m as shown. Find the area of the shaded portion.



Solution:

Let the figure be:



Area of the shaded portion,
=Area(ABCD) + Area(EFGH) - Area(IJKL)…(1)
Dimensions of ABCD = 45m x 15 m
Then area of ABCD = 675 m^2
Dimensions of EFGH = 80m x 8 m
Then area of EFGH = 640 m^2
Dimensions of IJKL = 15m x 8 m
Then area of IJKL = 120 m^2
Therefore, Area = 675 + 640 – 120
= 1195 m^2

13. The rate for a 1.20 m wide carpet is Rs. 40 per metre; find the cost of covering a hall 45 m long and 32 m wide with this carpet. Also, find the cost of carpeting the same hall if the carpet, 80 wide, is at Rs. 25. Per metre.

Solution:

Area of the hall = 45 x 32

= 1440 m^2

Cost = $\frac{40}{1.20} \times 1440$

= 48000

Then, cost of carpeting of 80 cm = 0.8 m wide carpet, if the rate of carpeting is Rs. 25. Per metre.

$Cost = \frac{25}{0.8} \times 1440$

$= Rs.45,000$

14. Find the area and perimeter of a square plot of land, the length of whose diagonal is 15 metres. Given your answer correct to 2 places of decimals.

Solution:

Let the length of each side of the square = a

$2a^2 = (\text{diagonal})^2$

$a^2 = 15^2/2$

$a^2 = 112.5$

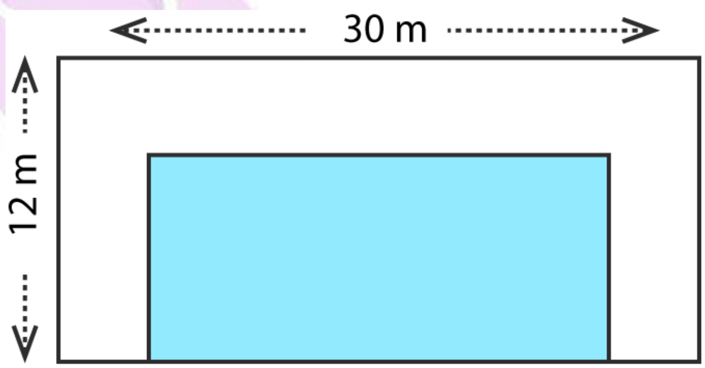
a= 10.60

Area= $a^2 = 10.60^2$

= 112.36

Perimeter= 4a = 42.43m

15. The shaded region of the given diagram represents the lawn in the form of a house. On the three sides of the lawn there are flowerbeds having a uniform width of 2 m.

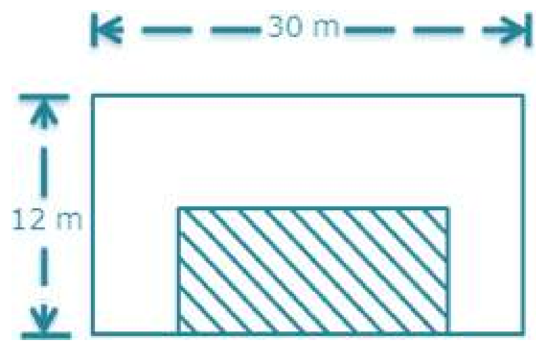


(i) Find the length and the breadth of the lawn.

(ii) Hence, or otherwise, find the area of the flower-beds.

Solution:

Let the figure be:



(i)

The length of the lawn = 30 - 2 - 2 = 26 m
The breadth of the lawn = 12 - 2 = 10 m

(ii)

The shaded area in the figure is the required area.
Then, the area of the flower bed is,
A = 10 x 2 + 10 x 2 + 30 x 2
= 20 + 20 + 60
= 100 m^2

16. A floor which measures 15 m × 8 m is to be laid with tiles measuring 50 m × 25 cm. Find the number of tiles required.
Further, if a carpet is laid on the floor so that a space of 1 m exists between its edges and the edges of the floor, what fraction of the floor is uncovered?

Solution:

Area of the floor = 15 x 8 = 120 m^2
Area of one tile = 0.50 x 0.25 = 0.125 m^2
Number of tiles required

$$n = \frac{\text{Area of floor}}{\text{Area of tiles}}$$

$$= \frac{120}{0.125}$$

$$= 960$$

Area of carpet uncovered = 2(1 x 15 + 1 x 6)
= 42 m^2

Fraction of floor uncovered= $\frac{42}{120} = \frac{7}{20}$

17. Two adjacent sides of parallelogram are 24 cm and 18 cm. If the distance between the longer sides is 12 cm; find the distance between the shorter sides.

Solution:

We know that,
Area = Base x Height
= 24 x 12
= 18 x h

$h = \frac{24 \times 12}{18}$

$h = 16$

18. Two adjacent sides of parallelogram are 28 cm and 26 cm. If one diagonal of it is 30 cm long; find the area of the parallelogram. Also, find the distance between its shorter sides.

Solution:

At first we have to calculate the area of the triangle having sides, 28cm, 26cm and 30cm.

Let the area be S.

$$S = \frac{28 + 26 + 30}{2}$$
$$= \frac{84}{2}$$
$$= 42 \text{ cm}$$

By Heron's Formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{42(42-28)(42-26)(42-30)}$$
$$= \sqrt{42 \times 14 \times 16 \times 12}$$
$$= \sqrt{112896}$$
$$= 336 \text{ cm}^2$$

Area of a Parallelogram = 2 × Area of a triangle

= 2 × 336

= 672 cm^2

We know that,

Area of a parallelogram = Height × Base

⇒ 672 = Height × 26

⇒ Height = 25.84 cm

∴ the distance between its shorter sides is 25.84 cm.

19. The area of a rhombus is 216 sq. cm. If its one diagonals is 24 cm; find:

(i) Length of its other diagonal,

(ii) Length of its side,

(iii) Perimeter of the rhombus.

Solution:

(i)

$$\text{Area of Rhombus} = \frac{1}{2} \times AC \times BD$$

Here A=216sq.cm

AC=24cm

BD=?

$$A = \frac{1}{2} \times AC \times BD$$

$$216 = \frac{1}{2} \times 24 \times BD$$

$$BD = 18\text{cm}$$

(ii) Let length of each side of the rhombus = a

$$a^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$a^2 = 12^2 + 9^2$$

$$a^2 = 225$$

$$a = 15\text{cm}$$

(iii) Perimeter of the rhombus=4a=60cm.

20. The perimeter of a rhombus is 52 cm. If one diagonal is 24 cm; find:
(i) The length of its other diagonal,
(ii) Its area.

Solution:

Let a be the length of each side of the rhombus.
Perimeter = 4a
4a= 52
a=13 cm

(i)
It is given that,
AC=24cm

$$a^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$13^2 = 12^2 + \left(\frac{BD}{2}\right)^2$$

$$\left(\frac{BD}{2}\right)^2 = 5^2$$

$$BD = 10\text{cm}$$

Hence the other diagonal is 10cm.

(ii)

$$\text{Area of the rombus} = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 24 \times 10$$

$$= 120 \text{sq.cm}$$

21. The perimeter of a rhombus is 46 cm. If the height of the rhombus is 8 cm; find its area.

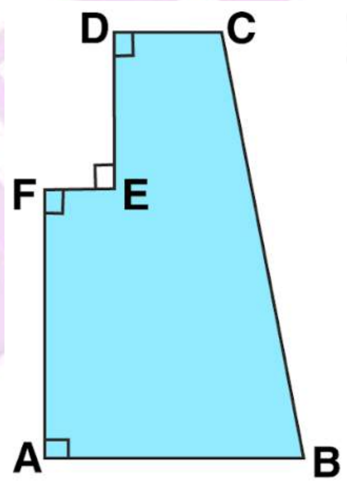
Solution:

Let a be the length of each side of the rhombus.

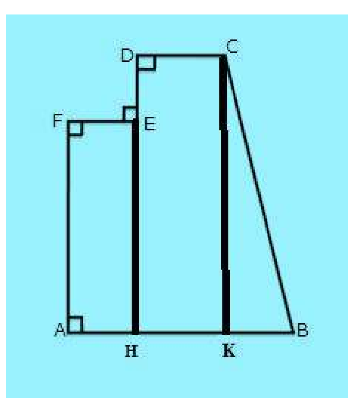
Perimeter = 4a
4a= 46
a=11.5 cm

Area = Base x Height
= 11.5 x 8
= 92 cm^2

22. The figure given below shows the cross-section of a concrete structure. Calculate the area of cross-section if AB = 1.8 cm, CD = 0.6 m, DE = 0.8 m, EF = 0.3 m and AF = 1.2 m.



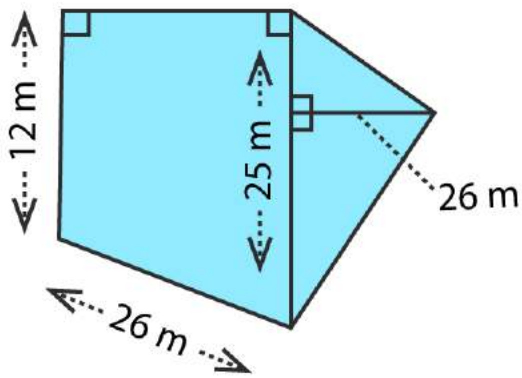
Solution:



AF=1.2m,EF=0.3m,DC=0.6m,BK=1.8-0.6-0.3=0.9m
Hence,

$$\text{Area of } ABCDEF = \text{Area of } AHEF + \text{Area of } HKCD$$
$$+\Delta KBC$$
$$= 1.2 \times 0.3 + 2 \times 0.6 + \frac{1}{2} \times 2 \times 0.9$$
$$= 2.46 \text{sq.m}$$

23. Calculate the area of the figure given below: which is not drawn scale.

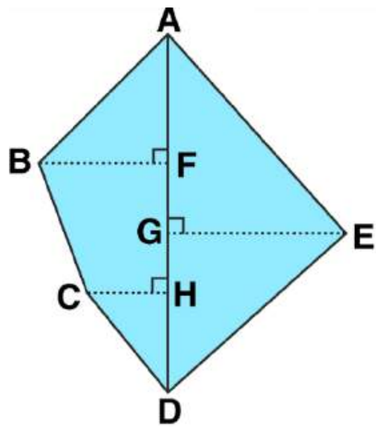


Solution:

$$\text{Area of the triangle} = \frac{1}{2} \times 12 \times 25$$
$$= 150 \text{sq.m}$$
$$\text{Area of the trapezium} = \frac{1}{2} \times (25+15) \times \left(\sqrt{26^2 - (25-15)^2}\right)$$
$$= 20 \times 24$$
$$=480 \text{sq.m}$$

Area of the whole figure=150+240=630sq.m

24. The following diagram shows a pentagonal field ABCDE in which the lengths of AF, FG, GH and HD are 50 m, 40 m, 15 m and 25 m respectively; and the lengths of perpendiculars BF, CH and EG are 50 m, 25 m ad 60 m respectively. Determine the area of the field.



Solution:

Divide the field into three triangles and one trapezium.

Let A,B,C be the three triangular region and D be the trapezoidal region.

$$\text{Area of } A = \frac{1}{2} \times AD \times GE$$

$$= \frac{1}{2} \times (50 + 40 + 15 + 25) \times 60$$

$$= 3900 \text{sq.m}$$

$$\text{Area of } B = \frac{1}{2} \times AF \times BF$$

$$= \frac{1}{2} \times 50 \times 50$$

$$= 1250 \text{sq.m}$$

$$\text{Area of } B = \frac{1}{2} \times HD \times CH$$

$$= \frac{1}{2} \times 25 \times 25$$

$$= 312.5 \text{sq.m}$$

$$\text{Area of } D = \frac{1}{2} \times (BF + CH) \times (FG + GH)$$

$$= \frac{1}{2} \times (50 + 25) \times (40 + 15)$$

$$= \frac{1}{2} \times 75 \times 55$$

$$= 2062.5 \text{sq.m}$$

Area of the figure=Area of A+ Area of B+ Area of C+ Area of D
=3900+1250+312.5+2062.5
=7525sq.m

25. A footpath of uniform width runs all around the outside of a rectangular field 30 m long and 24 m wide. If the path occupies an area of 360 m², find its width.

Solution:

Let the width of the footpath = x

$$\text{Area of footpath} = 2\times(30+24)x + 4x^2$$
$$= 4x^2 + 108x$$

Area of the footpath is 360sq.m.

$$4x^2 + 108x = 360$$
$$x^2 + 27x - 90 = 0$$
$$(x-3)(x+30) = 0$$
$$x = 3$$

Hence, width of the footpath is 3m.

26. A wire when bent in the form of a square encloses an area of 484 m². Find the largest area enclosed by the same wire when bent to from:

(i) An equilateral triangle.

(ii) A rectangle of length 16 m.

Solution:

Area of the square = 484.
Let the length of each side of the square = a.
$a^2 = 484$
a = 22
Hence, length of the wire is= 4x22=88m.
(i)

$$\text{Side of the triangle} = \frac{88}{3}$$
$$= 29.3m$$
$$\text{Area of the triangle} = \frac{\sqrt{3}}{4}\times(\text{side})^2$$
$$= \frac{\sqrt{3}}{4}\times(29.3)^2$$
$$= 372.58\text{m}^2$$

(ii)
Let breadth of the rectangle = x

$2(l+b)=88$

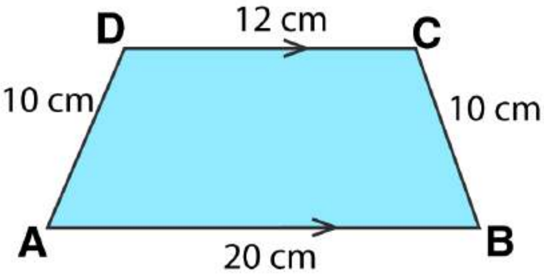
$16+x=44$

$x=28\text{m}$

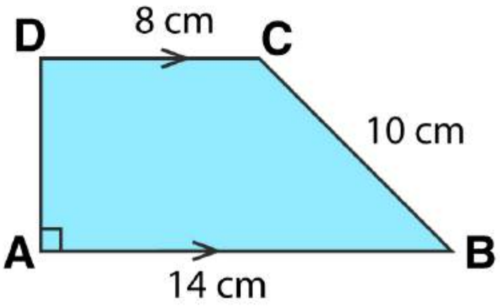
Hence area=16x28=448m^2

27. For each trapezium given below; find its area.

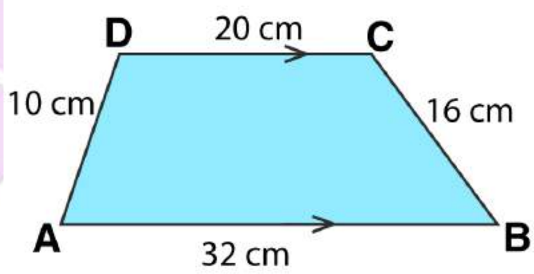
(i)



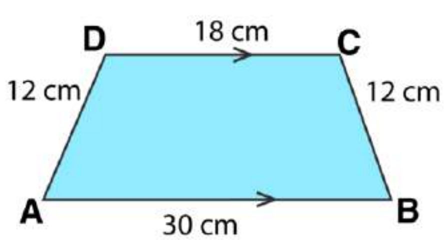
(ii)



(iii)

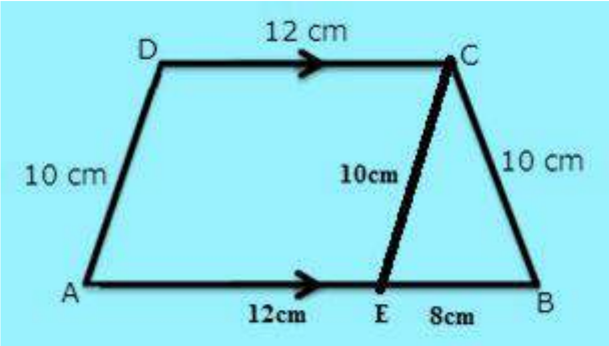


(iv)



Solution:

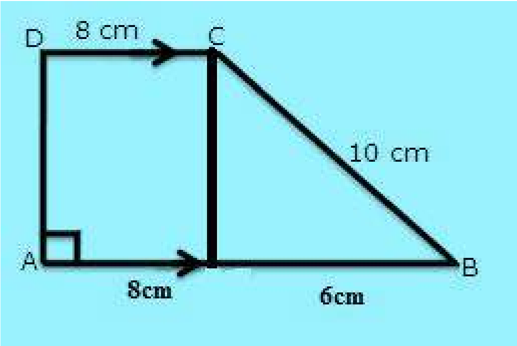
(i)



$$Area\ of\ \triangle EBC = \frac{1}{4} \times 8 \times \sqrt{4 \times 10^2 - 8^2}$$
$$= \frac{1}{4} \times 8 \times 18.3$$
$$= 36.6\ cm^2$$

$$\text{Area of } \Delta\text{EBC} = \frac{1}{2} \times 8 \times h$$
$$36.6 = 4h$$
$$h = 9.15$$
$$\text{Area of ABCD} = \frac{1}{2} \times (12 + 20) \times 9.15$$
$$= \frac{1}{2} \times 32 \times 9.15$$
$$= 146.64 \text{sq.cm}$$

(ii)

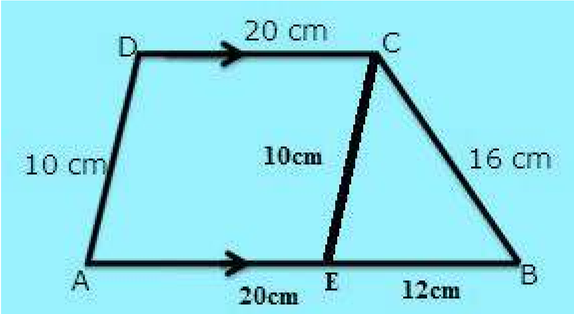


$$\text{Area of ABCD} = \frac{1}{2} \times (8+14) \times \left(\sqrt{10^2 - 6^2}\right)$$

$$= \frac{1}{2} \times 22 \times 8$$

$$= 88 \text{sq.cm}$$

(iii)



For the triangle EBC,
S=19cm

$$\text{Area of } \Delta EBC = \sqrt{19 \times (19-16) \times (19-12) \times (19-10)}$$

$$= \sqrt{19 \times 3 \times 7 \times 9}$$

$$= 59.9 \text{ sq.cm}$$

Let h be the height.

$$\text{Area of } \triangle EBC = \frac{1}{2} \times 12 \times h$$

$$\Rightarrow 59.9 = 6h$$

$$\Rightarrow h = \frac{59.9}{6} = 9.98 \ cm$$

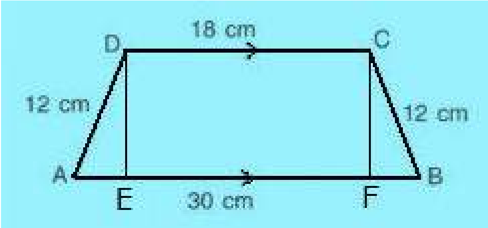
$$\text{Area of } ABCD = \frac{1}{2} \times (20+32) \times 9.98$$

$$= \frac{1}{2} \times 52 \times 9.98$$

$$= 259.48 \ cm^2$$

(iv)

Draw DE and CF perpendiculars to AB.



Area of the parallelogram is

$30 = AE + 18 + FB$

$\Rightarrow 30 = AE + 18 + AE$

$\Rightarrow 2AE + 18 = 30$

$\Rightarrow 2AE = 30 - 18$

$\Rightarrow 2AE = 12$

$\Rightarrow AE = 6\text{ cm}$

Now, consider the right angled triangle ADE.

$AD^2 = AE^2 + DE^2$

$\Rightarrow 12^2 = 6^2 + DE^2$

$\Rightarrow 144 = 36 + DE^2$

$\Rightarrow DE^2 = 144 - 36$

$\Rightarrow DE^2 = 108$

$\Rightarrow DE = \sqrt{36 \times 3}$

$\Rightarrow DE = 6\sqrt{3}$

$\text{Area}(\square ABCD) = \text{Area}(\triangle ADE) + \text{Area}(\square DEFC) + \text{Area}(\triangle CFB)$

$\Rightarrow \text{Area}(\square ABCD) = \frac{1}{2} \times 6 \times 6\sqrt{3} + 18 \times 6\sqrt{3} + \frac{1}{2} \times 6 \times 6\sqrt{3}$

$\Rightarrow \text{Area}(\square ABCD) = 6 \times 6\sqrt{3} + 18 \times 6\sqrt{3}$

$\Rightarrow \text{Area}(\square ABCD) = 144\sqrt{3} = 249.41\text{cm}^2$

28. The perimeter of a rectangular board is 70 cm. taking its length as x cm, find its width in terms of x.
If the area of the rectangular board is 300 cm^2; find its dimensions.

Solution:

Let b be the breadth of rectangle. then its perimeter

2(x+b) = 70

x+b = 35

b= 35-x

$$x \times b = 300$$
$$x(35 - x) = 300$$
$$x^2 - 35x + 300 = 0$$
$$(x - 15)(x - 20) = 0$$
$$x = 15, 20$$

Hence, length is 20cm and width is 15cm.

29. The area of a rectangular is 640 m^2. Taking its length as x cm; find in terms of x, the width of the rectangle. If the perimeter of the rectangle is 104 m; find its dimensions.

Solution:

Let b be the width of the rectangle.

$$x \times b = 640$$

$$b = \frac{640}{x}$$

Perimeter of the rectangle is 104m.

$$2\left(x + \frac{640}{x}\right) = 104$$

$$x^2 - 52x + 640 = 0$$

$$(x - 32)(x - 20) = 0$$

$$x = 32, 20$$

length=32m
width=20m.

30. The length of a rectangle is twice the side of a square and its width is 6 cm greater than the side of the square. If area of the rectangle is three times the area of the square; find the dimensions of each.

Solution:

Let the length of the sides of the square = a

$$2a \times (a + 6) = 3a^2$$

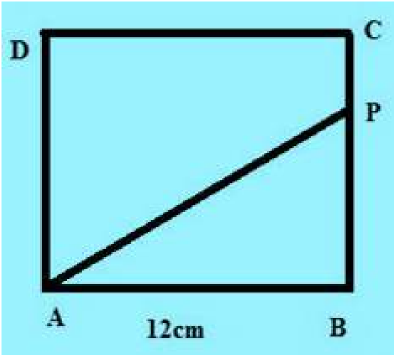
$$2a^2 + 12a = 3a^2$$

$$a = 12$$

Sides of the square = 12cm each
Length of the rectangle =2a=24cm
Width of the rectangle=a+6=18cm.

31. ABCD is a square with each side 12 cm. P is a point on BC such that area of triangle ABP: area of trapezium APCD = 1: 5. Find the length of CP.

Solution:



$$\frac{\text{Area of } \Delta ABP}{\text{Area of trapezium } APCD} = \frac{1}{5}$$

$$\Rightarrow \frac{\frac{1}{2} \times 12 \times (12 - CP)}{\frac{1}{2} \times (12 + CP) \times 12} = \frac{1}{5}$$

$$\Rightarrow 60 - 5CP = 12 + CP$$

$$\Rightarrow 6CP = 48$$

$$\Rightarrow CP = 8 \ cm$$

32. A rectangular plot of land measures 45 m $\times$ 30 m. A boundary wall of height 2.4 m is built all around the plot at a distance of 1 m from the plot. Find the area of the inner surface of the boundary wall.

Solution:

Length of the wall=45+2=47m
Breath of the wall=30+2=32m
Hence area of the inner surface of the wall is given by

$$A = (47 \times 2 \times 2.4) + (32 \times 2 \times 2.4)$$

$$= 225.6 + 153.6$$

$$= 379.2 \ m^2$$

33. A wire when bent in the form of a square encloses an area = 576 cm^2. Find the largest area enclosed by the same wire when bent to form;

(i) an equilateral triangle.

(ii) A rectangle whose adjacent sides differ by 4 cm.

Solution:

Let the length of each side = a

$a^2 = 576$
$a = 24$ cm
$4a = 96$ cm

Hence length of the wire = 96cm

(i)

From the equilateral triangle,

$$side = \frac{96}{3} = 32\text{cm}$$

$$Area = \frac{\sqrt{3}}{4}(side)^2$$

$$= \frac{\sqrt{3}}{4} \times 32^2$$

$$= 256\sqrt{3}\text{sq.cm}$$

(ii)

Let the adjacent side of the rectangle be x and y cm respectively.
Perimeter = 96 cm, we have,
2(x+y) = 96
x + y = 48
x – y = 4
x = 26
y = 22

$$x + y = 48$$

$$x - y = 4$$

$$x = 26$$

$$y = 22$$

Area of the rectangle is = 26 x 22 = 572 sq.cm

34. The area of a parallelogram is y cm^2 and its height is h cm. The base of another parallelogram is x cm more than the base of the first parallelogram and its area is twice the area of the first. Find, in terms of y, h and x, the expression for the height of the second parallelogram.

Solution:

Let 'y' and 'h' be the area and the height of the first parallelogram respectively.

Base of the first parallelogram= $\frac{y}{h}$ cm

Base of second parallelogram= $\left(\frac{y}{h} + x\right)$ cm

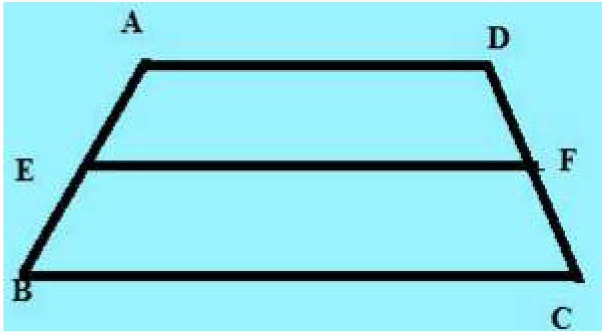
$$\left(\frac{y}{h} + x\right) \times height = 2y$$

$$height = \frac{2hy}{y + hx}$$

35. The distance between parallel sides of a trapezium is 15 cm and the length of the line segment

joining the mid-points of its non-parallel sides is 26 cm. Find the area of the trapezium.

Solution:



$$EF = \frac{1}{2} \times (AD + BC) = 26 \text{ cm}$$

$$\text{Area of the trapezium} = \frac{1}{2} \times (AD + BC) \times h$$

$$= 26 \times 15$$

$$= 390 \text{ cm}^2$$

36. The diagonal of a rectangular plot is 34 m and its perimeter is 92 m. Find its area.

Solution:

Let a and b be the sides of the rectangle

Since the perimeter is 92 m, we have,

$2(a + b) = 92$

$\Rightarrow a + b = 46\ m....(1)$

Also given that diagonal of a trapezium is 34 m.

$\Rightarrow a^2 + b^2 = 34^2....(2)$

We know that

$(a + b)^2 - a^2 - b^2 = 2ab$

From equations (1) and (2), we have,

$46^2 - 34^2 = 2ab$

$\Rightarrow 2ab = 960$

$\Rightarrow ab = \frac{960}{2}$

$\Rightarrow ab = 480\ m^2$

EXERCISE 20(C) PAGE:261

1. The diameter of a circle is 28 cm. Find its:
(i) Circumference
(ii) Area.

Solution:

Let r be the radius of the circle.

(i)

$$2r = 28\text{cm}$$
$$\text{circumference} = 2\pi r$$
$$= 28\pi\text{cm}$$
$$= 88 \text{ cm}$$

(ii)

$$\text{area} = \pi r^2$$
$$= \pi\left(\frac{28}{2}\right)^2$$
$$= 196\pi\text{cm}^2$$
$$= 616 \text{ cm}^2$$

2. The circumference of a circular field is 308 m. Find is:
(i) Radius
(ii) Area.

Solution:

Let r be the radius of the circular field

(*i*)

$$2\pi r = 308$$
$$\Rightarrow r = \frac{308}{2\pi}$$
$$\Rightarrow r = \frac{308}{2} \times \frac{7}{22}$$
$$\Rightarrow r = 49 \text{ m}$$

(ii)

$$\text{area} = \pi r^2$$
$$= \frac{22}{7} \times (49)^2$$
$$= 7546 \text{ m}^2$$

3. The sum of the circumference and diameter of a circle is 116 cm. Find its radius.

Solution:

Let the radius of the circle = r

$2\pi r + 2r = 116$

$2r(\pi + 1) = 116$

$r = \dfrac{116}{2(\pi + 1)}$

$= 14\text{cm}$

4. **The radii of two circles are 25 cm and 18 cm. Find the radius of the circle which has circumference equal to the sum of circumferences of these two circles.**

Solution:

Circumference of the first circle

$S_1 = 2\pi \times 25$

$= 50\pi\text{cm}$

Circumference of the second circle

$S_2 = 2\pi \times 18$

$= 36\pi\text{cm}$

Let r be the radius of the resulting circle.

$2\pi r = 50\pi + 36\pi$

$2\pi r = 86\pi$

$r = \dfrac{86\pi}{2\pi}$

$= 43\text{cm}$

5. **The radii of two circles are 48 cm and 13 cm. Find the area of the circle which has its circumference equal to the difference of the circumferences of the given two circles.**

Solution:

Circumference of the first circle

$S_1 = 2\pi \times 48$

$= 96\pi\text{cm}$

Circumference of the second circle

$S_2 = 2\pi \times 13$

$= 26\pi\text{cm}$

Let radius of the resulting circle = r

$$2\pi r = 96\pi - 26\pi$$

$$2\pi r = 70\pi$$

$$r = \frac{70\pi}{2\pi}$$

$$= 35\text{cm}$$

Hence area of the circle

$$A = \pi r^2$$

$$= \pi \times 35^2$$

$$= 3850\text{cm}^2$$

6. **The diameters of two circles are 32 cm and 24 cm. Find the radius of the circle having its area equal to sum of the areas of the two given circle.**

Solution:

Let the area of the resulting circle = r.

$$\pi \times (16)^2 + \pi \times (12)^2 = \pi \times r^2$$

$$256\pi + 144\pi = \pi \times r^2$$

$$400\pi = \pi \times r^2$$

$$r^2 = 400$$

$$r = 20\text{cm}$$

7. **The radius of a circle is 5 m. find the circumference of the circle whose area is 49 times the area of the given circle.**

Solution:

Area of the circle having radius 85m is

$$A = \pi \times (85)^2$$

$$= 7225\pi\text{m}^2$$

Let the radius of the circle whose area is 49times of the given circle = r

$$\pi r^2 = 49 \times (\pi \times 5^2)$$

$$r^2 = (7 \times 5)^2$$

$$r = 35$$

$S = 2\pi r$

$= 2\pi \times 35$

$= 220\text{m}$

8. A circle of largest area is cut from a rectangular piece of card-board with dimensions 55 cm and 42 cm. Find the ratio between the area of the circle cut and the area of the remaining card-board.

Solution:

Area of the rectangle is given by

$A = 55 \times 42$

$= 2310\text{cm}^2$

For the largest circle, the radius of the circle will be half of the sorter side of the rectangle.

r=21cm.

$\text{Area of the circle} = \pi \times (21)^2$

$= 1384.74\text{cm}^2$

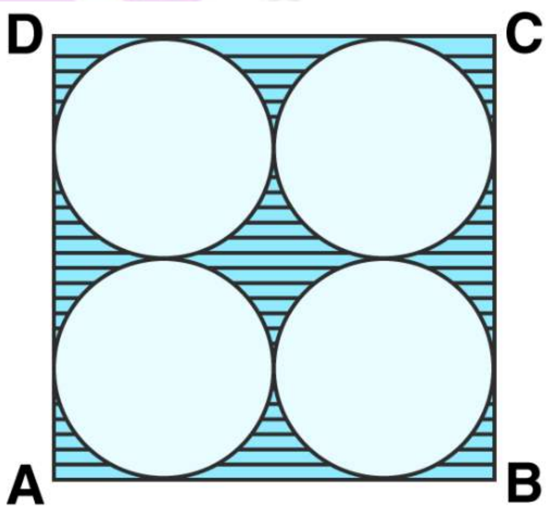
$\text{Area remaining} = 2310 - 1384.74$

$= 925.26$

$\text{the volume of the circle: area remaining} = 1384.74 : 915.26$

$= 3:2$

9. The following figure shows a square cardboard ABCD of side 28 cm. Four identical circles of largest possible size are cut from this card as shown below.



Find the area of the remaining card-board.

Solution:

Area of the square is given by

$A = 28^2$

$= 784\text{cm}^2$

Since there are four identical circles inside the square.

Hence radius of each circle is one fourth of the side of the square.

$$\text{Area of one circle} = \pi \times 7^2$$
$$= 154\text{cm}^2$$
$$\text{Area of four circle} = 4 \times 154\text{cm}^2$$
$$= 616\text{cm}^2$$
$$\text{Area remaining} = 784 - 616$$
$$= 168\text{cm}^2$$

Area remaining in the cardboard is $= 168\text{cm}^2$

10. The radii of two circles are in the ratio 3 : 8. If the difference between their areas is 2695 π cm2, find the area of the smaller circle.

Solution:

Let the radius of the two circles be 3r and 8r.

$$\text{area of the circle having radius 3r} = \pi(3r)^2$$
$$= 9\pi r^2$$
$$\text{area of the circle having radius 8r} = \pi(8r)^2$$
$$= 64\pi r^2$$
$$64\pi r^2 - 9\pi r^2 = 2695\pi$$
$$55r^2 = 2695$$
$$r^2 = 49$$
$$r = 7\text{cm}$$

Hence radius of the smaller circle is 3 x 7 = 21 cm

Area of the smaller circle is given by

$$A = \pi r^2 = \frac{22}{7} \times 21^2 = 1386 \text{ cm}^2$$

11. The diameters of three circles are in the ratio 3 : 5 : 6. If the sum of the circumferences of these circles be 308 cm; find the difference between the areas of the largest and the smallest of these circles.

Solution:

Let the diameter of the three circles be 3d, 5d and 6d.

$$\pi \times 3d + \pi \times 5d + \pi \times 6d = 308$$
$$14\pi d = 308$$
$$d = 7$$

$$\text{radius of the smallest circle} = \frac{21}{2} = 10.5$$

$$\text{Area} = \pi \times (10.5)^2$$

$$= 346$$

$$\text{radius of the largest circle} = \frac{42}{2} = 21$$

$$\text{Area} = \pi \times (21)^2$$

$$= 1385.5$$

$$\text{difference} = 1385.5 - 346$$

$$= 1039.5$$

12. Find the area of a ring shaped region enclosed between two concentric circles of radii 20 cm and 15 cm.

Solution:

$$\text{Area of the ring} = \pi(20)^2 - \pi(15)^2$$

$$= 400\pi - 225\pi$$

$$= 175\pi$$

$$= 549.7\text{cm}^2$$

13. The circumference of a given circular park is 55 m. It is surrounded by a path of uniform width 3.5 m . Find the area of the path.

Solution:

Let r be the radius of the circular park.

$$2\pi r = 55$$

$$r = \frac{55}{2\pi}$$

$$= 8.75\text{m}$$

$$\textit{area of the park} = \pi \times (8.75)^2 = 240.625 \text{ m}^2$$

Radius of the outer circular region including the path is given by

$$R = 8.75 + 3.5$$

$$= 12.25 \ m$$

Area of that circular region is

$$A = \pi \times (12.25)^2 = 471.625 \text{ m}^2$$

Hence area of the path is given by

$$\textit{Area of the path} = 471.625 - 240.625 = 231 \ m^2$$

14. There are two circular gardens A and B. The circumference of garden A is 1.760 km and the area of garden B is 25 times the area of garden A. Find the circumference of garden B.

Solution:

Let r be the radius of the circular garden A.

Since the circumference of the garden A is 1.760 Km = 1760m, we have,

$2\pi r = 1760$ m

$\Rightarrow r = \frac{1760 \times 7}{2 \times 22} = 280$ m

Area of garden A $= \pi r^2 = \frac{22}{7} \times 280^2$ m^2

Let R be the radius of the circular garden B.

Since the area of garden B is 25 times the area of garden A, we have,

$\pi R^2 = 25 \times \pi r^2$

$\Rightarrow \pi R^2 = 25 \times \pi \times 280^2$

$\Rightarrow R^2 = 1960000$

$\Rightarrow R = 1400$ m

Thus circumference of garden B $= 2\pi R = 2 \times \frac{22}{7} \times 1400 = 8800$ m $= 8.8$ Km

15. A wheel has diameter 84 cm. Find how many completer revolutions must it make to cover 3.168 km.

Solution:

Diameter of the wheel = 84 cm

Thus, radius of the wheel = 42 cm

Circumference of the wheel $= 2 \times \frac{22}{7} \times 42 = 264$ *cm*

In 264 cm, wheel is covering one revolution.

Thus, in 3.168 Km = 3.168 × 100000 cm, number of revolutions

covered by the wheel $= \frac{3.168}{264} \times 100000 = 1200$

16. Each wheel of a car is of diameter 80 cm. How many completer revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Solution:

the car travells in 10minutes $= \frac{66}{6}$

$= 11$km

$= 1100000$cm

Circumference of the wheel = distance covered by the wheel in one revolution

Thus, we have,

Circumference $= 2 \times \frac{22}{7} \times \frac{80}{2} = 251.43$ *cm*

Thus, the number of revolutions covered

by the wheel in 1100000 cm $= \frac{1100000}{251.43} \approx 4375$

17. An express train is running between two stations with a uniform speed. If the diameter of each wheel of the train is 42 cm and each wheel makes 1200 revolutions per minute, find the speed of the train.

Solution:

$$\text{radius of the wheel} = \frac{42}{2}$$
$$= 21\text{cm}$$
$$\text{circumference of the wheel} = 2\pi \times 21$$
$$= 132\text{cm}$$
$$\text{Distance travelled in one minute} = 132 \times 1200$$
$$= 158400\text{cm}$$
$$= 1.584\text{km}$$
$$\text{hence the speed of the train} = \frac{1.584\text{km}}{\frac{1}{60}\text{hr}}$$
$$= 95.04\text{km/hr}$$

18. The minute hand of a clock is 8 cm long. Find the area swept by the minute hand between 8.30 a.m. and 9.05 a.m.

Solution:

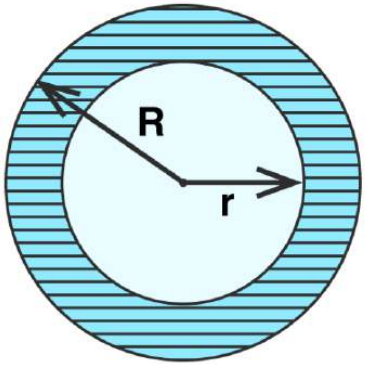
Time interval is 9.05 am – 8.30 am = 35 minutes

Area covered in one 60 minutes= $\pi \times 8^2 = 201\text{cm}^2$

Hence area swept in 35 minutes is given by

$$A = \frac{201}{60} \times 35 = 117\frac{1}{3}\ cm^2$$

19. The shaded portion of the figure, given alongside, shows two concentric circles. If the circumference of the two circles be 396 cm and 374 cm, find the area of the shaded portion.



Solution:

Let R and r be the radius of the big and small circles respectively.

Given that the circumference of the bigger circle is 396 cm

Thus, we have,

$2\pi R = 396$ cm

$\Rightarrow R = \frac{396 \times 7}{2 \times 22}$

$\Rightarrow R = 63$ cm

Thus, area of the bigger circle $= \pi R^2$

$$= \frac{22}{7} \times 63^2$$

$$= 12474 \text{ cm}^2$$

Also given that the circumference of the smaller circle is 374 cm

$\Rightarrow 2\pi r = 374$

$\Rightarrow r = \frac{374 \times 7}{2 \times 22}$

$\Rightarrow r = 59.5$ cm

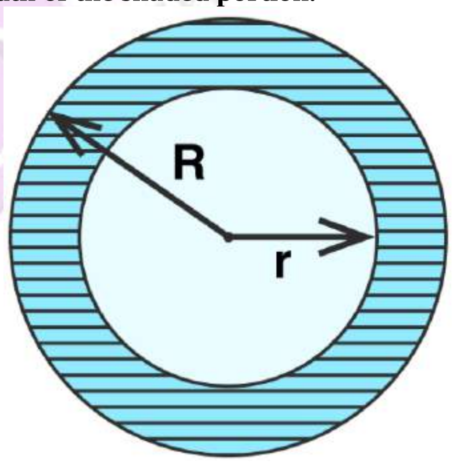
Thus, the area of the smaller circle $= \pi r^2$

$$= \frac{22}{7} \times 59.5^2$$

$$= 11126.5 \text{ cm}^2$$

Thus the area of the shaded portion $= 12474 - 11126.5 = 1347.5 \text{ cm}^2$

20. In the given figure, the area of the shaded portion is 770 cm2. If the circumference of the outer circle is 132 cm, find the width of the shaded portion.



Solution:

Given that the circumference of the outer circle is 132 cm

Thus, we have, $2\pi R = 132$ cm

$\Rightarrow R = \frac{132 \times 7}{2 \times 22}$

$\Rightarrow R = 21$ cm

Area of the bigger circle $= \pi R^2$

$= \frac{22}{7} \times 21^2$

$= 1386 \text{ cm}^2$

Also given the area of the shaded portion.

Thus the area of the inner circle = Area of the outer circle − Area of the shaded portion

$= 1386 - 770$

$= 616 \text{ cm}^2$

$\Rightarrow \pi r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{22}$

$\Rightarrow r^2 = 196$

$\Rightarrow r = 14$ cm

Thus, the width of the shaded portion $= 21 - 14 = 7$ cm

21. The cost of fencing a circular field at the rate of ₹ 240 per meter is ₹ 52,800. The field is to be ploughed at the rate of ₹ 12.50 per m2. Find the cost of pouching the field.

Solution:

Let the radius of the field is *r* meter.

Therefore circumference of the field will be: $2\pi r$ meter.

Now the cost of fencing the circular field is 52,800 at rate 240 per meter.

Therefore

$2\pi r \cdot 240 = 52800$

$r = \frac{52800 \times 7}{2 \times 240 \times 22}$

$= 35$

Thus the radius of the field is 35 meter.

Now the area of the field will be:

$\pi r^2 = \left(\frac{22}{7}\right) \cdot 35^2$

$= 3850 \text{ m}^2$

Thus the cost of ploughing the field will be:

$3850 \times 12.5 = 48,125$ rupees

22. Two circles touch each other externally. The sum of their areas is 58π cm2 and the distance between their centers is 10 cm. Find the radii of the two circles.

Solution:

Let r and R be the radius of the two circles.

$r + R = 10 \qquad \ldots(1)$

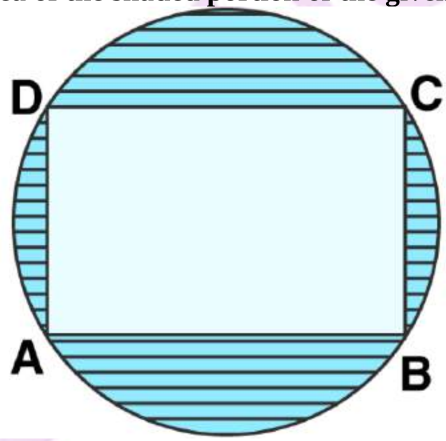
$\pi r^2 + \pi R^2 = 58\pi \qquad \ldots(2)$

Putting the value of r in (2)

$$r^2 + R^2 = 58$$
$$(10 - R)^2 + R^2 = 58$$
$$100 - 20R + R^2 + R^2 = 58$$
$$2R^2 - 20R + 42 = 0$$
$$R^2 - 10R + 21 = 0$$
$$(R-3)(R-7) = 0$$
$$R = 3, 7$$

Hence the radius of the two circles is 3cm and 7cm respectively.

23. The given figures shows a rectangle ABCD inscribed in a circle as shown alongside. If AB = 28 cm and BC = 21 cm, find the area of the shaded portion of the given figure.



Solution:

$$AB = 28\,\text{cm}$$
$$BC = 21\,\text{cm}$$
$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{28^2 + 21^2}$$
$$= 35\,\text{cm}$$

$$\text{Area} = \pi \times \left(\frac{35}{2}\right)^2$$
$$= 962.5 \ \text{cm}^2$$

$$\text{Area of the rectangle} = 28 \times 21$$
$$= 588\text{cm}^2$$

Therefore, the area of the shaded portion,

$$A = 962 - 588 = 374.5 \ \text{cm}^2$$

24. A square is inscribed in a circle of radius 7 cm. Find the area of the square.

Solution:

Since the diameter of the circle is the diagonal of the square inscribed in the circle.

Let a be the length of the sides of the square.

Hence

$$\sqrt{2}a = 2 \times 7$$
$$a = \sqrt{2} \times 7$$
$$a^2 = 98$$

Hence the area of the square is 98sq.cm.

25. A metal wire, when bent in the form of an equilateral triangle of largest area, encloses an area of 484 $\sqrt{3}$ cm2. If the same wire is bent into the form of a circle of largest area, find the area of this circle.

Solution:

Let 'a' be the length of each side of an equilateral triangle formed.

Now, area of equilateral triangle formed $= 484\sqrt{3}\ \text{cm}^2$

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 484\sqrt{3}$$
$$\Rightarrow a^2 = 4 \times 484$$
$$\Rightarrow a = 2 \times 22 = 44 \text{ cm}$$

Then, perimeter of equilateral triangle $= 3a = 3 \times 44 = 132$ cm

Now, length of wire = perimeter of equilateral triangle = circumference of circle

$\Rightarrow$ circumference of circle = 132 cm

$\Rightarrow 2\pi r = 132$ (r is radius of circle)

$$\Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

$$\therefore \text{Area of circle} = \pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386 \text{ cm}^2$$

EXERCISE 20(D) PAGE:263

1. **The perimeter of a triangle is 450 m and its side are in the ratio 12 : 5 : 13. Find the area of the triangle.**

Solution:

Let the sides of the triangle equals

a = 12x

b = 5x

c = 13x

Perimeter = 450 cm

⇒ 12x + 5x + 13x = 450

⇒ 30x = 450

⇒ x = 15

Then the sides of a triangle are

a = 12x = 12(15) = 180 cm

b = 5x = 5(15) = 75 cm

c = 13x = 13(15) = 195 cm

Now,

$$\text{semi-perimeter of a triangle, } s = \frac{a+b+c}{2} = \frac{180+75+195}{2} = \frac{450}{2} = 225 \text{ cm}$$

$$\begin{aligned}\therefore \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{225(225-180)(225-75)(225-195)}\\ &= \sqrt{225 \times 45 \times 150 \times 30}\\ &= \sqrt{15 \times 15 \times 9 \times 5 \times 25 \times 6 \times 5 \times 6}\\ &= \sqrt{15 \times 15 \times 3 \times 3 \times 25 \times 25 \times 6 \times 6}\\ &= 15 \times 3 \times 25 \times 6\\ &= 6750 \text{ cm}^2\end{aligned}$$

2. **A triangle and a parallelogram have the same base and the same area. If the side of the triangle are 26 cm, 28 cm and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.**

Solution:

Let the sides of the triangle equal

a = 26 cm, b = 28 cm and c = 30 cm

$$\text{semi-perimeter of a triangle, } s = \frac{a+b+c}{2} = \frac{26+28+30}{2} = \frac{84}{2} = 42 \text{ cm}$$

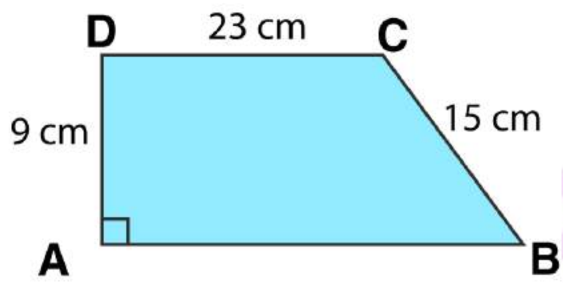
$$\begin{aligned}\therefore \text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{42(42-26)(42-28)(42-30)}\\ &= \sqrt{42 \times 16 \times 14 \times 12}\\ &= \sqrt{7 \times 6 \times 4 \times 4 \times 7 \times 2 \times 6 \times 2}\\ &= \sqrt{7 \times 7 \times 4 \times 4 \times 6 \times 6 \times 2 \times 2}\\ &= 7 \times 4 \times 6 \times 2\\ &= 336 \text{ cm}^2\end{aligned}$$

Base of parallelogram= 28 cm

According to the question,

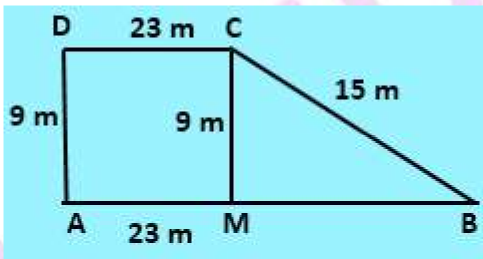
Area of parallelogram = Area of triangle
Base x Height = 336
28 x height = 336
Height = 12cm

3. Using the information in the following figure, find its area.



Solution:

Draw CM ⊥ AB



In right-angled triangle CMB,
$BM^2 = BC^2 - CM^2 = (15)^2 - (9)^2 = 225 - 81 = 144$
$\Rightarrow BM = 12$ m
Now, AB = AM + BM = 23 + 12 = 35 m

$$\therefore \text{ Area of trapezium ABCD} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{Height}$$
$$= \frac{1}{2} \times (AB + CD) \times AD$$
$$= \frac{1}{2} \times (23 + 35) \times 9$$
$$= \frac{1}{2} \times 58 \times 9$$
$$= 261 \text{ m}^2$$

4. Sum of the areas or two squares is 400 cm². If the difference of their perimeters is 16 cm, find the sides of the two squares.

Solution:

Let the sides of two squares be a and b.
According to the question,
Area of one square, $S_1 = a^2$
Area of other square, $S_2 = b^2$
Given, $S_1 + S_2 = 400$ cm^2

$\Rightarrow a^2 + b^2 = 400\ cm^2$ …..(1)
Also, difference in perimeter = 16 cm
$\Rightarrow 4a - 4b = 16$ cm
$\Rightarrow a - b = 4$
$\Rightarrow a = (4 + b)$
Substituting the value of 'a' in (1), we get
$(4 + b)^2 + b^2 = 400$
$\Rightarrow 16 + 8b + b^2 + b^2 = 400$
$\Rightarrow 2b^2 + 8b - 384 = 0$
$\Rightarrow b^2 + 4b - 192 = 0$
$\Rightarrow b^2 + 16b - 12b - 192 = 0$
$\Rightarrow b(b + 16) - 12(b + 16) = 0$
$\Rightarrow (b + 16)(b - 12) = 0$
$\Rightarrow b + 16 = 0$ or $b - 12 = 0$
$\Rightarrow b = -16$ or $b = 12$
[Here, the side of a square cannot be negative, so, -16 is not possible.]
Thus, b = 12
$\Rightarrow a = 4 + b = 4 + 12 = 16$
Hence, the sides of a square are 16 cm and 12 cm.

5. Find the area and the perimeter of a square with diagonal 24 cm. [Take $\sqrt{2}$=1.41]

Solution:

Given that, diagonal of a square = 24 cm

Now, diagonal of a square = side of a square $\times \sqrt{2}$

$\Rightarrow 24 =$ side of a square $\times \sqrt{2}$

$$\Rightarrow \text{Side of a square} = \frac{24}{\sqrt{2}} = \frac{12 \times \sqrt{2} \times \sqrt{2}}{\sqrt{2}} = 12\sqrt{2}$$

$$\therefore \text{Area of a square} = (\text{Side})^2 = \left(12\sqrt{2}\right)^2 = 288\ cm^2$$

And, perimeter of a square = $4 \times \text{Side} = 4 \times 12\sqrt{2} = 48 \times 1.41 = 67.68$ cm

6. A steel wire, when bent in the form of a square, encloses an area of 121 cm². The same wire is bent in the form of a circle. Find area the circle.

Solution:

Area of a square= side x side
Side x side =121 cm^2
$(\text{Side})^2 = 121$
Side = $\sqrt{121}$
Side = 11 cm

We know that,
Perimeter of a square = 4 x side
4 x side = 4 x 11
= 44 cm
According to the question,
Perimeter of a circle = perimeter of a square
Perimeter of a circle = 44

$\Rightarrow 2\pi r = 44$ (r is radius of a circle)

$\Rightarrow r = \frac{44}{2\pi} = \frac{44}{2 \times \frac{22}{7}} = 7$ cm

$\therefore$ Area of a circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ cm^2$

7. The perimeter of a semi-circular plate is 108 cm. find its area.

Solution:

Perimeter of a semi-circular plate = 108 cm

$\Rightarrow \pi r + 2r = 108$ (r is radius of a circle)

$\Rightarrow r(\pi + 2) = 108$

$\Rightarrow r = \frac{108}{\pi + 2} = \frac{108}{\frac{22}{7} + 2} = \frac{108 \times 7}{36} = 21$ cm

Now, area of a semi-circular pleate $= \frac{\pi r^2}{2} = \frac{\frac{22}{7} \times 21 \times 21}{2} = 693\ cm^2$

8. Two circles touch externally. The sum of their areas is 130π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

Solution:

Let the radii of two circles be r_1 and r_2 respectively.

Sum of the areas of two circles = 130π sq. cm

$\Rightarrow \pi r_1^2 + \pi r_2^2 = 130\pi$

$\Rightarrow r_1^2 + r_2^2 = 130$ ….(i)

Given that,

Distance between two radii = 14 cm

$\Rightarrow r_1 + r_2 = 14$

$\Rightarrow r_1 = (14 - r_2)$

Substituting the value of r_1 in equation (i), we get

$(14 - r_2)^2 + r_2^2 = 130$

$\Rightarrow 196 - 28r_2 + r_2^2 + r_2^2 = 130$

$\Rightarrow 2r_2^2 - 28r_2 + 66 = 0$

$\Rightarrow r_2^2 - 14r_2 + 33 = 0$

$\Rightarrow r_2^2 - 11r_2 - 3r_2 + 33 = 0$

$\Rightarrow r_2 (r_2 - 11) - 3 (r_2 - 11) = 0$

$\Rightarrow (r_2 - 11) (r_2 - 3) = 0$

$\Rightarrow r_2 = 11$ or $r_2 = 3$

$\Rightarrow r_1 = 14 - 11 = 3$ or $r_1 = 14 - 3 = 11$

Hence, the radii of two circles are 11 cm and 3 cm respectively.

9. The diameters of the front and rear wheels of a tractor are 63 cm and 1.54 m respectively. The rear wheel is rotating at $24\frac{6}{11}$ revolutions per minute. Find :

(i) the revolutions per minute made by the front wheel.

(ii) the distance travelled by the tractor in 40 minutes.

Solution:

According to the question,
The diameter of the front and real wheels of tractor are 63 cm = 0.63 m and 1.54 m respectively.

$$\text{Radius of the rear wheel} = \frac{1.54}{2} = 0.77 \text{ m}$$

$$\text{and radius of the front wheel} = \frac{0.63}{2} = 0.315 \text{ m}$$

Distance travelled by tractor in one revolution of rear wheel
= circumference of the rear wheel
$= 2\pi r$

$$= 2 \times \frac{22}{7} \times 0.77 = 4.84 \text{ m}$$

The rear wheel rotates at $24\frac{6}{11}$ revolutions per minute

$$= \frac{270}{11} \text{ revolutions per minute}$$

Since in one revolution the distance travelled by the rear wheel = 4.84 m

So, in $\frac{270}{11}$ revolutions, the tractor travels $\frac{270}{11} \times 4.84 = 118.8$ m

Let the number of revolutions made by the front wheel be x.
(i) Now, number of revolutions made by the front wheel in one minute
× circumference of the wheel
= the distance travalled by the tractor in one minute

$$\Rightarrow x \times 2 \times \frac{22}{7} \times 0.315 = 118.8$$

$$\Rightarrow x = \frac{118.8 \times 7}{2 \times 22 \times 0.315} = 60$$

(ii) Distance travelled by the tractor in 40 minutes
= Number of revolutions made by the rear wheel in 40 minutes
× circumference of the rear wheel

$$= \frac{270}{11} \times 40 \times 4.84 = 4752 \text{ m}$$

10. Two circles touch each other externally. The sum of their areas is 74π cm^2 and the distance between their centres is 12 cm. Find the diameters of the circle.

Solution:

Let the radius of the circle be r_1 and r_2.

According to the question,

$r_1 + r_2 = 12$

Then,

$r_2 = 12 - r_1$

Sum of the areas of the circles $= 74\pi$

$\Rightarrow \pi r_1^2 + \pi r_2^2 = 74\pi$

$\Rightarrow r_1^2 + r_2^2 = 74$

$\Rightarrow r_1^2 + (12 - r_1)^2 = 74$

$\Rightarrow r_1^2 + 144 - 24r_1 + r_1^2 = 74$

$\Rightarrow 2r_1^2 - 24r_1 + 70 = 0$

$\Rightarrow r_1^2 - 12r_1 + 35 = 0$

$\Rightarrow (r_1 - 7)(r_1 - 5) = 0$

$\Rightarrow r_1 = 7$ or $r_1 = 5$

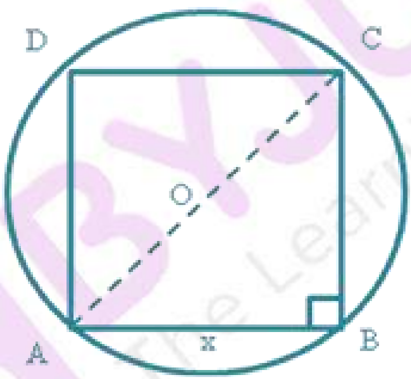
If $r_1 = 7$ cm, then $r_2 = 5$ cm

If $r_1 = 5$ cm, then $r_2 = 7$ cm

So, the diameters of the circles will be 10 cm and 14 cm.

11. If a square in inscribed in a circle, find the ratio of the areas of the circle and the square.

Solution:



If AB $= x$, AC $= x\sqrt{2}$

Diameter of the circle = diagonal of the square

$\Rightarrow 2r = x\sqrt{2}$

$\Rightarrow r = \frac{x\sqrt{2}}{2}$

Area of the circle $= \pi r^2$

$$= \pi\left(\frac{x\sqrt{2}}{2}\right)^2$$

$$= \pi\left(\frac{x^2 2}{4}\right)$$

$$= \frac{\pi x^2}{2}$$

Area of the square = x^2

$$\text{Required ratio} = \frac{\frac{\pi x^2}{2}}{x^2}$$

$$= \frac{\pi}{2}$$

$$= \frac{22}{7} \times \frac{1}{2}$$

$$= \frac{11}{7}$$

Hence, the required ratio is 11 : 7.